Mixed normal-superconducting states in the presence of strong electric currents

Yaniv Almog, Louisiana State University, USA

Abstract. We study the time-dependent Ginzburg-Landau equations in the presence of strong currents, but weaker than the critical current where the normal state losses its stability. In the large κ limit, we prove that the superconductivity order parameter is exponentially small in a significant part of the domain, and small in the rest of it. Some results in the large domain limit will be presented as well.

Joint work with Bernard Helffer and Xingbin Pan.

Curves and surfaces with constant nonlocal mean curvature: meeting Alexandrov and Delaunay

Xavier Cabré, ICREA and UPC, Barcelona

Abstract. This is a joint work with Mouhamed M. Fall, Joan Sol-Morales and Tobias Weth. It concerns hypersurfaces of $\mathbb{R}^N$ with constant nonlocal (or fractional) mean curvature. This is the equation associated to critical points of the fractional perimeter under a volume constraint. Our results are twofold. First we prove the nonlocal analogue of the Alexandrov result characterizing spheres as the only closed embedded hypersurfaces in $\mathbb{R}^N$ with constant mean curvature. Here we use the moving planes method. Our second result establishes the existence of periodic bands or "cylinders" in $\mathbb{R}^2$ with constant nonlocal mean curvature and bifurcating from a straight band. These are Delaunay type bands in the nonlocal setting. Here we use a Lyapunov-Schmidt procedure for a quasilinear type fractional elliptic equation.

Stability of equilibria and existence of pullback attractors for delay 2D Navier-Stokes equations

Tomás Caraballo, Universidad de Sevilla, Spain

Abstract. In this talk we will show several methods to analyze the long time behaviour of solutions to 2D Navier-Stokes models when some hereditary characteristics (constant, distributed or variable delay, memory, etc) appear in the formulation. First the existence, uniqueness and local stability analysis of steady-state solutions is studied by using several methods: the theory of Lyapunov functions, the Razumikhin-Lyapunov technique, by constructing appropriate Lyapunov functionals and finally by using a method based in Gronwall-like inequalities. Then the global asymptotic behaviour of solutions can be analyzed by using the theory of attractors. As the delay terms are allowed to be very general, the statement of the problem becomes nonautonomous in general. For this reason, the theory of nonautonomous pullback attractors appears to be appropriate.

Non homogeneous boundary value problems for the stationary Navier-Stokes equations in 2-d symmetric semi-infinite outlets

Michel Chipot, University of Zurich, Switzerland

Abstract. We would like to present existence results for the stationary non homogeneous Navier-Stokes problem in symmetric domains having a semi-infinite outlet. We assume for this Leray problem the so called general outflow condition.

This is a joint work with K. Kaulakyte, K. Pileckas and W. Xue.

Symplectic decomposition, Darboux theorem and ellipticity

Bernard Dacorogna, EPFL, Switzerland

Abstract. Our first result concerns the classical Darboux theorem. We prove that if ω_m is the standard symplectic form and f is a symplectic form, then we can find a diffeomorphism φ, with optimal regularity, satisfying

$$\varphi^*(\omega_m) = f \quad \text{and} \quad \delta[\varphi \lrcorner \omega_m] = 0$$

provided that f is a small perturbation of ω_m. Moreover we show that the above system is elliptic and that we have uniqueness, when coupled with a Dirichlet datum.

We then apply the above result to the so-called symplectic decomposition. We show that any map φ, satisfying appropriate assumptions, can be written as

$$\varphi = \psi \circ \chi$$

where

$$\psi^*(\omega_m) = \omega_m \quad \text{and} \quad \delta[\chi \lrcorner \omega_m] = d\chi \lrcorner \omega_m = 0.$$

The analogy with mass transportation and the Monge-Ampère equation, as well as with the polar decomposition, will be emphasized.

This is a joint work with Wifrid Gangbo and Olivier Kneuss.

