BIRS WORKSHOP

"LIFTING PROBLEMS AND GALOIS THEORY" AUGUST 16–21, 2015

Schedule

Sunday, August 16

16:00	Check-in begins (Front Desk - Professional Development Center - open 24 hours)
17:30 - 19:30	Buffet Dinner, Sally Borden Building
19:30	Informal gathering (Corbett Hall, 2nd floor lounge)

Monday, August 17

7:00 - 8:45	Breakfast
8:45 - 9:00	Introduction and welcome by BIRS station manager
9:00 - 10:00	Oort: *Lifting questions.*
10:00 - 10:30	Coffee break
10:30 - 11:30	Wewers: *Swan conductors and differential obstructions.*
11:45 - 13:00	Lunch
13:00 - 13:50	Guided Tour of The Banff Centre (meet in the 2nd floor lounge, Corbett Hall)
13:50 - 14:00	Group Photo (meet in foyer of TCPL)
14:00 - 14:45	Bary-Soroker: *Geometric versus arithmetic ramification.*
14:45 - 15:15	Coffee break
15:15 - 16:00	Marques: *Holomorphic differentials for Galois towers of function fields.*
16:15 - 17:00	Park: *Faithful realizability of tropical curves.*
17:15 - 18:00	Introduction Session
18:00 - 19:30	Dinner

Tuesday, August 18

7:00 - 9:00	Breakfast
9:00 - 10:00	Kedlaya: *Combinatorial constraints on lifting problems via p-adic differential equations.*
10:00 - 10:30	Coffee break
10:30 - 11:30	Guralnick: *Groups and curves.*
11:45 - 13:15	Lunch
13:15 - 14:00	Zieve: *Monodromy groups in Galois theory.*
14:15 - 15:00	Cadoret: *Structure of the image of the geometric étale fundamental group on étale cohomology with $\mathbb{F}_\ell$-coefficients.*
15:00 - 15:30	Coffee break
15:45 - 17:20	Problem Session
17:30 - 19:30	Dinner

Wednesday, August 19

7:00 - 9:00	Breakfast
9:00 - 10:00	Dèbes: *Specializations of covers and inverse Galois theory.*
10:00 - 10:30	Coffee break
10:30 - 11:15	Kontogeorgis: *Representations of automorphisms and deformation of curves.*
11:30 - 12:15	Scherr: *Separated Belyi Maps.*
12:30 - 13:30	Lunch
	Free Afternoon
17:30 - 19:30	Dinner

Thursday, August 20

7:00 - 9:00	Breakfast
9:00 - 10:00	Harbater: *Galois group schemes over arithmetic curves.*
10:00 - 10:30	Coffee break
10:30 - 11:30	Bouw: *Computing L-functions of superelliptic curves.*
11:45 - 13:15	Lunch
13:15 - 14:00	Razafindramahatsiaro: *Deuring's constant reductions theory and lifting problems.*
14:15 - 15:00	Neftin: *Monodromy and ramification of rational functions.*
15:00 - 15:30	Coffee break
15:30 - 16:15	Davis: *Galois theory of a quaternion origami.*
16:30 - 17:30	Oort: CM *liftings.*
17:45 - 19:30	Dinner

Friday, August 21

7:00 - 8:45	Breakfast
8:45 - 9:30	Liedtke: *Good reduction of* K3 *surfaces.*
9:45 - 10:30	Holschbach: *Étale contractible varieties in positive characteristic.*
10:30 - 11:00	Coffee break
11:00 - 11:45	Sijsling: *On descent of marked curves and maps.*
12:00 - 13:30	Lunch

Remarks:

- All lectures will be held in the TransCanada Pipelines Pavilion (TCPL).
- All meals will be served in the Sally Borden Building:
  Breakfast: 7:00 - 9:30 (Monday - Friday),
  Lunch: 11:30 - 13:30 (Monday - Friday),
  Dinner: 17:30 - 19:30 (Sunday - Thursday).
- The coffee breaks are in the foyer of TCPL.
- **All participants are required to check out by 12 noon on Friday, August 21.**

Abstracts

Lior Bary-Soroker (Tel Aviv University).

Geometric versus arithmetic ramification.

