Tensegrities, flexibility of discrete and semidiscrete surfaces

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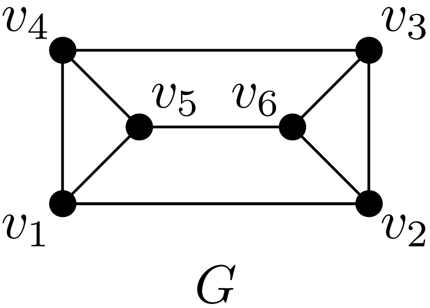
13 July 2015

Introduction

- **I. Stratification of tensegrities**
- **II. Flexibility of polyhedral surfaces**
- **III. Flexibility of semidiscrete surfaces**

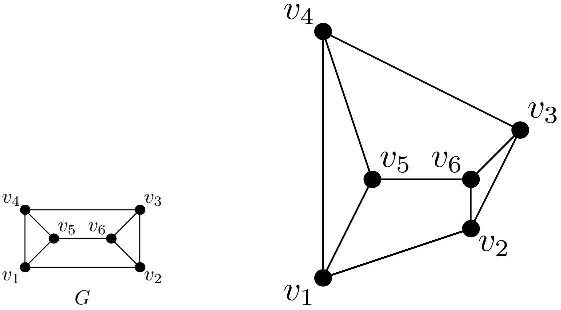
I. Stratification of tensegrities

When does tensegrities exist?-I



The condition is:

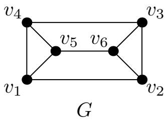
When does tensegrities exist?-I

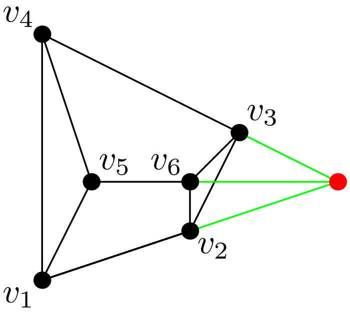


The condition is:

The lines v_1v_2, v_3v_4, and v_5v_6 intersect in a common point.

When does tensegrities exist?-I





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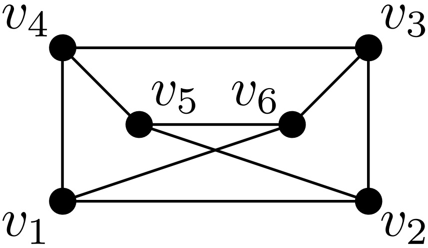
When does tensegrities exist?-II



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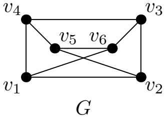
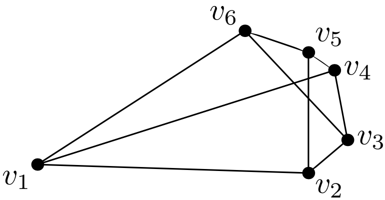
When does tensegrities exist?-II





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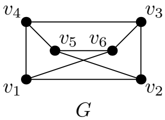
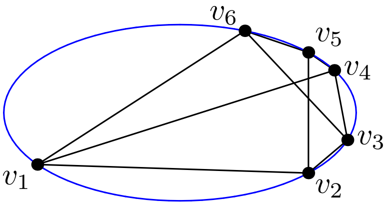
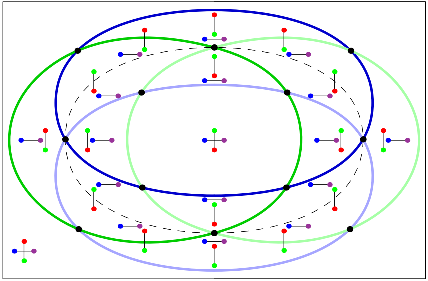




The condition is:

The points v_1, v_2, v_3, v_4, v_5, and v_6 are on a conic.

