Banff Workshop 15w5164: Developments in the Theory of Homogenization
Talk titles and abstracts
(in alphabetic order by speaker's last name)

Speaker: **Scott Armstrong** (Universite Paris-Dauphine)
Title: *Additive structure of elliptic homogenization*
Abstract: I will explain some recent progress in using subadditive methods to obtain quantitative results for stochastic homogenization of elliptic equations in divergence form and the important role played by mesoscopic regularity estimates.

Speaker: **Pierre Cardaliaguet** (Universite Paris-Dauphine)
Title: *Stochastic homogenization of quasilinear Hamilton-Jacobi equations and geometric motions*
Abstract: We study random homogenization of second-order, degenerate and quasilinear Hamilton-Jacobi equations which are positively homogeneous in the gradient. Included are the equations of forced mean curvature motion and others describing geometric motions of level sets as well as a large class of viscous, non-convex Hamilton-Jacobi equations. The main results include the first proof of qualitative stochastic homogenization for such equations. We also present quantitative error estimates which give an algebraic rate of homogenization. Joint work with S. Armstrong.

Speaker: **Adina Ciomaga Ralea** (Paris Diderot University, Paris, LJLL)
Title: *Stochastic homogenization of interfaces moving with changing sign velocity*
Abstract: I will present a result on stochastic homogenization of interfaces moving, in stationary ergodic environments, with oscillatory normal velocity which changes sign. The problem can be formulated as the homogenization of a Hamilton-Jacobi equation with a 1-positively homogeneous, non-coercive and non-convex Hamiltonian. The periodic setting was studied earlier by Cardaliaguet, Lions and Souganidis (2009). Here we concentrate in the random media and show that the solutions of the oscillatory

Hamilton-Jacobi equation homogenize only weakly to a linear combination of the initial datum and the solutions of several initial value problems with deterministic effective Hamiltonian(s). The result apply for a wide class of domains, such as percolation structures from probability. This is joint work with Takis Souganidis and Hung Tran.

Speaker: **Jean-Dominique Deuschel** (Technische Universitat Berlin)
Title: *Quenched invariance principle for random walks in time-dependent balanced random environment*
Abstract: We prove an almost sure functional limit theorem for a random walk in an space-time ergodic balanced environment under certain moment conditions. The proof is based on the maximal principle for parabolic difference operators. We also deal with the non-elliptic case, where the corresponding limiting diffusion matrix can be random in higher dimensions. This is a joint work with N. Berger, X. Guo and A. Ramirez.

Speaker: **Nicolas Dirr** (Cardiff University)
Title: *Homogenisation for mean field games*
Abstract: This is joint work with Analisa Cesdaroni and Claudio Marchi. We investigate dynamical mean field games as introduced by Lasry and Lions in the small noise limit when the Hamilton-Jacobi equation of the system has a rapidly varying dependence on the state variable x.

Speaker: **William Feldman** (University of Chicago)
Title: *Homogenization of Oscillating Boundary Conditions*
Abstract: I will discuss the homogenization of periodic oscillating Dirichlet boundary problems in general domains for second order uniformly elliptic equations. These problems are connected with the study of boundary layers in fluid mechanics and with the study of higher order asymptotic expansions in interior homogenization theory. The talk will be aimed at a general audience. I will explain some recent progress about the continuity properties of the homogenized problem which displays a sharp contrast between the case of linear and nonlinear interior equations. This is based on joint work with Inwon Kim.

Speaker: **Hongwei Gao** (Unversity of California at Irvine)
Title: *Random homogenization of coercive Hamilton-Jacobi equations in 1d*
Abstract: In this talk, I will present the random homogenization of general coercive Hamilton-Jacobi equations

$$u_t^\epsilon + H(u_x^\epsilon, \frac{x}{\epsilon}, \omega) = 0$$

in 1d. This extends a result of Armstrong, Tran and Yu when the Hamiltonian has a separable form $H(p, x, \omega) = H(p) + V(x, \omega)$ for any coercive $H(p)$.

Speaker: **David Gerard-Varet** (Université Denis Diderot Paris 7)
Title: *Homogenization of rough boundaries for rotating fluid flows*
Abstract: Our goal is to describe the effect of a rough boundary on a viscous and rotating fluid flow. When the boundary is flat, a boundary layer called Ekman layer developps near the wall. When roughness is taken into account (through a small amplitude/small wavelength oscillation), the dynamics of this boundary layer becomes nonlinear. We shall discuss its qualitative properties and associated mathematical difficulties (notably when the oscillation of the boundary is not periodic).The talk is based on joint works with Emmanuel Dormy and Anne-Laure Dalibard.

