Compressible quantum matter: connecting field theories and holography

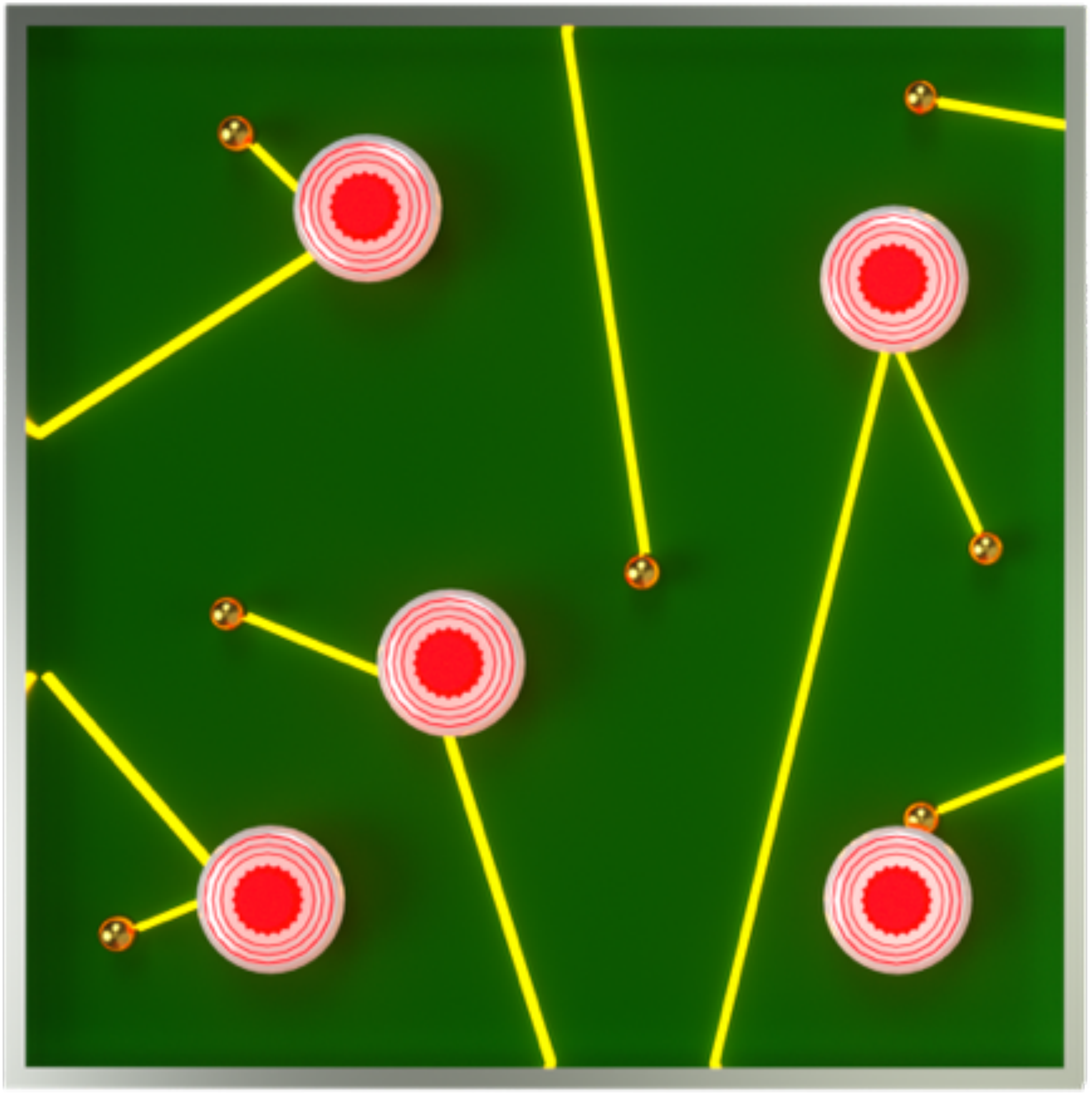
Gauge/Gravity Duality and Condensed Matter Physics
Banff International Research Station
February 29, 2016

Subir Sachdev

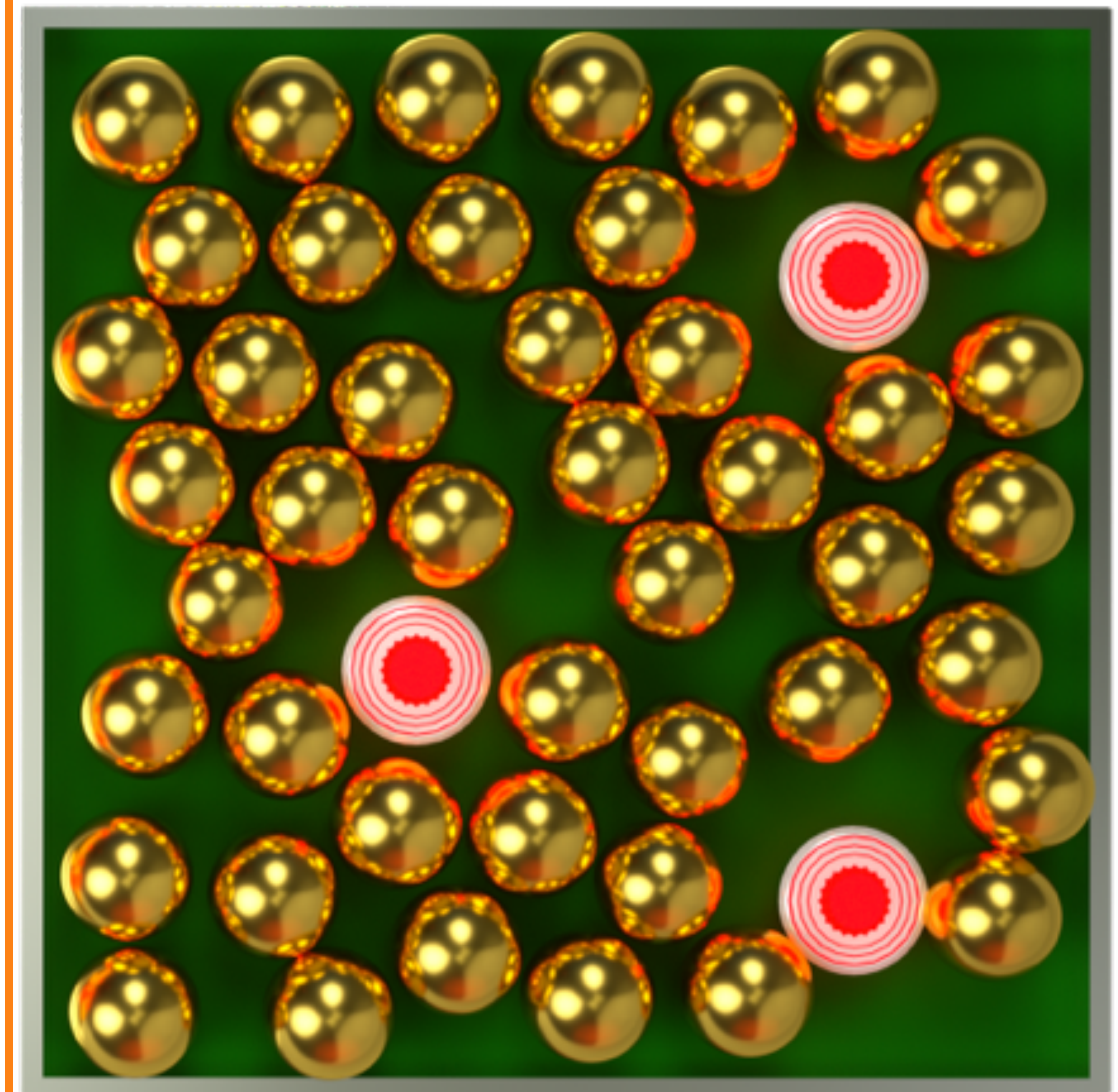
Talk online: sachdev.physics.harvard.edu