Blow up for harmonic map flow

Juan Diego Davila, Universidad de Chile, Chile

Abstract. We study singularity formation for the harmonic map flow from a two dimensional domain into the sphere. We show that for suitable initial conditions the flow develops a type 2 singularity at some point in finite time, and that this is stable under small perturbations of the initial condition. This phenomenon and the rate of blow up were studied formally by van den Berg, Hulshof and King (2003) and proved by Raphael and Schweyer (2013) in the class of radial and 1-corrotationally symmetric maps. Our results hold without any symmetry assumptions.

Constant solutions, ground-state solutions and radial terrace solutions

Yihong Du, University of New England, Australia

Abstract. In many applications, one is interested in the nonlinear parabolic problem

$$u_t - \Delta u = f(u)(x \in R^N, t > 0),\ u(x,0) = u_0(x)(x \in R^N),$$

where $u_0 \in L^\infty(R^N)$ is nonnegative and has compact support, f is a smooth function satisfying $f(0) = 0$. One wants to know how much of the long-time dynamics of this problem is determined by the corresponding elliptic problem

$$-\Delta u = f(u),\ u \geq 0(x \in R^N).$$

We show that, if $u(\cdot, t)$ stays bounded in $L^\infty(R^N)$ for all $t > 0$, then as $t \to \infty$, $u(\cdot, t)$ converges to a stationary solution in $L^\infty_{loc}(R^N)$, provided that all the zeros of $f(u)$ in $[0, \infty)$ are nondegenerate (i.e., $f(u) = 0$ and $u \geq 0$ imply $f'(u) \neq 0$). Moreover, this stationary solution is either a stable constant solution (hence a stable zero of f), or a ground-state solution based on a stable zero of f (namely a solution $v(x)$ of the elliptic problem which is radially symmetric about some point $x_0 \in R^N$, decreases in $|x - x_0|$, and $\lim_{|x|\to\infty} v(x)$ is a stable zero of f). Thus, viewed in the space $L^\infty_{loc}(R^N)$, the long-time behavior of $u(\cdot, t)$ is determined by two simplest types of solutions of the corresponding elliptic problem.

Furthermore, we show that, viewed in the space $L^\infty(R^N)$, the long-time behavior of $u(\cdot, t)$ resembles that of a radial terrace solution $v(|x|, t)$, whose limit as $t \to \infty$ is determined completely by a propagating terrace of 1-space dimension, which by definition is a set of traveling wave solutions $\{w_i\}_{i=1}^k$, with each w_i solving the elliptic equation

$$-w_{zz} + c_i w_z = f(w)(z \in R^1),$$

where c_i is a certain constant, called the wave speed of w_i.

This talk is based on joint works with Peter Polacik and with Hiroshi Matano.

Rigidity results for elliptic PDEs

Mostafa Fazly, University of Alberta, Canada

Abstract. This talk provides classification and symmetry results for certain local and nonlocal elliptic PDEs with power type nonlinearities. We start with a brief background on standard methods and ideas developed over the past couple of decades. Then we present some new developments as well as new challenges. Monotonicity formulas, Liouville theorems and one-dimensional symmetry properties stand as our main focus. This talk covers a series of joint work with Nassif Ghoussoub, Juncheng Wei and Yannick Sire.

A higher-order large-scale regularity theory for random elliptic operators

Julian Fischer, Max Planck Institute, Germany

Abstract. We develop a large-scale regularity theory of higher order for divergence-form elliptic equations with heterogeneous coefficient fields a in the context of stochastic homogenization. Under the assumptions of stationarity and slightly quantified ergodicity of the ensemble, we derive a $C^{k,\alpha}$-"excess decay" estimate on large scales and a $C^{k,\alpha}$-Liouville principle for any $k \geq 2$: For a given a-harmonic function u on a ball B_R, we show that its energy distance on some ball B_r to the space of a-harmonic functions that grow at most like a polynomial of degree k has the natural decay in the radius r, at least above some minimal (random) radius r_0. Our Liouville principle states that the space of a-harmonic functions that grow at most like a polynomial of degree k has (almost surely) the same dimension as in the constant-coefficient case. Our results rely on the existence of higher-order correctors for the homogenization problem, which we establish by an iterative construction.

