



# Banff International Research Station

for Mathematical Innovation and Discovery

## Hybrid Methods in Imaging

June 14–19 2015

### MEALS

\*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

\*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

\*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

\*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.

### MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

### SCHEDULE

		<b>Sunday</b>
16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)	
17:30–19:30	Buffet Dinner, Sally Borden Building	
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired) Beverages and a small assortment of snacks are available on a cash honor system.	
		<b>Monday</b>
7:00–8:45	———— Breakfast ————	
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL	
9:00–9:40	<b>Kui Ren</b> , <i>Recent theoretical and numerical results on some inverse transport problems with internal data</i>	
9:45–10:25	<b>Sebastian Acosta</b> , <i>Multiwave imaging in an enclosure with variable wave speed</i>	
10:25–10:40	———— Coffee Break, TCPL ————	
10:40–11:20	<b>Todd Quinto</b> , <i>A paradigm to classify added artifacts in limited data tomography</i>	
11:30–13:00	———— Lunch ————	
13:00–14:00	———— Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall	
14:00–14:15	———— Group Photo; meet in foyer of TCPL	
14:15–14:55	<b>Linh Nguyen</b> , <i>On the artifacts in a limited data spherical Radon transform</i>	
14:55–15:15	———— Coffee Break, TCPL ————	
15:15–15:55	<b>Alexander Oraevsky</b> , <i>Challenges of Quantitative Photoacoustic Tomography</i>	
16:00–16:40	<b>Mark A. Anastasio</b> , <i>Iterative Image Reconstruction Methods for Photoacoustic Computed Tomography with Application to Experimental Data</i>	
16:45–17:25	TBA	
17:30–19:30	———— Dinner ————	

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**Tuesday**

7:00–9:00 ——— Breakfast ———  
9:00–9:40 **Armando Manduca**, *Magnetic Resonance Elastography: A Signal Processing Perspective*  
9:45–10:25 **Joyce McLaughlin**, *Stability and Statistics for Shear Stiffness Imaging*  
10:25–10:40 ——— Coffee Break, TCPL ———  
10:40–11:20 **Gen Nakamura**, *Data analysis for micro-MRE and PVS*  
11:30–13:30 ——— Lunch ———  
13:30–14:10 **Otmar Scherzer**, *Edge detection for solving hybrid inverse problems*  
14:15–14:55 **Giovanni Alberti**, *Disjoint sparsity for signal separation and applications to hybrid imaging inverse problems*  
14:55–15:15 ——— Coffee Break, TCPL ———  
15:15–15:55 **Yang Yang**, *Thermoacoustic Tomography in Closed Domains*  
16:00–16:40 **Amir Moradifam**, *Uniqueness of minimizers of general least gradient problems arising in hybrid inverse problems*  
16:45–17:25 TBA  
17:30–19:30 ——— Dinner ———

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**Wednesday**

7:00–9:00 ——— Breakfast ———  
9:00–9:40 **David Isaacson**, *A method to image the ventilation perfusion ratio using EIT*  
9:45–10:25 **Yves Capdeboscq**, *On Proper Data Sets for Elliptic Hybrid Inverse Problems*  
10:25–10:40 ——— Coffee Break, TCPL ———  
10:40–11:20 **Rakesh**, *The Inverse Backscattering Problem*  
11:30–13:30 ——— Lunch ———  
13:30–17:30 ——— Free Afternoon —  
17:30–19:30 ——— Dinner ———