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

PHYSICS
HARVARD

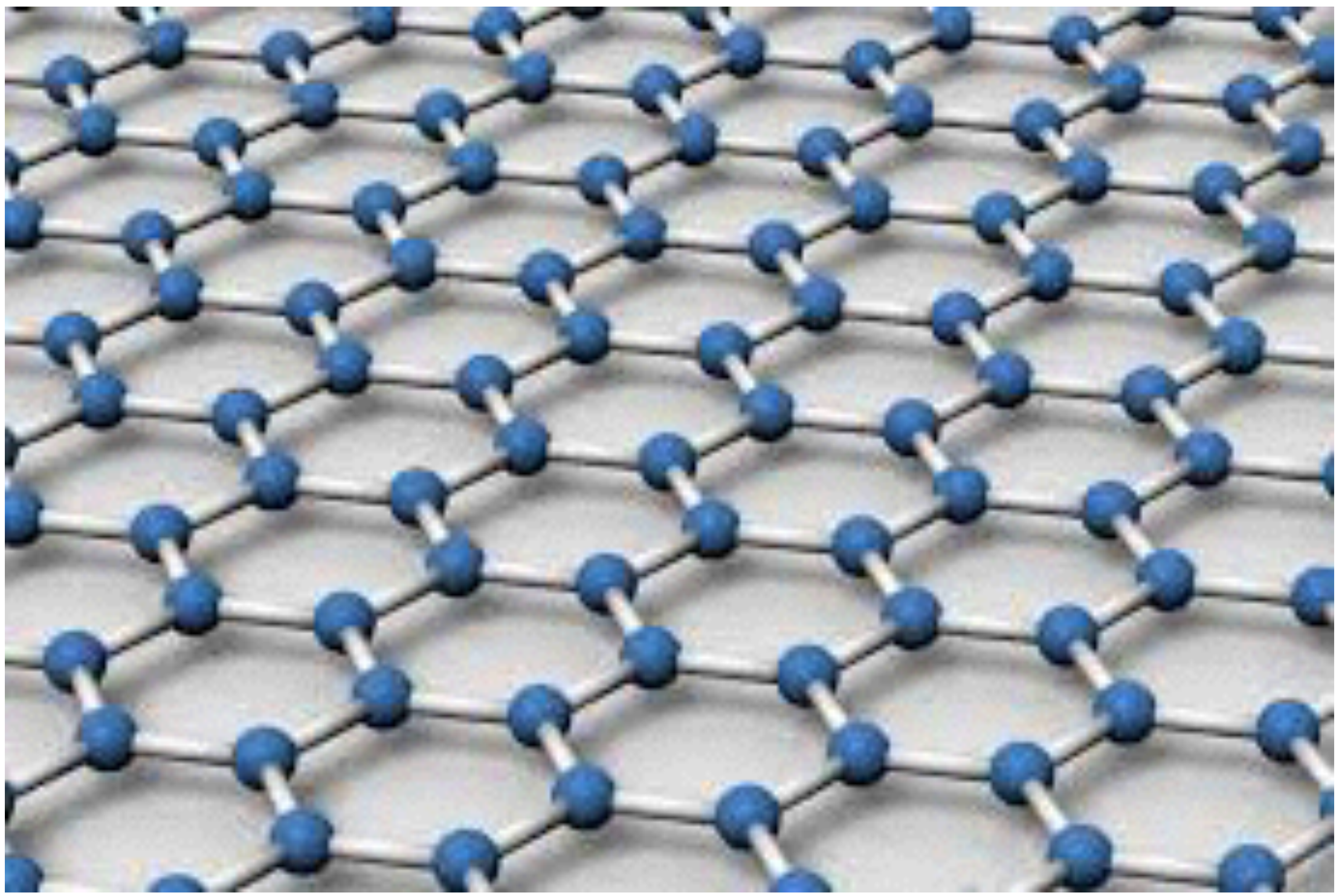


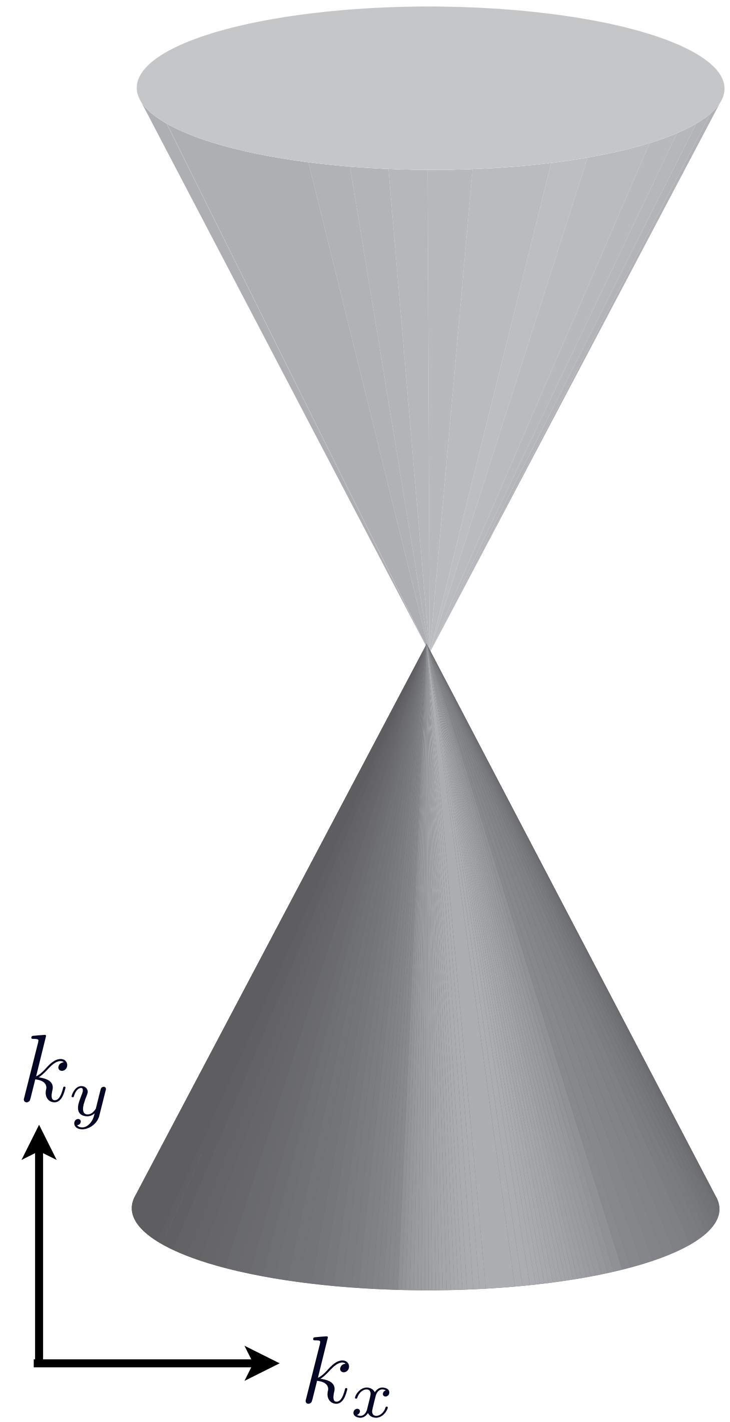
Fermi liquids: quasiparticles moving ballistically between impurity (red circles) scattering events

