Discrete Geometry and Symmetry
February 8–13, 2015

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

Sunday

16:00	**Check-in begins** (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	**Buffet Dinner** (Sally Borden Building)
19:30	**Informal gathering** (in 2nd floor lounge of Corbett Hall) Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	**Breakfast**
8:45–9:00	**Introduction and Welcome by BIRS Station Manager** (TCPL)
9:00	**Brief Welcome by Organizers**
9:05	**Marston Conder** – Arc-types of vertex-transitive graphs
9:35	**Maria Elisa Carrancho Fernandes** – Regular and chiral hypertopes
10:05	**Coffee Break**
10:30	**Undine Leopold** – Euclidean symmetry of closed surfaces immersed in 3-space, Part I
11:00	**Thomas Tucker** – Euclidean symmetry of closed surfaces immersed in 3-space, Part II
11:30–13:00	**Lunch**
13:00–14:00	**Guided Tour of The Banff Centre** (meet in the 2nd floor lounge, Corbett Hall)
14:00	**Group Photo** (outdoors, meet in foyer of TCPL).
14:30	**Coffee Break**
15:00	**Eric Ens** – Consistent colourings of polytopes
15:30	**Micael Toledo** – On automorphism group of maniplexes and their symmetry type graphs
16:00	**Isabel Hubard** – Products of abstract polytopes
16:30–17:30	**Informal Discussions**
17:30–19:30	**Dinner**

Tuesday

7:00–9:00	**Breakfast**
9:00	**Dimitri Leemans** – Abstract polytopes and projective lines
9:30	**Eugenia O'Reilly-Regueiro** – Abstract polytopes and projective lines, the chiral case
10:00	**Coffee Break**
10:30	**Nikolai Matteo** – Three-orbit convex polytopes
11:00	**Kyle Meyer** – Counting faces of colorful associahedra
11:30–13:30	**Lunch**
14:00	**Nikolay Abrosimov** – Volume of hyperbolic octahedra with $\overline{3}$-symmetry
14:30	**Coffee Break**
15:00	**Wendy Finbow-Singh** – Low dimensional neighbourly simplicial polytopes
15:30	**Ryan Telford** – X-raying 3-dimensional convex bodies with mirror symmetry
16:00–17:30	**Informal Discussions**
17:30–19:30	**Dinner**

Wednesday **Morning program only!**

7:00–9:00	**Breakfast**
9:00	**Janos Pach** – On the number of crossings between curves
9:30	**Deborah Oliveros** – Helly's Theorem over subgroups and other additive subsets of $\mathbb{R}^d$
10:00	**Coffee Break**
10:30	**Vladislav Yaskin** – Stability results for sections of convex bodies
11:00	**Abigail Williams** – Uniform skeletal polyhedra
11:30–13:30	**Lunch**
13:30–	**Free afternoon**
17:30–19:30	**Dinner**

Thursday

7:00–9:00	**Breakfast**
9:00	**Karoly Bezdek** – Contact numbers - old and new
9:30	**Marton Naszodi** – Coverings in Euclidean space and on the sphere
10:00	**Coffee Break**
10:30	**Frieder Ladisch** – Affine symmetries of orbit polytopes
11:00	**David Richter** – Algebraic universality of parallel drawings
11:30–13:30	**Lunch**
14:00	**Alexander Litvak** – On the isotropic constant of random polytopes
14:30	**Coffee Break**
15:00	**Muhammad Khan** – The covering index of convex bodies
15:30–16:30	**Problem Session**
16:30–17:30	**Informal Discussions**
17:30–19:30	**Dinner**

Friday	**Morning program only!**
7:00–9:00	**Breakfast**
9:00	**Javier Bracho** – A full-rank chiral polytope in dimension 4
9:30	**Egon Schulte** – Colorful polytopes, associahedra and cyclohedra
10:00	**Coffee Break**
10:30–11:30	**Informal Discussions**
11:30–13:30	**Lunch**

Checkout by 12 noon.

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Discrete Geometry and Symmetry
February 8–13, 2015

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Nikolay Abrosimov** (Sobolev Institute of Mathematics)

Title: *Volume of hyperbolic octahedra with $\overline{3}$-symmetry*

Abstract: The talk is based on our joint paper with Alexander Mednykh and Ekaterina Kudina, accepted for publication in Proceedings of Steklov Institute of Mathematics.

