Distribution of Rational and Holomorphic Curves in Algebraic Varieties

March 15 to March 20, 2015

ABSTRACTS

(in alphabetic order by speaker surname)

Speaker: **Damian Brotbek**
Title: *Symmetric differential forms on complete intersection varieties*
Abstract: We study the space of holomorphic symmetric differential forms on complete intersection varieties in projective space. We give an explicit description of these spaces in terms of the equations defining the complete intersection under consideration. Our main application is the construction of varieties with ample cotangent bundle, providing new results towards a conjecture of Olivier Debarre.

Speaker: **Frederic Campana**
Title: *h-principle and specialness*
Abstract: A complex space X is said to satisfy the h-principle if, for any Stein manifold S, any continuous map from S to X is homotopic to a holomorphic map from S to X. This notion was introduced by Gromov, inspired by work of Grauert and Oka. Let X be a connected complex projective manifold satisfying the h-principle. We show (among other things) that every holomorphic map from X to a Brody-hyperbolic space is constant. And also that X is 'special' (this is an algebro-geometric birational notion, opposite to being of general type, which implies that X has no dominant rational map to a positive dimensional variety of general type). The converse implication is very much open, even for surfaces. This is a joint work with J. Winkelmann.

Speaker: **William Cherry**
Title: *Non-Archimedean Analytic Curves in Algebraic Varieties*
Abstract: I will discuss degeneracy of non-Archimedean analytic maps from the affine line into quasi-projective varieties. This is a subject intermediate

to the themes of the workshop, namely rational and holomorphic curves. In my talk, I will mention older work about non-Archimedean maps to Abelian and semi-Abelian varieties, as well as some newer work on the degeneracy of non-Archimedean analytic maps omiting divisors with sufficiently many ample components.

Speaker: **Pietro Corvaja**
Title: *Torsion subvarieties*
Abstract: Given an algebraic family of abelian varieties, i.e. a map $A \to S$ whose fibers are abelian varieties, and a section $s : S \to A$, we study the subvarieties $X \subset S$ where s takes a torsion value; these are called torsion subvarieties. Among some new results, we prove the finiteness of torsion hypersurfaces, outside trivial cases. This is a joint work with D. Masser & U. Zannier

Speaker: **Gerd Dethloff**
Title: *Holomorphic Curves into Algebraic Varieties Intersecting Moving Hypersurface Targets*
Abstract: In 2009 Min Ru proved a Second Main Theorem for algebraically non-degenerate holomorphic curves in smooth complex projective varieties intersecting fixed hypersurface targets. In joint work with William Cherry and Tan Van Tran and by using a different proof method, we generalize this result to possibly singular varieties and to moving hypersurface targets.

Speaker: **Souvik Goswami**
Title: *A note on height paring*
Abstract: For a smooth projective variety X (of dimension d) defined over $\mathbb{Q}$, Beilinson (and independently Bloch) constructed a height pairing

$$CH^r_{hom}(X;\mathbb{Q}) \times CH^{d-r+1}_{hom}(X;\mathbb{Q}) \to \mathbb{R},$$

under very reasonable assumptions and with a number of conjectural properties. A folklore conjecture related to this pairing states that the Griffiths Abel-Jacobi map

$$\Phi_r : CH^r_{hom}(X;\mathbb{Q}) \to J^r(X) \otimes \mathbb{Q}$$

is injective (BBC). But if X is defined over a field of finite transcendence degree over $\mathbb{Q}$, then the injectivity of the Abel-Jacobi map doesn't hold any more. Instead we have the concept of a conjectural Bloch-Beilinson filtration,

a candidate for which was given by James Lewis. Under some assumptions, specially BBC, the main point of this talk would be to generalize the height pairing to the graded pieces of this candidate Bloch-Beilinson filtration using cohomological machinery

Speaker: **Gordon Heier**
Title: *On semi-definite holomorphic sectional curvature and rational connectedness*
Abstract: The main theorem presented in this talk, obtained jointly with B. Wong, is the following Conjecture of S.-T. Yau. Let X be a projective Kaehler manifold of positive holomorphic sectional curvature (HSC). Then X is rationally connected, i.e., any two points of X can be joined by a chain of rational curves. This theorem complements the well-known result of Campana and Kollar-Miyaoka-Mori which states that a Kaehler manifold of positive Ricci curvature is rationally connected.