In this talk we will consider a branched covering $f : C \to \mathbb{P}^1$ defined over $\mathbb{Q}$. For $a \in \mathbb{Q}$, the fiber $f^{-1}(a)$ gives rise to a number field (in fact, étale algebra) which, loosely speaking, is generated by the coordinates of the points in the fiber. The main focus of this talk is the study of the number of ramified prime numbers in these number fields. I will present two results: (1) a central limit theorem, obtained with François Legrand, which answers the question what the typical number of ramification is, and (2) sharp upper bounds, which is joint work with Tomer Schlank. The underlying idea behind these results is that the geometric branch locus "controls" the arithmetic one. If time permits, some applications, e.g. to the minimal ramification problem will be discussed.

Irene Bouw (Universität Ulm).

Computing L-functions of superelliptic curves.

Let Y be a superelliptic curve defined over a number field, i.e. Y is a cyclic cover of the projective line. In this talk I report on algorithmic results for computing the local L-factor and the conductor exponent of Y at the primes of bad reduction. The key ingredient is the calculation of the stable reduction of Y at the bad primes. As an application, we verify the functional equation numerically for a large class of examples. In particular, we consider a class of hyperelliptic curves of genus $g \geq 2$ defined over $\mathbf{Q}$ which have semistable reduction everywhere. This is joint work with Stefan Wewers and Michel Börner.

Anna Cadoret (École Polytechnique).

Structure of the image of the geometric étale fundamental group on étale cohomology with $\mathbb{F}_\ell$-coefficients.

When studying representations of the arithmetic étale fundamental group on étale cohomology, the knowledge of the structure of the image geometric étale fundamental group plays a crucial part. In particular, when the ring of coefficients is a field, it is conjectured that this image is semi simple. This is known for $\mathbb{Q}_\ell$-coefficients and in characteristic 0. In this talk, I will focus on the case of $\mathbb{F}_\ell$-coefficients in positive characteristic. This is a joint work with Akio Tamagawa.

Rachel Davis (Purdue University).

Galois theory of a quaternion origami.

This is joint work with Edray Herber Goins. Let X be equal to an elliptic curve over $\mathbb{Q}$ minus its origin. Let $f : Y \to X$ be an étale cover and let $\overline{x} \in X$ be a geometric point. Grothendieck and others consider Galois representations arising from the action of $G_{\mathbb{Q}}$ on $\left\{f^{-1}(\overline{x})\right\}$. We study a particular map f with deck transformation group equal to the quaternion group.

Pierre Dèbes (Université Lille 1).

Specializations of covers and inverse Galois theory.

I will present a series of problems and results from a program that I have been pursuing in the last years about the specializations of covers of the line in connection with inverse Galois theory. The main topics include Hilbert's irreducibility theorem, the Inverse Galois Problem and its regular version and the Malle conjecture.

Robert Guralnick (University of Southern California).
Groups and curves.

We will give a survey talk discussing how finite group theory is useful in studying various problems related to Brauer groups, coverings of curves, automorphism groups of curves and liftings of curves with group actions from positive characteristic to characteristic zero.

David Harbater (University of Pennsylvania).
Galois group schemes over arithmetic curves.

Much of modern Galois theory takes place in the context of function fields of curves defined over complete discretely valued fields. A common strategy is to choose a projective model of the function field and consider Galois branched covers over the closed fiber, which one then attempts to lift to the whole model. This has been used for the inverse Galois problem, embedding problems, and lifting problems, often with the help of patching methods in order to work locally. Traditionally one considers finite Galois groups (or profinite groups in the limit), but one can also treat non-constant finite Galois group schemes via torsors, as in work of Moret-Bailly. This talk considers more general linear algebraic groups as Galois group schemes over such function fields, in two contexts: the inverse differential Galois problem, and obstructions to local-global principles.

Armin Holschbach (Universität Heidelberg).
Étale contractible varieties in positive characteristic.

By Artin-Schreier theory, the affine line $\mathbb{A}^1_k$ over an algebraically closed field k of characteristic $p > 0$ has an infinite fundamental group. This is in contrast to the situation in characteristic 0, where the affine line can be thought of as an algebraic equivalent of the unit interval in topology: Not only is it simply connected, but it is indeed contractible in the sense of étale homotopy theory. Since the affine line as natural candidate does not work, the question arises whether there is any étale contractible variety in positive characteristic. In this talk, we show that there are no non-trivial smooth varieties over an algebraically closed field of characteristic p that are étale contractible, and discuss some consequences for the decomposition theory of fundamental groups of varieties in positive characteristic. This talk is based on joint work with Johannes Schmidt and Jakob Stix.