We consider an octahedron with $\overline{3}$-symmetry, i.e. an octahedron admitting order 3 rotation and antipodal involution. To switch on the intuition we start with Euclidean case. We establish the existence criterion for such an octahedron and get the volume formula. Then we put the Euclidean octahedron in projective Cayley-Klein model of the hyperbolic space. We compute a hyperbolic edge lengths and dihedral angles in terms of coordinates of vertices, which are non-invariant parameters depending on the choice of coordinate chart. Then we eliminate the coordinates and obtain relations between edge lengths and dihedral angles. Using this relations we are able to solve a Schläfli equation and get an explicit volume formula for a hyperbolic octahedron with $\overline{3}$-symmetry. We also establish the existence criterion for such a hyperbolic octahedron.

Speaker: **Karoly Bezdek** (University of Calgary)

Title: *Contact numbers - old and new*

Abstract: Contact numbers are natural extensions of kissing numbers. The talk is a survey-type on estimating the number contacts in a packing of $n > 1$ unit balls in Euclidean d-space with $d > 1$.

Speaker: **Javier Bracho** (UNAM)

Title: *A full-rank chiral polytope in dimension 4*

Abstract: An unexpected example of a finite regular polytope of rank 4 with a chiral embedding in R^4 will be presented. Pictures will be displayed and some particularities of its chiral facet will be discussed.

Speaker: **Maria Elisa Carrancho Fernandes** (University of Aveiro)

Title: *Regular and chiral hypertopes*

Abstract: In 1983, Aschbacher proved that string C-groups are thin, residually connected, regular geometries. Here we will consider C-groups with nonlinear Coxeter diagrams. We show that thin, residually connected regular geometries are C-groups, but the converse is not true. Nevertheless flag-transitivity is a sufficient condition if we want to go the other way around. That is, flag-transitive C-groups are thin, residually connected regular geometries (we use Tits algorithm to get incidence geometry from a C-group). Abstract regular polytopes are string C-groups, as described by McMullen and Schulte in their book (2002). For this reason we use the term (regular) hyper- tope to designate a thin, residually connected (regular) geometry. Abstract regular polytopes are regular hypertopes with linear Coxeter diagram. Guided by the ideas of chirality in the abstract polytope theory, we extend the concept to a more general setting of incidence geometries. Indeed, when the geometry is thin, it is possible to define chirality, as in the case of polytopes. We give characterisations of automorphism groups of thin residually connected chiral geometries and we show how to construct such chiral objects group-theoretically. One of our focus is the classification of hypertopes of a certain type. Here we consider spherical, locally spherical and locally toroidal hypertopes (hypertopes having all parabolic subgroups either spherical or toroidal).

Speaker: **Marston Conder** (University of Auckland)

Title: *Arc-types of vertex-transitive graphs*

Abstract: Let X be vertex-transitive graph of valency d, and let A be its full automorphism group. Then the *arc-type* of X is defined in terms of the lengths of the orbits of the action of the stabiliser A_v of a given vertex v on the set of arcs incident with v. Specifically, the arc-type is the partition of d as the sum $n_1 + n_2 + \cdots + n_t + (m_1 + m_1) + (m_2 + m_2) + \cdots + (m_s + m_s)$, where $n_1, n_2, \ldots, n_t$ are the lengths of the self-paired orbits, and $m_1, m_1, m_2, m_2, \ldots, m_s, m_s$ are the lengths of the non-self-paired orbits, in ascending order. For example, if X is arc-transitive then its arc-type is d, while if X is half-arc-transitive then its arc-type is $d/2 + d/2$. In this talk I will explain how it can be shown that there are vertex-transitive graphs with every possible arc-type, except $1 + 1$ and $(1 + 1)$.

Speaker: **Eric Ens** (York University)

Title: *Consistent colourings of polytopes*

Abstract: We call a colouring of the facets of a polytopes consistent if colouring is respected (though not necessarily preserved) by the automorphism group. We can trivially colour any polytope with a different colour for each facet or by colouring every facet the same colour. We will look at few interesting examples and then examine regular and chiral toroidal polytopes of type $\{4, 4\}$ in more depth.

Speaker: **Wendy Finbow-Singh** (St. Mary's University)

Title: *Low dimensional neighbourly simplicial polytopes*

Abstract: Amongst the d-polytopes with v vertices, the neighbourly polytopes have the greatest number of facets. This maximum property has prompted researchers to compose lists of them. In this talk, we will discuss an algorithm for generating the list of simplicial neighbourly d-polytopes with v vertices, for a given dimension d and number of vertices, v.