Effective energy of nearly-parallel Ginzburg-Landau vortex filaments

Robert L. Jerrard, University of Toronto, Canada

Abstract. Starting from the Ginzburg-Landau model, we derive an effective free energy functional for nearly-parallel vortex filaments. As a consequence, we establish the existence of solutions of the Ginzburg-Landau equations, in certain scaling regimes, possessing a collection of vortex filaments minimizing this effective energy.

This is joint work with Andres Contreras.

On the first nonconstant Neumann-eigenfunction for the p-Laplacian

Bernd Kawohl, Universität zu Köln, Germany

Abstract. The first nonconstant Neumann-eigenfunction for the p-Laplacian operator leads to interesting questions when p approaches 1 or infinity.

I shall report on joint results with L. Esposito, V. Ferone, C. Nitsch and C. Trombetti.

Remarks about the Poisson-Nernst-Planck equations

David Kinderlehrer, Carnegie Mellon, USA

Abstract. The Poisson-Nernst-Planck system of equations used to model ionic transport is interpreted as a gradient flow for the Wasserstein distance and a free energy in an appropriate space of probability measures. The interaction term between the species arising from the Gauss law is singular which gives rise to some challenging issues. We give a description of this situation attempting to maintain a minimal technical level including the basic format of the Wasserstein-type implicit scheme.

This is joint work with Léonard Monsaingeon and Xiang Xu.

On the prescribed Jacobian inequality $\det \nabla \varphi \geq f$ in Sobolev spaces in the plane

Olivier Kneuss, Federal University of Rio de Janeiro, Brasil

Abstract. I will speak about the prescribed Jacobian inequality coupled with a Dirichlet condition, namely

$$\begin{cases} \det \nabla \phi \geq f & \text{a.e. in } \Omega \\ \phi = \mathrm{id} & \text{on } \partial\Omega \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^2$ is a bounded smooth connected open set, $f : \Omega \to \mathbb{R}$ and where $\phi : \Omega \to \mathbb{R}^2$ is the unknown. I will prove that for every $1 < p \leq \infty$ and every $f \in L^p(\Omega; [0, \infty))$ with $\int_\Omega f < |\Omega|$ there exists a bi-Sobolev solution ϕ of (1); more precisely, for every $\epsilon > 0$, there exists an homeomorphism $\phi : \overline{\Omega} \to \overline{\Omega}$ with

$$\phi, \phi^{-1} \in W^{1,(p+1)/2-\epsilon}(\Omega; \Omega).$$

This is a joint work with Julian Fischer.

Extremum problems for Laplacian eigenvalues and a Generalized Polya Conjecture

Fanghua Lin, New York University, USA

Abstract. In this lecture I shall discuss a simple class of extremum problems for the Laplacian Dirichlet eigenvalues on an Euclidian domain and its connection to a generalized version of the Polya Conjecture. This would be an introductory lecture in which various classical and elementary properties of the Laplacian Dirichlet eigenvalues will be reviewed. Then we shall describe a few relatively recent progress on a class of extremum problems associated with these eigenvalues and its connections to the classical Polya conjecture.

The Bingham flow in periodic domains

Sorin Mardare, University of Rouen, France

Abstract. We consider a Bingham flow in a domain which is periodic in one direction and we are interested in the asymptotic behaviour of the solution to the stationary Bingham problem as the length 2ℓ of the domain (in the periodic direction) goes to infinity. In order to do that we follow the methods used in [CM], where the same study has been done for the Stokes system. However, the techniques in [CM] need some important adaptations in order to treat the difficulties related to the nonlinearity of the problem.

Our main result states that the velocity of the fluid converges strongly in the H^1-norm to the solution of a Bingham problem in the infinite periodic domain. Nevertheless, the speed of the convergence is much lower than the one obtained for the (linear) Stokes problem. More specifically, for a Bingham fluid, the rate of convergence is of the order of ℓ^{-a} with $0 < a < \frac{1}{2}$, while being exponential for the Stokes problem, i.e. of the order of $e^{-\alpha\ell}$ for some positive α.

