Banff International Research Station

for Mathematical Innovation and Discovery

Computability in Analysis, Geometry, and Dynamics
March 8-13, 2015

MEALS

*Breakfast (Buffet): 7:00 – 9:30 am, Sally Borden Building, Monday – Friday
*Lunch (Buffet): 11:30 am – 1:30 pm, Sally Borden Building, Monday – Friday
*Dinner (Buffet): 5:30 – 7:30 pm, Sally Borden Building, Sunday – Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

Sunday

16:00 Check-in begins (Front Desk – Professional Development Centre - open 24 hours)
17:30-19:30 Buffet Dinner
20:00 Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
Beverages and small assortment of snacks are available on a cash honor system.

Monday

7:00-8:45 Breakfast
8:45-9:00 Introduction and Welcome by BIRS Station Manager, TCPL
9:00-9:45 Lackenby
9:50-10:35 Dunfield
10:35-10:50 Coffee Break
10:50-11:35 Sapir

11:30-13:00 Lunch

13:00-14:00 Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00 Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).

14:15-15:00 Basu
15:00-15:30 Coffee Break
15:30-16:15 Pak

17:30-19:30 Dinner

Tuesday

7:00-9:00 Breakfast

9:00-9:45 Hirschfeldt
9:50-10:35 Rojas
10:35-10:50 Coffee Break
10:50-11:35 Avigad

11:30-13:30 Lunch

Afternoon -- discussions in groups

17:30-19:30 Dinner

Wednesday

7:00-9:00 Breakfast
9:00-9:45 Rohde
9:50-10:35 Binder
10:35-10:50 Coffee Break
10:50-11:35 Selinger

11:30-13:30 Lunch
Free Afternoon
17:30-19:30 Dinner

Thursday

7:00-9:00 Breakfast
9:00-9:45 Nabutovsky
9:50-10:35 Nekrashevych
10:35-10:50 Coffee Break
10:50-11:35 Garg

11:30-13:30 Lunch

Afternoon -- discussions in groups

17:30-19:30 Dinner

Friday

7:00-9:00 Breakfast
9:00 Informal Discussions
11:30-13:30 Lunch

Checkout by 12 noon.

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Abstracts to follow (if desired) in alphabetical order by last name of speaker.

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ABSTRACTS

(in alphabetic order by speaker surname)

Speaker: J. Avigad (Carnegie Mellon University)

Title: Computability and uniformity in ergodic theory

Abstract: Countless theorems of analysis assert the convergence of sequences of numbers, functions, or elements of an abstract space. Classical proofs often establish such results without providing explicit rates of convergence, and, in fact, it is often impossible to compute the limiting object or a rate of convergence from the given data. This results in the curious situation that a theorem may tell us that a sequence converges, but we have no way of knowing how fast it converges, or what it converges to.
On the positive side, it is often possible to "mine" quantitative and computational information from a convergence theorem, even when a rate of convergence is generally unavailable. Moreover, such information can often be surprisingly uniform in the data. In this talk, I will discuss examples that illustrate the kinds of information that can and cannot be obtained, focusing on results in ergodic theory.

Speaker: S. Basu (Purdue University)

Title: A complexity theory of constructible functions and sheaves.

Abstract: Constructible functions and more generally constructible sheaves play a very important role in algebraic geometry with many applications, including in theory of *D*-mocules, algebraic theory of partial differential equations, and even in more applied areas such as computational geometry and signal processing.
In this talk I will describe an approach towards developing a complexity theory for these objects, which generalizes the Blum-Shub-Smale model over $\mathbb{R}$. More precisely, we introduce a class of sequences simple constructible sheaves, that could be seen as the sheaf-theoretic analog of the Blum-Shub-Smale class $\mathbf{P}_{\mathbb{R}}$. We also define a hierarchy of complexity classes of sheaves mirroring the polynomial hierarchy, $\mathbf{PH}_{\mathbb{R}}$ in the B-S-S theory. We prove a singly exponential upper bound on the topological complexity of the sheaves in this hierarchy mirroring a similar result in the B-S-S setting. We obtain as a result an algorithm with singly exponential complexity for a sheaf-theoretic variant of the real quantifier elimination problem. Finally, we pose the natural sheaf-theoretic analogs of the classical $\mathbf{P}$ vs $\mathbf{NP}$ question, and also discuss a connection with Toda's theorem from discrete complexity theory in the context of constructible sheaves.
I will try to keep the talk self-contained as much as possible.

Speaker: I. Binder (University of Toronto)

Title: Computability properties of conformal maps.

Abstract: I will give an overview of the recent research on the computational properties of univalent maps. I will discuss the computability and complexity of computing a conformal map onto a given domain, both for the interior and boundary points. I will also talk about computability of harmonic measure. Based on joint projects with M. Braverman, C. Rojas, and M. Yampolsky

Speaker: N. Dunfield (UIUC)

Title: Computational complexity of problems in 3-dimensional topology.

Abstract: Almost all computation questions about high-dimensional manifolds (e.g. deciding if a manifold is S^n) are undecidable. In contrast, in 3-dimensions many natural questions are algorithmic solvable, including whether two given manifolds are homeomorphic, though often these algorithms are based on deep theories (Geometrization, Floer homology). The question then becomes how hard is it to solve these questions, both in theory and in practice. I will survey these issues of computational complexity, focusing on the question of computing the genus of a knot and its special case of deciding if a knot is knotted, including the foundational work of Haken from the 1960s as well as more recent work of Agol-Hass-Thurston, Kuperberg, Lackenby, Dunfield-Hirani, and Dunfield-Friedl-Jackson, from both theoretical and practical perspectives. I will conclude with some examples of trying to compute the genus of random knots with several hundred crossings.

