Perspectives on Parabolic Points in Holomorphic Dynamics
March 29 – April 3.

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

Sunday	
16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
	Beverages and a small assortment of snacks are available on a cash honor system.
Monday	
7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00–10:00	Cheritat, Parabolic Implosion minicourse II
	Coffee Break, TCPL
10:30–11:30	Shishikura, Dynamical charts for irrationally indifferent fixed points and Denjoy odometer
11:30–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).
14:15–15:05	Bedford, Introduction to diffeomorphisms of C^2 with semi-parabolic fixed point I
	Coffee Break, TCPL
15:35–16:25	Uhre, An inequality for the *résidu itératif* on $\mathrm{Per}_1(1) \setminus \mathcal{M}_1$.
	Legs stretching
16:40–17:30	Yampolsky, Beyond cylinder renormalization
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–10:00	Hubbard, Parabolic Implosion Intro lecture.
	Coffee Break, TCPL
10:30–11:30	Epstein, Geometric Limits in Holomorphic Dynamics I.
11:30–13:30	Lunch
14:00–14:50	Bouillot, Computation of analytical invariants of tangent-to-identity diffeomorphisms.
	Coffee Break, TCPL
15:20–16:10	Lomonaco, Parabolic-like mappings and correspondences.
	Legs stretching
16:30-17:20	Yampolsky, Fixed point of parabolic renormalization.
17:30–19:30	Dinner
19:45–20:30	Han Peters, Polynomial maps with wandering components.

Wednesday

7:00–9:00	Breakfast
9:00–10:00	Cheritat, Parabolic Implosion minicourse III.
	Coffee Break, TCPL
10:20–11:10	Bedford, Introduction to diffeomorphisms of C^2 with semi-parabolic fixed point II
11:20–12:10	
11:30–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner
19:45–20:30	Dudko, Ecalle-Voronin invariants via resurgent theory.

Thursday

7:00–9:00	Breakfast
9:00–10:00	Hubbard, Parabolic Implosion minicourse IV
	Coffee Break, TCPL
10:30–11:30	Rousseau, Modulus of analytic classification of a generic germ of analytic family unfolding a parabolic point of co-dimension k.
11:30–13:30	Lunch
14:00–14:50	Roesch, Cubic parabolic slice.
	Coffee Break, TCPL
15:20–16:10	Petersen, Double Parabolic Implosion.
	Legs stretching
16:30-17:20	Epstein, Geometric Limits in Holomorphic Dynamics II.
17:30–19:30	Dinner
19:45–	Poster Session

Friday

7:00–9:00	Breakfast
9:00–9:50	Inou, Parabolic implosion in anti-holomorphic family.
	Coffee Break, TCPL
10:10–11:00	Mukherjee, Discontinuity of the straightening map in anti-holomorphic dynamics.
	Legs stretching
11:10–12:00	Open Problems Session
	NB Checkout of rooms by 12 noon.
12::15–13:30	Lunch
	Free Afternoon

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Perspectives on Parabolic Points in Holomorphic Dynamics
March 29 – April 3.

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Eric Bedford** (SUNY StonyBrook)
Title: *Introduction to diffeomorphisms of C^2 with semi-parabolic fixed point I, II*
Abstract: We give an introduction to the work of Ueda which describes the local behavior of a 2-dimensional diffeomorphism at a semi-parabolic fixed point. This describes the semi-parabolic basin and gives the analogue of the repelling petal. As time permits, we discuss two sorts of perturbations of such maps. One of them is the perturbation for which the fixed point splits into a saddle/attractor pair. The other is in the direction of implosion, for which we will describe a paper with Smillie and Ueda: arXiv:1208.2577.

Speaker: **Olivier Bouillot** (Université Paris Est - Marne-la-Vallée)
Title: *Computation of analytical invariants of tangent-to-identity diffeomorphisms*
Abstract: One of the classic question in holomorphic dynamics is the study of the analytic conjugacy of parabolic germs of $(\mathcal{C}\,;0)$, that is

$$\mathcal{T} = \left\{ f \in \mathcal{C}\{x\} \ ; \ f(x) = \lambda x + \mathcal{O}(x^2) \ , \text{where } \lambda \in \mathcal{C}^* \right\}.$$

Let us recall that when λ is a root of unity, a parabolic germ has two formal invariants, but an infinity of analytic invariants. The main goal of the talk will be to show explicit formula (coming from resurgent analysis) for the horn map which leads us to understand the combinatorial structure of the analytical invariants as well as to develop an algorithm to compute these invariants.

Speaker: **Arnaud Cheritat** (Université de Bordeaux)
Title: *Implosion and Vector Fields.*
Abstract: I will compare parabolic points and their perturbation to vector fields. This will be applied to : - a normalization convention for Fatou coordinates for a parabolic point with several petals - estimates on perturbed Fatou coordinates (parabolic implosion).

