Applied Probability Frontiers: Computational and Modeling Challenges.
Report

1 General Overview

The workshop concentrated on applied probability, broadly defined, but with an emphasis in engineering and statistical applications, and having in mind both algorithmic approaches and theoretical insights. The workshop included two sessions of open problems, which, as we shall discuss, provide a good indication of research directions that the field is taking. These directions are centered at the core of contemporary computational advances and challenges in applied probability.

The areas that were covered by the workshop and which we singled out during our proposal submission are the following:

1) Stability and steady-state performance measures (including, but not restricted to, Markov chain Monte Carlo techniques),

2) Numerical methods for SDEs (stochastic differential equations) and related continuous time continuous space models,

3) Extremes and rare events,

4) Stochastic optimization and control,

5) Robust performance analysis of stochastic systems.

We will provide a brief overview of the current research interests and advances in these areas and also explain how the workshop's content covered some of these advances.

In addition to the presentations and posters, the open problem sessions were also related to these four areas and were key to expose some questions of current interest.

2 Computation of steady-state performance measures

Simulation methodology is the primary computational tool for steady-state analysis of stochastic systems. (In this document we use the terms stochastic simulation and Monte Carlo interchangeably.) There are literally thousands of documented applications of the Monte Carlo method for steady-state computations, motivated by different scientific areas such as Computer Science, Operations Research, Physics, and Statistics.

Some of the main challenges here includes: acceleration of simulation schemes for exploration of a high dimensional, multi-modal, target steady state distribution, a problem that often arises in the fields of Physics and Statistics. **Professor Gareth Roberts** delivered a lecture which discusses precisely these types challenges in the context of Bayesian analysis for very large data sets, giving rise to a likelihood which cannot be evaluated practically. **Professor Roberts'** approach, based on so-called pseudo-likelihood methods, successfully exploits unbiased estimation techniques for continuous time processes, which we shall discuss in the next section.

Another challenge in the area involves quantification of rates of convergence to stationary of specific Markov chains. This topic is often studied in Statistics and Computer Science. Motivated once again by applications to Bayesian statistics, **Professor Eric Molines**' talk concentrated precisely on the estimation of rates of convergence to stationarity for the continuous-time Metropolis chain (also known as Langevin diffusion) assuming a high dimensional, strongly convex, potential. He demonstrated a positive result (i.e. gracious rate of convergence scaling as the dimension increases) for such types of Metropolis chains.

Yet another type of challenge in the area involves the detection and deletion of the initial transient bias in the steady-state simulation of semi-Markov models, arising frequently in Operations Research, and for which typically very little is known about the equilibrium distribution. **Professor Soren Asmussen** discussed advances and open problems in areas such as exact simulation (i.e. simulation without bias) for regenerative processes observing that generic approaches might yield infinite expected termination time. In this vein, **Professor Karl Sigman** presented the first class of exact simulation algorithms for semi-Markov multiserver queues, using coupling techniques which guarantee, under natural stability assumptions, finite expected termination time for the procedure.

The talk delivered by **Professor Jim Dai** introduced the use of Stein's method for steady-state approximations of queues in heavy traffic. In his talk, **Professor Dai** demonstrated how Stein's method can be used to address rates of convergence in the so-called "limit interchange" problem discussed in work of **Professor David Gamarnik** and **Professor Assaf Zeevi**. Such problem concerns heavy-traffic approximations of steady-state distributions by using the stationary distribution of the finite-time heavy-traffic limit and has attracted a significant attention in the Operations Research literature.

Now, although steady-state distributions of complex systems can be quite challenging to compute, sometimes it is possible to obtain quite precise characterizations of steady-state distributions. This was illustrated by the talk given by **Professor Onno Boxma** who discussed closed form expressions involving various classical special functions in problems in the context of inventory, queueing, and insurance problems.

Finally, **Professor Thorisson's** talk which concentrated on fundamental theoretical questions in the context of two sided stationarity processes. For instance, consider a two-sided Brownian motion centered at zero. Characterize the random times that, after centering the process at one of such times, one obtains a two-sided Brownian motion. He provided non-trivial examples of such times involving a construction based on the local time at zero and suitable randomizations. Although his talk was not computational, his results gave rise to discussions connected to the topic of perfect simulation.