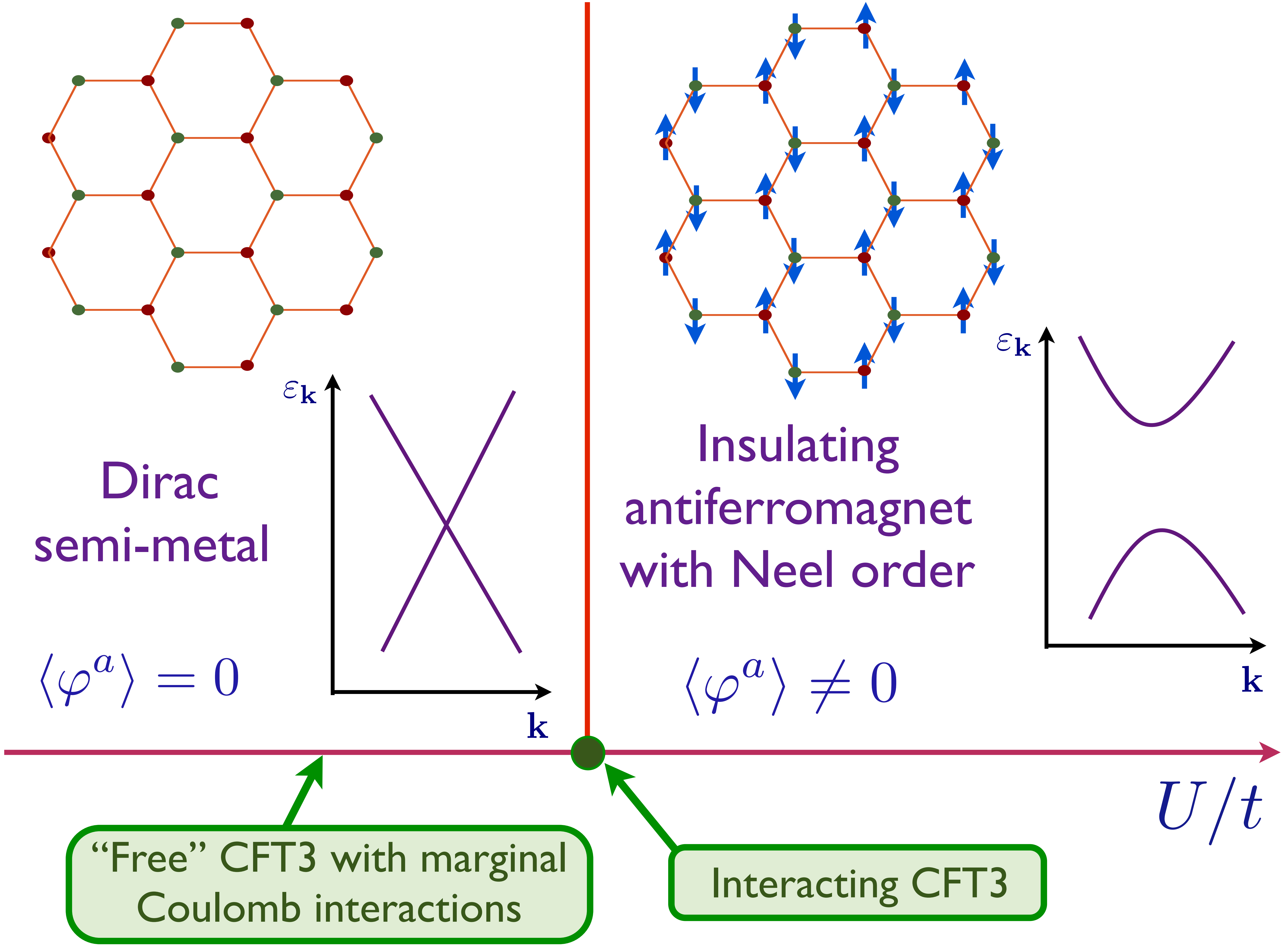


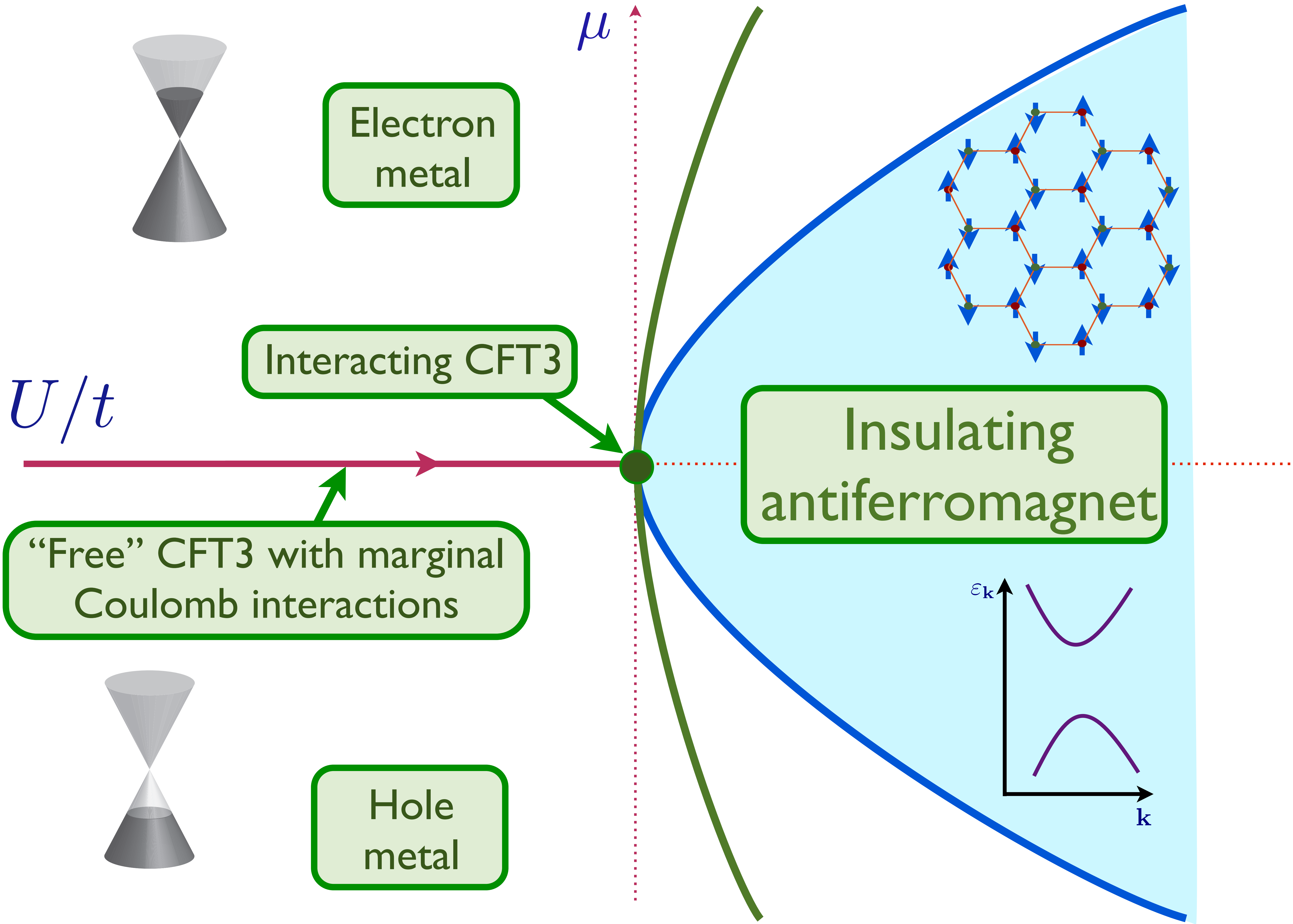
Strange metals: electrons scatter frequently off each other, so there is no regime of ballistic quasiparticle motion. The electron "liquid" then "flows" around impurities

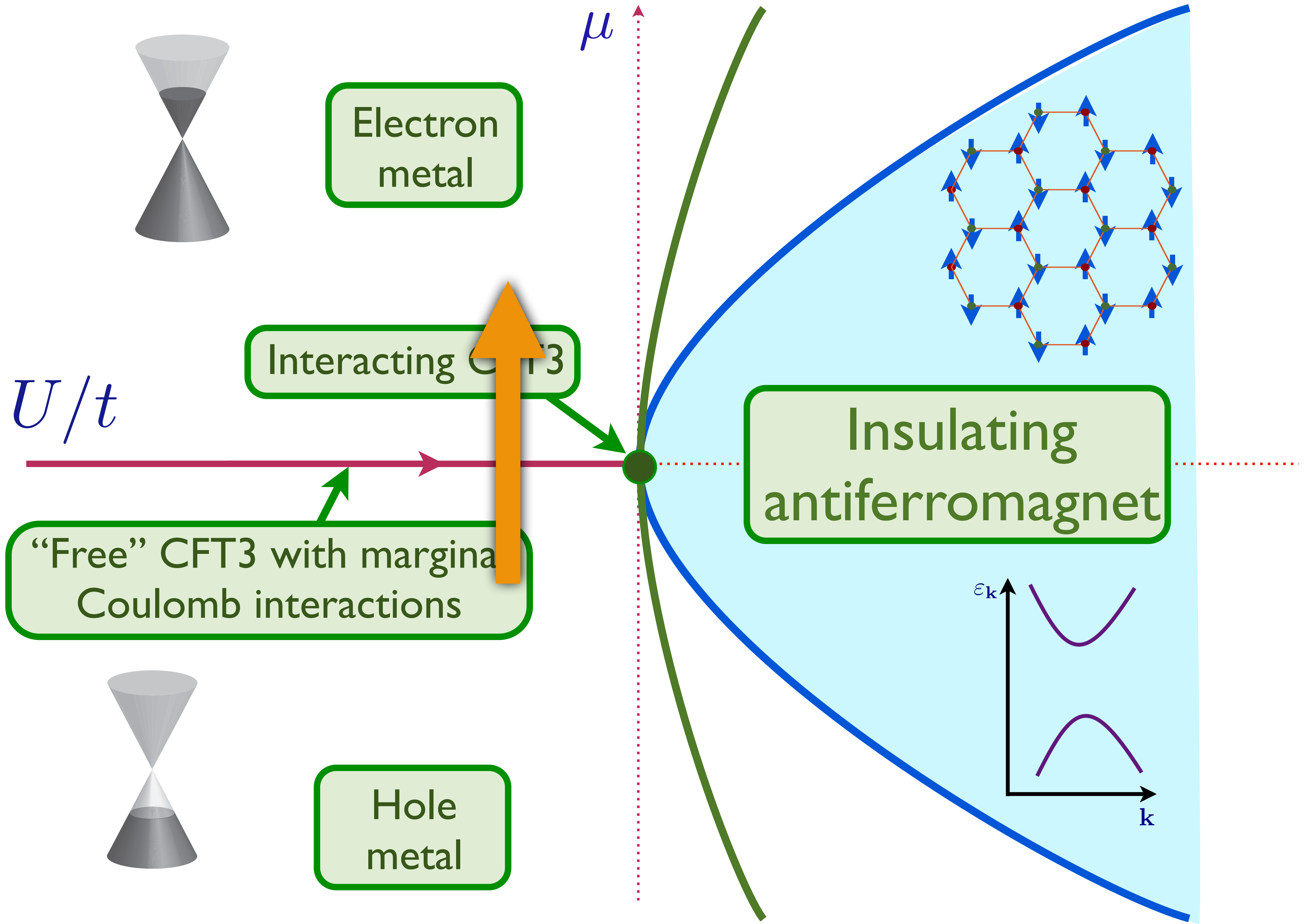
Graphene






$\varepsilon_{\mathbf{k}}$
$\mathbf{k}$
Dirac semi-metal
$\langle \varphi^a \rangle = 0$
Insulating antiferromagnet with Neel order
$\varepsilon_{\mathbf{k}}$
$\mathbf{k}$
$\langle \varphi^a \rangle \neq 0$
U/t
"Free" CFT3 with marginal Coulomb interactions
Interacting CFT3


μ
Electron metal
Interacting CFT3
U/t
"Free" CFT3 with marginal Coulomb interactions
Insulating antiferromagnet
$\varepsilon_{\mathbf{k}}$
$\mathbf{k}$
Hole metal