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**Thursday**

7:00–9:00 ——— Breakfast ———  
9:00–9:40 **John Schotland**, *Inverse problems in acoustic-optic imaging*  
9:45–10:25 **Alexandru Tamasan**, *Current Density Impedance imaging with Complete Electrode Model*  
10:25–10:40 ——— Coffee Break, TCPL ———  
10:40–11:20 **Victor Palamodov**, *On reconstruction of strain fields from scattering data*  
11:30–13:30 ——— Lunch ———  
13:30–14:10 **Alexander Mamonov**, *Nonlinear seismic imaging via reduced order model backprojection*  
14:15–14:55 **Jie Chen**, *The inverse problem for electroseismic conversion*  
14:55–15:15 ——— Coffee Break, TCPL ———  
15:15–15:55 **Kaloyan Marinov**, *Inverse boundary-value problems in an infinite slab with partial data*  
16:00–16:40 TBA  
16:45–17:25 TBA  
17:30–19:30 ——— Dinner ———

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**Friday**

7:00–9:00 ——— Breakfast ———  
9:00–10:25 ——— Informal Discussions  
10:25–10:40 ——— Coffee Break, TCPL ———  
10:40–11:30 ——— Informal Discussions  
11:30–13:30 ——— Lunch ———

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**Checkout by noon 12pm** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon.

# Hybrid Methods in Imaging

## June 14–19 2015

### ABSTRACTS (in alphabetic order by speaker surname)

Speaker: **Sebastian Acosta** (Baylor College of Medicine)

Title: *Multiwave imaging in an enclosure with variable wave speed*

Abstract: This talk considers the problem of photoacoustic and thermoacoustic tomography in the presence of physical boundaries such as reflectors or interfaces, which reflect some wave energy back into the domain. To model the physical boundaries we consider the wave problem where an impedance boundary condition is imposed. We relate the inverse problem with a statement in boundary observability and stabilization of waves. We present uniqueness and stability of the inverse problem and propose two different reconstruction methods. In both cases, if well-known geometrical conditions are satisfied, the approaches are naturally suited for variable wave speed and for measurements on a subset of the boundary.

This is a joint work with Carlos Montalto from University of Washington.

Speaker: **Giovanni S. Alberti** (Ecole Normale Supérieure)

Title: *Disjoint sparsity for signal separation and applications to hybrid imaging inverse problems*

Abstract: The main focus of this talk is the reconstruction of the signals  $f$  and  $g_i$ ,  $i = 1, \dots, N$ , from the knowledge of their sums  $h_i = f + g_i$ , under the assumption that  $f$  and the  $g_i$ s can be sparsely represented with respect to two different dictionaries  $A_f$  and  $A_g$ . This generalises the well-known “morphological component analysis” to a multi-measurement setting. The main result states that  $f$  and the  $g_i$ s can be uniquely and stably reconstructed by finding sparse representations of  $h_i$  for every  $i$  with respect to the concatenated dictionary  $[A_f, A_g]$ , provided that enough incoherent measurements  $g_i$ s are available. The incoherence is measured in terms of their mutual disjoint sparsity.

This method finds applications in the reconstruction procedures of several hybrid imaging inverse problems, where internal data are measured. These measurements usually consist of the main unknown multiplied by other unknown quantities, and so the disjoint sparsity approach can be directly applied. As an example, I will show how to apply the method to the reconstruction in quantitative photoacoustic tomography, also in the case when the Grüneisen parameter, the optical absorption and the diffusion coefficient are all unknown.

Speaker: **Mark A. Anastasio** (Washington University in St. Louis)

Title: *Iterative Image Reconstruction Methods for Photoacoustic Computed Tomography with Application to Experimental Data*

Abstract: Photoacoustic computed tomography (PACT) is an emerging soft-tissue imaging modality that has great potential for a wide range of preclinical and clinical imaging applications. It can be viewed as a hybrid imaging modality in the sense that it utilizes an optical contrast mechanism combined with ultrasonic detection principles, thereby combining the advantages of optical and ultrasonic imaging while circumventing their primary limitations. In this talk, we review our recent advancements in practical image reconstruction approaches for PACT. Such advancements include physics-based models of the measurement process and associated optimization-based inversion methods for reconstructing images from limited data sets in acoustically heterogeneous media. Applications of PACT to transcranial brain imaging and breast cancer detection will be discussed.