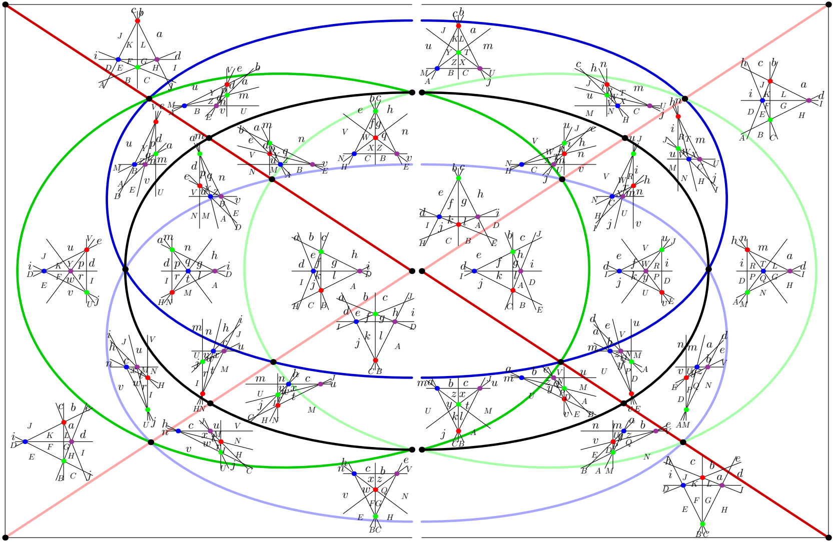
Stratification of $B_2(K_4)$



- Light blue (6 strata): v_1, v_2, v_3 in a line.
- Dark blue (6 strata): v_1, v_2, v_4 in a line.
- Light green (6 strata): v_1, v_3, v_4 in a line.
- Dark green (6 strata) v_2, v_3, v_4 in a line.

- The dashed black line is the projection of the equator. It corresponds to the degenerate case of parallel segments.

Stratification of $B_2(K_5)$ (K., Schepers, B. Servatius, '13)



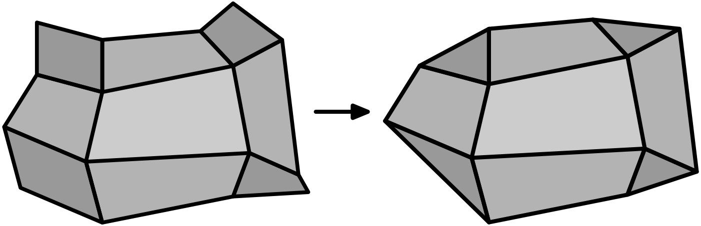
Directions for further studies.

- Adjacency of different strata.
- Surgeries on graphs that preserves the strata.

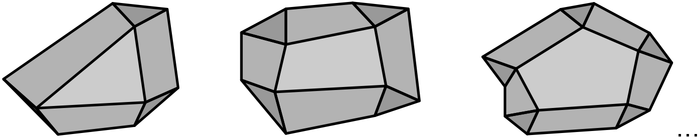
II. Flexibility of polyhedral surfaces

(3×3)-meshes and other Kokotsakis meshes

- (3×3)-meshes (I.Izmestyev):



- Kokotsakis meshes:



Why are the flexibility questions for Kokotsakis meshes so important?

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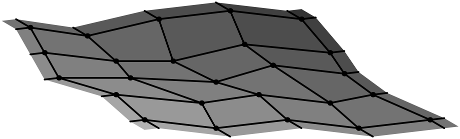
Definition

A *conjugate net* is a mesh corresponding to a mapping of a $\mathbb{Z}^2$-lattice cell complex of the plane.



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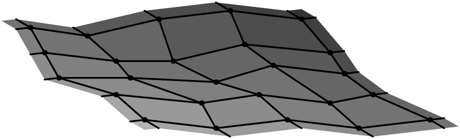
A. I. Bobenko, T. Hoffmann, W. K. Schief [2008]. *A discrete conjugate net in general position is isometrically deformable if and only if all its* 3×3 *subcomplexes are isometrically deformable* (3×3)*-meshes.*



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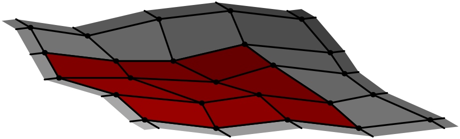