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"Free" CFT3 with marginal Coulomb interactions
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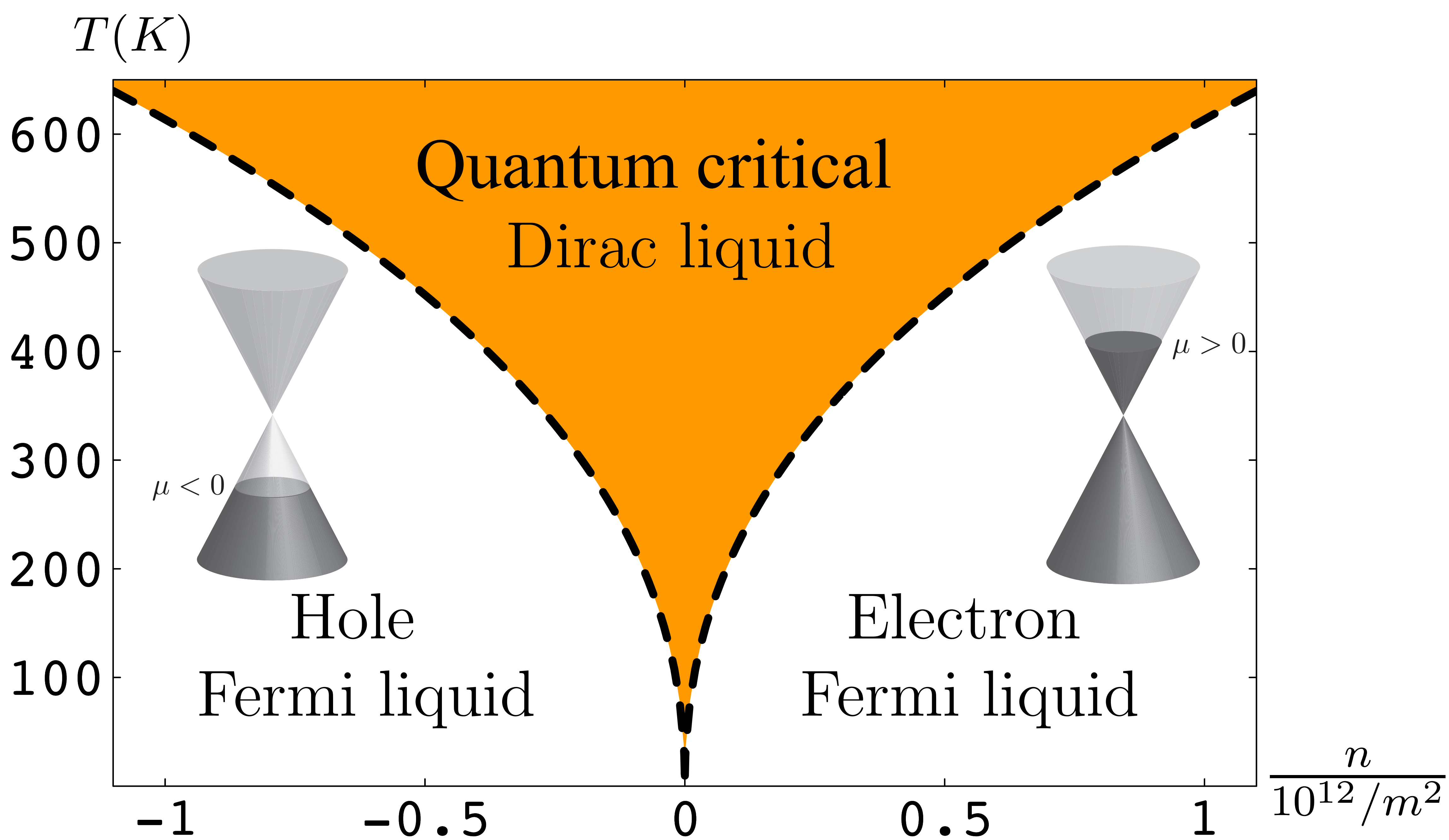
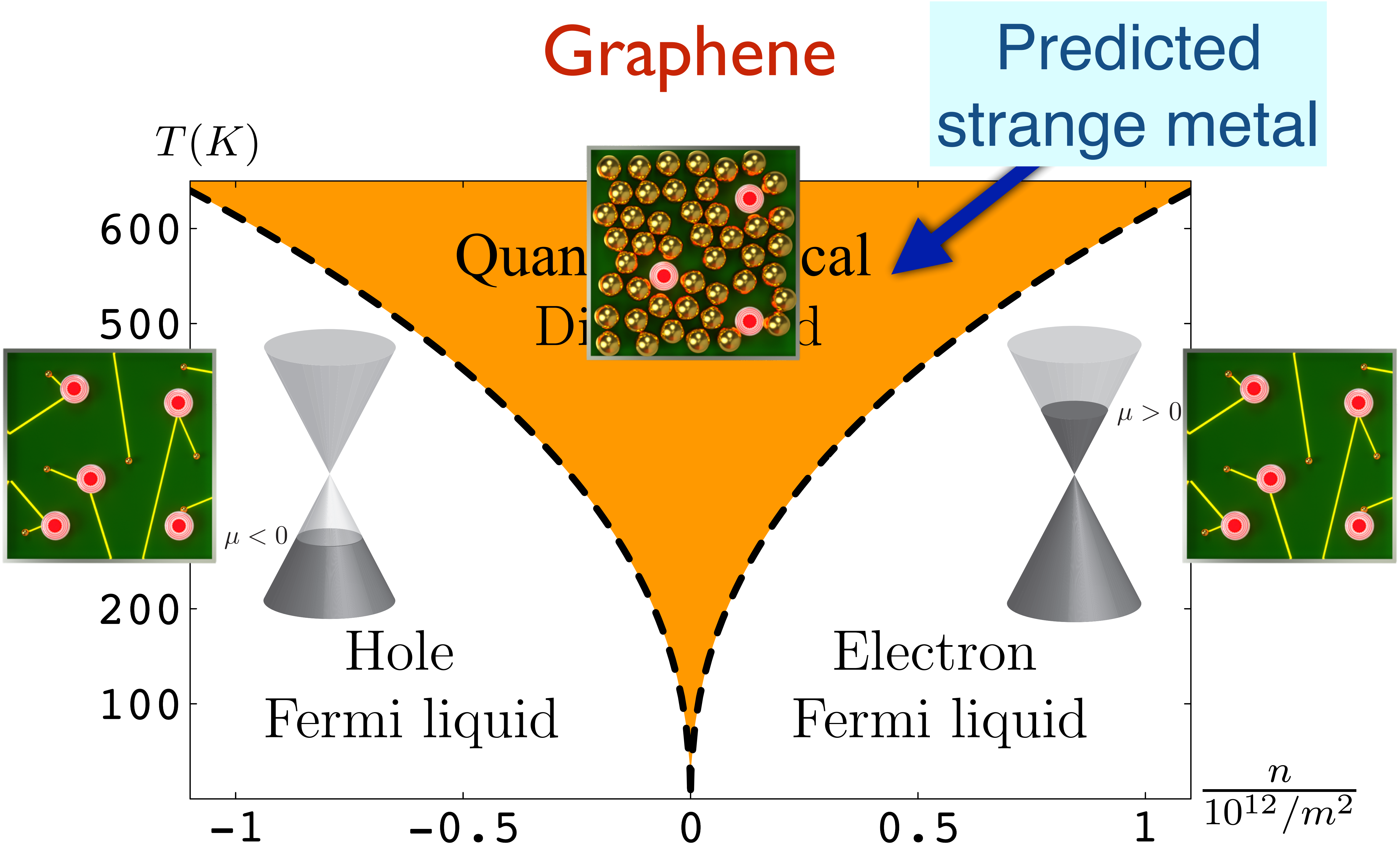
$\varepsilon_{\mathbf{k}}$
$\mathbf{k}$
Hole metal

