Change point and trend analyses of annual expectile curves of tropical storms

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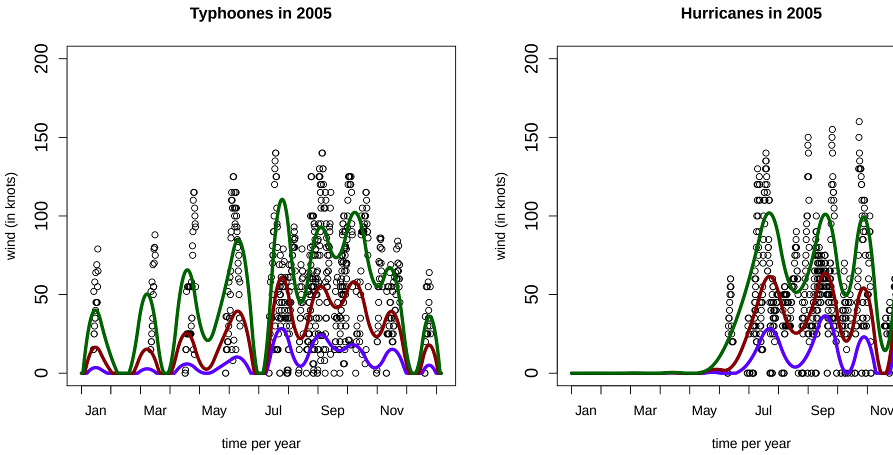
Outline of the talk:

1. Expectile curves of tropical storm strength
2. A functional regression model
3. Change point and trend tests
4. Application to hurricane and typhoon data

2005 expectile curves



Expectile curves for $\tau = 0.1, 0.5$ and 0.9.

Expectiles, background

Y is a square integrable random variable

$$E_-(e) = E\left[(X-e)^2 \mathbf{I}\{X \le e\}\right];$$
$$E_+(e) = E\left[(X-e)^2 \mathbf{I}\{X > e\}\right].$$

$$E_\tau(e) = (1-\tau)E_-(e) + \tau E_+(e),$$

The τth expectile e_τ minimizes $E_\tau(e)$.
(τ close to 1, $E_+(e)$ must be small, e_τ must be large.)
Estimation through empirical expectations.

Idea can be extended to the scatter plot of points (t_i, x_i), $t_i \in I$.
Our application: $I =$ year, x_i wind speed at time t_i.
Using spline smoothing, one obtains expectile curves.

change point test

We observe curves $X_1, X_2, \ldots, X_N$, $\mu_i(t) = \mathrm{E}X_i(t)$.

$$H_0: \ \mu_1 = \mu_2 = \ldots = \mu_N.$$

Test based on projections of

$$P_k(t,\tau) = \frac{k(N-k)}{N} \left\{ \hat{\mu}_k(t,\tau) - \tilde{\mu}_k(t,\tau) \right\},$$

Scores:

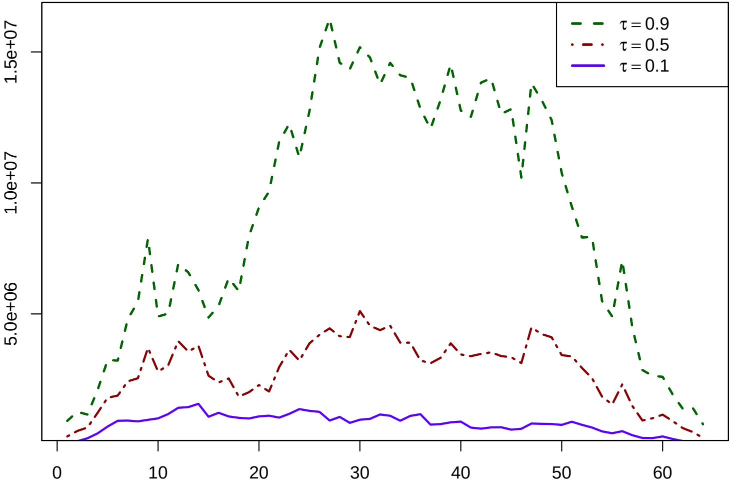
$$\hat{\xi}_{j,n} = \int \left\{ X_n(t) - \bar{X}_N(t) \right\} \hat{v}_j(t) dt.$$

Test statistics:

$$\widehat{S}_d = \frac{1}{N^2} \sum_{j=1}^{d} \frac{1}{\hat{\lambda}_j} \sum_{k=1}^{N} \left(\sum_{1 \le i \le k} \hat{\xi}_{j,i} - \frac{k}{N} \sum_{1 \le i \le k} \hat{\xi}_{j,i} \right).$$

Limit distribution of Kiefer's type
(sum of integrals of squared Brownian bridges).

$P_n(t;\tau)$ functions, typhoons



Trend model

$$X_n(t) = \alpha(t) + \beta(t)n + \varepsilon_n(t).$$

$$H_0 : \ \beta = 0, \quad \text{vs.} \quad H_A : \ \beta \neq 0.$$

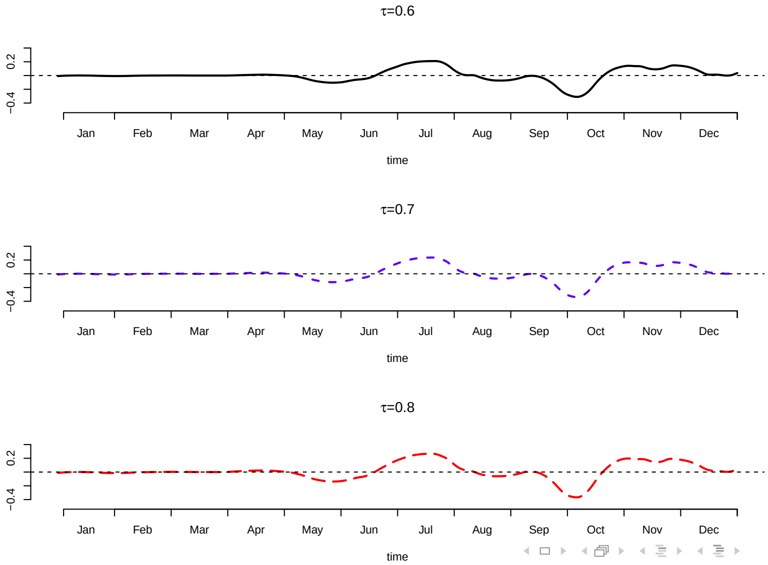
LSE:

$$\hat{\beta}(t) = \frac{6}{N(N+1)(N-1)} \sum_{k=1}^{N} (2k - N - 1) X_k(t)$$

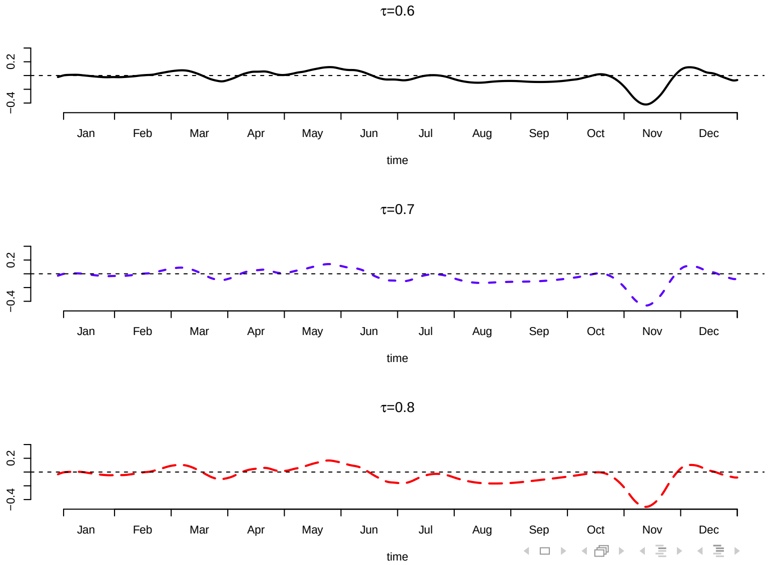
If the function $\hat{\beta}$ is large, we reject H_0.

λ_j eigenfunctions of the error curves ε_i
$\hat{\lambda}_j$ sample eigenvalues of the residual curves $\hat{\varepsilon}_i$
$\hat{v}_j$ corresponding eigenfunctions (sample FPC's)

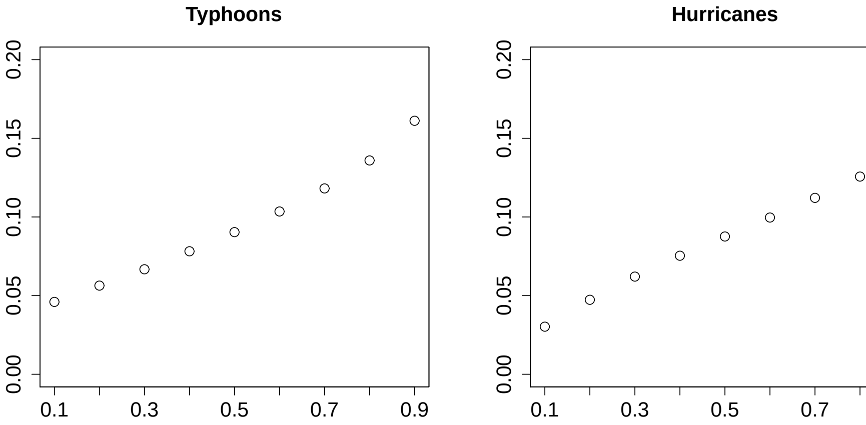
Hurricanes coefficient functions



Typhoons coefficient functions



Norms of the slope functions $\hat{\beta}$



Trend tests

Monte Carlo test:

Under H_0,

$$\widehat{\Lambda}_N = \frac{N^3}{12} \int_0^1 \left(\hat{\beta}(t) \right)^2 dt \xrightarrow{\mathcal{L}} \Lambda_\infty \stackrel{def}{=} \sum_{j=1}^{\infty} \lambda_j Z_j^2,$$

(Z_j independent standard normal)

Chi–square test:

Under H_0,

$$\widehat{T}_N = \frac{N^3}{12} \sum_{j=1}^{q} \hat{\lambda}_j^{-1} \left\langle \hat{\beta}, \hat{v}_j \right\rangle^2 \xrightarrow{\mathcal{L}} \chi_q^2.$$

Both tests are shown to be consistent.

Application to typhoon and hurricane data

Change point test rejects H_0 for all values of $\tau \in .1, .2, \ldots, .9$

Monte Carlo test P–values

τ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.
typhoons	0.365	0.537	0.545	0.495	0.438	0.381	0.329	0.3
hurricanes	0.439	0.239	0.133	0.081	0.062	0.047	0.038	0.0

Chi–square test P–values

τ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.
q	10	11	12	12	12	12	12	1
typhoons	0.534	0.705	0.722	0.688	0.587	0.466	0.382	0.3
q	5	5	5	6	6	6	7	7
hurricanes	0.069	0.024	0.015	0.006	0.003	0.003	0.004	0.0

Main conclusions

- Simulations show that the Monte Carlo test is more accurate if DGP's resemble actual data.
- The annual pattern of wind speeds of both hurricanes and typhoons has been changing at all wind speed levels over the last 60 years.
- There is a significant trend in the shape of this pattern for upper wind speed levels of hurricanes.