Dispersive Hydrodynamics
The Mathematics of Dispersive Shock Waves and Applications
May 17-22, 2015

MEALS

*Breakfast (Buffet): 7:00–9:00 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE at a glance

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall
	Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:30–8:45	Welcome by BIRS Station Manager, TCPL
8:45–9:00	Introduction and Opening Remarks
9:00-10:00	Plenary Lecture (Boris Dubrovin)
10:00-10:30	Coffee Break, TCPL
10:30-12:00	Lectures
12:00–13:30	Lunch
13:30	Group Photo; meet in foyer of TCPL
14:00–15:00	Lectures
15:00–15:30	Coffee Break, TCPL
15:30–17:00	Lectures
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–10:00	Plenary Lecture (Stefano Trillo)
10:00-10:30	Coffee Break, TCPL
10:30-12:00	Lectures
12:00–13:30	Lunch
13:30–15:00	Lectures
15:00–15:30	Coffee Break, TCPL
15:30–17:00	Lectures
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00–10:00	Plenary Lecture (Philippe LeFloch)
10:00–10:30	Coffee Break, TCPL
10:30–12:00	Lectures
12:00–13:30	Lunch
13:30–17:30	Free Afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–10:00	Plenary Lecture (Alfred R. Osborne)
10:00-10:30	Coffee Break, TCPL
10:30-12:00	Lectures
12:00–13:30	Lunch
13:30–15:00	Lectures
15:00–15:30	Coffee Break, TCPL
15:30–17:00	Lectures
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00–10:30	Informal Discussions
10:30–11:00	Coffee Break, TCPL
11:30–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS
(in presentation order)

Monday

Time: **9:00–10:00** Speaker: **Boris Dubrovin** (SISSA)
Title: *Hamiltonian partial differential equations and Painlevé transcendents*
Abstract: I will explain basic notions of geometry of weakly dispersive Hamiltonian partial differential equations (PDEs) including a perturbative approach to integrability. There are various types of phase transitions in solutions to such PDEs. Conjecturally the critical behaviour of a generic solution can be described by certain Painlevé transcendents. I will explain motivations for such Universality Conjecture and formulate rigorous results and open problems.

Time: **10:30–11:00** Speaker: **Sergey Gavrilyuk** (Aix-Marseille University), L. Truskinovsky (Ecole Polytechnique)
Title: *Modulation equations for a dispersive p-system: application to the "hard spheres" case*
Abstract: We study a prototypical dispersive model describing dynamics of a generic continuum with internal time and length scales. The model is based on the hyperbolic p -system regularized by both spatial and temporal dispersive terms. For this system we derive the Whitham modulation equations and study the hyperbolicity of these equations for the maximally simplified case of the continuum constitutive model of a "hard spheres" type.

Time: **11:00–11:30** Speaker: **Sylvie Benzoni-Gavage** (University of Lyon 1), Colin Mietka (University of Lyon 1), Pascal Noble (University of Toulouse), Miguel Rodrigues (University of Lyon 1)
Title: *On the stability of periodic waves in Hamiltonian PDEs*
Abstract: Hamiltonian PDEs are known to admit rich families of planar, periodic traveling waves. These are in particular the building blocks in the modulation theory of dispersive hydrodynamics. However, the stability theory of periodic traveling waves is much less advanced than for solitary waves, which were first studied by Boussinesq 140 years ago, and have received a lot of attention in the last decades.

We have addressed the stability of periodic waves without relying on any integrability argument. We have derived and investigated stability criteria in a fairly general abstract framework for one-dimensional Hamiltonian PDEs, and then applied our theory to three basic examples that are very closely related, and ubiquitous in dispersive hydrodynamics, namely, a quasilinear version of the generalized Korteweg-de Vries equation (qKdV), and the Euler–Korteweg system in both Eulerian coordinates (EKE) and in mass Lagrangian coordinates (EKL). In particular, we have shown that a single tool, the abbreviated action integral along the orbits of waves in the phase plane, encodes several types of stability, namely, spectral, modulational, and co-periodic orbital stability. Up to now, this integral had been used from place to place for some specific equations, in particular the generalized Korteweg-de Vries equation - which is special case of (qKdV) - but not from a general point of view. Our abbreviated action integral turns out to be the same for (EKE) and (EKL), and to be connected to the one for (qKdV) in a simple way. From the most abstract point of view, this integral is the counterpart for periodic waves of the Boussinesq moment of instability for solitary waves. Since it has more degrees of freedom, the stability criteria that it yields are more complicated than for solitary waves, but they can at least be tested numerically. Various numerical experiments have been conducted which clearly discriminate between stable cases and unstable cases for (qKdV), (EKE) and (EKL).