2.1 Numerical methods for Stochastic Differential Equations (SDEs) and Related Continuous Time Continuous Space Models

Continuous-time models, such as stochastic (partial or even ordinary) differential equations (driven by Brownian motion or other type of noise such as Levy or long range dependent processes) are ubiquitous in scientific applications primarily because, owing to the theory of analysis (both classical and stochastic), these models are amenable to mathematical manipulation. Nevertheless, computational tasks associated with these models are often challenging given the fact that they ideally require a continuum of information to be stored in a computer's memory. Moreover, basic probabilistic quantities, such as for example, transition probability functions, are virtually impossible to compute analytically in almost all cases.

In recent years, there have been a number of major advances in the theory of simulation that have enhanced our ability to access the solution to SDEs with an increasingly accurate assessment of the control error. For example, the work of [7] has allowed us to simulate, essentially without any bias, a range of diffusion processes. This work has enabled the possibility of providing estimates for the underlying transition density and, consequently, the direct application of explicit likelihood ratio methods in statistical analysis of diffusion models ([8]). In fact, an application of these methods was highlighted, as mentioned earlier, by the talk of **Professor Gareth Roberts.** The approach of ([8]) is also applicable as a means to implement the class of algorithms analyzed by **Professor Moulines**.

Another important recent advance in numerical methods for SDEs corresponds to multilevel Monte Carlo methods, (see [21]). This approach has enabled the estimation of a large class of expectations with an optimal rate of convergence. **Professor Mike Giles** delivered a comprehensive lecture which explained how the multilevel Monte Carlo method can be applied not only to SDEs, but also to nested simulation problems, optimal stopping problems, and chemical reaction simulations.

More recently, there has also been work that allows the simulation of piecewise linear processes which are strong sample path approximations to SDEs, in the sense they are subject to a prescribed (deterministic) error with probability one ([10]). These constructions enable the estimation of sample path expectations of complex objects such as multidimensional local-time-like processes, for example those arising in the solution of the so-called Skorokhod problem studied in stochastic networks ([32], [17]). The lecture delivered by **Professor Kavita Ramanan**, which discussed sample path derivatives of the solutions to the Skorokhod problem, brought up a potential avenue for using path approximations for numerical evaluation of unbiased estimators of derivatives of expectations of SDEs.

There are still many outstanding challenges in analyzing multidimensional diffusions, in SPDEs, and in SDEs driven by multidimensional Levy and long-range dependent processes. A problem related to local times and maxima of Levy process was posed by **Professor Mike Giles** at the beginning of the workshop, involving the existence of a density under certain conditions, and motivated by the development of efficient Monte Carlo methods for SDEs driven by Levy processes. Significant progress towards the solution of the problem (including a partial solution which is applicable to the examples of interest to **Professor Giles**) was reported by **Professor Jose Blanchet** during the last day of the workshop.

While, as mentioned before, there remain many outstanding challenges in the area, it was also evident from the discussions in the workshop that are also opportunities for cross fertilization among communities that tend to have little intersection, e.g., statistical inference (exact sampling of diffusions, [23]), mathematical finance and chemical reaction networks (multilevel Monte Carlo, [22], [2]), and stochastic queueing networks (strong couplings, [10]). As an additional example of such cross fertilization, we mention the poster presented by **Dr. Chang-han Rhee**, which explained how to use an extra randomization step on top of multilevel Monte Carlo ideas in order to estimate without bias steady-state expectations of a positive recurrent Harris Markov chain (see [36] and also [34]).

2.2 Extremes and rare events

This area has been traditionally investigated in the context of communication networks, finance, and insurance. **Professor Soren Asmussen** lecture and an open problem posed discussed some recent problems involving loss rates (i.e. the probability of packages being dropped) in the context of communication-transmission model involving potentially long range dependence.