Speaker: **Antoine Gloria** (Université Libre de Bruxelles)
Title: *Quantitative homogenization and regularity theory in the large via weighted functional inequalities*
Abstract: In this talk I shall prove some optimal quantitative estimates in stochastic homogenization when the coefficient field displays strong correlations. Previous results were based on spectral gap estimates, which are not necessarily satisfied by examples of practical interest and essentially require fast decay of correlations. The aim of the talk is to present a new family of functional inequalities (weaker than spectral gap) with the following three properties:

- flexible enough to cover examples of practical interest
- weak enough to handle non-integrable correlations
- strong enough to yield quantitative and optimal results

After introducing a class of weighted functional inequalities, I'll discuss the typical examples we have in mind, shall show how these functional inequalities imply the validity of an improved regularity theory in the large for elliptic equations with random coefficients, and shall conclude with sharp bounds on the growth of the corrector and on the two-scale expansion. This talk is based on joint works with M. Duerinckx (ULB), S. Neukamm (TU Dresden), and F. Otto (MPI Leipzig).

Speaker: **Wenjia Jing** (University of Chicago)
Title: *Homogenization of Hamilton-Jacobi equations in dynamic random environments*
Abstract: We consider homogenization problems of Hamilton-Jacobi equations in time-dependent (dynamic) random environments, where the Hamiltonian and the diffusion coefficients (for second order equation) are highly

oscillatory in the space and time variables. We focus on second order equations with general diffusion coefficients and with Hamiltonian that has super quadratic growth in the momentum, and discuss a couple of approaches to prove stochastic homogenization.

Speaker: **Frédéric Legoll** (Ecole Nationale des Ponts et Chaussees)
Title: *Special Quasirandom Structures: a selection approach for stochastic homogenization*
Abstract: We consider the homogenization of linear elliptic PDEs with random stationary coefficients. As is well-known, the deterministic homogenized coefficients are obtained through a corrector problem that is set on the entire space. This problem is thus intractable from the practical viewpoint. A standard approximation consists in considering large but bounded domains, and solve the corrector problem (complemented with appropriate boundary conditions) on these domains. The obtained apparent effective coefficients are random. It is thus natural to consider several realizations of the microstructure, in a Monte Carlo fashion.

In this talk, we describe a selection method, where we a priori select the microstructures for which we solve the corrector problem. This selection is performed on the basis of some appropriate criteria. In that spirit, the expensive corrector problem is solved only for microstructures that are "more representative" than generic microstructures of the materials on the entire space. We will discuss this approach, both from the theoretical and numerical standpoints.

Joint work with C. Le Bris and W. Minvielle.

Speaker: **Jessica Lin** (University of Wisconsin-Madison)
Title: *On the Stochastic Homogenization of Parabolic Equations*
Abstract: We will present an overview of the study of stochastic homogenization of parabolic equations in spatio-temporal media. This will include a discussion of the qualitative theory, the regularity theory for parabolic equations, and error estimates. In particular, we obtain quenched error estimates which decay at an algebraic rate. This talk is based on joint work with Charles Smart.

Speaker: **Pierre Mathieu** (Universite d'Aix-marseille)
Title: *FDT for reevrsible diffusions in a random environment.*
Abstract: In a previous paper with N. Gantert and A. Piatnitski (Comm. Pure Appl. Math. 2012) we proved the so-called Einstein relation for reversible diffusions in a random environment with bounded range correlations. Namely we showed that the derivative of the effective drift of the diffusion

perturbed by a small fixed external force is given by the effective diffusivity matrix. In a work in preparation, still with A. Piatnitski, we are obtaining a more complete picture proving that similar relations (so-called FDT relations) hold for 'general' additive functionals. In a sense, we thus establish that the derivative of the steady state of a perturbed reversible diffusion is given by its corrector.

Speaker: **Hiroyoshi Mitake** (Hiroshima university)
Title: *On asymptotic speed of solutions to level-set mean curvature flow equations with driving and source term*
Abstract: This is a joint work with Y. Giga (U. Tokyo), and H. V. Tran (U. Chicago). In this talk, we will discuss the asymptotic speed of solutions to a model equation appearing in the crystal growth. We show a new type nonlinear phenomenon in terms of the asymptotic speed, which is sensitive to a shape of source terms.

Speaker: **Jean-Christophe Mourrat** (ENS Lyon)
Title: *Describing the fluctuations in stochastic homogenisation*
Abstract: Consider the solution of a divergence-form problem with random coefficients. Under suitable assumptions on the law of the coefficients, homogenisation theory ensures that as the correlation length of the random coefficients is sent to 0, this solution converges to the solution of a similar problem with constant, "homogenised" coefficients. The problem of quantifying the error in this convergence has witnessed tremendous progress recently. The goal of this talk is to explain how one can go beyond error bounds, and describe precisely the statistics of the fluctuations in terms of a finite number of new effective parameters.

Speaker: **James Nolen** (Duke University)
Title: *Gaussian approximation for the effective conductance and the corrector in stochastic homogenization*
Abstract: I will talk about solutions to a linear, divergence-form elliptic PDE with conductivity coefficient that varies randomly with respect to the spatial variable. I'll describe a Gaussian fluctuation theory (central limit theorem) for the effective conductivity of a finite sample, assuming finite-range dependence in the coefficients. The technique can also be applied to the corrector showing that the rescaled corrector (in the analogous discrete model, in dimension 3 or more) has a scaling limit which is a Gaussian field. The basic analytical tool is normal approximation based on Stein's method. Some of this is joint work with Antoine Gloria and with Jean-Christophe Mourrat.

