Multiscale Asymptotics for
MJO Skeleton & Tropical–Extratropical Interactions

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Tropical convection – extratropic interactions

Motivation

- Teleconnection between midlatitude dynamics and tropical climate

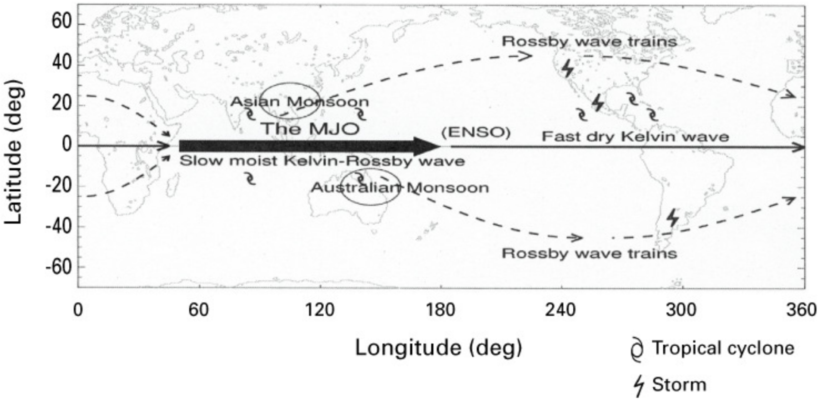

Figure from Moncrieff et al 2007 *WMO Bulletin*

Extratropical waves initiate MJO

- Observation: Matthews & Kiladis 1999 ...
- Model: Ray *et al.* 2009, Ray & Zhang 2010 ...

MJO excites extratropical waves

- Observation: Kiladis & Weickmann 1992 ...
- Model: Jin & Hoskins 1995, Matthews *et al.* 2004 ...

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- Observation: Kiladis & Weickmann 1992 ...
- Model: Jin & Hoskins 1995, Matthews *et al.* 2004 ...

MJO termination

- Observation: Stachnik *et al.* 2015
- others??

Model setup

- Vertical: barotropic – first baroclinic modes:

$$\mathbf{v}_{\text{full}}(x, y, z, t) = \overline{\mathbf{v}} + \cos(z)\mathbf{v}$$

- Dry dynamic interactions
  - Majda & Biello 2003, Khouider & Majda 2005 ...
- MJO skeleton model
  - Majda & Stechmann 2009, Majda & Stechmann 2011 ...

Barotropic–first baroclinic–MJO skeleton model

Barotropic dynamics

$$\frac{\partial \overline{\mathbf{v}}}{\partial t} + \overline{\mathbf{v}} \cdot \nabla \overline{\mathbf{v}} + y \overline{\mathbf{v}}^{\perp} + \nabla \overline{p} = -\frac{1}{2} \nabla \cdot (\mathbf{v} \otimes \mathbf{v})$$

$$\nabla \cdot \overline{\mathbf{v}} = 0$$

Baroclinic dry dynamics

$$\frac{\partial \mathbf{v}}{\partial t} + \overline{\mathbf{v}} \cdot \nabla \mathbf{v} - \nabla \theta + y \mathbf{v}^{\perp} = -\mathbf{v} \cdot \nabla \overline{\mathbf{v}}$$

$$\frac{\partial \theta}{\partial t} + \overline{\mathbf{v}} \cdot \nabla \theta - \nabla \cdot \mathbf{v} = \delta^2 (\bar{H} a - S^{\theta})$$

Moisture equation

$$\frac{\partial q}{\partial t} + \overline{\mathbf{v}} \cdot \nabla q + \tilde{Q} \nabla \cdot \mathbf{v} = -\delta^2 (\bar{H} a - S^{q})$$

Convection equation

$$\frac{\partial a}{\partial t} = \Gamma q a$$

Variables:

$\overline{\mathbf{v}}$ – barotropic velocity

$\mathbf{v}$ – first baroclinic velocity

θ – potential temperature

q – low tropospheric moisture

a – convective envelope

Coefficients:

Γ, $\tilde{Q}$, $\bar{H}$ – downscaling parameterization

S^{θ}/S^{q}– radiative cooling/moisture source

δ – small paramters

Energy conservation

$$\mathcal{E} = \overbrace{\mathcal{E}_T}^{\text{barotropic energy}} + \overbrace{\mathcal{E}_C}^{\text{baroclinic energy}} + \overbrace{\mathcal{E}_M}^{\text{moisture energy}} + \overbrace{\mathcal{E}_A}^{\text{convective energy}}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{E} = 0$$

$$\mathcal{E}_T = \frac{1}{2}\int |\overline{\mathbf{v}}|^2 \mathrm{d}x\mathrm{d}y$$

$$\mathcal{E}_C = \frac{1}{4}\int |\mathbf{v}|^2 + \theta^2 \mathrm{d}x\mathrm{d}y$$

$$\mathcal{E}_M = \frac{1}{4}\int \frac{1}{\tilde{Q}(1-\tilde{Q})}(q + \tilde{Q}\theta)^2 \mathrm{d}x\mathrm{d}y$$

$$\mathcal{E}_A = \frac{\delta^2}{2}\int \frac{1}{\tilde{Q}\Gamma}\left[\bar{H}a - S^\theta \log(a)\right]\mathrm{d}x\mathrm{d}y$$

Asymptotic expansion: small parameter δ

Small convective forcing and source terms:

$$\delta^2(\bar{H}a - S^\theta), \quad -\delta^2(\bar{H}a - S^q)$$

Long wave assumption:

$$x' = \delta x, \quad t' = \delta t, \quad v' = \delta v$$

Multiscale in time:

$$T_1 = \delta t', \quad T_2 = \delta^2 t'$$

Small amplitude:

$$\vec{U} = \delta^2 \vec{U}_1 + \delta^3 \vec{U}_2 + \delta^4 \vec{U}_3 + O(\delta^5)$$

From 2D to 1D: meridional truncation

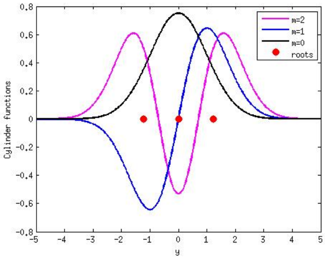
$$\psi = B(x,t)\sin(Ly) \qquad \psi\text{: barotropic stream function}$$

$$(l,r,q) = \left(l^{(0)}(x,t), r^{(0)}(x,t), q^{(0)}(x,t)\right)\Phi_0(y),$$

$$+\left(l^{(2)}(x,t), r^{(2)}(x,t), q^{(2)}(x,t)\right)\Phi_2(y),$$

$$v' = v^{(1)}\Phi_1(y),$$

$$a' = a^{(0)}(x,t)\Phi_0(y) \qquad l, r : \text{Riemann invariants}$$



$\Phi_m(y)$: parabolic cylinder functions

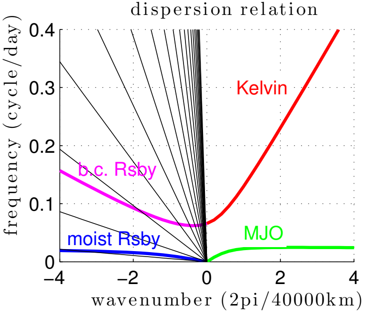
$$\langle \Phi_m(y), \Phi_n(y)\rangle = \delta_{mn}$$

$$\Phi_0(y) = \pi^{-1/4}\exp(-y^2/2)$$

$$\Phi_1(y) = \pi^{-1/4}\sqrt{2}y\exp(-y^2/2)$$

$$\Phi_2(y) = \pi^{-1/4}2^{-1/2}(2y^2-1)\exp(-y^2/2)$$

Dispersion relation at leading order



Barotropic dispersion relation depends on meridional wavelength $2Y$

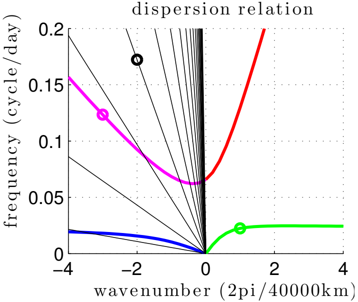
$$\omega_{\mathrm{T}}(k) = -\frac{1}{\pi^2} k Y^2$$