Graphene



D. E. Sheehy and J. Schmalian, PRL **99**, 226803 (2007)
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

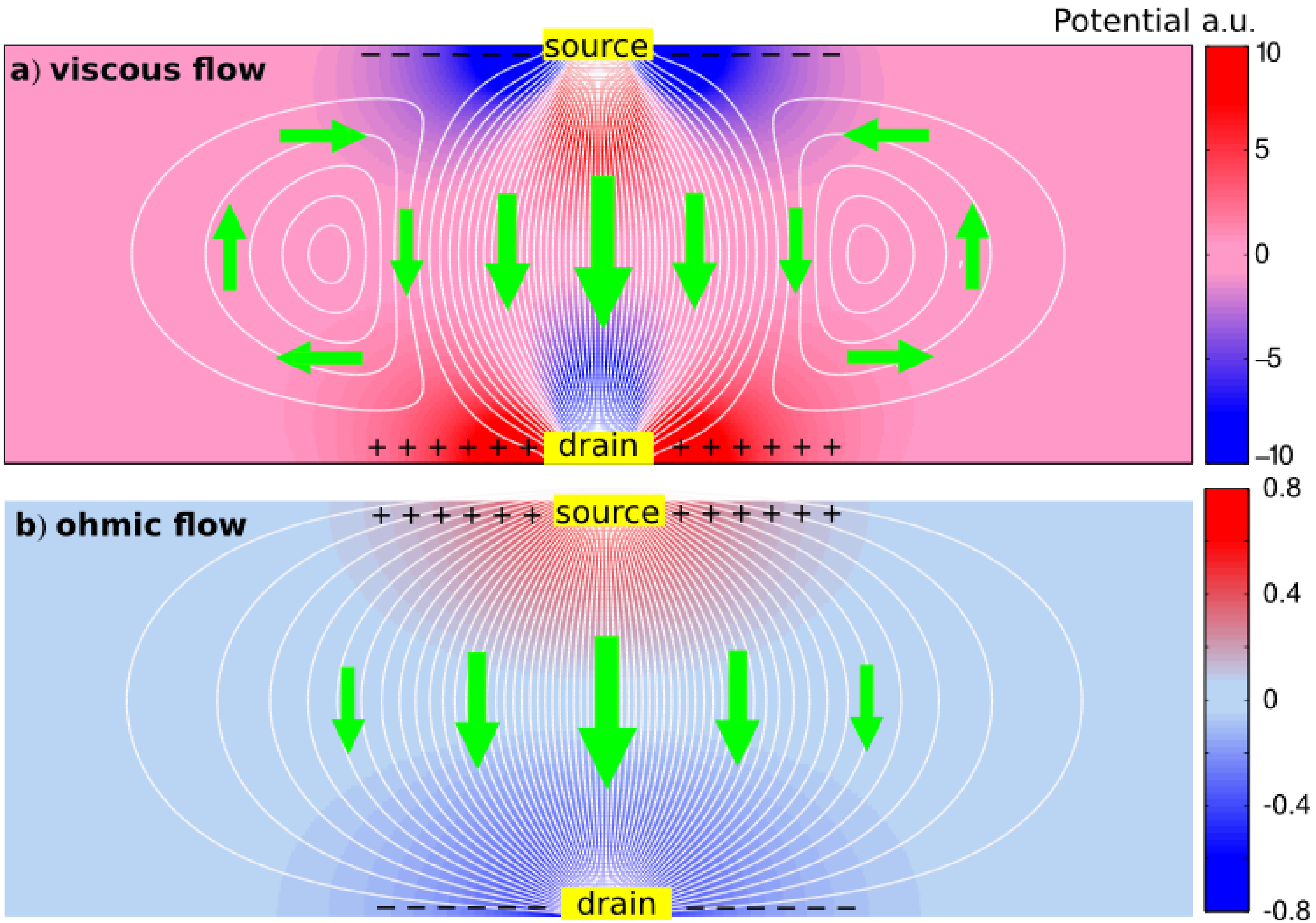
Graphene

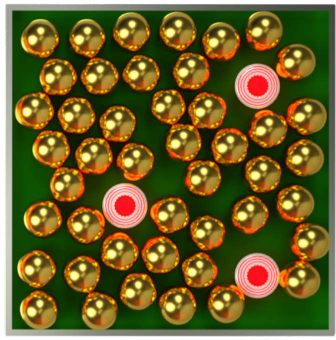


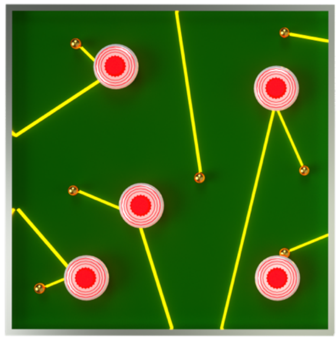
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Strange metal in graphene

Negative local resistance due to viscous electron backflow in graphene







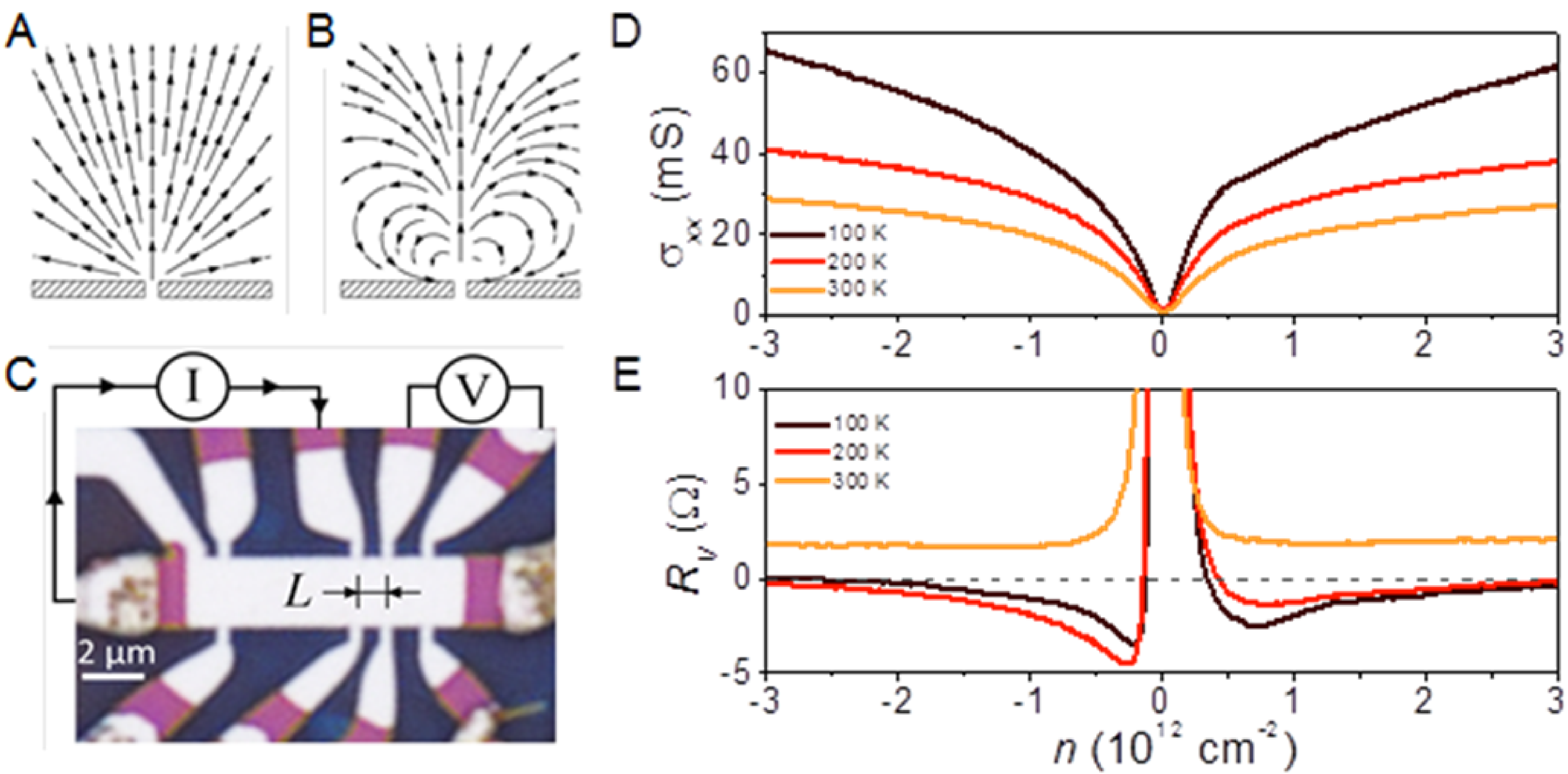
L. Levitov and G. Falkovich, arXiv:1508.00836, *Nature Physics online*

Strange metal in graphene

arXiv:1509.04165
Science, to appear

Negative local resistance due to viscous electron backflow in graphene

D. A. Bandurin[1], I. Torre[2,3], R. Krishna Kumar[1,4], M. Ben Shalom[1,5], A. Tomadin[6], A. Principi[7], G. H. Auton[5], E. Khestanova[1,5], K. S. Novoselov[5], I. V. Grigorieva[1], L. A. Ponomarenko[1,4], A. K. Geim[1], M. Polini[3,6]



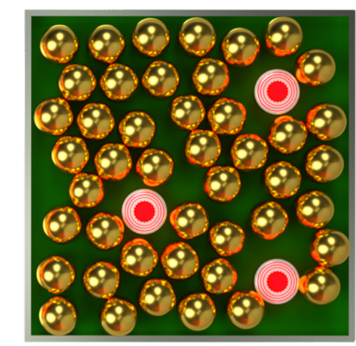
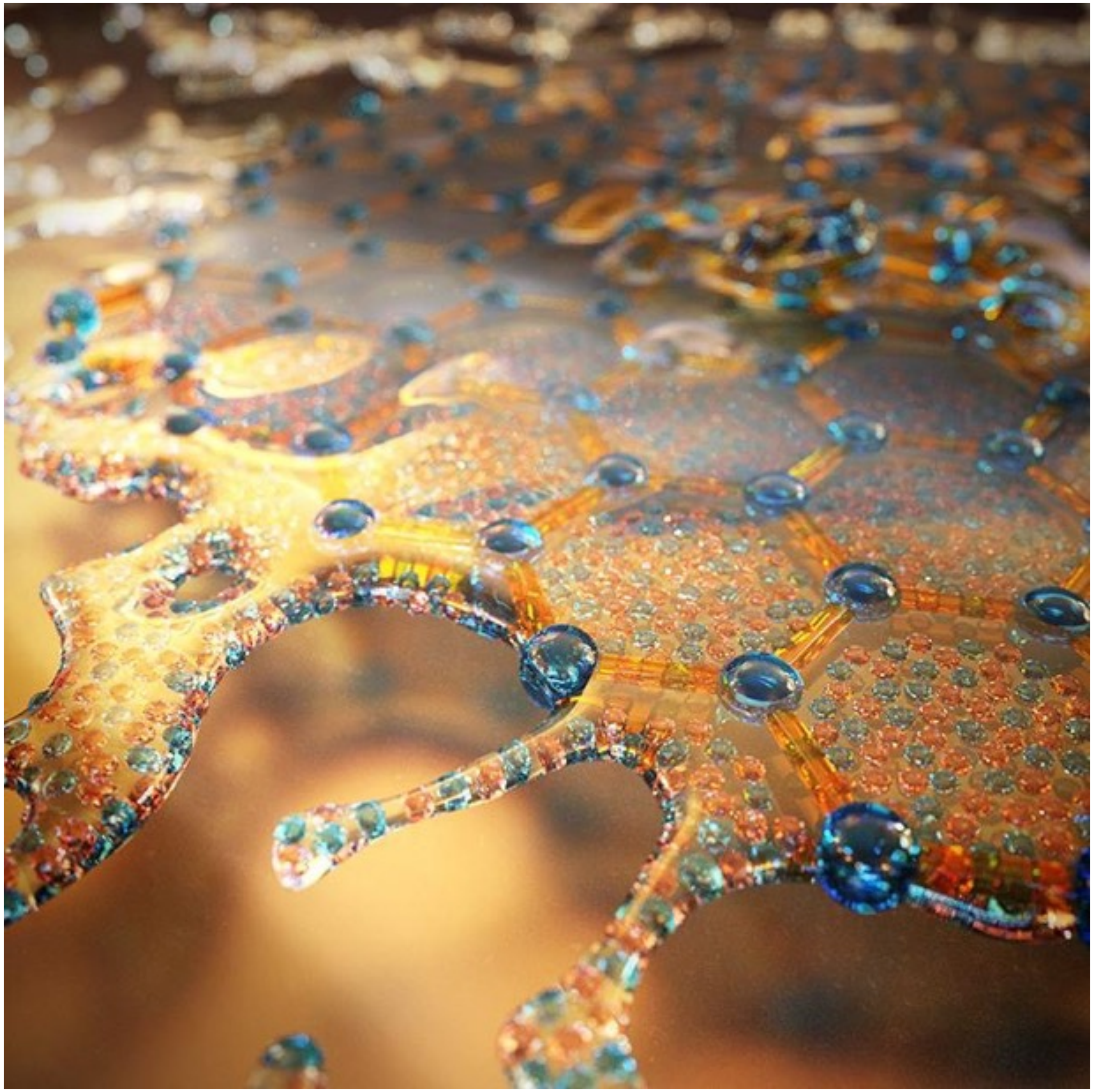