Time: **11:30–12:00** Speaker: **Peter Miller**, Alfredo Wetzel (University of Michigan, Ann Arbor)
Title: *Exact Direct Scattering for the Benjamin-Ono Equation and Applications to Small Dispersion Theory*
Abstract: Building on an example calculation described by Kodama, Ablowitz, and Satsuma, we present a general scheme for explicitly solving the direct scattering problem appearing in the solution of the Benjamin-Ono equation by an inverse scattering transform under the dense assumption that the initial condition is a rational function. The resulting explicit formulae for the scattering data exhibit a complexity that grows with the degree of the rational initial data, but that significantly is independent of the dispersion parameter in the Benjamin-Ono equation. It therefore becomes possible to calculate the scattering data for the Benjamin-Ono equation in the small-dispersion limit with great accuracy. This is an essential first step toward a complete understanding of the dynamics of the Benjamin-Ono equation in the small-dispersion limit, for which previous work at various levels of accuracy and rigor suggests the formation of dispersive shock waves that are described by the multi-sheeted solution of the inviscid Burgers equation.

Time: **14:00–14:30** Speaker: **Ted Johnson** (University College London)
Title: *The reduced Ostrovsky equation: integrability and breaking*
Abstract: The reduced Ostrovsky equation is a modification of the Korteweg-de Vries equation, in which the usual linear dispersive term with a third-order derivative is replaced by a linear non-local integral term, which represents the effect of background rotation. This equation is integrable provided a certain curvature constraint is satisfied. It is shown, through theoretical analysis and numerical simulations, that when this curvature constraint is not satisfied at the initial time, then wave breaking inevitably occurs. Similar results are obtained for the modified reduced Ostrovsky equation, arising analogously from the modified KdV equation, where the condition dividing integrability from breaking becomes a constraint on the initial slope.

Time: **14:30–15:00** Speaker: **Robert Jenkins** (University of Arizona)
Title: *Regularization of a shard shock by the defocusing nonlinear Schrödinger equation*
Abstract: I'll discuss the defocusing nonlinear Schrodinger (NLS) equation for a family of step-like initial data with piecewise constant amplitude and phase velocity with a single jump discontinuity at the origin. Riemann-Hilbert and steepest descent techniques will be used to study the long-time/zero-dispersion limit of the solutions to NLS associated to this family of initial data. We'll see that the initial discontinuity is regularized in the long time/zero-dispersion limit by the emergence of five distinct regions in the (x, t) half-plane. These are left, right, and central plane waves separated by a rarefaction wave on the left and a slowly modulated elliptic wave on the right. The Riemann-Hilbert machinery allows us to rigorously derive the leading order asymptotic solutions predicted by modulation theory while also capturing the phase corrections and gives explicit error bounds.

Time: **15:30–16:00** Speaker: **Gino Biondini**, Dionyssis Mantzavinos (SUNY Buffalo)
Title: *The nonlinear stage of modulational instability*
Abstract: We first show how the modulational instability for the focusing nonlinear Schrodinger (NLS) equation manifests itself within the context of the inverse scattering transform. We then characterize the nonlinear stage of modulational instability by discussing the long-time asymptotic behavior of solutions of the focusing NLS equation with initial conditions that are a small perturbation of a constant background. In the long-time asymptotics, the solution decomposes the xt-plane into three regions: a left far field and a right far field, in which the solution equals the boundary condition to leading order, and a central region in which the asymptotic behavior is described by a slowly modulated elliptic solution.

Time: **16:00–16:30** Speaker: **Dionyssis Mantzavinos**, Gino Biondini (SUNY Buffalo)
Title: *Long-time asymptotics for the focusing nonlinear Schrödinger equation with constant boundary conditions at infinity*
Abstract: We characterise the long-time asymptotics for the focusing nonlinear Schrödinger (NLS) equation with constant boundary conditions at infinity. We employ the recently-developed inverse scattering

transform for the focusing NLS equation with nonzero boundary conditions and we use the Deift-Zhou (DZ) nonlinear steepest descent method for oscillatory Riemann-Hilbert problems (RHPs) to compute the long-time asymptotic behaviour. In particular, we show how the growing jumps in the RHP – which are the signature of the modulational instability – are regularised via the DZ method by appropriate deformations of the RHP in different regions of the xt-plane. The various kinds of deformations correspond to different asymptotic behaviour for the solution. In particular, we show that in the long-time asymptotics the xt-plane decomposes into three regions: a left far field and a right far field, in which the solution equals the boundary condition to leading order, and a central region in which the asymptotic behaviour is described by slowly modulated periodic oscillations. The expression of the solution in this central region in terms of elliptic solutions of the focusing NLS equation is also explicitly discussed.