Kiran Kedlaya (UC San Diego).
Combinatorial constraints on lifting problems via p-adic differential equations.

We describe an approach to recovering the standard combinatorial constraints arising in the study of local lifting problems as a corollary of the properties of convergence polygons of p-adic connections. Based on `http://arXiv.org/abs/1505.01890`.

Aristides Kontogeorgis (National and Kapodistrian University of Athens).
Representations of automorphisms and deformation of curves.

We will give applications of the representation theory of automorphism groups of curves to the theory of deformations of curves with automorphisms.

Christian Liedtke (Technische Universität München).
Good reduction of K3 *surfaces.*

By a classical theorem of Serre and Tate, extending previous results of Nron, Ogg, and Shafarevich, an Abelian variety over a p-adic field has good reduction if and only if the Galois action on its first ℓ-adic cohomology is unramified. In this talk, we show that if the Galois action on second ℓ-adic cohomology of a K3 surface over a p-adic field is unramified, then the surface admits an "RDP model" over that field, and good reduction (that is, a smooth model) after a finite and unramified extension. (Standing assumption: potential semi-stable reduction for K3's.) Moreover, we give examples where such an unramified extension is really needed. On our way, we establish existence and termination of certain semistable flops, and study group actions of models of varieties. This is joint work with Yuya Matsumoto.

Sophie Marques (New York University).
Holomorphic differentials for Galois towers of function fields.

This is joint work with Kenneth Ward. First, we will recall the necessary conditions which permitted Boseck to obtain an explicit basis for the space of the holomorphic differentials for Kummer and Artin-Schreier extensions of a rational field. For this, we will go through the basics about Kummer and Artin-Schreier extensions, particularly the existence of standard forms. Then, we will explain how it was possible to obtain a basis for a Galois tower of function fields of a rational field, provided the existence of a global standard form using Boseck's method. We will describe the Galois action on the basis and present the Galois module structure of the Holomorphic differentials for cyclic function field over a perfect field (natural extension of the results done previously by Sotiris Karanikolopoulos and Aristides Kontogeorgis over an algebraically closed field). Finally, we will present encountered problems for possible further developments/applications.

Danny Neftin (University of Michigan).
Monodromy and ramification of rational functions.

The monodromy group and ramification type are two fundamental invariants associated to every rational function. We shall discuss the accumulating work towards describing all possibilities for both the monodromy group and the ramification type of an indecomposable rational function.

Frans Oort (Universiteit Utrecht).
Lifting questions.

I will explain questions, discuss examples, but give almost no solutions, leaving that to the rest of the week.... Hope the audience will get an impression of techniques and questions. The main emphasis will be on methods available. A.o. I will introduce liftings of algebraic curves (with automorphisms), lifting higher dimensional varieties, and (CM-)liftings of abelian varieties. I will try to make all details of my talk understandable to everyone.

CM *liftings.*

In this talk I will explain the full story from Deuring (1941), via Weil, Tate, Honda-Tate, isogenies (1992) and finally results of the recent book (2014) Ching-Li Chai - Brian Conrad - Frans Oort, 'Complex multiplication and lifting problems', giving full answers to possible CM lifting questions. I will give several proofs: explain the "residual reflex condition", explain "CM types" and formulate complete answers. Most proofs will be given in the case of the crucial "toy model".

Files containing material of both talks, with references, will be on my homepage:
`http://www.staff.science.uu.nl/~oort0109/`

Jennifer Park (McGill University).
Faithful realizability of tropical curves.

Every algebraic curve over a nontrivially valued field has a corresponding tropical curve (through a process called "tropicalization"), where tropical curves are defined as balanced weighted 1-dimensional rational polyhedral complexes. It is then natural to ask whether tropical curves can be realized as the tropicalization of a smooth, complete and connected algebraic curve. Further, we ask whether the tropicalization can be faithful. In this talk, I will outline the basics of the related topics in tropical geometry, then answer the above question of faithful realizability for a large class of tropical curves. This work is joint with Man Wai Cheung, Lorenzo Fantini and Martin Ulirsch.

