The Geometry, Algebra and Analysis of Algebraic Numbers
Oct 4 - Oct 9, 2015

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
Please remember to scan your meal card at the host/hostess station in the dining room for each meal.

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
	Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:05	Marie José Bertin: "Mahler measure, regulators and modular units"
9:40	Matilde Lalin: "The Mahler measure of elliptic curves"
10:10	Coffee Break, TCPL
10:40	Jean-Louis Verger-Gaugry: "The method of asymptotic expansions of Poincaré and Mahler measures of univariate polynomials in the Conjecture of Lehmer"
11:15	Charles Samuels: "Using continued fractions to study metric Mahler measures"
11:45–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).
14:15	Christopher Sinclair: "Root statistics of random (real) polynomials with bounded Mahler measure"
14:45	Coffee Break, TCPL
15:15	Fabrizio Barroero: "Counting algebraic integers of fixed degree and bounded height"
15:50	Arturas Dubickas: "Counting dominant and degenerate polynomials"
16:25	Martin Widmer: "Around the Northcott property"
17:00	Michael Coons: "Asymptotics of Mahler functions"
17:30–19:30	Dinner

Tuesday

7:00–8:45	Breakfast
8:45	Michael Filaseta: "Regions containing roots of polynomials"
9:20	Michael Mossinghoff: "Barker polynomials"
9:55	Stephen Choi: "Littlewood polynomials with L_4 norms "invariant" under rotations of the coefficients, preliminary report"
10:25	Coffee Break, TCPL
10:55	Tamas Erdelyi: "Coppersmith-Rivlin type inequalities and the order of vanishing of polynomials at 1"
11:30	Jonas Jankauskas: "Simple linear relations between conjugate algebraic numbers"
12:00–13:30	Lunch
13:30	Yann Bugeaud: "On the approximation of transcendental numbers by algebraic numbers of bounded degree"
14:05	Andrej Dujella: "Root separation for reducible integer polynomials"
14:40	Cameron Stewart: "Multiplicatively dependent vectors of algebraic numbers"
15:10	Coffee Break, TCPL
15:30	Robert Grizzard: "Remarks on diophantine approximation in the multiplicative group and generalized Lehmer problems"
16:05	Problem Session
17:30–19:30	Dinner

Wednesday

7:00–8:45	Breakfast
8:45	Andrzej Schinzel: "On ternary linear recurrences"
9:20	Shabnam Akhtari: "Rational approximation to algebraic numbers and the Diophantine equation $ax^n - bx^n = c$"
9:55	Emanuel Carneiro: "Extremal Fourier analysis and some applications to number theory"
10:25	Coffee Break, TCPL
10:55	Lukas Pottmeyer: "On Narkiewicz's property (P)"
11:30	Paulius Drungilas: "On the degree of compositum of two number fields"
12:00–13:30	Lunch
	Free Afternoon: Hiking, etc.
17:30–19:30	Dinner

Thursday

7:00–8:45	Breakfast
8:45	James McKee: "Salem numbers of trace -2 and a conjecture of Estes and Guralnick"
9:20	George Rhin: "Salem numbers with negative trace"
9:55	Pavlo Yatsyna: "A trace bound for positive definite connected integer symmetric matrices"
10:25	Coffee Break, TCPL
10:55	Qiang Wu: "The auxiliary function and the reciprocal algebraic integers"
11:30	Igor Pritsker: "Means of algebraic numbers"
12:00–13:30	Lunch
13:30	Robert Rumely: "A survey of arithmetic applications of capacity theory"
14:05	Paul Fili: "Effective bounds for unlikely intersections in arithmetic dynamics"
14:40	Adam Hughes: "Galois and Functional Analyses Via Weil Height"
15:10	Coffee Break, TCPL
15:30	Problem Session
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00	Problem Session
10:00	Group Discussions
10:30	Coffee Break, TCPL
11:00	Group Discussions
11:30–13:30	Lunch

Checkout by 12 noon.

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Fabrizio Barroero** (Scuola Normale Superiore di Pisa)
Title: *Counting algebraic integers of fixed degree and bounded height*
Abstract: We will give an asymptotic formula for the number of algebraic integers of fixed degree over a fixed number field and bounded height. We will show how this problem reduces to counting lattice points in certain bounded domains defined by the Mahler measure and introduce the counting theorem (developed in collaboration with M. Widmer) we used to deal with it.