Time: **16:30–17:00** Speaker: **Barbara Prinari** (University of Colorado, Colorado Springs)
Title: *Dark-bright soliton solutions with nontrivial polarization interactions for the three-component defocusing nonlinear Schrodinger equation*
Abstract: In this talk we will present novel dark-bright soliton solutions for the three-component defocusing nonlinear Schrodinger equation with nonzero boundary conditions. The solutions are obtained within the framework of a recently developed inverse scattering transform for the underlying nonlinear integrable PDE, and unlike dark-bright solitons in the two component (Manakov) system in the same dispersion regime, their interactions display non-trivial polarization shift for the two bright components.

Tuesday

Time: **9:00–10:00** Speaker: **Stefano Trillo** (University of Ferrara)

Title: *Shock Waves in dispersive systems: a survey of recent experimental results*

Abstract: Dispersive shock waves (DSWs) have importance in regimes that range from the deterministic propagation of water waves in a tank to the turbulent propagation of an incoherent field. I will overview very recent experiments aimed at the characterisation of these phenomena. First, I will report on the observation of DSWs in shallow water described in terms of weakly dispersing KdV equation with nonsoliton initial condition (with M. Onorato and M. Klein). Then I will discuss the conditions under which a DSW can resonantly radiates energy into linear dispersive waves. Taking the NLS equation with higherorder dispersion as a reference model I will show that such resonant transfer can be strongly enhanced in the presence of a periodic perturbation along the evolution coordinate. The demonstration of such an effect has been recently carried out in a dispersion oscillating fiber (with M. Conforti, A. Kudlinski, and A. Mussot). Finally I will consider the turbulent dynamics of an incoherent field in a deterministic nonlinear medium described by a nonlocal equation of the NLS type. In this case different regimes are accessible including a noteworthy transition from a disordered sea of DSWs to a global collective shock which exhibits a regularisation mechanism that differs from the ordinary oscillations featured by a coherent DSW (with A. Picozzi, G. Xu, J. Garnier, D. Faccio, D. Vocke, T. Roger).

Time: **10:30–11:00** Speaker: **Peter Engels** (Washington State University, Pullman, WA)

Title: *Artificial gauge fields in dilute-gas Bose-Einstein Condensates*

Abstract: Dilute-gas Bose-Einstein Condensates (BECs) provide a very flexible platform to investigate nonlinear dynamics of few- and many-body systems. A major focus of current experimental and theoretical research with ultracold quantum gases is the creation of interesting dispersion relations. For example, if a dispersion relation has a local minimum at a nonzero quasimomentum, it can be interpreted as being effected by an artificial gauge field. If the gauge field is made time dependent or spatially dependent, artificial electric or magnetic fields can be created for the electrically neutral atoms in a BEC. This is an important step towards the simulation of complicated condensed matter systems using well-controlled Bose-Einstein condensates that can be manipulated using a plethora of atomic physics tools. In this talk I will present recent and ongoing experiments conducted at Washington State University that investigate interesting dispersion relations, including dispersion relations with a roton-like minimum created by Raman dressing schemes and Floquet-Bloch lattice structures created by a time-dependent Hamiltonian.

Time: **11:00–11:30** Speaker: **Arnaldo Gammal** (University of Sau Paulo)

Title: *Shock waves in 2D NLS supersonic flow past oscillating attractive-repulsive obstacle*

Abstract: We performed numerical simulations of a supersonic flow of the Nonlinear Schrödinger equation past an oscillating attractive repulsive obstacle. We show different pattern formation in the transition from slow to high frequencies, as oblique solitons, vortices, vortex-dipoles and fragments. Also, bow waves in front of the obstacle are analytically treated.

Time: **11:30–12:00** Speaker: **Boaz Ilan** (University of California, Merced), Mark Hoefer (University of Colorado, Boulder)

Title: *Transverse instabilities of confined dark solitary waves*

Abstract: It is well known that line dark soliton solutions of the defocusing NLS equation are unstable to small transverse perturbations. On the other hand, when sufficiently confined, the dynamics are stable. This raises the questions: what is the threshold confinement width for stability and what is the nature of the instability? To investigate these questions, we study dark solitary waves of the defocusing NLS equation confined to two-dimensional channels. These bound states have non-vanishing boundary conditions along the unconfined direction. The onset of their transverse instability is characterized analytically in terms of a spectral bifurcation. Computations reveal multi-lobed modes and the onset of vortex pairs. This research has implications to the control of ultracold matter waves in Bose-Einstein condensates and laser propagation in defocusing nonlinear optical media.