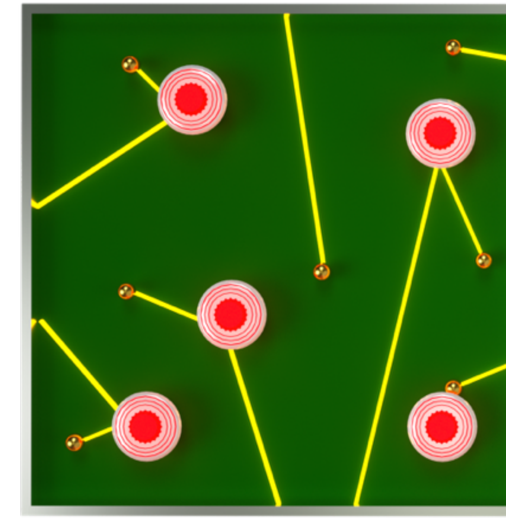
Figure 1. Viscous backflow in doped graphene. (**a,b**) Steady-state distribution of current injected through a narrow slit for a classical conducting medium with zero ν (a) and a viscous Fermi liquid (b). (**c**) Optical micrograph of one of our SLG devices. The schematic explains the measurement geometry for vicinity resistance. (**d,e**) Longitudinal conductivity σ_{xx} and R_V for this device as a function of n induced by applying gate voltage. $I = 0.3\ \mu A$; $L = 1\ \mu m$. For more detail, see Supplementary Information.

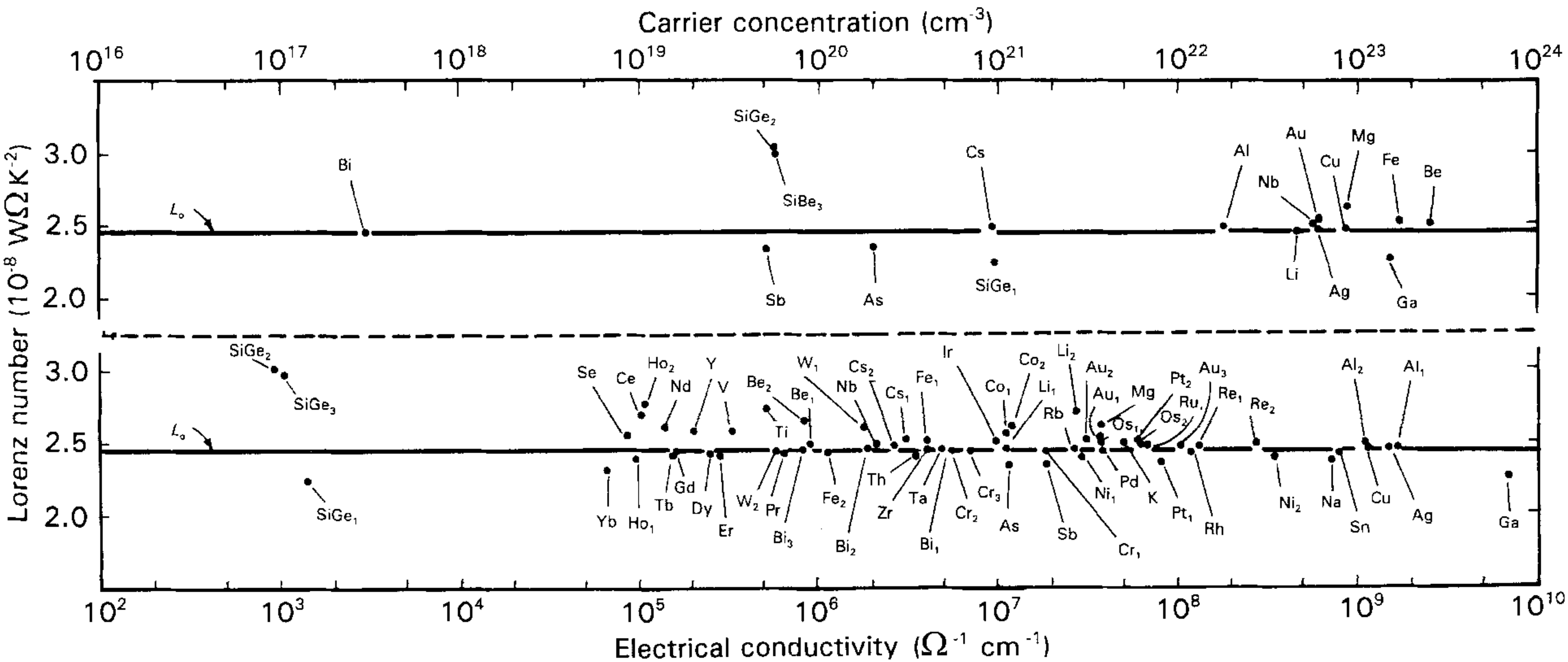
Graphene: "a metal that behaves like water"



- Wiedemann-Franz law in a Fermi liquid:

$$\frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8} \,\frac{\mathrm{W}\cdot\Omega}{\mathrm{K}^2}.$$





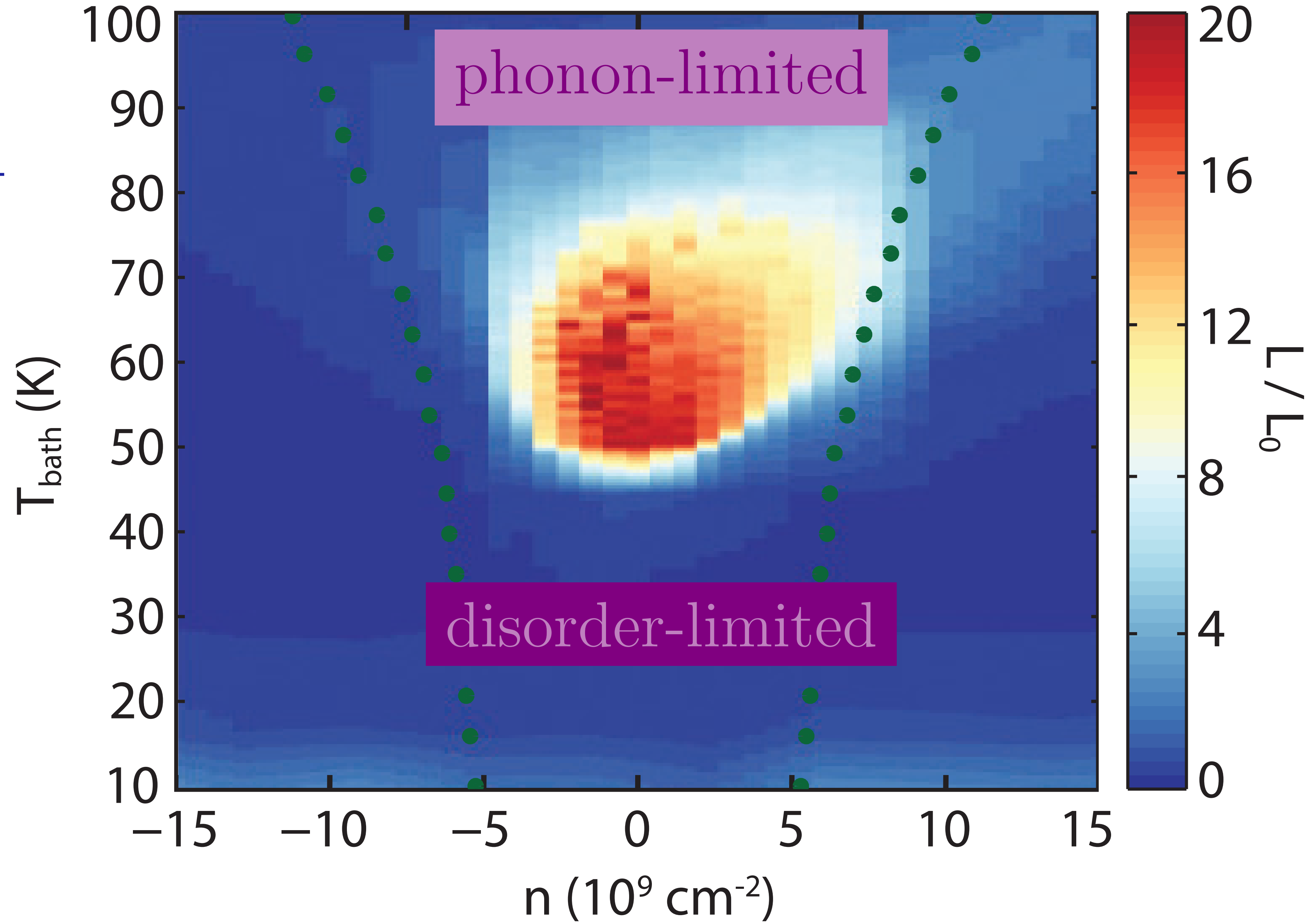
G. S. Kumar, G. Prasad, and R.O. Pohl, J. Mat. Sci. **28**, 4261 (1993)