Christalin Razafindramahatsiaro (African Institute of Mathematical Sciences).
Deuring's constant reductions theory and lifting problems.

Let X be a stable curve over a Dedekind scheme S, with smooth generic fiber X_η. It is well known (from Deligne and Mumford) that there exists a natural injective homomorphism between the full automorphism group of X_η and any special fibre of X. In this talk, first, we give a generalisation of this theorem in function fields of one variable version. Then, we will try to solve a "weak lifting problem" for cyclic curves. In particular, we will give the complete list of all full automorphism groups of hyperelliptic curves in odd prime characteristic that we can lift to characteristic 0.

Zachary Scherr (University of Pennsylvania).
Separated Belyi Maps.

Let C be a smooth, projective and geometrically irreducible algebraic curve defined over $\mathbb{C}$. In 1980, G. V. Belyi gave an unexpected necessary and sufficient condition for C to be isomorphic to a curve defined over $\overline{\mathbb{Q}}$. Namely, that there should exist a *Belyi map* $\varphi\colon C \to \mathbb{P}^1$. That is, a finite morphism which is ramified only over the three point set $\{0, 1, \infty\}$. This talk is concerned with how much flexibility there is in constructing Belyi maps. For a fixed curve $C/\overline{\mathbb{Q}}$, it is known that for each positive integer n there are, up to automorphism, finitely many n-element subsets of $C(\overline{\mathbb{Q}})$ occurring as the preimage of $\{0, 1, \infty\}$ under a Belyi map. While it is extremely difficult to try to describe all such subsets, we prove a result in this direction. We prove that given finite, disjoint subsets $S, T \subseteq C(\overline{\mathbb{Q}})$ there is always a Belyi map φ which is ramified on S and for which $\varphi(T) \cap \{0, 1, \infty\} = \emptyset$, refining a theorem of Mochizuki. In this talk we will discuss this theorem as well as a comparable theorem in positive characteristic. This is joint work with Michael Zieve.

Jeroen Sijsling (Dartmouth College).
On descent of marked curves and maps.

Let F be a field with separable closure F^{sep}, and let X be a curve over F^{sep} that is isomorphic with all its $\text{Gal}(F^{\text{sep}} \,|\, F)$-conjugates. Then one can wonder whether there exists a *descent* of X, that is, a curve X_0 over F that is isomorphic with X over F. Surprisingly, counterexamples due to Shimura and Earle show that such a descent need not always exist. However, classical results by Dèbes and Emsalem imply that the statement does hold for *smoothly marked* curves. More precisely, let X be a curve as above and let $P \in X(F^{\text{sep}})$ be a smooth point on X. Then if for all $\sigma \in \text{Gal}(F^{\text{sep}} \,|\, F)$ there exists an isomorphism $\sigma X \to X$ taking σP to P, then there exists a curve X_0 over F and a point $P_0 \in X_0(F)$ such that (X_0, P_0) is isomorphic to (X, P) over F^{sep}. We discuss a constructive version of this classical result that uses the *branches* of a morphism between algebraic curves. This allows us to remove some superfluous hypotheses and to give explicit descent constructions for marked curves and Belyĭ maps. After showing these examples and their applications, we give some counterexamples for singular curves. This is joint work with John Voight.

Stefan Wewers (Universität Ulm).
Swan conductors and differential obstructions.

Swan conductors measure ramification of Galois extensions with respect to a valuation. They exist in many different forms. In my talk I will try to explain how a certain Swan conductor with 'differential value' due to Kazuya Kato can be used to define obstructions against lifting Galois covers from characteristic p to characteristic zero. Proving that this obstruction vanishes in certain cases was an important ingredient in the proof of the Oort conjecture on lifting of cyclic Galois covers (by Obus, Pop and myself). By recent work of Andrew Obus, proving a more general vanishing result is the only obstacle left in proving a 'generalized Oort conjecture' on lifting covers with cyclic p-Sylow subgroup.

Michael Zieve (University of Michigan).
Monodromy groups in Galois theory.

I will present several types of results obtained by means of monodromy groups. These include refinements of Hilbert's irreducibility theorem, results about images of morphisms of curves over finite fields, results about reducibility of fibered products, and solutions to certain functional equations in rational or meromorphic functions.