Time: **13:30–14:00** Speaker: **Luca Salasnich** (University of Padua)
Title: *Shock waves in the unitary Fermi gas*
Abstract: We investigate shock waves in the unitary Fermi gas [1] by using the zero-temperature equations of superfluid hydrodynamics. We obtain analytical solutions for the dynamics of a localized perturbation of the uniform gas [2]. Moreover, motivated by a recent experiment [3], we simulate the collision between two clouds of cold Fermi gas at unitarity conditions by using an extended Thomas-Fermi density functional. We find that a quantitative agreement with the experimental observation of the dynamics of the cloud collisions is obtained within our superfluid effective hydrodynamics approach, where density variations during the collision are controlled by a purely dispersive quantum gradient term [4].

[1] W. Zwerger (Ed.), The BCS-BEC Crossover and the Unitary Fermi Gas (Springer, Berlin, 2012).
[2] L. Salasnich, EPL 96, 40007 (2011).
[3] J. Joseph, J. Thomas, M. Kulkarni, and A. Abanov, Phys. Rev. Lett. 106, 150401 (2011).
[4] F. Ancilotto, L. Salasnich, and F. Toigo, Phys. Rev. A 85, 063612 (2012).

Time: **14:00–15:00** Speaker: **Gennady El** (Loughborough University), **Alex Tovbis** (University of Central Florida), Eduardo Khamis (National Institute for Space Research, Brazil)
Title: *Small-dispersion focusing nonlinear Schrödinger equation and the generation of rogue waves I and II*
Abstract: We consider two analytically tractable scenarios of the rogue wave formation in the framework of the semi-classical (small dispersion) focusing nonlinear Schrödinger equation (NLSE). The first scenario is typified by the evolution of an analytical, bell-shaped profile (e.g. sech or gaussian) leading to the formation of a 'focussing' gradient catastrophe dispersively regularized via the generation of Peregrine breathers, and further, modulated finite-gap potentials of increasing genus. In the second scenario, realized most prominently for the initial profile in the form of a real-valued rectangular pulse (a 'box'), the formation of rogue waves occurs due to the interaction of two counter-propagating single-phase dispersive dam break flows – the 'elliptic' counterparts of dispersive shock waves (DSWs). We show that the DSW interaction dynamics results in the emergence of modulated large-amplitude breather lattice which is closely approximated by the sequence of Akhmediev and Peregrine breathers within certain time-space domain. Further evolution leads to the solution genus increase, but in a drastically different way compared to the 'bell' case.

We use a combination of the nonlinear modulation (Whitham) theory, careful numerical simulations and elements of the Riemann-Hilbert problem (RHP) steepest descent approach to the semiclassical NLSE, to quantitatively describe the two fundamental scenarios. Our semi-classical analytical results are shown to be in excellent agreement with the results of direct numerical simulations of the focusing NLSE with small dispersion parameter.

In the first part of the talk (G. El) the main attention will be given to the box problem and the DSW interaction scenario in the framework of the Whitham modulation theory.

The second part of the talk (A. Tovbis) will concentrate on the mathematical underpinnings of both scenarios via the RHP techniques.

Time: **15:30–16:00** Speaker: **Mark Ablowitz** (University of Colorado, Boulder)
Title: *Remarks on interactions of shock waves and long time asymptotics*
Abstract: The Korteweg-deVries (KdV) equation is the universal model for systems with weak dispersion and weak, quadratic nonlinearity. In this talk the long time asymptotic solution of the KdV equation for general step-like data will be discussed. It is found that while multi-step data evolve to have multiphase dynamics at intermediate times, these interacting DSWs eventually merge to form a single-phase DSW at large time. In this sense it is like Burgers equation where viscous shock interactions eventually merge into a single shock

Time: **16:00–16:30** Speaker: **Guo Deng** (SUNY Buffalo)
Title: *How many solitons are there in the Zabusky-Kruskal experiment?*
Abstract: We revisit the Zabusky-Kruskal experiment within the context of the inverse scattering transform

(IST). Namely, we study the IST for the KdV equation in the small dispersion limit with a cosine initial condition. Using a WKB expansion for the solutions of the scattering problem, we derive an asymptotic expression for the trace of the monodromy matrix. In turn, we use this expression to obtain upper and lower bounds on the number of solitons, as well as asymptotic values for their amplitude. We validate the theory by comparing the WKB results with the numerical results of the Zabusky-Kruskal experiment and with the numerical solution of the scattering problem.

Time: **16:30–17:00** Speaker: **Naum Gershenzon**, Gust Bambakidis, Thomas Skinner (Wright State University)
Title: *Sine-Gordon modulation solutions: application to macroscopic friction*
Abstract: The Frenkel-Kontorova (FK) model and its continuum approximation, the sine-Gordon (SG) equation, are widely used for modeling of various phenomena. In many practical applications the wave-train solution, which includes many solitons, is required. In such cases the system of Whitham's modulation equations, superimposed on the SG equation, is an appropriate tool.