J. Crossno, Jing K. Shi, Ke Wang, Xiaomeng Liu, A. Harzheim, A. Lucas, S. Sachdev, Philip Kim, Takashi Taniguchi, Kenji Watanabe, T. A. Ohki, and Kin Chung Fong arXiv:1509.04713; Science online

Strange metal in graphene

$$L = \frac{\kappa}{T\sigma}$$

$$L_0 = \frac{\pi^2 k_B^2}{3e^2}$$



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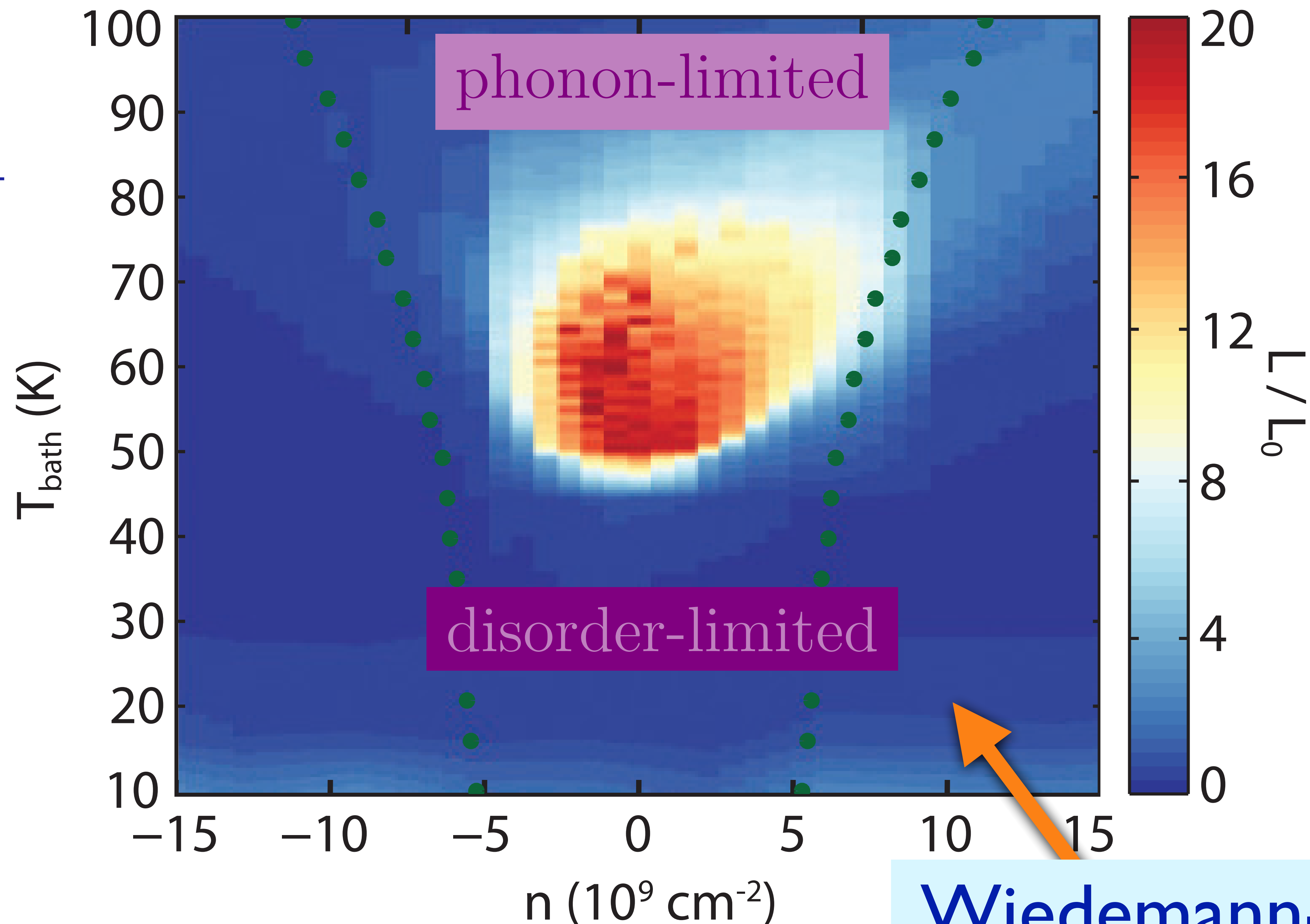
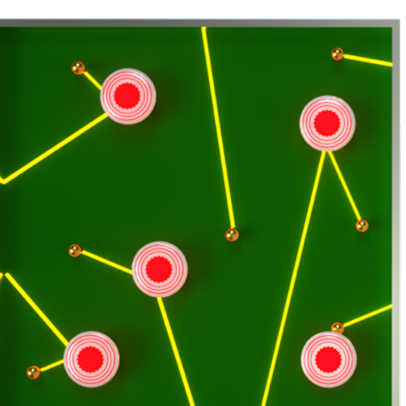