This is a joint work with Patrizia Donato and Bogdan Vernescu.

References

[CM] M. Chipot, S. Mardare, *Asymptotic behaviour of the Stokes problem in cylinders becoming unbounded in one direction*, J. Math. Pures Appl. 90 (2008), 133-159.

Circle-valued maps: bubbles and singularities

Petru Mironescu, University of Lyon 1, France

Abstract. We consider maps $u : \Omega \to \mathbb{S}^1$ having some Sobolev regularity $u \in W^{s,p}$. For values of s and p relevant for applications,
a) either such maps need not have a phase φ with the same regularity as u
b) or the phase φ exists but is not controlled by the norm of u.

In case a), "factorization allows to write each such u as $u = v\,w$, where v lifts and w is "smoother than u. In the first part of the talk, I will present a new, very simple proof of this result, based on the theory of weighted Sobolev spaces.

Case b) occurs only in dimension one. In that case, there is an explicit example of loss of control (a "kink), and it turns out that only kinks lead to loss of control. This translates into a description of weakly convergent sequences which complements the one given by the theory of Cartesian currents. A quantitative result concerning kinks leads to the existence of minimal maps winding once around the unit circle.

The common theme of the proofs of the above results is the geometric detection of the energy concentration of manifold-valued maps.

Infinite time bubbling in the critical heat equation: the role of Green's Function

Monica Musso, Pontificia Universidad Católica de Chile, Chile

Abstract. We construct global unbounded solutions for the critical nonlinear heat equation on a bounded smooth domain satisfying zero Dirichlet boundary conditions. Given an integer k, and given any set of k distinct points of the domain, which satisfy a certain condition involving Green's function of the domain, we find a positive solution for the critical heat equation blowing up at exactly those k points as time goes to infinity.

This work is in collaboration with C. Cortazar and M. del Pino.

On the Helmholtz equations with sign changing coefficients

Hoai-Minh Nguyen, EPFL, Switzerland

Abstract. This talk is devoted to various properties and applications of the Helmholtz equations with sign changing coefficients. These equations are used to model negative index materials which are artificial structures whose refractive index are negative over some frequency range. These materials were first investigated theoretically by Veselago in 1964 and their existence was confirmed experimentally by Shelby et al. in 2001. The study of these equations faces two difficulties. First the ellipticity and the compactness are lost in general due to the changing sign coefficients. Second, the localized resonance, i.e., the fields blow up in some regions and remain bounded in some others as the loss (the viscosity) goes to 0, might occur. In this talk, I will present several remarkable properties of these equations and their applications such as cloaking and superlensing using complementary media and cloaking an object via anomalous localized resonance. Various conditions under which the equations are stable are also presented. Some useful techniques dealing with these equations such as reflecting and removing localized singularity techniques are mentioned.

On non-topological solutions for planar Liouville Systems of Toda-type

Arkady Poliakovsky, Ben-Gurion University of the Negev, Israel

Abstract. Motivated by the study of non-abelian Chern Simons vortices of non-topological type in Gauge Field Theory we analyse the solvability of some Liouville-type system in presence of singular sources We identify necessary and sufficient conditions which ensure the radial solvability of this system.

Two scale Gamma-convergence in random nonconvex homogenisation

Etienne Sandier, Université Paris 12 Val de Marne, France

Abstract. This is a joint work with L. Berlyand and S. Serfaty, where we bring together approaches of Dal Maso-Modica and Alberti-Müller to provide a framework we believe to be efficient for the analysis of multiscale problems. As a test we apply it to a random version of a problem studied by Alberti-Müller. The approach generalizes the one in previous work of S. Serfaty and myself on the Ginzburg-Landau functional.

Distances between classes of S^1-valued maps

Itai Shafrir, Technion, Israel

Abstract. Certain Sobolev spaces of S^1-valued functions can be written as a union of disjoint classes. It is interesting to study the distances between these classes. Previous work (Rubinstein-Shafrir 2007) studied the distances between classes in $W^{1,2}(\Omega, S^1)$, where Ω is a multiply connected domain in $\mathbb{R}^2$. In this talk, based on a joint work with Brezis and Mironescu, we shall concentrate on classes in $W^{1,1}(\Omega, S^1)$, where Ω is a simply connected domain in $\mathbb{R}^N$, $N \geq 2$. We will present estimates for the minimal distance as well as the Hausdorff distance between different classes.