Speaker: **Marie José Bertin** (Université PARIS 6)
Title: *Mahler measure, regulators and modular units*
Abstract: One of the many Boyd's conjectures expresses the logarithmic Mahler measure of a polynomial in two variables defining an elliptic curve E of conductor 20 to the L-series of E at $s = 2$. We shall explain the role of regulators and modular units in the proof of such a conjecture.

Speaker: **Yann Bugeaud** (Université de Strasbourg)
Title: *On the approximation of transcendental numbers by algebraic numbers of bounded degree*
Abstract: Throughout, the height H denotes the naïve height. Let n be a positive integer and ξ be a transcendental real number. We denote by $w_n(\xi)$ the supremum of the real numbers w for which the inequality

$$0 < |P(\xi)| \leq H(P)^{-w}$$

is satisfied for infinitely many polynomials $P(X)$ with integer coefficients and degree at most n. We denote by $w_n^*(\xi)$ the supremum of the real numbers w^* for which the inequality

$$0 < |\xi - \alpha| \leq H(\alpha)^{-w^*-1}$$

is satisfied for infinitely many algebraic numbers α of degree at most n. It easily follows from Minkowski's theorem that $w_n(\xi) \geq n$. In his seminal paper, Wirsing proved in 1961 that $w_n^*(\xi) \geq (n+1)/2$ and conjectured that $w_n^*(\xi) \geq n$. This has been confirmed for $n = 2$ (Davenport of Schmidt, 1967) but it still open for $n \geq 3$: Wirsings result has been only marginally improved since 1961. We survey recent and not so recent around Wirsings conjecture, including known relations between the exponent of approximation w_n^* and other exponents of approximation.

Speaker: **Emanuel Carneiro** (Instituto Nacional de Matematica Pura e Aplicada)
Title: *Extremal Fourier analysis and some applications to number theory*
Abstract: In this talk I plan to survey how some extremal problems in Fourier analysis involving approximations by functions of exponential type relate to improved bounds in problems in number theory, for instance: (i) for the argument of the Riemann zeta-function and general L-functions on the critical line, under RH (or GRH); (ii) for the antiderivative of argument of the Riemann zeta-function and general L-functions on the critical line, under RH (or GRH); (iii) for bounds on the pair correlation of the zeros of the Riemann zeta-function on RH, (iv) and a few others, if time permits.

Speaker: **Paulius Drungilas** (Vilnius University)
Title: *On the degree of compositum of two number fields*
Abstract: We study the set of triplets $(a, b, c) \in \mathbb{N}^3$ for which there exist three algebraic numbers α, β, γ,

with degrees a, b, c (over $\mathbb{Q}$), respectively, such that $\alpha + \beta + \gamma = 0$. Such triplets are called *sum-feasible*. In particular, we describe all the sum-feasible triplets (a, b, c) satifying $a \le b \le c$, $b \le 7$. We also investigate a related problem of describing the set of triplets $(a, b, c) \in \mathbb{N}^3$ for which there exist number fields K and L of degrees a and b, respectively, such that the degree of the compositum KL equals c.

Speaker: **Arturas Dubickas** (Vilnius University)
Title: *Counting dominant and degenerate polynomials*
Abstract: We present some estimates on the number of polynomials with integer coefficients of fixed degree and bounded height that have some special property. In particular, we investigate dominant polynomials (those which have one root whose modulus is strictly greater than the moduli of the remaining roots) and degenerate polynomials (those which have a pair of distinct roots whose quotient is a root of unity).

The motivation for this study arose from linear recurrence sequences and all the results are joint with Min Sha (Sydney). Finally, we give an asymptotic formula for the number of reducible polynomials. The later problem is an old one: in the monic case the corresponding formula was established by Chela in 1963.

Speaker: **Andrej Dujella** (University of Zagreb)
Title: *Root separation for reducible integer polynomials*
Abstract: We consider the question how close to each other can be two distinct roots of an integer polynomial $P(X)$ of degree d. We compare the distance between two distinct roots of $P(X)$ with its height $H(P)$, defined as the maximum of the absolute values of its coefficients. The first result in this direction in due to Mahler, who proved that the distance is $> c(d) \cdot H(P)^{-d+1}$, for an explicit constant $c(d)$, depending only on d. We will present some recent results in the opposite direction, obtained by constructing explicit parametric families of (monic) reducible polynomials having two roots very close to each other. This is joint work with Yann Bugeaud.