Analysis of extreme events traditionally have relied on the classical theory of large deviations theory for light tailed systems, see for example [16]. More recently, in part motivated by some of the applications described earlier, there have been advances in the development of techniques for systems with heavy-tailed characteristics ([9], [5], [30]). Some of these methods are cross disciplinary, relying, for example, on the application of statistical extreme value theory, [20], combined with the theory of stability of Markov chains and Lyapunov inequalities (typically used in applications involving Markov chain Monte Carlo). **Professor Sigman**'s lecture gave an example of a perfect sampling algorithm which uses large deviations techniques for steady-state analysis.

A new set of applications, in the context of page rank algorithms, which require the use of heavy-tailed approximations arising from the analysis of tree-like models with light-tailed characteristics was illustrated by the talk of **Professor Mariana Olvera-Cravioto**.

Finally, we mention the talk of **Professor Ton Dieker,** who described the first exact sampling algorithm for so-called max stable random fields. These processes characterize the extreme behavior of a sequence of suitably scaled spatial processes and are natural in applications such as climate modeling. **Professor Dieker**'s talk combined ideas from large deviations and stationary concepts and also connected to topics simulation of continuous processes and large deviations ideas for Gaussian fields (see [1]).

Important open problems which were discussed, for example towards the end of **Professor Olvera-Cravioto**'s talk include the fact that the majority of the Monte Carlo estimators for rare events that are shown to achieve desirable optimality properties (in terms of achieving an optimal convergence rate) require the existence of asymptotic approximations. A challenge in the area is to design estimators that are both efficient in some sense and that are applicable in environments that do not require these types of approximations.

In addition, it remains to be seen how the theory and algorithms developed in the rare event simulation literature can aid the development of statistical inference procedures for the likelihood of rare events, for example, using a Bayesian perspective.

2.3 Stochastic Optimization and Control

Stochastic optimization entails the minimization of a function that is represented as an expectation, possibly subject to constraints that might also involve expectations. These problems can quickly become quite challenging from a computational standpoint and Monte Carlo methods are often used to find approximate solutions in a reasonable amount of time; this approach is known as simulation optimization. Examples in which simulation optimization arises in practical applications include the design of emergency-service systems [33] and service-system staffing [13].

The area is advancing rapidly, partly due to advances in hardware, and partly through advances in software enabled by recent research. In addition, the trend towards parallel computing, as evidenced through multicore architectures, the use of graphical processing units in scientific computing, cloud computing, and the increasing availability of high-performance computing (supercomputing), provides a rich environment for the development of new and exciting work in this area. **Professor Barry Nelson**'s poster presented an algorithm aimed at exploiting meta models for large scale optimization problems of discrete systems. **Professor Sandeep Juneja**'s lecture discussed lower bounds related to the best possible performance that can be expected from an optimal simulation scheme for optimization of discrete simulation optimization problems. And adaptive optimal allocation policies for such discrete optimization problems was discussed in **Professor Zeevi'**s poster.

Another new an exciting application involving a stochastic model for optimal allocation of bikes across New York City was given in the poster of **Professor Shane Henderson**; the optimal policy in his poster combines coupling of Markov chains and combinatorial optimization ideas in a novel way.

Another type of application in the context of stochastic control for staffing problems was given in the talk of **Professor Amy Ward**, where an asymptotically optimal solution to a complex stochastic control problem was provided. These types of approximate solutions have been studied in the last decade in heavy-traffic theory of queueing systems.

Finally, yet another application to the context of discrete stochastic optimization, in the context of assortment selection to maximize the revenue of a vendor, was illustrated by an open problem posed by **Professor Jose Blanchet**.

We mentioned problems involving discrete / ordinal optimization. In the setting of stochastic optimization problems with convex / differentiable structures, one must cope with noisy estimates of function values and derivatives. There is a substantial literature on methods for attacking such problems; see, e.g., [4], [35], Chapters 17-21 of [26]. The poster of **Professor Shanbhag** related to this problem via efficient use stochastic approximation methods. Also in this context, advances in stochastic optimization by means of sample average approximation methods with constraints were presented in the talk of **Professor Sigrun Andradottir**. **Professor Jose Blanchet** presented a poster showing the first class of estimators which achieve complete deletion of the systematic bias arising in virtually any application of sample average approximations.