Speaker: **Isabel Hubard** (UNAM)

Title: *Products of abstract polytopes*

Abstract: Given two convex polytopes, the join, the cartesian product and the direct sum of them are well understood. In this talk we shall extend these three kinds of products to abstract polytopes and introduce a new product, called the topological product, which also arises in a natural way from geometry. One of our interests is to understand the automorphism group of a product of $\mathcal{P}$ and $\mathcal{Q}$, in terms of the automorphism groups of $\mathcal{P}$ and $\mathcal{Q}$. To this end, we introduce the concept of a *prime* polytope, for a given product. We shall see that highly symmetric non-prime polytopes are sparse; in fact, for the join product the only regular non-prime polytopes are the simplices, for the cartesian product the only regular non-prime polytopes are the hypercubes, for the direct sum the only regular non-prime polytopes are the cross polytopes and for the topological product the only regular non-prime polytopes are toroidal polytopes.

Speaker: **Muhammad Khan** (University of Calgary)

Title: *The covering index of convex bodies*

Abstract: Covering a convex body by its homothets is a classical notion in discrete geometry that has resulted in a number of interesting and long standing problems. Swanepoel introduced the covering parameter of a convex body as a means of quantifying its covering properties. Here, we introduce a relative of the covering parameter called covering index, which turns out to have a number of nice properties. Intuitively, the covering index measures how well a convex body can be covered by a relatively small number of homothets having a relatively small homothety ratio. We show that the covering index provides a useful upper bound for well-studied quantities like the illumination number, the illumination parameter, the vertex index and the covering parameter of a convex body. We obtain upper bounds on the covering index and investigate its optimizers. Furthermore, we show that the covering index satisfies a nice compatibility with the operations of direct vector sum and vector sum that helps in determining the covering index of several convex bodies. Joint work with Karoly Bezdek.

Speaker: **Frieder Ladisch** (University of Rostock)

Title: *Affine symmetries of orbit polytopes*

Abstract: An orbit polytope is the convex hull of an orbit under a finite group $G \leq \mathrm{GL}(d, \mathbb{R})$. We consider the possible affine symmetry groups of orbit polytopes. For every group, there is an open and dense set of "generic points" such that the orbit polytopes of generic points have conjugated affine symmetry groups and are minimal in a certain sense. The symmetry group of a generic orbit polytope is again G if G is itself the affine symmetry group of some orbit polytope, or if G is absolutely irreducible. On the other hand, there are some general cases where the affine symmetry group grows, for example representation polytopes (the convex hull of a finite matrix group). Their affine symmetries can be computed effectively from a certain character. This is joint work with Erik Friese.

Speaker: **Dimitri Leemans** (University of Auckland)

Title: *Abstract polytopes and projective lines*

Abstract: We will discuss the classification of abstract polytopes whose automorphism group is an almost simple group of PSL(2,q) type. We will detail the classification of the regular polytopes that we started together with Egon Schulte for the groups PSL(2,q) and PGL(2,q) and that we later extended with Thomas Connor and Julie De Saedeleer. We will also explain where we stand, together with Eugenia OReilly-Regueiro and Jeremie Moerenhout, for the chiral polytopes related to these groups.

Speaker: **Undine Leopold** (Technische Universität Chemnitz)

Title: *Euclidean symmetry of closed surfaces immersed in* 3*-space, Part I*

Abstract: Given a finite group G of orientation-preserving euclidean isometries and a closed surface S, an immersion $f : S \to E^3$ is in G-general position if $f(S)$ is invariant under G, points of S have disk neighborhoods whose images are in general position, and no singular points of $f(S)$ lie on an axis of rotation of G. For such an immersion, there is an induced action of G on S whose Riemann-Hurwitz equation satisfies certain natural restrictions. In the first part of this talk, we introduce these restrictions and present how models arise from the quotient surface S/G in the orbifold E^3/G. It may be particularly surprising that an orientable symmetric surface can lead to a nonorientable quotient. We also point out that the problem of classifying which of the restricted Riemann-Hurwitz equations are realizable becomes intractable outside of G-general immersions. (Joint work with Tom Tucker, who gives Part II of the talk.)

Speaker: **Alexander Litvak** (University of Alberta)

Title: *On the isotropic constant of random polytopes*

Abstract: Let $X_1, \dots, X_N$ be independent random vectors uniformly distributed on an isotropic convex body $K \subset \mathbb{R}^n$, and let K_N be the symmetric convex hull of X_i's. We show that with high probability $L_{K_N} \leq C\sqrt{\log(2N/n)}$, where C is an absolute constant. This result closes the gap in known estimates in the range $Cn \leq N \leq n^{1+\delta}$. Furthermore, we extend our estimates to the symmetric convex hulls of vectors $y_1X_1, \dots, y_NX_N$, where $y = (y_1, \dots, y_N)$ is a vector in $\mathbb{R}^N$. Finally, we discuss the case of a random vector y. Joint work with D. Alonso-Gutierrez and Nicole Tomczak-Jaegermann.

