Optical Rogue Waves in Integrable Turbulence

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http://math.univ-lille1.fr/~cempi/index_eng.php
### Statistical Optics (1950-…)
- Linear problems
- Coherence of sources
- Speckle
- Fluctuations and noise of lasers


### Nonlinear Optics (1964-…)
- Second Harmonics Generation, Sum Frequency Generation
- New coherent sources
- Coherent structures (solitons)


### Nonlinear Statistical Optics (2000-…)
- Wave Turbulence Theory
- Only a few experiments

Picozzi *et al.*, Physics Reports, 2014
Nonlinear Optical fibers and 1D NLSE

Analogy Hydrodynamics / nonlinear optics
   Focusing 1D NLS / Defocusing 1D NLS dispersionless limit

Tabletop hydrodynamical like experiments
   Rogue waves, Undular bores...

JM Dudley et al., Nat. Phot, 2014

Very well known, versatile and controled systems
   Optical Fibers : Peregrine soliton, Kuznetsov Ma solitons...

B Kibler et al., Nature Physics, 2010
B Kibler et al., Scientific reports, 2012

⇒ Integrable Turbulence

Overview

Integrable Turbulence
Propagation of partially coherent waves in (close to) Integrable System
Focusing regime

Outline

I Experiments
Probability Density Function measurement $\Delta \nu \simeq \text{THz}$
Propagation of partially incoherent waves in single mode fibers

II Numerical simulations
One Dimensionnal Nonlinear Schrödinger Equation (1D NLS)
Comparison with experiments
Nonlinear Optics: various and versatile setups

Spatial (localized) experiments

Ex: 1D-speckle

Y. Bromberg et al., Nature Photonics, 4, 721 (2010)

Temporal experiments

Optical Rogue Waves

Pulsed temporal experiments: Coherent pulse + very small noise


Influence of noise (pulse to pulse)

**Generalized** Nonlinear Schrödinger Equation

Spectral filtering
Hydrodynamical rogue waves

1D water-tank + Random initial condition

PHYSICAL REVIEW E 70, 067302 (2004)

Observation of strongly non-Gaussian statistics for random sea surface gravity waves in wave flume experiments

M. Orontio, A. R. Osborne, and M. Serio
Dipartimento di Fisica Generale, Università di Torino, Via Pietro Giuria 1, 10125 Torino, Italy

L. Cavedon
ISMAR, S. Polo 1364, 30125 Venezia, Italy

C. Brandini
LaMMA, Regione Toscana, Via Madonna del Piano, 50019 Sesto Fiorentino, Italy

C. T. Stansberg
Norwegian Marine Technology Research Institute A.S (Marintek), P.O. Box 4125 Valenstads, N-7450 Trondheim, Norway
(Received 5 May 2004; published 30 December 2004)
Equivalent distributions

\[ \psi = a + jb \]

- Gaussian statistics of the field \( \psi \)
  
  \[ a = \Re(\psi) \text{ and } b = \Im(\psi) : \text{independant random processes} \]

  \[ P(a) = \frac{1}{\sqrt{\pi}} e^{-a^2} = P(b) = \frac{1}{\sqrt{\pi}} e^{-b^2} \]  \( (1) \)

- Exponential distribution of the power

  \[ P(|\psi|^2) = e^{-|\psi|^2} \]  \( (2) \)

- Rayleigh distribution of \( |\psi| \)

  \[ P(|\psi|) = 2|\psi|e^{-|\psi|^2} \]  \( (3) \)
Optical Experiments with Partially Coherent Waves

Experiments well described by the 1DNLSE (Integrable one-dimensionnal Nonlinear Schrödinger Equation)

Partially coherent initial condition

Detection of Temporal Dynamics: low pass filtering

Electric field (complex) \( E(x, y, z, t) = A(x, y) \psi(z, t) e^{i(k_0z - \omega_0 t)} \)

Bandwidth of |\( \psi |^2 \): \( \Delta \nu \approx 100 \text{GHz} - 10 \text{THz} \)

Bandwidth of detectors: \( \Delta \nu < 50 \text{GHz} \)

\[ |\psi(t)|^2 \]
Measurement of Probability Density Function of Optical Power

Asynchronous Optical Sampling
Temporal accuracy : 250 fs
Optical Sampling Partially Coherent Signal
Probability Density Function (PDF) of the initial optical power
What is occurrence of $|\psi|^2 = 50\langle |\psi|^2 \rangle$?