Speaker: **Michael Filaseta** (University of South Carolina)
Title: *Regions containing roots of polynomials*
Abstract: A monic irreducible non-cyclotomic polynomial with integer coefficients must have a root outside the unit disk $\{z \in \mathbb{C} : |z| \le 1\}$. The literature is filled with results giving estimates on how far from the unit disk such roots must lie. In this talk, we discuss some analogous regions that are not disks for which similar estimates could be sought. These regions, associated with the locations of zeroes of polynomials, give rise to applications on the irreducibility of polynomials with non-negative coefficients, and the connection to these will be described. In particular, questions will be asked concerning roots in these regions that would lead to further developments in the applications to irreduciblity.

Speaker: **Paul Fili** (Oklahoma State University)
Title: *Effective bounds for unlikely intersections in arithmetic dynamics*
Abstract: In 2009, Baker and DeMarco, motivated by a question of Zannier, proved a finiteness result for unlikely intersections of certain dynamical systems by using potential theoretic techniques. In this talk, we discuss how one can make results of this sort effective. In order to do so, we use discrete energy approximation techniques and we introduce a new metric of mutual energy between measures which satisfies a triangle inequality. As an application we give an effective degree bound on the set of parameters c for which $z = 0, 1$ are both preperiodic under the map $f_c(z) = z^2 + c$.

Speaker: **Robert Grizzard** (University of Wisconsin)
Title: *Remarks on diophantine approximation in the multiplicative group and generalized Lehmer problems*
Abstract: For a subfield k of $\overline{\mathbb{Q}}$ and an algebraic number α, we can consider the "distance" from α to $k^\times \otimes \mathbb{Q}$ as

$$\inf_{\beta \in k^\times, \ m \in \mathbb{N}} h\left(\beta/\alpha^{1/m}\right).$$

We'll explain a simple way to estimate this distance from below, and how this estimate says something about the generalized Lehmer problems introduced by G. Rémond. This is joint work in progress with Jeffrey Vaaler.

Speaker: **Adam Hughes** (University of Texas)
Title: *Galois and Functional Analyses Via Weil Height*
Abstract: We discuss a formulation of a result due to Allcock and Vaaler to reframe the analysis of non-zero algebraic numbers in the context of periodic functions on the absolute Galois group of the rational numbers. This form of analysis yields a generalized embedding of idèle groups of global fields into a Banach space normed by the Weil height in such a way that this space can be naturally endowed with the structure of a Banach algebra with the Banach multiplication inherited from convolution of functions on L^1 of the absolute Galois group of the base field. Time permitting we will mention applications to equidistribution in idèle class groups.

Speaker: **Matilde Lalin** (Université de Montrèal)
Title: *The Mahler measure of elliptic curves*
Abstract: The Mahler measure of a multivariable polynomial P is given by the integral of $log|P|$ where each of the variables moves on the unit circle and with respect to the Haar measure. In 1998 Boyd made a systematic numerical study of the Mahler measure of many polynomial families and found interesting conjectural relationships to special values of L-functions of elliptic curves. Recently, many of Boyd's conjectures have been proved by Burnault, Mellit, Rogers, and Zudilin. I will discuss some of those results and present new ones (in collaboration with D. Samart and W. Zudilin.)

Speaker: **James McKee** (University of London)
Title: *Salem numbers of trace -2 and a conjecture of Estes and Guralnick*
Abstract: In 1993 Estes and Guralnick conjectured that any totally separable monic polynomial with rational integer coefficients will occur as the minimal polynomial of some integer symmetric matrix. We show that there are counterexamples to this conjecture for ALL degrees d ¿= 6. One of the ingredients in the proof is to show that there are Salem numbers of degree 2d and trace -2 for every even degree 2d ¿= 24. This is joint work with Pavlo Yatsyna.

Speaker: **Michael Mossinghoff** (Davidson College)
Title: *Barker polynomials*
Abstract: A *Barker sequence* is a finite sequence of integers $\{a_i\}$, each ± 1, for which the sum $\sum_i a_i a_{i+k} \in \{-1, 0, 1\}$ for every positive integer k. Very few Barker sequences are known, and it is widely conjectured that none exists with length larger than 13. A *Barker polynomial* is a univariate polynomial whose coefficient sequence is a Barker sequence: such polynomials are unusually flat over the unit circle in the complex plane. We describe some relationships between Barker polynomials and a number of problems in number theory, analysis, and combinatorics, and we report on some recent research on some of these problems, including some joint work with P. Borwein and B. Logan.

