Global Rigidity

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1 Overview of the Field

The rigidity and flexibility of a structure, either man-made in buildings, linkages, and lightweight deployable forms, or found in nature ranging from crystals to proteins, is critical to the form, function, and stability of the structure. A strong form of rigidity is "Global Rigidity" when the given lengths permit only one realization, up to congruence. The mathematical theory of "Global Rigidity" is developing methods for the analysis and design of man-made structures , such as sensor networks, as well as for natural structures such as proteins.

We live in 3-dimensions, and a fundamental problem is to develop results for global rigidity in 3-dimensions which are as good, and as efficient, as the recently developed theory for global rigidity of structures in 2-dimensions. The mathematical methods also give insights into fundamental mathematical systems of constraints and computations, with even wider application in areas of computer aided design and manufacturing, CAD/CAM.

2 Recent Developments and Open Problems

Connelly-Gortler [2] extended the characterisation of universal rigidity in terms of PSD stress matrices [5] from generic frameworks to arbitrary frameworks.

Jordán, Kiraly and Tanigawa [7] extended the, d-dimensional, characterisation of global rigidity for body-bar frameworks [4] to body-hinge frameworks. Resulting from this work they identified a new family of counterexamples to Hendrickson's conjecture [6] in 3-dimensions. Previously only a single counterexample, $K_{5,5}$, was known [1]. Following this a key substantial open problem is to develop an appropriate conjecture for 3D global rigidity.

Tanigawa [8] gave a new sufficient condition for generic global rigidity in terms of vertex redundant rigidity. He used his more elaborate geometric arguments to simplify the combinatorial steps in the characterisation of generic global rigidity in the plane.

3 Presentation Highlights

Shin-Ichi Tanigawa presented his sufficient condition for global rigidity mentioned above.

There were two interesting talks on universal rigidity. Steven Gortler talked about the universal rigidity of complete bipartite graphs [3] and Anthony Man-Cho So discussed facial reduction techniques and degree of singularity.

There were two prominent student talks. Katie Clinch talked about progress towards characterising global rigidity of direction-length frameworks and Hakan Guler presented a new necessary condition for global rigidity in terms of covers.

There were also a number of 5 minute talks and presentations of open problems.

4 Scientific Progress Made

The session on Sunday Morning on points at infinity was a direct result of extending the workshop. It was a great way to pull together a number of connections among these topics. In particular it was a very clear connection between the 5-day workshop and the 2-day.

Bob Connelly, Steven Gortler and Tony Nixon had an interaction about global rigidity for frameworks in the plane with the l^p metric following on from Stephen Powers presentation of this problem. It may happen that rather than use a stress matrix to determine global rigidity, it may be more useful to some of the more combinatorial methods of Szabadka and Jordán and Tanigawa.

Connelly and Gortler also had an interaction with Simon Guest and Louis Theran concerning the global rigidity of complete bipartite graphs, where some subset of the edges have been removed. Again many of these graphs can be proven to be generically globally rigid by some standard edge splitting methods.

Tony Nixon and Bill Jackson had some discussions about their ongoing work on global rigidity on surfaces. They also benefitted from a discussion with Steven Gortler about a stress matrix characterisaton in this context.

5 Outcome of the Meeting

Tibor Jordán and Walter Whiteley talked for several hours, using information from the workshop to work on a revision on the Handbook Chapter on Global Rigidity.

References

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