Speaker: **Yves Capdeboscq** (University of Oxford)

Title: *On Proper Data Sets for Elliptic Hybrid Inverse Problems*

Abstract: In this talk I will discuss some analysis questions arising in hybrid inverse problems. The focus of this talk will be on the determination of suitable boundary conditions a priori, from general properties of elliptic second order boundary value problems, rather than the resolution of the inverse problem itself.

Speaker: **Jie Chen** (Purdue University)

Title: *The inverse problem for electroseismic conversion*

Abstract: In fluid-saturated porous media, electromagnetic fields couple with seismic waves through the electroseismic conversion. Indeed, at the contact interface of pore fluid and solid rock, an electrical double layer (EDL) is formed. When electric or magnetic fields impinge on the EDL, the electrokinetic phenomenon causes movement of the fluid relative to rock frame and thus emits seismic waves, which can be remotely detected.

In this project, we study the inverse problem of the electroseismic conversion. We divide the problem in two steps. The first step is the inverse source problem of Biot's equations. With low frequency assumption, we derive a quantitative estimate of the Gassmann approximation which reduces Biot's equations to the elastic wave equation. We then prove a stability result of recovering the seismic sources from the boundary seismic measurements. The second step studies the inversion of Maxwell's equations with internal data. According to the coupling effects between seismic waves and EM waves, we consider the seismic sources as internal data for Maxwell's equations, from which we recover electrical parameters and achieve a Lipschitz-type stability estimate of the reconstruction. CGO solutions play an important role in the proof.

Speaker: **David Isaacson** (Rensselaer Polytechnic Institute)

Title: *A method to image the ventilation perfusion ratio using EIT*

Abstract: The ventilation perfusion ratio is the ratio of the volume of air entering a region of the lungs per breath divided by the volume of blood entering the same region per heart beat.

It will be shown how to define, measure, and image an approximation to this VQ ratio using electrical impedance imaging.

Images, movies, and data from human subject studies will be presented using the RPI and GE electrical impedance imaging systems.

Speaker: **Alexander V. Mamonov** (University of Houston)

Title: *Nonlinear seismic imaging via reduced order model backprojection*

Abstract: We introduce a novel nonlinear seismic imaging method based on model order reduction. The reduced order model (ROM) is an orthogonal projection of the wave equation propagator operator on the subspace of the snapshots of the solutions of the wave equation. It can be computed entirely from the knowledge of the measured time domain seismic data using the block Cholesky decomposition. The image is a backprojection of the ROM using the subspace basis for the known smooth kinematic velocity model. The implicit orthogonalization of solution snapshots is a nonlinear procedure that differentiates our approach from the conventional linear methods (Kirchhoff, reverse time migration). It allows for the removal of multiple reflection artifacts. It also enables us to estimate the magnitude of the reflectors similarly to the true amplitude migration algorithms. We present the numerical results for the standard Marmousi model and a synthetic high contrast hydraulic fracture example.

Co-authors: Vladimir Druskin and Mikhail Zaslavsky.

Speaker: **Armando Manduca** (Mayo Clinic)

Title: *Magnetic Resonance Elastography: A Signal Processing Perspective*

Abstract: Magnetic Resonance Elastography (MRE) is a non-invasive, phase contrast MRI technique that detects mechanically-induced motion in tissue, and uses that information to construct spatial maps of tissue stiffness. The best performance of any MRE inversion strategy is fundamentally determined by the quality of the harmonic motion information that is provided to it. In this work, we develop a statistical signal processing framework for steady-state MRE that enables rigorous characterization of the accuracy, precision, and uniqueness of harmonic motion information estimated from the raw data collected by an MRI scanner. After deriving and demonstrating the utility of this framework, we discuss statistical strategies for optimally estimating MRE harmonic information directly from raw MRI data, overview several unique mathematical aspects of this problem, and present a robust computational strategy for solving this problem.