Description of the transitional process from a static to a dynamic frictional regime is a fundamental problem of modern physics. We showed that Whitham's modulation equations are suitable apparatus for describing this transition. The model provides relations between kinematic (rupture and slip velocities) and dynamic (shear and normal stresses) parameters of the transition process. The advantage of the model is ability to describes frictional processes over a wide range of rupture and slip velocities (up to 7 orders of magnitude) allowing, in particular, the consideration of seismic events ranging from regular earthquakes, with rupture velocity of order a few km/s, to slow slip events, with rupture velocity of order a few km/day.

Wednesday

Time: **9:00–10:00** Speaker: **Philippe LeFloch** (University of Paris 6 and CNRS)
Title: *The mathematical theory of small-scale dependent shock waves*
Abstract: Small-scale dependent shock waves arise in many problems of continuum physics, especially in the dynamics of complex fluids and phase interfaces. In particular, shock waves my be driven by viscosity and capillarity terms, which can not be neglected at the hyperbolic level of modeling. In this talk, I will review the theoretical and numerical methods that, in the past twenty years, were developed to tackle such problems.

[1] N. Bedjaoui and P.G. LeFloch, Diffusive-dispersive traveling waves and kinetic relations. An hyperbolic-elliptic model of phase transitions, Proc. Royal Soc. Edinburgh 132 (2002), 545-565.

[2] B.T. Hayes and P.G. LeFloch, Nonclassical shocks and kinetic relations. Finite difference schemes, SIAM J. Numer. Anal. 35 (1998), 2169-2194.

[3] P.G. LeFloch, Kinetic relations for undercompressive shock waves. Physical, mathematical, and numerical issues, Contemporary Mathematics 526 (2010), 237-272.

[4] P.G. LeFloch and S. Mishra, Numerical methods with controlled dissipation for small-scale dependent shocks, Acta Numerica 23 (2014), 743-816.

Time: **10:30–11:00** Speaker: **Michael Shearer** (North Carolina State University), Gennady El (Loughborough University), Mark Hoefer (University of Colorado, Boulder)
Title: *Riemann Problems for the Modified KdV-Burgers Equation*
Abstract: The modified KdV-Burgers equation

$$u_t + (u^3)_x = \mu u_{xx} + \beta u_{xxx},$$

in which $\mu \geq 0$ and β are constant, is both dissipative and dispersive. Moreover, the flux u^3 is non-convex. Much can be learned from the structure of solutions of initial value problems with Riemann initial data, in which $u(x,0)$ is piecewise constant with a single jump. When $\mu > 0$ the solutions are easily related to shock waves and rarefaction waves for the conservation law $u_t + (u^3)_x = 0$. However, with $\mu = 0$, the solutions involve dispersive shock waves (DSWs). I show how the two cases are related, discuss the limit $\mu \to 0+$, and demonstrate time scales over which different wave structures appear. The construction of the DSWs turns out to contain subtleties related to the presence of undercompressive traveling waves for the $\mu > 0$ case, and to the construction of shock-rarefaction wave solutions of the conservation law, due to the non-convex flux.

Time: **11:00–11:30** Speaker: **Antonio Moro** (Northumbria University)
Title: *Shock waves, mean field models and critical phenomena*
Abstract: A phase transition denotes a drastic change of state of a thermodynamic system due to a continuous change of thermodynamic parameters. Inspired by the theory of nonlinear conservation laws and shock waves we develop a novel approach to thermodynamic phase transitions based on the solution of Maxwell relations for a generalised family of entropy functions. This theory provides an exact mathematical description of discontinuities of the order parameter within the phase transition region, it explains the universal form of the equations of state and the occurrence of triple points in terms of the dynamics of nonlinear shock wave fronts.

We also formulate the problem in terms of a mean field model that allows to naturally extend partition functions and equations of state to critical region. We show how under rather general assumptions on the form of the potential, combined with the hypothesis on the existence of a unique solution to the Stiltjes moment problem, we can explicitly construct the natural extension of the partition function inside the

critical region for finite number of particles given the equation of state outside the criticality. The theory is shown at work in the case of a class of Curie-Weiss type magnetic models and the celebrated van der Waals model. The role of dispersive shock waves in the context of mean field models for complex systems will be also discussed.

Time: **11:30–12:00** Speaker: **Christian Klein** (University of Bourgogne)
Title: *Dispersive shocks in 2+1 dimensional systems*
Abstract: We present a numerical study of dispersive shocks and blow-up in solutions to 2+1 dimensional systems. In particular we discuss the Kadomtsev-Petviashvili, the Davey-Stewartson and the Toda equations.