Speaker: **Ljudmila Kamenova**
Title: *Kobayashi pseudometric on hyperkähler manifold and Kobayashi's conjecture*
Abstract: The Kobayashi pseudometric on a complex manifold M is the maximal pseudometric such that any holomorphic map from the Poincare disk to M is distance-decreasing. Kobayashi has conjectured that this pseudometric vanishes on Calabi-Yau manifolds. Using ergodicity of complex structures, we prove this conjecture for any hyperkähler manifold that admits a deformation with a Lagrangian fibration, and its Picard rank is not maximal. We shall discuss the proof of Kobayashi's conjecture for K3 surfaces and for certain hyperähler manifolds. These results are joint with S. Lu and M. Verbitsky.

Speaker: **Ryoichi Kobayashi**
Title: *Metrization of Osserman's theory on the Gauss map of minimal surfaces with finite total curvature*
Abstract: We propose a Nevanlinna Theory for the Gauss map of algebraic and pseudo-algebraic minimal surfaces whose basic domain M is a compact Riemann surface finitely many points removed and is uniformized by the disk $D = \{|z| < 1\}$. The point is to couple the Nevanlinna theory on D with the action of the fundamental group $\pi_1(M)$ on D. The first feature of the theory is to establish an effective version of the Lemma on Logarithmic

Derivative (LLD) for the lifted Gauss map on D, which is interpreted as a Nevanlinna theoretic analogue of the Cohn-Vossen inequality. The second one is to translate the period condition of algebraic minimal surfaces into certain relations in the Nevanlinna theory on D. As an application, we give the optimal estimate for the totally ramified value number of the lifted Gauss map of algebraic and pseudo-algebraic minimal surfaces. In particular, we prove that the Gauss map of any algebraic minimal surface can omit at most two values. (joint work with Reiko Miyaoka).

Speaker: **David McKinnon**
Title: *Rational curves and rational points*
Abstract: The existence of rational curves on varieties defined over a number field is closely tied to the existence of rational points on those same varieties. I will discuss some of these connections, including the implications of joint work with Mike Roth relating all this to Seshadri constants.

Speaker: **Atsushi Moriwaki**
Title: *Semiample invertible sheaves with semipositive continuous hermitian metrics*
Abstract: Let X be a proper algebraic variety over the complex number field. Let L be an invertible sheaf on X and h a continuous metric of L. S. Zhang conjectured that if L is ample and h is smooth and semipositive, then the pair (L, h) is semiample metrized. In this talk, I will talk on a generalization of the above conjecture. Actually I will discuss the following result: if L is semiample and h is semipositive, then (L, h) is semiample metrized. Moreover, if it is possible, I will talk on an adelic analogue on a Berkovich space.

Speaker: **Bruno de Oliveira**
Title: *Surfaces that are multi-isotropic.*
Abstract: I would talk about on going work with Fedor Bogomolov about the space of holomorphic 2-forms on the Albanese variety of X that pullback trivially to the variety X, with a special focus to the surface case. We describe potential dimensional bounds on the space of such 2-forms of rank higher than 2 (in the case no rank 2 are present in the space) and describe the geometrical significance.

Speaker: **Erwan Rousseau**
Title: *Curves in Hilbert modular varieties.*

Abstract: We study curves in Hilbert modular varieties from the point of view of the Green-Griffiths-Lang conjecture claiming that entire curves in complex projective varieties of general type should be contained in a proper subvariety. Using holomorphic foliations theory, we establish the Second Main Theorem in this context as well as a function field analogue of Vojta's conjecture. We also establish the strong Green-Griffiths-Lang conjecture for Hilbert modular varieties up to finitely many possible exceptions. (Joint work with F. Touzet)

Speaker: **Bernard Shiffman**
Title: *Value distribution of holomorphic sections on algebraic manifolds*
Abstract: We look at the divisors of powers of an ample line bundle on a projective algebraic manifold. How large can we expect these divisors to be inside a fixed domain? For large degree, the average volume of the divisor is approximately proportional to the volume of the domain times the degree of the divisor. We showed (in a 2008 joint work with Zelditch and Zrebiec) that the probability that the volume deviates from this average, and in particular the "hole probability" that the divisor misses the domain, decays at a faster than exponential rate as the degree increases. The methods involve bounds on the Nevanlinna proximity term and construction of almost orthonormal sections using the asymptotics of the Bergman kernel. These methods were recently used to construct uniformly bounded, L^2 normalized holomorphic sections of all degrees, and by Junyan Zhu to obtain precise decay rates for the hole probability for zero divisors of polynomials in a polydisk.