A particular outstanding challenge is the design of adaptive procedures that adjust tuning sequences that calibrate the stochastic approximation algorithm, to various information that is a prior unknown concerning the target function that is to be optimized, and the environment in which it is observed/simulated (see, e.g., [14]). A related open problem involving adaptive root finding algorithms was posed by **Professor Shane Henderson.** These adaptive techniques have a strong connection with the theory of stochastic stability in Markov chains, analysis of Markov chain Monte Carlo algorithms, and stochastic recursions (discrete time analogues of SDE's). Given these, and other links stated above, we believe there is ample potential for cross fertilization and synergies to be realized in this area as well.

2.4 Robust Performance Analysis of Stochastic Systems

The last that was covered during the workshop relates to the issue of inaccurate model assumptions in calculating expectations of interest. From a methodological standpoint, robust performance analysis is closely related to stochastic optimization since a natural approach consists in solving a optimization problem to obtain a worst-case bounds among a class of models of interest.

Such worst-case analysis approach, subject to constraints, is popular in methodological areas such as robust control [25], and robust optimization, [28], [6]. And the approach has been used in application domains such as economics and finance [24], among others.

From a computational standpoint, this area brings a wide number of challenges and opportunities. For example, the choice of feasible regions from which to optimize often gives rise to infinite dimensional optimization problems. Some times those problems can be tractable if one ignores natural constraints (such as stochastic independence or Markovianity), but tractability comes at the expense of ending up with too pessimistic upper bounds. This is one of several open problems in the area, which were addressed in some of the talks in the workshop. **Professor Henry Lam** discussed recent advances involving techniques to approximate the solution to such optimization problems (including non-convex constraints) assuming that the feasible region becomes a small neighborhood around a baseline model. His talk also included model uncertainty considerations for rare events and steady-state analysis. Finally, in the context of robustness, **Professor Sean Meyn** posed an open problem involving the stability of robust non-linear filtering.

3 Conclusions

The workshop promoted the identification unresolved problems that, thanks to recent advances in other areas, are within reach of what we currently know. At the same time we believe that the workshop planted seeds for significant cross fertilization among the different allied areas. In fact, we discussed already some of this potential in the body of this report.

References

[1] R.J. Adler, J. Blanchet and J. Liu (2012), "Efficient Monte Carlo for high excursions of Gaussian random field", *Annals of Applied Probability*, Vol. 22, No. 3, pp 1167-1214.

[2] J.B. Anderson (2007), *Quantum Monte Carlo: Origins, Development, Applications*, Oxford University Press.

[3] S. Asmussen, P.W. Glynn and H. Thorisson (1992), "Stationary detection in the initial transient problem", *TOMACS*, Vol 2, No. 2, pp 130-157.

[4] S. Asmussen and P. W. Glynn (2007), *Stochastic Simulation: Algorithms and Analysis*, 1st ed. Spinger, New York, NY.

[5] S. Asmussen and D. Kortscha (2013), "Error rates and improved algorithms for rare event simulation with heavy Weibull tails", *Methodology and Computing in Applied Probability*, published online.

[6] A. Ben-Tal, D. Bertsimas, and D. B. Brown (2010) "A soft robust model for optimization under ambiguity", *Operations Research*, 58 (4):1220-1234.

[7] A. Beskos and G. Roberts (2005), "Exact simulation of diffusions", *Annals of Applied Probabilit*, Vol. 15, No. 4, pp 2422-2444.

[8] A. Beskos, O. Papaspiliopoulos and G. Roberts (2006), "Retrospective exact simulation of diffusion sample paths with applications", *Bernoulli*, 12, 1077-1098.

[9] J. Blanchet and J. C. Liu (2014), "Total variation approximations for multivariate regularly varying random walks", 20, 416-456.

[10] J. Blanchet and X. Chen (2015), "Steady-state simulation of reflected Brownian motion and related stochastic networks", Annals of Applied Probability, To appear.

[11] M. Boue and P. Dupuis (1999), "Multidimensional stochastic approximation: adaptive algorithms and applications", *SIAM J. Numer. Anal.*, Vol 36, pp 667-695.

[12] P. Brémaud (1999), *Markov Chains: Gibbs Fields, Monte Carlo Simulation and Queues*, Spinger, New York, NY.