Speaker: **Artem Dudko** (SUNY StonyBrook)
Title: *Ecalle-Voronin invariants via resurgent theory*
Abstract: Given a holomorphic germ with a simple parabolic fixed point at the origin the Fatou coordinates have common asymptotic expansion. Using Ecalle's alien operators I will show how to reconstruct Ecalle-Voronin analytic invariants from the formal Borel transform of this expansion and write each of the invariants as a convergent numerical series.

Speaker: **Adam L. Epstein** (University of Warwick)
Title: *Geometric Limits in Holomorphic Dynamics I + II*
Abstract: We present a formal definition of the notion of geometric limit of conformal dynamical systems, and outline a proof of a Structure Theorem for the geometric limits of algebraically convergent sequences of rational maps.

Speaker: **John Hamal Hubbard** (Cornell University)
Title: *TBA*
Abstract:

Speaker: **Hiroyuki Inou** (Kyoto University)
Title: *Parabolic implosion in anti-holomorphic family.*
Abstract: Some anti-holomorphic families can be considered as a real-analytic sub-family of a two or more dimensional family of holomorphic dynamics with a special symmetric structure. In such a family, one and two-dimensional phenomena coexist. We study the family of quadratic (or unicritical) anti-holomorphic polynomials and how parabolic implosion in this family is related to some two-dimensional phenomena.

Speaker: **Luna Lomonaco** (University of Sao Paulo)
Title: *Parabolic-like mappings and correspondences.*
Abstract: I will report on joint work with S. Bullett. The classical Mandelbrot set $\mathcal{M}$ is the subset of parameter space for which the Julia set of the quadratic polynomial $Q_c(z) = z^2 + c$ is connected. Two analogous connectivity loci are $\mathcal{M}^1$ for the family $\mathrm{Per}_1(1) = \{P_A(z) = z + 1/z + A | A \in \mathcal{C}\}$ of quadratic rational maps with a parabolic fixed point of multiplier 1 (normalized by having critical points at ± 1), and $\mathcal{M}^{\mathrm{corr}}$ for the family of quadratic holomorphic correspondences which are matings between polynomials Q_c and the modular group $PSL(2, \mathcal{Z})$.

Conjecture 1. $\mathcal{M}^{\mathrm{corr}}$ is homeomorphic to $\mathcal{M}^1$.

In my talk I will outline the main steps in the (by now pretty detailed) strategy that Shaun and I have to prove Conjecture 1, focussing in particular on the role parabolic-like mappings (which are objects similar to polynomial- like mappings, but with a parabolic external map) have in this picture. In the 1994 article introducing matings between Q_c and the modular group, Chris Penrose and Shaun Bullett conjectured that $\mathcal{M}^{\mathrm{corr}}$ is homeomorphic to the classical Mandelbrot set $\mathcal{M}$. A proof of Conjecture 1, and of the well supported conjecture that $\mathcal{M}^1$ is homeomorphic to $\mathcal{M}$, would resolve this question.

Speaker: **Sabyasachi Mukherjee** (Jacobs University Bremen)
Title: *Discontinuity of the straightening map in anti-holomorphic dynamics.*
Abstract: It is well known by classical works of Douady and Hubbard that the 'straightening map' from a baby Mandelbrot set to the original one is a homeomorphism (which explains why the Mandelbrot set contains infinitely many small copies of itself). The corresponding situation for parameter spaces of higher degree polynomials is much more complicated; there exists a wider variety of parameter space configurations, and the corresponding straightening maps are typically not as well-behaved as in the unicritical case. Inou showed that the straightening map is typically discontinuous in the presence of critical orbit relations. Inou's proof of discontinuity of the straightening map, however, makes essential use of two complex dimensional bifurcations, and can not be applied to one-parameter families. In this talk, we will consider the tricorn, the connectedness locus of quadratic anti-holomorphic polynomials $\bar{z}^2 + c$. Our main goal is to demonstrate that every odd period hyperbolic component of the tricorn is the basis of a 'baby tricorn' (much like the Mandelbrot set), but the straightening map from any 'baby tricorn' to the original one is discontinuous. This is achieved by proving that all non-real 'umbilical cords' of the tricorn wiggle, which generalizes a theorem of Hubbard and Schleicher, and settles a conjecture of Hubbard, Milnor and Schleicher. Time permitting, we will outline some other stark topological/analytic differences between the Mandelbrot set and the tricorn.