Thursday

Time: **9:00–10:00** Speaker: **Alfred R. Osborne** (Nonlinear Waves Research Corp.)
Title: *Soliton turbulence and breather gas dynamics in ocean surface waves*
Abstract: Recent experimental results demonstrate the simultaneous occurrence of soliton turbulence and breather gas dynamics in the same data set of ocean surface waves. Thus two types of dense, complex, interacting coherent structures suggests a new paradigm for ocean surface waves. I discuss a rather complete data analysis that provides some important insights about this amazing, highly nonlinear situation. These results support the idea that the soliton turbulence is governed by Korteweg-deVries (KdV) dynamics and the breather gas dynamics are governed by the nonlinear Schrdinger (NLS) equation. An additional data analysis provides insight into the interactions between these two types of dynamics.

From a theoretical point of view I investigate the behavior of soliton turbulence in the KdV equation and breather gas dynamiucs in the NLS equation using finite gap theory (FGT) for each of these equations. I am able to formulate in this way the exact correlation functions for these equations and their temporally evolving power spectra: These mathematical quantities are of course written explicitly in terms of the FGT Riemann matrix elements, the wavenumbers, frequencies and phases. Furthermore the results are all to infinite order, as I make no closure approximation. I give several numerical examples to illustrate the method and to interpret the ocean wave data.

Time: **10:30–11:00** Speaker: **Pierre Suret**, Stéphane Randoux, Pierre Walczak (University of Lille)
Title: *Optical Rogue Waves in integrable turbulence*
Abstract: We report optical experiments and numerical simulations allowing to investigate integrable turbulence in the focusing regime of 1D-NLSE [1]. In analogy with broad spectrum excitation of one-dimensional water-tank [2], we launch random initial waves in a single mode optical fiber. Using an original optical sampling setup, we measure precisely the probability density function (PDF) of optical power of the partially coherent waves rapidly fluctuating with time. The PDF of optical power is found to evolve from the exponential distribution to a strong heavy-tailed distribution. The exponential distribution of the power corresponds to a gaussian statistics for the field. Our experiments thus reveal the occurence of extreme events (RWs) in integrable turbulence with a probability of occurence much higher than precicted by the normal law.

Numerical simulations of 1D-NLSE with stochastic initial conditions reproduce quantitatively our experiments. The stationnary state of the statistics dramatically depends on the statistics of the initial conditions. In particular, if our incoherent initial condition is replaced by a wave plane with additionnal noise, the stationnary statistics follows the normal law [3]. Our numerical investigations suggest finally that the heavy-tailed PDF experimentally observed rely on the stochastic generation of coherent analytic solutions of 1D-NLSE such as Akhmediev breathers.

[1] P. Walczak, S. Randoux, and P. Suret, Phys. Rev. Lett. **114,** 143903 (2015).

[2] M. Onorato, A. R. Osborne, M. Serio, L. Cavaleri, C. Brandini, and C. T. Stansberg, Phys. Rev. E **70,** 067302 (2004).

[3] D. S. Agafontsev and V. E. Zakharov, arXiv:1409.4692 (2014).

Time: **11:00–11:30** Speaker: **Stéphane Randoux**, Pierre Suret, Pierre Walczak (University of Lille), Miguel Onorato (University of Torino)
Title: *Intermittency in integrable turbulence*
Abstract: In this work, we present an optical fiber experiment accurately ruled by the one-dimensional integrable and *defocusing* nonlinear Schrödinger equation (1D-NLSE) [1]. The experiment has been designed to investigate a situation of integrable turbulence [2] and it has been dimensioned in order to capture the entire dynamics of a partially-coherent optical wave having initially a gaussian statistics. This is usually

never achieved in optics where the spectrum of the incoherent wave is often much broader than the detector bandwidth. Using fast detection techniques, we explore the changes in the global statistics of the incoherent wave occurring in the nonlinear regime and we observe that the probability density function (PDF) of intensity of light fluctuations is characterized by tails that are lower than those predicted by a Gaussian distribution. Implementing an optical filtering technique, we also examine the statistics of intensity of light fluctuations on different scales and we observe that the PDFs show heavy tails that strongly depend on the scales. This reveals an unexpected phenomenon of intermittency that is similar to the one reported in several other wave systems, though they are fundamentally far from being described by an integrable wave equation [3, 4, 5]. The asymptotic evolution of the wave system has been studied from numerical simulations of the 1D-NLSE for a propagation distance that is one order of magnitude greater than the propagation distance reached in the experiment. After a transient evolution of a few kilometers, the wave system reaches a statistically stationary state in which the mean optical power spectrum and the PDFs no longer change with propagation distance. The theoretical description of the statistical features observed in the experiment is still a complex and open problem in the field of integrable turbulence.

