Conditions for Globally Rigid Graphs

Shin-ichi Tanigawa

CWI (Netherlands) & Kyoto University (Japan)

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Characterizing Globally Rigid Graphs

- Thm(Gortler-Healy-Thurston10). characterization in terms of the rank of stress matrices
- Open: combinatorial characterization for $d \geq 3$
  - Understanding (local) rigidity is already challenging...
- Can we find a necessary/sufficient condition for graphs to be global rigidity written in terms of local rigidity plus combinatorial properties (e.g. connectivity)??

Combinatorial Characterization of Global Rigidity

Theorem (Hendrickson 92)

If G is globally rigid in $\mathbb{R}^d$, then either G is a complete graph or G is $(d+1)$-connected and redundantly rigid in $\mathbb{R}^d$.

- $d = 1$: the converse is true: G is globally rigid in $\mathbb{R}^1$ iff G is 2-connected.
- $d = 2$: the converse is true: Thm (Jackson-Jordán 05). G is globally rigid in $\mathbb{R}^2$ iff G is 3-connected and redundantly rigid in $\mathbb{R}^2$.
- $d \geq 3$: the converse is not true:

- Theorem(Jackson-Jordán 05) G is globally rigid in $\mathbb{R}^2$ iff G is 3-connected and redundantly rigid in $\mathbb{R}^2$.
  - Algebraic Part (Connelly 05) 1-extension preserves the global rigidity in $\mathbb{R}^d$
  - Combinatorial Part (Berg-Jordán 03, Jackson-Jordán 05) If G is a "minimally" 3-connected and redundantly rigid graph in $\mathbb{R}^2$, then G has a vertex of degree three at which 1-reduction is admissible.

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- A simpler proof (T14)
  - Algebraic Part (Jackson-Jordán-Szabadka06, T.14) Suppose that $G - v$ is rigid and $G - v + K(N_G(v))$ is globally rigid in $\mathbb{R}^d$. Then G is globally rigid in $\mathbb{R}^d$.
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- Theorem(T14) If G is vertex redundantly rigid, then G is GR.
- Q. Find a graph class A (defined in terms of local rigidity and some combinatorial properties) such that the minimality in A implies the existence of a vertex v for which $G - v$ is rigid in $\mathbb{R}^d$.