Three-wave interactions

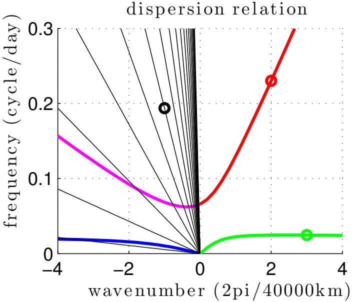
Resonance condition at higher order with nonlinear terms:

$$k_1 + k_2 + k_{\mathrm{T}} = 0,$$
$$\omega_1 + \omega_2 + \omega_{\mathrm{T}} = 0.$$

MJO–b.c. Rossby–b.t. Rossby



MJO–Kelvin–b.t. Rossby



Reduced asymptotic model

$$
\begin{aligned}
&-\partial_{T_2}\beta & &+ id_2\beta + id_3\alpha_1^*\alpha_2^* = 0\\
&-\partial_{T_2}\alpha_1 + id_4\alpha_1^2\alpha_1^* & &+ id_5\alpha_1 + id_6\beta^*\alpha_2^* = 0\\
&-\partial_{T_2}\alpha_2 + id_7\alpha_2^2\alpha_2^* & &+ id_8\alpha_2 + id_9\beta^*\alpha_1^* = 0
\end{aligned}
$$

- Leading order amplitudes and phases:

  β – b.t. wind; α_1 – MJO; α_2 – b.c. Rossby/Kelvin

- Coefficients ds are in terms of eigenvectors

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  - Cubics: nonlinear Γqa
  - Linear terms: dispersive terms
  - Quadratics: barotropic–baroclinic interactions

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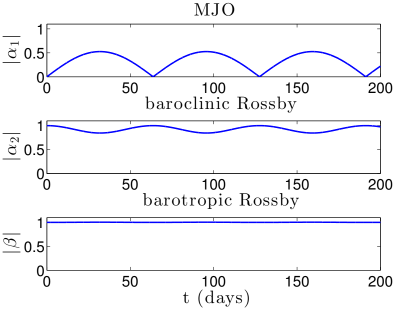
- Coefficients ds are in terms of eigenvectors
- Connection with the full system:
  - Cubics: nonlinear $\Gamma q a$
  - Linear terms: dispersive terms
  - Quadratics: barotropic–baroclinic interactions
- Energy conservation: $\frac{\mathrm{d}}{\mathrm{d}T_2}(|\alpha_1|^2 + |\alpha_2|^2 + |\beta|^2) = 0$

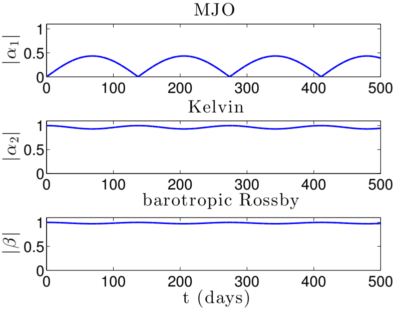
Numerical results

MJO–b.c. Rossby–b.t. Rossby



$\delta^2 = 0.1$, $\overline{u} \approx 3.0$ m/s

MJO–Kelvin–b.t. Rossby



$\delta^2 = 0.1$, $\overline{u} \approx 2.3$ m/s

Observations:

- Periodic behavior in results
- Barotropic Rossby wave has almost no energy exchange
- Different phase relation at $t = 0$ day does not affect energy relation

Conclusions and ongoing work

Conclusions

- Setup of a multiscale asymptotic model for tropical – extratropical and MJO interactions
- Derived an ODE system for three-wave interactions
- Numerical periodic solution predicts initiation/termination for MJO event

Ongoing explorations

- Further analytical prediction from ODE system
  - e.g., the period of solution, the ratio of energy exchange, maximum amplitudes...
- Comparisons with observation/other models