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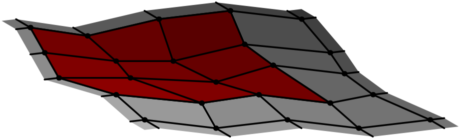

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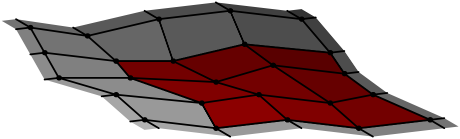

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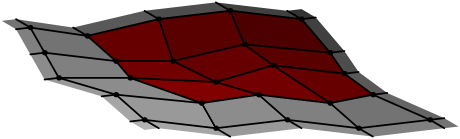


Polyhedral surfaces

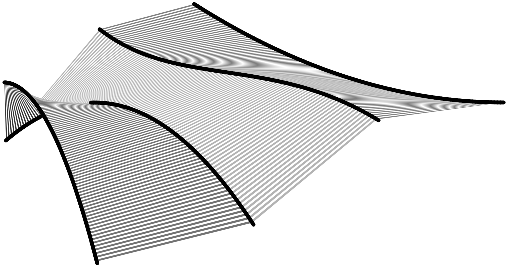
Directions for further studies.

- Flexibility of pentagonal, hexagonal, etc. Kokotsakis meshes.
- Flexibility of meshes with non-planar faces.

III. Flexibility of semidiscrete surfaces

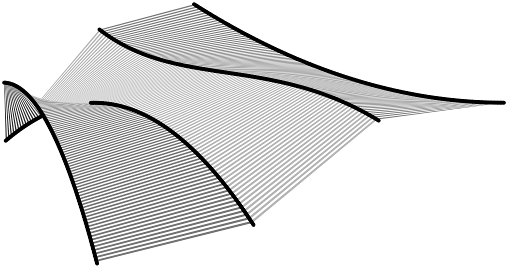
Semidiscrete surfaces

- Semidiscrete surface: $f : \mathbb{R} \times \mathbb{Z} \to \mathbb{R}^3$.

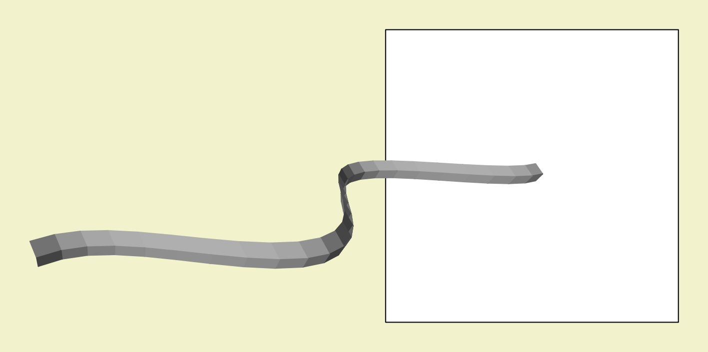


Semidiscrete surfaces

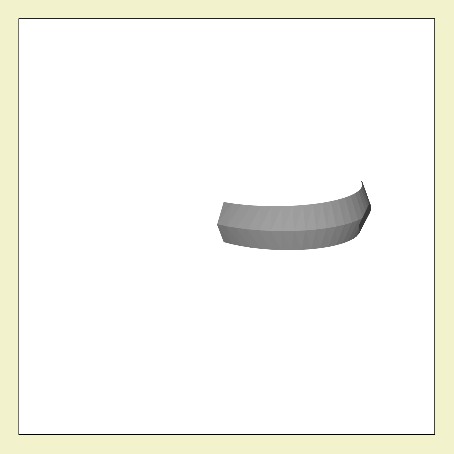
- Semidiscrete surface: $f : \mathbb{R} \times \mathbb{Z} \to \mathbb{R}^3$.
- n-ribbon surface: $f : [a, b] \times \{0, \ldots, n\} \to \mathbb{R}^3$.



Flexibility of semidiscrete surfaces



Snake



Butterfly Trunk

Directions for further studies.

- Flexibility of 3-ribbon surfaces.
- Flexions for non-generic surfaces.
- Compare with a discrete case.
- Applications.