Uniqueness of absolute minimizers of L-infinity functional involving Hamiltonian functions $H(x,p)$

Changyou Wang, University of Kentucky, USA

Abstract. In this talk, I will describe a uniqueness result of absolute minimizers of Hamiltonian functions $H(x,p)$, provided

(i) H is lower semicontinuous, and $H(x,p)$ is convex in p;
(ii) $0 = H(x,0) \le H(x,p)$ and $\cup_x \{p : H(x,p) = 0\}$ is contained in a hyperplane of R^n;
(iii) $H(x,p)$ is uniformly coercive in p.

On Serrin's overdetermined problem and a conjecture of Berestycki, Caffarelli and Nirenberg

Juncheng Wei, University of British Columbia, Canada

Abstract. In 1971, Serrin proved that the only bounded domain for which the overdetermined problem

$$\Delta u + f(u) = 0, u > 0 \text{ in } \Omega$$

$$u = 0 \text{ on } \partial\Omega$$

$$\partial_\nu u = C \text{ on } \partial\Omega$$

admits a solution is the bounded ball. In 1997, Berestycki, Caffarelli and Nirenberg considered the unbounded domain case, and proposed the following conjecture: If Serrin's problem admits a solution and Ω^c is connected, then Ω is either a half space, a cylinder $B \times R^{N-k}$, or complement of a ball or cylinder. In this talk, I shall discuss positive and negative answers to this conjecture.In particular, when Ω is an epigraph $\Omega = \{x_N > \varphi(x^{'})\}$, we show that (1) BCN conjecture is always true when $N = 2$, (2) BCN conjecture is true when $3 \leq N \leq 8$ if $\frac{\partial u}{\partial x_N} > 0$ (3) BCN conjecture is false when $N \geq 9$. A key observation is the connection between this problem and a one-phase free boundary problem.

Chemotactic systems in the presence of conflicts: a new functional inequality

Gershon Wolansky, Technion, Israel

Abstract. The evolution of a chemotactic system involving a population of cells attracted to self-produced chemicals is described by the Keller-Segel system in dimension 2, this system demonstrates a balance between the spreading effect of diffusion and the concentration due to self-attraction. As a result, there exists a critical "mass" (i.e. total cell's population) above which the solution of this system collapses in a finite time, while below this critical mass there is global existence in time. In particular, sub critical mass leads under certain additional conditions to the existence of steady states, corresponding to the solution of an elliptic Liouville equation. The existence of this critical mass is related to a functional inequality known as the Moser-Trudinger inequality.

An extension of the Keller-Segel model to several cells populations was considered before in the literature. Here we review some of these results and, in particular, consider the case of conflict between two populations, that is, when population one attracts population two, while, at the same time, population two repels population one. This assumption leads to a new functional inequality which generalizes the Moser-Trudinger inequality. As an application of this inequality we derive sufficient conditions for the existence of steady states corresponding to solutions of an elliptic Liouville system.

Interfaces in the Fisher equation and a Hamilton-Jacobi equation

Eiji Yanagida Tokyo Institute of Technology, Japan

Abstract. We consider the dynamics of interfaces in the Fisher-KPP equation. It is known that solutions of this equation exhibit interfaces that correspond to transition layers from the trivial steady state to a positive steady state. If an initial value decays rapidly in space, then the interface moves with a constant speed that is equal to the minimal speed of traveling fronts in one-dimensional space. On the other hand, it is known that if an initial value decays slowly, the interface may move in a rather irregular way. In this talk, we show that the dynamics of interfaces for slowly decaying initial data can be described as a level set of a Hamilton-Jacobi equation. We also discuss properties of solutions of the Hamilton-Jacobi equation.

This is a joint work with Hirokazu Ninomiya (Meiji University).