Speaker: **Carsten Lunde Petersen** (Roskilde University)
Title: *Double Parabolic Implosion*
In this talk I will discuss the parabolic bifurcation unfolding near the map $z + 1/z$ with a view towards the unfolding of a general parabolic bifurcation near a parabolic fixed point of multiplier 1 and degeneracy 2.
Abstract:

Speaker: **Pascale Roesch** (Université de Marseille)
Title: *Cubic parabolic slice.*
Abstract:

Speaker: **Christiane Rousseau** (Université de Montréal)
Title: *Modulus of analytic classification of a generic germ of analytic family unfolding a parabolic point of co-dimension k.*
Abstract: We present here a complete modulus of analytic classification for a germ of generic k-parameter family unfolding a parabolic point of co-dimension k. The modulus is an unfolding of the modulus of the parabolic point. It is described for generic values of the parameter. The first step is to show that the parameters are canonical. The generic values of the parameters where the fixed points are simple are covered with C_k sectoral domains, where C_k is the k-th Catalan number. Canonical changes of coordinates to the formal normal form are constructed on sectors in the variable space for parameters in each sectoral domain. The construction has a bounded limit for the other parameter values.

Speaker: **Mitsuhiro Shishikura** (Kyoto University)
Title: *Dynamical charts for irrationally indifferent fixed points and Denjoy odometer*
Abstract: We introduce the notion of dynamical charts for holomorphic functions with a irrationally indifferent fixed point. Dynamical charts will be constructed recursively corresponding to the steps of near-parabolic renormalization. The combinatorics of the charts is governed by a symbolic dynamics called (modified) Denjoy odometer. As an application we describe the structure of hedgehogs.

Speaker: **Eva Uhre** (Roskilde Kathedralskole, Roskilde University)
Title: *An inequality for the* résidu itératif *on* $\mathrm{Per}_1(1) \setminus \mathcal{M}_1$.
Abstract: Let $\mathbf{rat}_2$ denote the moduli space of all quadratic rational maps up to Möbiusconjugacy, and let

$$\mathrm{Per}_1(\lambda) = \{[f] \in \mathbf{rat}_2 : \text{some fixed point of } f \text{ has eigenvalue } \lambda\}$$

For $\lambda \in \mathcal{D} \cup \{1\}$, let $\mathcal{M}^\lambda$ denote the connectedness locus in $\mathrm{Per}_1(\lambda)$. For $[f] \in \mathrm{Per}_1(1)$, let z_0 denote the fixed point of f with multiplier 1, and let ϕ denote a Fatou coordinate for f at z_0. For $[f] \in \mathrm{Per}_1(1) \setminus \mathcal{M}^1$, both critical values, v_1 and v_2, of f are in the parabolic basin of z_0. Let $h = |\Im(\phi(v_1) - \phi(v_2))|$.

The *résidu itératif* of f at z_0, denoted $\text{résit}_{z_0}(f)$, is the residue of the 1-form $\frac{1/2(1-f'(z))-1}{z-f(z)}dz$ at z_0. The résidu itératif was first introduced by Écalle and further developed by Buff & Epstein.

For certain components in $\mathrm{Per}_1(1) \setminus \mathcal{M}^1$ we prove the following inequality, relating the two conformal invariants $\text{résit}_{z_0}(f)$ and h of $[f]$:

$$-\frac{h}{2\pi} \leq \Re(\text{résit}_{z_0}(f)) \leq \frac{1}{2} - \frac{h}{2\pi}.$$

The inequality is obtained from considering rescaling limits of sequences that diverge in $\mathbf{rat}_2$.

Speaker: **Michael Yampolsky** (Univerty of Toronto)
Title: *Fixed point of parabolic renormalization*
Abstract: I will present a summary of our work with O. Lanford on the Inou-Shishikura fixed point of the parabolic renormalization operator. In particular, I will describe a renormalization-invariant class analytic maps with a maximal domain of analyticity to which the fixed point belongs, and a numerical computation scheme for the fixed point which is based on resurgent asymptotic series for the Fatou coordinate.

Speaker: **Michael Yampolsky** (University of Toronto)
Title: *Beyond cylinder renormalization*
Abstract: I introduced the cylinder renormalization operator in the early 2000s to prove hyperbolicity of renormalization of critical circle maps. It has since appeared in many related contexts, most notably, in Inou-Shishikuras work on almost parabolic germs. Parabolic renormalization can be naturally seen as its limiting case. Recently, D. Gaydashev and I have studied higher-dimensional generalizations of the above renormalization problems. In this context, we had to replace cylinder renormalization with a different approach. In one complex dimension, we obtained new proofs of renormalization hyperbolicity results

for critical circle maps and Siegel disks – without using cylinder renormalization. In two dimensions, we obtained results on rotational attractors of dissipative maps. In particular, jointly with D. Gaydashev and R. Radu, we proved that for a highly dissipative Henon map a with semi-Siegel fixed point with the golden-mean rotation number, the boundary of the Siegel disk is a topological (in fact, quasi-) circle.