Speaker: **Lukas Pottmeyer** (Universität Basel)
Title: *On Narkiewicz's property (P)*
Abstract: We use a (non-archimedean) equidistribution theorem for points of small dynamical height to prove that $\mathbb{Q}^d$ - the compositum of all number fields of degree $\leq d$ - has property (P); i.e. there is no polynomial $f \in \mathbb{Q}^d[x]$, with $\deg(f) \geq 2$, s.t. $f(X) = X$ for any infinite subset $X \subseteq \mathbb{Q}^d$. If time permits, we also state a connection between dynamical heights and small Salem numbers.

Speaker: **Igor Pritsker** (Oklahoma State University)
Title: *Means of algebraic numbers*
Abstract: Schur (1918) studied limit points for the arithmetic means of conjugate algebraic numbers

contained in the closed unit disk and in the real line. The problem on the smallest limit point of the mean trace of totally positive algebraic integers was developed further by Siegel, Smyth and others, but still remains open. We show that the smallest limit point in question is 2 for many classes of algebraic numbers equidistributed in subsets of the real line. This includes algebraic numbers whose minimal polynomials satisfy various extremal properties, as well as those arising in polynomial dynamics. We also consider various generalizations of this problem, e,g,, to the symmetric means of algebraic numbers.

Speaker: **George Rhin** (University of Lorraine)
Title: *Salem numbers with negative trace*
Abstract: We found a Salem number with trace -3 and degree 34. The main tools are an explicit auxiliary function giving a lower bound of the absolute trace of a totally positive algebraic integer and an integer linear programming.

Speaker: **Robert Rumely** (University of Georgia)
Title: *A survey of arithmetic applications of capacity theory*
Abstract: This talk will summarize the current state of knowledge regarding arithmetic applications of capacity theory, including the Fekete and Fekete-Szegö theorems on existence of algebraic points, the Polya-Carlson rationality criterion, and Ferguson's theorem on functions taking algebraic integer values on a set. Both classical examples and modern versions of the theorems will be given.

Speaker: **Charles Samuels** (Christopher Newport University)
Title: *Using continued fractions to study metric Mahler measures*
Abstract: The metric Mahler measure was first studied by Dubickas and Smyth in 2001. While they used this construction to rephrase Lehmer's conjecture in topological language, the metric Mahler measure has exhibited only superficial connections to any classical areas of number theory. We explore a new connection between the metric Mahler measure and the theory of best rational approximation. In certain special cases, we show that the value of the metric Mahler measure can be determined by studying the continued fraction expansions for certain logarithms.

Speaker: **Andrzej Schinzel** (Polish Academy of Sciences)
Title: *On ternary linear recurrences*
Abstract: By ternary linear recurrence over a number field K I mean a sequence u_n, where $u_n = a_1 u_{n-1} + a_2 u_{n-2} + a_3 u_{n-3}$, where a_i and u_i $(0 < i < 4)$ are in K. A connection is studied between solubility of congruence $u_n = 0 \pmod{p}$ for almost all prime ideals p of K and solubility of equation $u_n = 0$.

Speaker: **Cameron Stewart** (University of Waterloo)
Title: *Multiplicatively dependent vectors of algebraic numbers*
Abstract: This is a report on joint work with F. Pappalardi, M. Sha and I. Shparlinski. Let K be a finite extension of the rationals and let n be a positive integer. We shall give estimates for the number of n-tuples whose entries are algebraic integers from K which have height below a given bound and which are muliplicatively dependent.

Speaker: **Martin Widmer** (University of London)
Title: *Around the Northcott property*
Abstract: We discuss some new problems around the Northcott property for infinite extensions of the rational numbers.

Speaker: **Qiang Wu** (Southwest University of China)
Title: *The auxiliary function and the reciprocal algebraic integers*
Abstract: In this talk we give a lower bound of the absolute trace of totally positive reciprocal algebraic integers and the smallest houses of reciprocal algebraic integers with degree up to 42 by using a new type auxiliary function for the trace of reciprocal algebraic integer.

Speaker: **Pavlo Yatsyna** (University of London)
Title: *A trace bound for positive definite connected integer symmetric matrices*

Abstract: In 1993, Estes and Guralnick conjectured that every totally real monic irreducible polynomial is the minimal polynomial of an integer symmetric matrix. The conjecture was shown to be incorrect, nevertheless we are still far from understanding for which polynomials the conjecture does not hold true. In this talk, we discuss a new method of finding counterexamples to the Estes and Guralnick conjecture via a trace bound for positive definite connected integer symmetric matrices. This is joint work with James McKee.