[13] M. T. Cezik and P. L'Ecuyer (2008), "Staffing multiskill call centers via linear programming and simulation", *Management Science*, Vol. 54, No. 2, pp. 310-323.

[14] D. Cicek, M. Broadie and A. Zeevi (2012), "Multidimensional stochastic approximation: adaptive algorithms and applications", *ACM TOMACS*.

[15] D. Crisan and B. Rosovsky (2008), "A Mean Field Theory of nonlinear filtering", in collections of *Oxford Handbook of Nonlinear Filtering*, Oxford University Press.

[16] Dembo, A. and Zeitouni, O. (1999), *Large Deviations: Techniques and Applications*, Springer, New York.

[17] P. Dupuis and K. Ramanan (2002), "A time-reversed representation for tail probabilities of stationary reflected brownian motion", *Stoch. Proc. and Their Appl.*, Vol 98, pp 253-287.

[18] P. Dupuis and H. Wang (2005), "Dynamic importance sampling for uniformly recurrent Markov Chain", *Annals of Applied Probability*, Vol. 15, pp 1-38.

[19] P. Dupuis, B. Kaynar, A. Ridder and R. Rubinstein (2012), "Counting with combined splitting and capture-recapture methods", *Stochastic Models*, Vol 28, Iss. 3, pp 478-502.

[20] Embrechts, P., Klüppelberg, C., and Mikosch, T. (1997), *Modelling Extreme Events for Insurance and Finance*, Springer, New York.

[21] M.B. Giles (2008), "Multilevel Monte Carlo path simulation", *Operations Research*, Vol. 56, No. 3, pp 607-617.

[22] M.B. Giles and L. Szpruch (2013), "Multilevel Monte Carlo methods for applications in finance", working paper.

[23] K. Giesecke and D. Smelov (2013), "Exact sampling of jump diffusions", *Operations Research*, Vol. 61, No. 4, pp 894-907.

[24] L. P. Hansen and T. J. Sargent (2001) "Robust control and model uncertainty", *The American Economic Review,* 91 (2):60-66.

[25] L. P. Hansen and T. J. Sargent. (2008) *Robustness*. Princeton University Press, 2008.

[26] S. G. Henderson and B. L. Nelson ed. (2006), *Handbooks in Operations Research and Management Science*, Elsevier, Amsterdam.

[27] M.A. Iglesias, K.J.H. Law and A.M. Stuart (2013), "Evaluation of Gaussian approximations for data assimilation in reservoir model", *Computational Geoscience* Vol. 17, pp 851-885.

[28] G. N. Iyengar (2005) "Robust dynamic programming", *Mathematics of Operations Research*, 30 (2):257-280.

[29] A. E. B. Lim, G. J. Shanthikumar, and M. Z. J. Shen (2006) *Model uncertainty, robust optimization and learning*, Tutorials in Operations Research.

[30] S. Juneja (2007), "Estimating tail probabilities of heavy tailed distributions with asymptotically zero relative error", *Queueing Systems*, Vol. 57, Iss. 2-3, pp 115-127.

[31] K. Latuszynski, G.O. Roberts and J.S. Rosenthal (2013), "Adaptive Gibbs samplers and related MCMC methods", *Annals of Applied Probability*, Vol. 23, No. 1, pp 66-98.

[32] A. Mandelbaum and K. Ramanan (2010), "Directional derivatives of oblique reflection map", *Mathematics of Operations Research*, Vol. 35, pp 527-558.

[33] M. S. Maxwell and M. Restrepo and S. G. Henderson and H. Topaloglu (2010), "Approximate dynamic programming for ambulance redeployment", *INFORMS Journal on Computing*, Vol. 22, pp. 266-281.

[34] D. McLeish (2011), “A general method for debiasing a Monte Carlo estimator", *Monte Carlo Methods and Application*, Vol. 17, pp. 301-315.

[35] G. Ch. Pflug (1996), *Optimization of Stochastic Models: The Interface Between Simulation and Optimization*, Kluwer, Boston, MA.

[36] C. Rhee and P. W. Glynn (2012), “A new approach to unbiased estimation for SDE's", in *Proceedings of the 2012 Winter Simulation Conference,* 201-207