[1] S. Randoux, P. Walczak, M. Onorato, and P. Suret, Phys. Rev. Lett. **113,** 113902 (2014).

[2] V. E. Zakharov, Stud. Appl. Math. **122,** 219 (2009).

[3] E. Falcon , S. Fauve and C. Laroche Phys. Rev. Lett. **98,** 154501 (2007)

[4] P. Denissenko, S. Lukaschuk and S. Nazarenko, Phys. Rev. Lett. **99,** 014501 (2007).

[5] E. Bosch and W. van de Water, Phys. Rev. Lett. **70,** 3420 (1993).

Time: **11:30–12:00** Speaker: **Benjamin Wetzel** (INRS-EMT, Montreal; University of Sussex), D. Bongiovanni, M. Kues, R. Morandotti (INRS-EMT, Montreal), Y. Hu (Nankai University), Z. Chen (Nankai University; San Francisco State University), J. M. Dudley (CNRS-Université de Franche-Comté), Stefan Wabnitz (Università degli Studi di Brescia)
Title: *Experimental Observation of Riemann Wave Shoaling in Nonlinear Optical Fibers*
Abstract: Nonlinear optical pulse propagation in optical fibers provides a convenient and controlled environment to study intriguing analogies with hydrodynamic wave phenomena. For optical pulses propagating in the normal dispersion regime, whenever nonlinearity is predominant over dispersion, the NLSE can be reduced to the Nonlinear Shallow Water Equation (NSWE)[1]-[2]. The NSWE exhibits nonlinear invariant solutions or Riemann waves, with the peculiar property of a chirp profile that is proportional to the wave amplitude. The dynamics of Riemann waves leads to a progressive and asymmetric steepening of the wave envelope, until it reaches a vertical front and, subsequently, the break-up of the pulse, in analogy with the shoaling of an ocean wave as it approaches the beach [3]. We report here the experimental demonstration of the shoaling of Riemann waves in nonlinear fiber optics. By considering a picosecond input Gaussian pulse, we calculated the ideal input chirp and power parameters to be used, so that the pulse will experience a maximal steepening of its trailing edge after 500m propagation in a highly nonlinear normal dispersive fiber. The appropriately pre-chirped pulse is generated experimentally by spectral reshaping of a broadband transform-limited pulse, and it is characterized at both the input and output ends of the fiber. Simulation results exhibit an excellent agreement with the experimental pulse signatures. Further experimental studies revealed the possibility to tune the shoaling distance of the quasi-Riemann wave over hundreds of meters of propagation by adding an extra quadratic spectral phase.

[1] G. B. Whitham, Linear and nonlinear waves, Wiley, New York, 1974.

[2] G. Biondini and Y. Kodama, J. Nonlinear Sci. 16 (2006) 435.

[3] S. Wabnitz, J. Opt. 15 (2013) 064002.

Time: **13:30–14:30** Speaker: **Michelle Maiden, Mark Hoefer** (University of Colorado, Boulder)
Title: *Dispersive hydrodynamics of viscous fluid conduit interfacial waves: experiments and theory*
Abstract: Viscous fluid conduits provide a cheap, accessible, versatile, and powerful system in which to study dissipationless, dispersive hydrodynamics. A dense, viscous fluid serves as the background media through which a less dense, less viscous fluid buoyantly rises. If this rising fluid is continuously injected, an interface forms that is similar to a deformable pipe. Through buoyancy, high viscosity contrast, and a long wave assumption, conduit interfacial dynamics can be described by a scalar, nonlinear, dispersive, nonlocal *conduit equation* with no assumption on amplitude.

In the first talk (Maiden), a live demonstration and laboratory experiments involving solitons, dispersive shock waves (DSWs), and their interactions will be presented. The results include the refraction and absorption of a soliton by a DSW and the refraction of a DSW by a second DSW, resulting in two-phase behavior. The simplicity and fidelity with which dispersive hydrodynamics can be accessed in this system will be shown.

The second talk (Hoefer) concerns the theory of dispersive hydrodynamics in the conduit system. Starting from a microscopic description (two-fluid Stokes equations with free boundary), proceeding to a mesoscopic description (the long wave conduit equation), and finally to a macroscopic description (the Whitham modulation equations), a multiscale hierarchy of theoretical models and their implications for dispersive hydrodynamics of this system will be presented.