Speaker: A. Garg (Princeton University)

Title: Communication and Information Complexity

Abstract: Communication complexity is one of the few models of computation in which we can prove strong unconditional lower bounds. It is roughly the amount of bits various parties involved need to communicate in order to compute a joint function of their inputs. It has been widely applicable in proving lower bounds in several other models of computation such as circuits, streaming algorithms, data structures etc. In the past 5-10 years, a continuous relaxation of communication complexity, information complexity, has been widely studied and has proven successful in tackling hard questions in communication complexity, such as direct sum and direct product theorems. I will define communication and information complexity, discuss what is known about them and mention open problems.

Speaker: D. Hirschfeldt (University of Chicago)

Title: An introduction to algorithmic randomness

Abstract: What does it mean to say that an individual object such as a real number is random, or to say that one real is more random than another? The theory of algorithmic randomness uses tools from computability theory to address questions such as these. Its modern study dates back to the 1960's, when Kolmogorov complexity and Martin-L\"of randomness were introduced, allowing us to make precise mathematical sense of the notion of randomness for individual objects, such as finite strings and real numbers, and to compare the levels of randomness of such objects. This talk will discuss some of the basic notions of this theory, as well as more recent developments, such as applications to "quantitative versions" of theorems in areas such as ergodic theory, which allow us to transform results that hold for almost all reals into ones that hold for all sufficiently random reals, where the necessary levels of randomness can often be precisely quantified.

Speaker: M. Lackenby (Oxford University)

Title: The complexity of knots

Abstract: Alan Turing asked whether there is an algorithm to decide whether two knots are equivalent. It is now known that there is such an algorithm. But the complexity of this problem is far from well understood. In my talk, I will give a survey of the current state of our knowledge. For example, I will explain why the problem of deciding whether a given knot is trivial is in NP and co-NP.

Speaker: A. Nabutovsky (University of Toronto)

Title: Finite presentations of $\mathbb{Z}$ and the trivial group and applications to four-dimensional geometry.

Abstract: First, we construct families of trivial 2-knots K_i in $\mathbb{R}^4$ such that the maximal complexity of 2-knots in any isotopy connecting K_i with the standard unknot grows faster than a tower of exponentials of any fixed height of the complexity of K_i. This is a joint work with Boris Lishak. To prove this result we used finite presentations of $\mathbb{Z}$ with certain remarkable combinatorial properties. Recently Boris Lishak proved the existence of similar ``tricky" finite presentations of the trivial group with just two generators and two relators. His work has a number of interesting implications for geometry of four-dimensional manifolds.

Speaker: V. Nekrashevych (Texas A&M)

Title: Topological conjugacy of expanding maps

Abstract: A locally expanding map of a compact metric space can be defined by a finite amount of data. Thus a natural problem is algorithmic decidability of topological conjugacy of such maps. This question remains to be open. I will discuss different algebraic reformulations of this question and possible approaches to its solution.

Speaker: I. Pak (UCLA)

Title: Words in linear groups, random walks, automata and P-recursiveness

Abstract: Fix a finite set S of integer k by k matrices. Denote by a_n the number of products of matrices in S of length n that are equal to 1. We show that the sequence {a_n} is not always P-recursive. This answers a question of Kontsevich.
We present two proofs. The first is based on computability approach going back to Mihailova who showed in 1960's that it is undecidable whether a_n is always zero. The second is ad hoc and uses a curious combination of deep results from Analysis, Number Theory, Probablity and Group Theory. While the former approach is amenable to generalizations, the latter is more elegant and leads to further questions.
Joint work with Scott Garrabrant.

Speaker: S. Rohde (University of Washington)

Title: How to draw trees

Abstract: This talk is based on joint work with Don Marshall. I will begin by describing the zipper algorithm for the numerical computation of conformal maps, together with the related Loewner equation and conformal welding. I will then consider convergence questions, both in the deterministic and in the probabilistic setting of SLE. After discussing a variant of the zipper algorithm that allows to approximate (the trees associated with) Shabat polynomials as well as dendritic Julia sets, I will conclude with speculations regarding conformal matings of continuum random trees.

Speaker: C. Rojas (Universidad Andres Bello)

Title: Algorithmic ergodic theory and randomness.

Abstract: We will discuss recent results surrounding computable aspects of ergodic theory and their connection to algorithmic randomness. We will further describe joint work with M. Hoyrup and S. Galatolo on the computability of invariant measures. We will see a general strategy allowing to establish computability of the relevant invariant measures. On the other hand, we will exhibit an example of a computable system for which every invariant measure is non computable, showing some subtleties on the problem. Finally we will survey recent joint work with M. Braverman and A. Grigo, supporting the conjecture that these kind of non computable phenomena are not robust to noise.

Speaker: M. Sapir (Vanderbilt University)

Title: Groups and machines

Abstract: I will discuss various computing devices (Turing machines, S-machines, Minsky machines) used in dealing with algorithmic problems in group theory.

Speaker: N. Selinger (Stony Brook)

Title: TBA