Speaker: **Kaloyan Marinov** (Technical University of Denmark)

Title: *Inverse boundary-value problems in an infinite slab with partial data*

Abstract: Partial-data inverse problems in an infinite slab have been studied by Li and Uhlmann for the case of the Schrödinger equation, and by Krupchyk, Lassas and Uhlmann for the case of the magnetic Schrödinger equation. In this talk, we will prove a log-log stability estimate for the inverse problems considered by Li and Uhlmann. The boundary measurements considered in these problems are modelled by partial knowledge of the Dirichlet-to-Neumann map: in the first inverse problem, the corresponding Dirichlet and Neumann data are known on different boundary hyperplanes of the slab; in the second inverse problem, they are known on the same boundary hyperplane of the slab. This is joint work with Pedro Caro.

Speaker: **Joyce R. McLaughlin** (Rensselaer Polytechnic Institute)

Title: *Stability and Statistics for Shear Stiffness Imaging*

Abstract: There are basically two time dependent experiments for shear stiffness imaging. For one, the tissue is excited with a time harmonic oscillation and then sequences of MR data (or less often sequences ultrasound RFQ data sets) are taken and processed to produce a movie of the oscillating tissue within the body. For this experiment we present stability results for a single elastic vector movie. In the second experiment one pulse or a sequence of pulses are imparted by focusing ultrasound; a wave with a front propagates away from the pulse position. The arrival time of one component of the wave is calculated from the movie created from a sequence of RFQ data sets. We establish statistical properties of the noise in the image when using the direct algorithm and show that even though the variance is infinite there are some favorable statistical properties.

Speaker: **Amir Moradifam** (University of California, Riverside)

Title: *Uniqueness of minimizers of general least gradient problems arising in hybrid inverse problems*

Abstract: Motivated by the weighted least gradient problems arising in the hybrid inverse problem of determining an isotropic conductivity from the knowledge of the magnitude of one current density vector field, I will discuss some existence, uniqueness, and comparison theorems for minimizers of the general least gradient problem

$$\inf_{u \in BV_f(\Omega)} \int_{\Omega} \varphi(x, Du),$$

where  $f : \partial\Omega \rightarrow \mathbb{R}$  is continuous,  $BV_f(\Omega) := \{v \in BV(\Omega) : v|_{\partial\Omega} = f\}$ , and  $\varphi(x, \xi)$  is a function that, among other properties, is convex and homogeneous of degree 1 with respect to the  $\xi$  variable.

In particular we prove that if  $a \in C^{1,1}(\Omega)$  is bounded away from zero, then minimizers of the weighted least gradient problem  $\inf_{u \in BV_f} \int_{\Omega} a|Du|$  are unique in  $BV_f(\Omega)$ . We construct counterexamples to show that the regularity assumption  $a \in C^{1,1}$  is sharp. This is a joint work with Robert I. Jerrard and Adrian Nachman.

Speaker: **Gen Nakamura** (Hokkaido University)

Title: *Data analysis for micro-MRE and PVS*

Abstract: The back ground and motivation of our study are as follows. The micro-MRE (magnetic resonance elastography) and PVS (pendulum-type viscoelastic spectrometer) are measurement devices to measure viscoelasticity of a medium. More precisely, micro-MRE provides an interior measurement which can measure time harmonic waves inside a medium by micro-MRI, and PVS provides a boundary measurement which can measure using laser the displacement under time harmonic torsion or bending of a subsurface of a medium which is generated by using Lorentz force. The frequency range of measurement for PVS is much larger than that of for micro-MRE, and the sample size for micro-MRE is much larger than that of for PVS. While MRE has been so successful for diagnosing liver diseases, the study of micro-MRE was to provide a bench mark test for MRE. Since there is few rheological measurement device which can measure in a wide frequency range, any recovered value of viscoelasticity has to be cross checked by using other rheological measurement devices.