Speaker: **Nikolai Matteo** (Northeastern University)

Title: *Three-orbit convex polytopes*

Abstract: I present a classification of the convex polytopes with three flag orbits under the symmetry group action. These exist only in eight dimensions or fewer.

Speaker: **Kyle Meyer** (Northeastern University)

Title: *Counting faces of colorful associahedra*

Abstract: The classical associahedra can be formulated in terms of flipping the diagonals of triangulations of convex polygons. Similarly the colorful associahedra, an abstract polytope, introduced by Araujo-Pardo, Hubard, Oliveros, and Schulte, is formulated in terms of flipping diagonals of triangulations whose

diagonals are colored (colored triangulations). In this talk we will give a modified formulation of the colorful associahedra in terms of partial colored triangulations, and using this formulation we will count the number of faces of the colorful associahedra by dimension.

Speaker: **Marton Naszodi** (Ecole Polytechnique Federale de Lausanne, and Eötvös Loránd University)
Title: *Coverings in Euclidean space and on the sphere*
Abstract: We present a method to obtain upper bounds on covering numbers. As applications of this method, we reprove and generalize results of Rogers on economically covering Euclidean n-space (resp. the sphere) with translates resp. rotated copies of a (spherically) convex body, or more generally, any measurable set. Using the same method, we sharpen an estimate by Artstein–Avidan and Slomka on covering a bounded set by translates of another.

The main novelty of our method is that it is not probabilistic. The key idea, which makes our proofs rather simple, is an algorithmic result of Lovász.

Speaker: **Deborah Oliveros** (UNAM)
Title: *Helly's Theorem over subgroups and other additive subsets of* $\mathbb{R}^d$.
Abstract: In the usual Helly-type theorems, the convex sets are required to intersect in a proper subset S of $\mathbb{R}^d$. For instance, in the classical Helly's theorem we have $S = \mathbb{R}^d$ and a Helly number of $d + 1$, for Doignon's theorem $S = \mathbb{Z}^d$ is the set of integer points and the Helly number is 2^d. In this talk we will present some extensions of these results where S is an arbitrary additive subgroup of $\mathbb{R}^d$, as well as some other interesting related results in dimension 2. Joint work with J.A. De Loera, R.N. La Haye and E. Roldán-Pensado.

Speaker: **Eugenia O'Reilly-Regueiro** (UNAM)
Title: *Abstract polytopes and projective lines, the chiral case*
Abstract: The classification of abstract polytopes with almost simple automorphism group of PSL(2,q) type has been addressed separately for the regular and the chiral cases. The regular case will be presented by Dimitri Leemans, it was completed jointly with Thomas Connor and Julie De Saedeleer following previous work with Egon Schulte. In this talk we present some results on the chiral case, from ongoing joint work with Dimitri Leemans and Jeremie Moerenhout.

Speaker: **Janos Pach** (Ecole Polytechnique Federale de Lausanne, and Renyi Institute)
Title: *On the number of crossings between curves*
Abstract:

Speaker: **David Richter** (Western Michigan University)
Title: *Algebraic universality of parallel drawings*
Abstract: Let Σ be a set of d fixed-point-free involutions on a given set $S = \{1, 2, 3, ..., 2n\}$. Graph-theoretically, this is the same as specifying a d-regular multigraph with vertex set S and an edge coloring by d colors. A *parallel drawing* of Σ is a drawing of the underlying graph in which every edge is represented by a segment and the segments sharing a common color are mutually parallel. The purpose of this talk is to explain "algebraic universality" for parallel drawings in the plane in the case when $|\Sigma| = 4$.

Speaker: **Egon Schulte** (Northeastern University)
Title: *Colorful polytopes, associahedra and cyclohedra*
Abstract: Every n-edge colored n-regular graph G naturally gives rise to a simple abstract n-polytope $P(G)$, called the colorful polytope of G, whose 1-skeleton is isomorphic to G. We describe colorful polytope versions of the associahedron and cyclohedron. Like their classical counterparts, the colorful associahedron and cyclohedron encode triangulations and flips, but now with the added feature that the diagonals of the triangulations are colored and adjacency of triangulations requires color preserving flips. The colorful

associahedron and cyclohedron are derived as colorful polytopes from the edge colored graph whose vertices represent these triangulations and whose colors on edges represent the colors of flipped diagonals. Joint work with G.Araujo-Pardo, I.Hubard and D.Oliveros.