Time: **14:30–15:00** Speaker: **Gavin Esler** (University College London)
Title: *Dispersive dam breaks and lock exchanges in a two-layer fluid*
Abstract: Dam-break and lock-exchange flows are considered in a Boussinesq two-layer fluid system in a uniform two-dimensional channel. The focus is on inviscid weak dam-breaks or lock-exchanges, for which waves generated from the initial conditions do not break, but instead disperse in an undular bore. It is shown that the details of a remarkable mapping between the two-layer and single-layer shallow water systems can be used to classify general initial conditions into dam-breaks and lock-exchanges. A dispersive two-layer shallow water model, namely the Miyata-Choi-Camassa (MCC) system, is used to investigate the subsequent evolution of each type of flow in detail. Within the MCC framework, Whitham modulation theory can be used to show that initial conditions of the dam-break type generate undular bores and rarefaction waves similar to those seen in the single-layer system. By contrast, initial conditions of the lock-exchange type can only be resolved by inclusion of a special nonlinear wave that is unique to the two-layer system, the so-called solibore.

Time: **15:30–16:00** Speaker: **Karima Khusnutdinova**, Xizheng Zhang (Loughborough University)
Title: *On a 2+1-dimensional equation for ring waves in a stratified fluid over a shear flow*
Abstract: Oceanic waves registered by satellite observations often have curvilinear fronts and propagate over various currents. We study long weakly - nonlinear ring waves in a stratified fluid in the presence of a depth-dependent horizontal shear flow, generalising the results obtained in [1, 2, 3]. It is shown that despite the clashing geometries of the waves and the shear flow, there exists a linear modal decomposition (different from the known decomposition in Cartesian geometry), which can be used to describe distortion of the wavefronts of surface and internal waves, and systematically derive a 2+1 - dimensional cylindrical Korteweg - de Vries - type equation for the amplitudes of the waves [4]. The general theory is applied to the case of the waves in a two-layer fluid with a piecewise - constant shear flow, with an emphasis on the effect of the shear flow on the geometry of the wavefronts. The distortion of the wavefronts is described by the singular solution (envelope of the general solution) of a nonlinear first-order differential equation. There exists a striking difference in the shape of the wavefronts of surface and interfacial waves propagating over the same shear flow. The new equation can be used to study multi-dimensional dispersive shock waves in the oceans and other fluids.

[1] R.S. Johnson. Water waves and Korteweg - de Vries equations, J. Fluid Mech., 97 (1980) 701–719.

[2] V.D. Lipovskii. On the nonlinear internal wave theory in fluid of finite depth, Izv. Akad. Nauk SSSR, Ser. Fiz. Atm. Okeana, 21 (1985) 864–871 (in Russian).

[3] R.S. Johnson. Ring waves on the surface of shear flows: a linear and nonlinear theory, J. Fluid Mech., 215 (1990) 145–160.

[4] K.R. Khusnutdinova, X. Zhang. Long ring waves in a stratified fluid over a shear flow, arXiv:1412.7095v1 [physics.flu-dyn] 22 Dec 2014, submitted to J. Fluid Mech.

Time: **16:00–16:30** Speaker: **Dimitrios Mitsotakis** (Victoria University of Wellington), Henrik Kalisch, Zahra Khorsand (University of Bergen)
Title: *Balance laws for the Serre equations*
Abstract: The Serre equations are used to model highly nonlinear and weakly dispersive waves propagating at the surface of a shallow layer of a perfect fluid. The Serre system has three associated conservation laws that describe the conservation of mass, momentum, and energy due to the surface wave motion. We present a systematic derivation of these conservation laws. The balance equations are of the same asymptotic order as the evolution equations over a flat bottom. A study of momentum balance, energy balance and the breaking limits are presented for the undular bores in the context of Serre equations. Numerical experiments for the study of undular bores and the shoaling of solitary waves are presented.

Time: **16:30–17:00** Speaker: **Nikola Stoilov** (University of Goettingen), Max Pavlov (Lebedev Physical Institute, Russian Academy of Sciences)
Title: *Dispersionful Version of WDVV Associativity Equations*
Abstract: The Witten-Dijkgraaf-Verlinde-Verlinde (WDVV) equations arise as the conditions of associativity of an algebra in an N dimensional space. These equations are fully integrable for any N. In the case $N = 3$, this system can be written as a hydrodynamic type system of PDEs, that is bi-Hamiltonian, but does not posses Riemann invariants. Applying a point transformation of the dependent and independent variables, we are able to construct dispersionfull version of the above equations, depending on a transformation parameter ϵ. This system possesses dispersionless limit $\epsilon \to 0$ which is a diagonalisable, bi Hamiltonian system of hydrodynamic type. The high - frequency limit at $\epsilon \to \infty$ coincides with the original WDVV system.