In order to get some approximate true value of viscoelasticity of a medium, it is very important to have a good model equation for the measurement and good inversion scheme to recover viscoelasticity from the measurement. In this talk we will report on the data analysis of these two rheological measurement devices along the line of the back ground and motivation of our study.

Speaker: **Linh Nguyen** (University of Idaho)

Title: *On the artifacts in a limited data spherical Radon transform*

Abstract: The spherical Radon transform arises in several imaging modalities, such as thermoacoustic/photoacoustic tomography, ultrasound tomography, and SONAR. In these modalities, detectors are located around an object of interest to detect ultrasound signals. These signals are closely related to the spherical Radon transform of the image of the object. Therefore, in order to reconstruct the image, one needs to invert the spherical Radon transform.

In many applications, the detector array does not totally enclose the object. This situation results in the limited data problem for spherical Radon transform. We will discuss a filtered-backprojection formula to approximately reconstruct the object. We show that some features of the image are well reconstructed, while some are not. Moreover, the reconstructed image contains some artifacts (i.e., added singularities). We will describe the geometry and strength of these artifacts.

Speaker: **Alexander Oraevsky** (TomoWave Laboratories, Inc.)

Title: *Challenges of Quantitative Optoacoustic Tomography*

Abstract: After almost two decades of active development optoacoustic tomography is entering the real world of clinical applications, with diagnostic imaging of breast cancer being the first major market niche for this technology where existing modalities have apparent drawbacks. We will present a number of system designs for optoacoustic mammography that show tissue structures in 2D and 3D with high sensitivity (contrast) / resolution, and discuss their advantages and limitations [1]. The main value of OAT is in its potential capability to provide functional and molecular information based on quantitatively accurate display of the optical absorption coefficient. However, quantitatively accurate OAT has not been demonstrated yet. We believe that only a full view three-dimensional tomography system that acquires complete set of forward data and uses rigorous solutions for inverse problem of image reconstruction have the potential for success in the breast cancer diagnostics. But even the full view design has challenges when providing quantitatively accurate information due to a large number of unknown parameters, separately distributed in the volume of tissue being examined. These parameters might be determined with sophisticated math-physics methods. As a step in this direction, we presently combine advantages of laser optoacoustics and ultrasound tomography as a hybrid dual-modality. The optoacoustic sub-system provides images based on distribution of molecular chromophores in the body absorbing near-infrared light, yielding functional images of the total hemoglobin [Hb] and blood oxygen saturation [SO<sub>2</sub>] as well as molecular images of targeted contrast agents. The ultrasound sub-system provides anatomical images of tissue structures and can also provide the speed of sound (SoS) and acoustic attenuation images, which can be used for iterative reconstruction of more accurate optoacoustic images. A review of our research efforts advancing biomedical applications of the three-dimensional optoacoustic tomography system in preclinical imaging using small laboratory animals and clinical application in diagnostic imaging of breast cancer will be presented and challenges of achieving further progress will be outlined.

[1] A.A. Oraevsky: “Optoacoustic Tomography: From Fundamentals to Diagnostic Imaging of Breast Cancer”, in Biomedical Photonics Handbook, 2nd edition, ed. by T. Vo-Dinh, CRC Press, Boca Raton, Florida, 2014, Chapter 21, pp. 715-757.

Speaker: **Victor Palamodov** (Tel Aviv University)

Title: *On reconstruction of strain fields from scattering data*

Abstract: The evaluation of the residual elastic strain in structural material requires imaging a six-component tensor quantity  $\varepsilon$  in three dimensions. A method of reconstruction of small residual strain fields in a body is based on data of diffraction pattern under penetrated X-ray or neutron radiation. The mathematical model is the longitudinal (axial) line transform of the tensor  $\varepsilon$ . These data are only sufficient for determination of the Saint-Venant tensor  $V\varepsilon$  which is a  $2 \times 2$ - symmetric-skewsymmetric tensor field. A complete reconstruction of a strain field by this method is impossible, since the Saint-Venant tensor vanishes for any potential strain field.