Speaker: **Alexei Novikov** (Pennsylvania State University)
Title: *Homogenization in stationary fluid flows*
Abstract: I will discuss recent results related to analysis of effective behavior of a passive scalar in incompressible stationary fluid flows in two dimensions. The particle is driven either by the classical Brownian motion or by the fractional Brownian motion.

Speaker: **Norbert Pozar** (Kanazawa University)
Title: *Homogenization of a Hele-Shaw-type problem in periodic spatiotemporal media*
Abstract: In this talk I will discuss the homogenization of the Hele-Shaw problem. The distinguishing feature of this generalization of the well-known free boundary problem is the periodic dependence of the coefficient in the free boundary velocity law on time as well as on position. In the homogenization limit, its solutions converge to the unique solution of a homogeneous Hele-Shaw-type problem with an anisotropic free boundary velocity law. Furthermore, the free boundaries converge with respect to the Hausdorff distance. The presented approach exploits the comparison principle structure of the Hele-Shaw problem and relies on the techniques of the theory of viscosity solutions.

Speaker: **Christophe Prange** (University of Chicago)
Title: *Improved Regularity in Bumpy Lipschitz Domains*
Abstract: "In this talk we will explain how to get Lipschitz regularity up to the microscale for elliptic systems over a bumpy boundary. The analysis relies on a compactness scheme and on an estimate in a space of non localized energy for a boundary layer corrector in the half-space. This is joint work with Carlos Kenig."

Speaker: **Fradoun Rezakhanlou** (University of California at Berkeley)
Title: *Generalized Smoluchowski Equations and Scalar Conservation Laws*
Abstract: By a classical result of Bertoin, if initially a solution to Burgers' equation is a Levy process without positive jumps, then this property persists at later times. According to a theorem of Groeneboom, a white noise initial data also leads to a Levy process at positive times. Menon and Srinivasan observed that in both aforementioned results the evolving Levy measure satisfies a Smoluchowski-type equation. They also conjectured that a similar phenomenon would occur if instead of Burgers' equation, we solve a general scalar conservation law with a convex flux function. Though a Levy process may evolve to a Markov process that in most cases is not Levy. The corresponding jump kernel would satisfy a generalized Smoluchowski equation. Along with Dave Kaspar, we show that a variant of this conjecture

is true for monotone solutions to scalar conservation laws. I also formulate some open question concerning the analogous questions for Hamilton-Jacobi PDEs in higher dimensions.

Speaker: **Russell Schwab** (Michigan State University)
Title: *Neumann Homogenization via Integro-Differential Operators*
Abstract: We use a recent result about the representation of the Dirichlet-to-Neumann operator for fully nonlinear equations as an integro-differential operator on the boundary of the domain to guide the analysis of the homogenization problem with oscillatory Neumann data. This allows us to attack the homogenization problem as a nonlocal homogenization on the boundary, which is amenable to methods already established for integro-differential equations. We will present the case of a infinite strip domain with almost periodic Neumann data. The emphasis will be on the method of converting the Neumann analysis into an auxiliary nonlocal problem which lives only on the boundary. This is joint work with Nestor Guillen.

Speaker: **Zhongwei Shen** (University of Kentucky)
Title: *Boundary Estimates in Elliptic Homogenization*
Abstract: In this talk I will discuss some recent work on sharp boundary estimates for a family of systems of linear elasticity with rapidly oscillating coefficients. We are interested in boundary estimates with either Dirichlet or Neumann conditions, down to the microscopic scale, without any smoothness assumptions on the coefficients. Under additional smoothness conditions, these estimates, combined with the corresponding local estimates, lead to the full Rellich type estimates in Lipschitz domains and Lipschitz estimates in $C^{1,\alpha}$ domains. The C^α, L^p, $W^{1,p}$ estimates in C^1 domains are also studied.

Speaker: **Antonio Siconolfi** (University of Rome)
Title: *Homological techniques for homogenization of Hamilton–Jacobi equations*
Abstract: We generalize periodic homogenization for Hamilton Jacobi equations replac- ing the torus by an arbitrary compact manifold or a bounded graph. This setting allows to reach a deeper understanding of the matter and unveils phenomena somehow hidden in the periodic case, for instance the fact that the ambient spaces of oscillating equations and that of the limit problem are different, and possess even different dimensions. Repetition structure for the base manifold, changes of scale in it and asymptotic analysis, which are the basic ingredients of homogenization, need substantial modification to work in the new frame, and this task is accomplished using tools from algebraic topology. An adapted no- tion of convergence allowing approximating entities and limit to lie in different spaces is also provided.

Speaker: **Hung Tran** (University of Wisconsin at Madison)
Title: *Some inverse problems in periodic homogenization of Hamilton–Jacobi equations.*
Abstract: We look at the effective Hamiltonian $\bar{H}$ associated with the Hamiltonian $H(p, x) = H(p) + V(x)$ in the periodic homogenization theory. Our central goal is to understand the relation between V and $\bar{H}$. We formulate some inverse problems concerning this relation. Such type of inverse problems are in general very challenging. I will discuss some interesting cases in both convex and nonconvex settings. Joint work with Songting Luo and Yifeng Yu.