Speaker: **Roberto Svaldi**
Title: *Hyperbolicity for log pairs*
Abstract: A classical result in birational geometry, Mori's Cone Theorem, implies that if the canonical bundle of a variety X is not nef then X contains rational curves. This is the starting point of the so-called Minimal Model Program. In particular, hyperbolic varieties are positive from the point of view of birational geometry. Very much in the same vein, one could ask what happens for a quasi projective variety, Y. Using resolution of singularity, then one is lead to consider pairs (X, D) of a variety and a divisor, such that $Y = X \setminus D$. I will show how to obtain a theorem analogous to Mori's Cone Theorem in this context. Instead of rational complete curves, algebraic copies of the complex plane will male their appearance. I will also discuss an ampleness criterion for hyperbolic pairs.

Speaker: **Misha Verbitsky**
Title: *Morrison-Kawamata conjecture for hyperkähler manifolds*
Abstract: We prove that the group of holomorphic automorphisms of a hyperkähler manifold acts on the set of faces of its Kahler cone with finitely many orbits. This is joint work with Ekaterina Amerik.

Speaker: **Julie Wang**
Title: *On non-Archimedean curves omitting few components and their arithmetic analogues*
Abstract: Similar to the correspondence between classical Nevanlinna theory and Diophantine approximation, a correspondence between non-Archimedean Nevanlinna theory and certain Diophantine statements over the integers $\mathbb{Z}$ or the rational numbers $\mathbb{Q}$ were discussed by Ta Thi Hoai An, Aaron Levin and J. Wang in 2011. Recently, this correspondence was further investigated in a joint work with Aaron Levin and the following is the abstract. Let k be an algebraically closed field complete with respect to a non-Archimedean absolute value of arbitrary characteristic. Let $D_1, \ldots, D_n$ be effective nef divisors intersecting transversally in an n-dimensional nonsingular projective variety X. We study the degeneracy of non-Archimedean analytic maps from k into $X \setminus \cup_{i=1}^n D_i$ under various geometric conditions. When X is a rational ruled surface and D_1 and D_2 are ample, we obtain a necessary and sufficient condition such that there is no non-Archimedean analytic map from k into $X \setminus D_1 \cup D_2$. Using the dictionary between non-Archimedean Nevanlinna theory and Diophantine approximation, we also study arithmetic analogues of these problems, establishing results on integral points on these varieties over $\mathbb{Z}$ or the ring of integers of an imaginary quadratic field.

Speaker: **Jorg Winkelmann**
Title: *Rationality and Growth conditions*
Abstract: This is joint work with Frédéric Campana. Let X be a kähler compact complex manifold and let $f : \mathbb{C}^n \to X$ be a differentiably non-degenerate meromorphic map. Our goal is to relate algebraic-geometric properties of X to the existence of such maps of small growth. One easily sees that for every unirational projective manifold X there is a rational such map, hence in particular a map of very small growth. We look for a result in the opposite direction. The main result is:

If there exists such a map of order $\rho_f < 2$, then X must be rationally connected. In particular, X is projective.

Here the order ρ_f is a tool to measure the growth of a meromorphic map $f : \mathbb{C}^n \to X$. For an algebraic map we have necessarily $\rho_f = 0$. On the other hand $\rho_\tau = 2$ for the universal covering map $\tau : \mathbb{C}^n \to T$ of a n-dimensional compact complex torus.

The order is defined in the following way: Let ω be a Kähler form on X and let α be the euclidean Kähler form on $\mathbb{C}^n$, i.e., $\alpha = dd^c||z||^2$. Define the characteristic function for $f : \mathbb{C}^n \to X$ as

$$T_f(r) = \int^r \frac{dt}{t^{2n-1}} \int_{B_t} (f^*\omega) \wedge \alpha^{n-1}.$$

Then the order is defined as

$$\limsup_{r\to\infty} \frac{\log T_f(r)}{\log r}.$$

A compact complex manifold is called "rationally connected" (RC) if for any two points there exists a chain of rational curves linking these two points. RC Kähler manifolds are automatically projective.

Our result improves on earlier work of Campana, Păun, Noguchi and Winkelmann.

The result does not hold without the Kähler assumption, in fact as shown by Noguchi and Winkelmann there is non-degenerate map $f : \mathbb{C}^2 \to S$ for a Hopf surface S with $\rho_f = 1$ although Hopf surfaces do not contain any rational curves.