Speaker: **Micael Toledo** (UNAM)

Title: *On automorphism group of maniplexes and their symmetry type graphs*

Abstract: Given a maniplex M, consider its set of flag orbits, O, under the action of its automorphism group. We can construct an edge-coloured graph having vertex set O, by joining to vertices o_1 and o_2 with an i-coloured edge, whenever there are two i-adjacent flags f and g, in o_1 and o_2, respectively. Such graph is called the symmetry type graph of M. In this talk we discuss some properties of symmetry type graphs of maniplexes as well as giving generators for the automorphism group of a maniplex, given its symmetry type graph.

Speaker: **Ryan Trelford** (University of Calgary, York University)

Title: *X-raying 3-dimensional convex bodies with mirror symmetry*

Abstract: Let K be a d-dimensional convex body. A point p on the boundary of K is said to be X-rayed along a line with direction vector $\mathbf{v}$ if the line through p with direction $\mathbf{v}$ intersects the interior of K. A collection of lines is said to X-ray K if every boundary point of K is X-rayed along one of the lines. The minimum number of lines required to X-ray K is called the X-ray number of K, and is denoted by $X(K)$. In 1994, Bezdek and Zamfirescu conjectured that $X(K) \leq 3 \cdot 2^{d-2}$ for any d-dimensional convex body K.

In this talk, we explain how the X-ray Conjecture is related to the famous Gohberg-Markus-Hadwiger Covering Conjecture. We briefly verify the X-ray conjecture for planar convex bodies, showing that three lines are needed if, and only if, the convex body is a triangle. Finally, we prove that any 3-dimensional convex body exhibiting mirror symmetry also satisfies the X-ray Conjecture.

Speaker: **Thomas W. Tucker** (Colgate University)

Title: *Euclidean symmetry of closed surfaces immersed in* 3*-space, Part II*

Abstract: Given a finite group G of orientation-preserving euclidean isometries and a closed surface S, an immersion $f : S \to E^3$ is in G-general position if $f(S)$ is invariant under G, points of S have disk neighborhoods whose images are in general position, and no singular points of $f(S)$ lie on an axis of rotation of G. For such an immersion, there is an induced action of G on S whose Riemann-Hurwitz equation satisfies certain natural restrictions. In the second part of this talk, we focus on additional group theoretic conditions that must be satisfied by G and the fundamental groups of the surface S and its quotient surface, before completing the classification of which restricted Riemann-Hurwitz equations are realized by a G-general position immersion of S. Exceptions arise, in particular, for low genus and little branching. We are then able to decide which genera of a surface allow a G-general immersion in 3-space. (Joint work with Undine Leopold, who gives Part I of the talk.)

Speaker: **Abigail Williams** (Northeastern University)

Title: *Uniform skeletal polyhedra*

Abstract: In this talk, I will discuss uniform skeletal polyhedra. When looking at skeletal polyhedra, we consider each face to be a set of edges which is not spanned by a membrane. The faces, and indeed the polyhedra themselves, are hollow. The uniformity condition signifies that the polyhedra are vertex transitive and have regular faces. I will show a construction which can be used to generate uniform skeletal polyhedra from the symmetry groups of the regular polyhedra. I will also discuss an extension of this construction which can be used to generate more uniform skeletal polyhedra.

Speaker: **Vlad Yaskin** (University of Alberta)

Title: *Stability results for sections of convex bodies*

Abstract: Let K be a convex body in $\mathbb{R}^n$. The *parallel section function* of K in the direction $\xi \in S^{n-1}$ is defined by

$$A_{K,\xi}(t) = \mathrm{vol}_{n-1}(K \cap \{\xi^{\perp} + t\xi\}), \quad t \in \mathbb{R}.$$

If K is origin-symmetric (i.e. $K = -K$), then Brunn's theorem implies

$$A_{K,\xi}(0) = \max_{t\in\mathbb{R}} A_{K,\xi}(t)$$

for all $\xi \in S^{n-1}$.

The converse statement was proved by Makai, Martini and Ódor. Namely, if $A_{K,\xi}(0) = \max_{t\in\mathbb{R}} A_{K,\xi}(t)$ for all $\xi \in S^{n-1}$, then K is origin-symmetric.

We provide a stability version of this result. If $A_{K,\xi}(0)$ is close to $\max_{t\in\mathbb{R}} A_{K,\xi}(t)$ for all $\xi \in S^{n-1}$, then K is close to $-K$. Joint work with Matthew Stephen.