1 every $10^{10}$ seconds at $z = 0m$
1 every $10^{-6}$ second at $z = 15m$
Numerical Simulations with one-dimensionnal Nonlinear Schrödinger Equation (1DNLS)

1D NLS

\[ i \frac{\partial \psi}{\partial z} = \frac{\beta_2}{2} \frac{\partial^2 \psi}{\partial t^2} - \gamma |\psi|^2 \psi \]  \hspace{2cm} (4)

Anomalous dispersion fiber (focusing)

- \( \beta_2 = -20 \text{ps}^2 \text{km}^{-1} \)
- \( \gamma = 60 \text{W}^{-1} \text{km}^{-1} \)
- \( < P > = 0.6 \text{W} \)

Typical lengths

- \( L_{NL} = 1/(\gamma P_0) \approx 30 \text{m} \)
- \( L_D = 1/(\beta_2 (2\pi \Delta \nu_0)^2) \approx 10 \text{m} \)
- \( \Delta \nu_0 = 340 \text{GHz} : \text{full width } @ 1/e^2 \)
Initial Conditions: Partially coherent Field

Central limit theorem: Normal law for the field

\[ N \gg 1 \text{ Fourier modes } \psi(t, z=0) = \sum_{p=0}^{N} \psi(\omega_p) e^{i\omega_p t} \]

Fitted Experimental spectrum

\[ n(\omega) = n_0 \text{sech}(\omega/\Delta\omega) \quad \text{with} \quad \Delta\omega \simeq 2\pi \times 64 \text{GHz} \quad (5) \]

Random Phases OR Random Phases and Amplitudes

1/ \[ \psi(\omega) = a(\omega) + i \ b(\omega) \]

\( a \) and \( b \) independent random processes with gaussian statistics

2/ OR \[ \psi(\omega) = \sqrt{n(\omega)} e^{i\phi_\omega} \]

with \( \phi_\omega \) random phase (white noise)
Initial Conditions

Normal law for the field

8192 points / 10000 realizations (ensembles average)

\[ PDF\left( P/\langle P \rangle \right) = e^{-P/\langle P \rangle} \quad \text{with} \quad P = |\psi(t, z = 0)|^2 \]
Evolution of the Probability Density Function

Emergence of numerous extreme events

\[ z = 15 \text{m} \]
Evolution of the Probability Density Function

Comparison with experiments

$z = 15m$

PDF [$P/<P>$]

P/<P>

$z = 0m$

$z = 15m$

Exp. : $z = 15m$
Statistical stationary state

\begin{align*}
H_{NL}/H_0 & \quad (z=0m) \\
H/H_0 & \quad (z=15m) \\
H_L/H_0 & \quad (z=100m)
\end{align*}
Stochastic generation of coherent structures

Akhmediev breathers, Peregrine and Kuznetsov-Ma solitons?

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Optical Rogue Waves in Integrable Turbulence
Influence of initial conditions: Condensate + Noise

D.S. Agafontsev and V.E. Zakharov
*Integrable turbulence and formation of rogue waves*, arXiv, 2014

Stationary state:

**Gaussian statistics!**

Influence of initial conditions: Incoherent wave

P. Walczak *et al.*

Stationary state:

**strongly NON Gaussian statistics**
Experimental measurements of “global” PDF of the Energy of Partially coherent waves

**CW laser with broad spectral bandwidth (≈THz)**

≠ Statistics of pulsed sources / ≠ Filtering

**Integrable Turbulence : 1D NLS eq.**
+ non localized Partially coherent initial conditions

**Focusing** : pdf with heavy tails, extreme events (rogue waves)
**Defocusing** : the talk of S. Randoux!
Localized initial conditions (zero boundary conditions)

1D-spatial optical experiments (1DNLS)


Non localized Initial conditions (our time domain experiments)

- Theory: periodic boundary conditions (non zero boundary conditions)
- Focusing case: no separation of solitons in the stationary state
- Defocusing case: dark solitons in the steady state are possible
Spectrum and PDF in 1D NLS with incoherent waves (theory)

Numerical simulations Ablowitz-Ladik system (focusing)
D. S. Agafontsev, JETP Letters, 98, 11, p826, (2013)

Wave Turbulence Theory (weakly nonlinear regime)
P. Suret et al., Opt. Express 19, 17852-17863 (2011)

**Perspectives / Open Questions**

*Link between statistics and the emergence of coherent structures*

*Collaboration with Gennady El (Loughborough, UK)*

Hydrodynamical model (small dispersion limit)

Inverse Scattering Transform

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