The method of polarization tomography is based on measurements of transformation of the polarization ellipse of the penetrating light through a weakly optically anisotropic material. The mathematical model is the line integral  $T\varepsilon$  of the traceless normal (truncated transverse) part of the stress field  $\varepsilon$ . A simple method of reconstruction of the displacement form from Tuy-complete data of  $T\varepsilon$  for any tensor whose axial line integrals vanish. Both methods can be combined for complete reconstruction of an arbitrary small strain tensor  $e$  from data of axial and traceless normal ray integrals of  $\varepsilon$ .

Speaker: **Eric Todd Quinto** (Tufts University)

Title: *A paradigm to classify added artifacts in limited data tomography*

Abstract: In this talk, we present a paradigm of Frikel and the speaker that explains the locations and properties of added artifacts that appear in limited angle tomography. We use microlocal analysis to understand the effect of data restriction, and we provide reconstructions from real and simulated data for X-ray CT, photoacoustic tomography, and the circular transform.

Speaker: **Rakesh** (University of Delaware)

Title: *The Inverse Backscattering Problem*

Abstract: We discuss the, still unsolved, problem of recovering a smooth, compactly supported potential on  $R^3$  from its backscattering data. We describe the known results and our partial result regarding the recovery of angularly controlled potentials and we state some simple sub-problems which are also unsolved. This is based on joint work with Gunther Uhlmann.

Speaker: **Kui Ren** (University of Texas at Austin)

Title: *Recent theoretical and numerical results on some inverse transport problems with internal data*

Abstract: We review here some theoretical and numerical results on inverse coefficient problems for the transport equation with multiple internal data sets. We will discuss applications of these problems in hybrid imaging modalities such as (fluorescence) photoacoustic tomography.

Speaker: **Otmar Scherzer** (University of Vienna)

Title: *Edge detection for solving hybrid inverse problems*

Abstract: In this talk we consider a problem of quantitative photoacoustic imaging. The problem decomposes into the photoacoustic imaging problem and an inverse problem (parameter estimation problem) for the fluence in a stationary diffusion equation. When one assumes piecewise constant diffusion, scattering, and Grueneisen parameters, respectively, then this problem can be decomposed into edge detection problem for the fluence and its derivatives and a parameter selection process based on the jump relations of the diffusion equation. Novel edge detection algorithms tuned to these problems are presented.

This is joint work with Elena Beretta (Milan), Markus Grasmair (Trondheim), Monika Muskieta (Wroclaw), Wolf Naetar (Vienna)

Speaker: **John Schotland** (University of Michigan)

Title: *Inverse problems in acoustic-optic imaging*

Abstract: A method to reconstruct the optical properties of a highly-scattering medium from acousto-optic measurements is proposed. The method is based on the solution to an inverse problem for the radiative transport equation with internal data. A stability estimate and a direct reconstruction procedure are described.

Speaker: **Alexandru Tamasan** (University of Central Florida)

Title: *Current Density Impedance imaging with Complete Electrode Model*

Abstract: This talk considers the inverse problem of recovering an isotropic conductivity from the interior knowledge of the magnitude one current density field generated by applying current on a set of electrodes. On the boundary we only require knowledge of the electrodes and their average input currents (one number only, for a pair of electrodes!). Mathematically, the forward problem is modeled by the Complete Electrode Model of Somersalo, Cheney and Isaacson. The inverse problem is modeled as a weighted minimum gradient problem. whose set of minimizers is completely characterized. We present a few applications. This is joint work with A. Nachman and J. Veras.

Speaker: **Yang Yang** (Purdue University)

Title: *Thermoacoustic Tomography in Closed Domains*

Abstract: We study the mathematical model of thermoacoustic tomography in bounded domains with perfect reflecting boundary conditions. We propose an averaged sharp time reversal algorithm which solves the problem with an exponentially converging Neumann series. Numerical reconstruction is implemented in both the full boundary and partial boundary data cases.