Rigidity in normed linear spaces

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Advances in Combinatorial and Geometric Rigidity
BIRS, 13-17th July 2015

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- What form does the infinitesimal flex condition take?
- Is infinitesimal rigidity a generic property?

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- How does the rigidity operator decompose?
- What can be deduced from the gain graph G/Γ?

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