Wiedemann-Franz obeyed



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Strange metal in graphene

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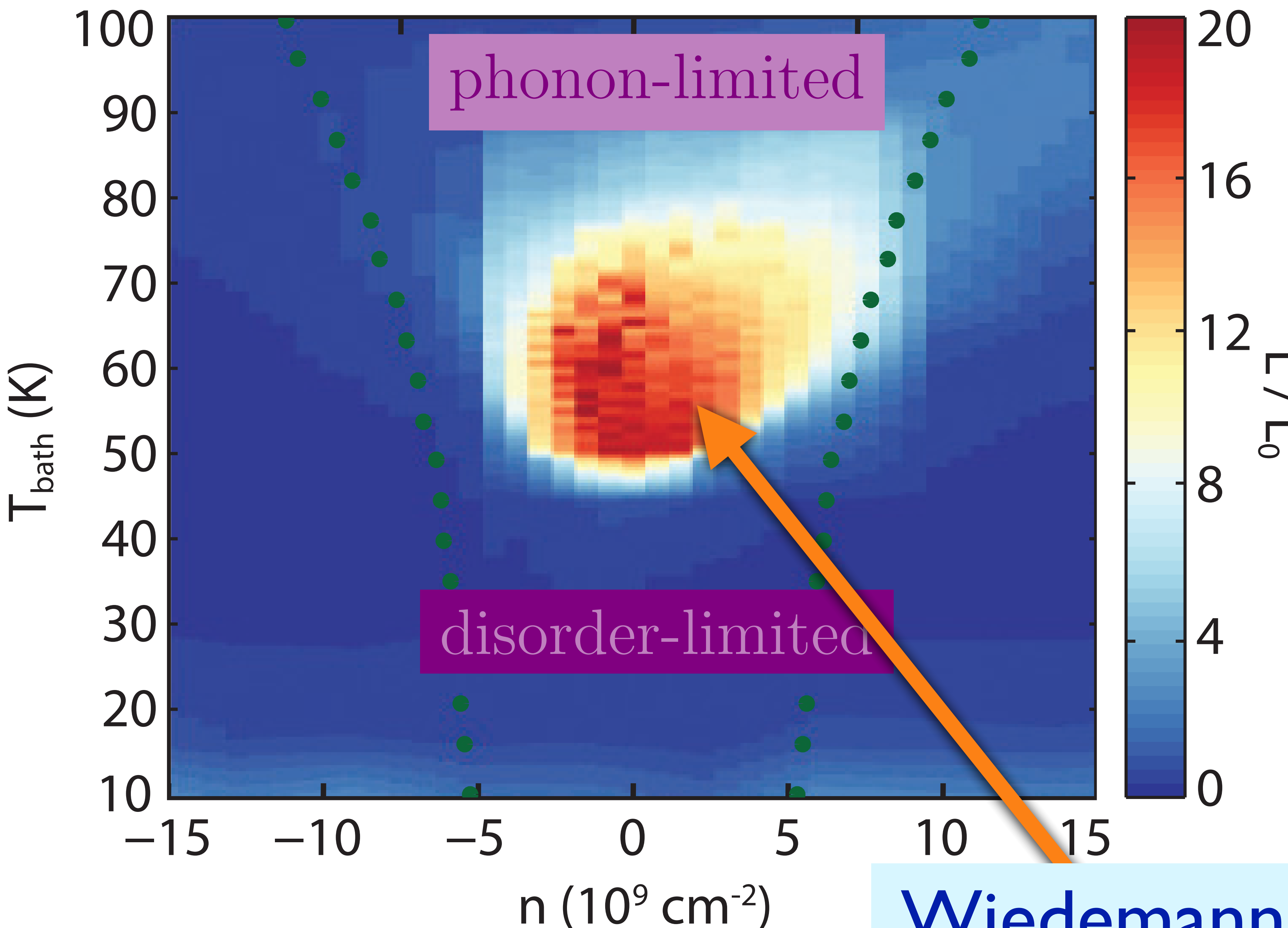
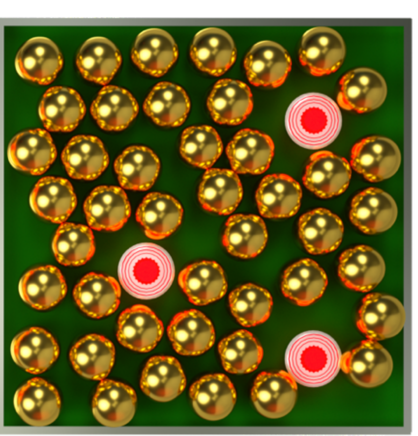
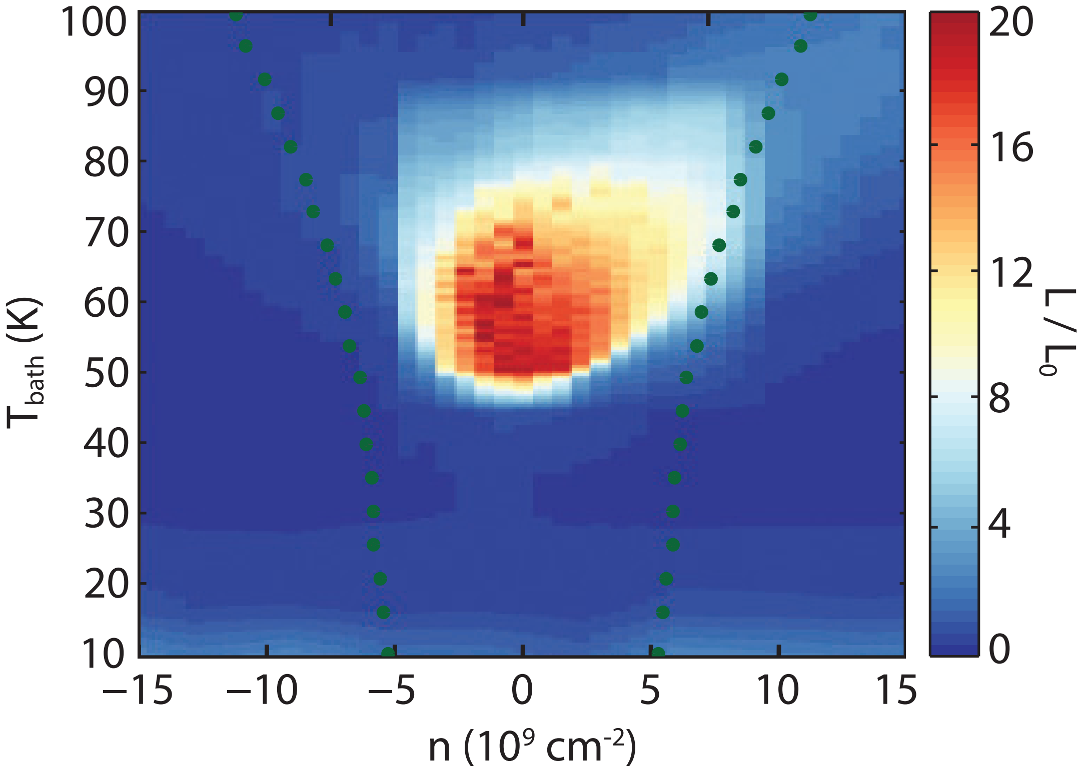
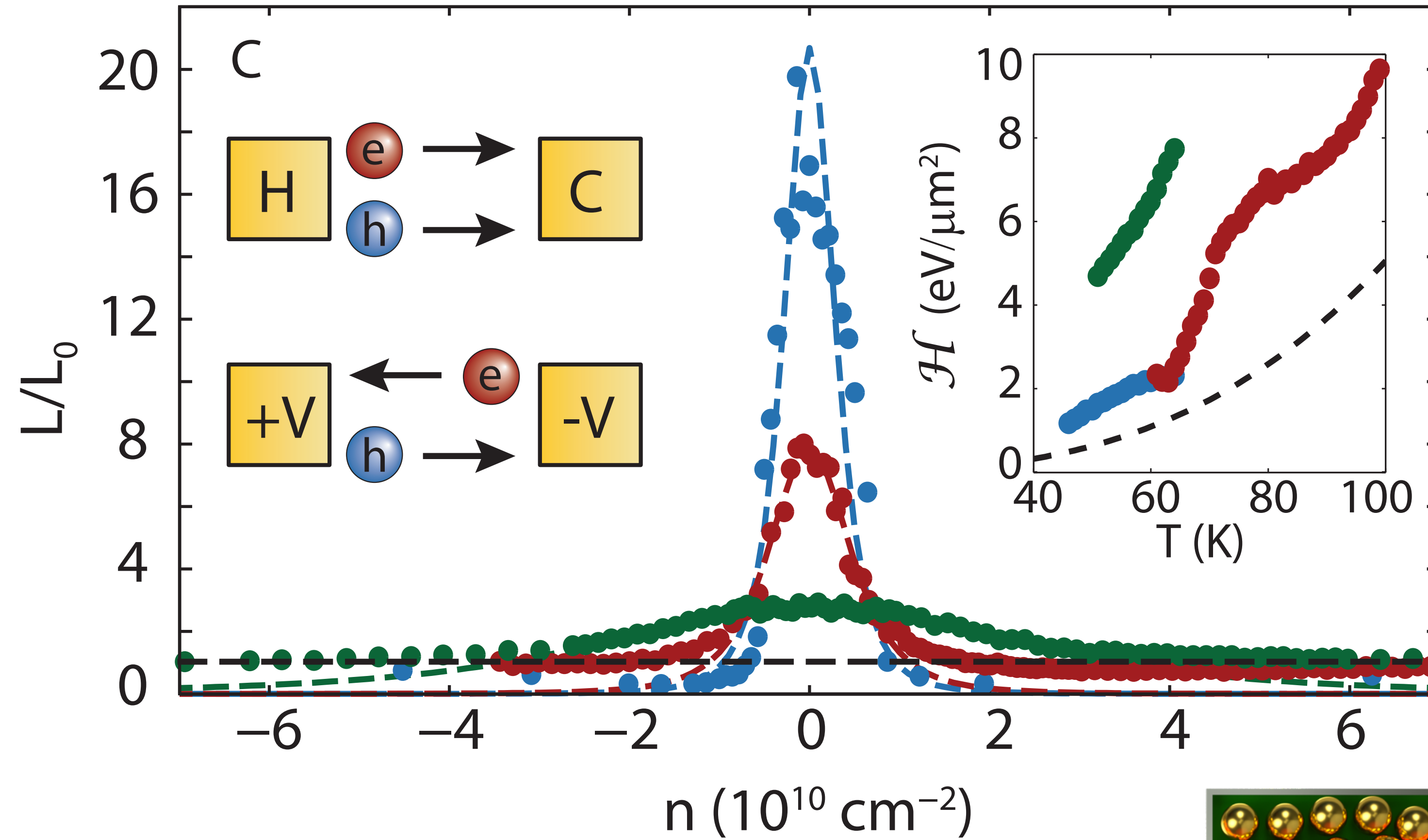
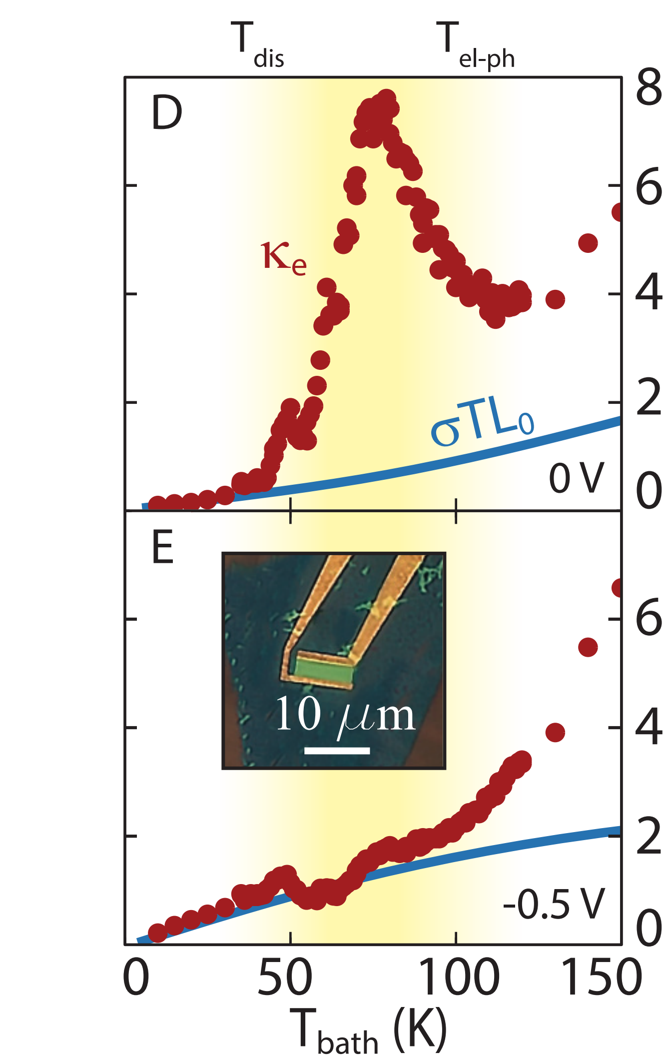


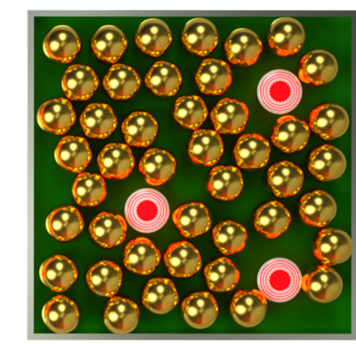
Wiedemann-Franz violated !











Lorentz ratio $L = \kappa/(T\sigma)$

$$= \frac{v_F^2 \mathcal{H} \tau_{\rm imp}}{T^2 \sigma_Q} \frac{1}{\left(1 + e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}/(\mathcal{H}\sigma_Q)\right)^2}$$

$\mathcal{Q} \rightarrow$ electron density; $\mathcal{H} \rightarrow$ enthalpy density

$\sigma_Q \rightarrow$ quantum critical conductivity

$\tau_{\rm imp} \rightarrow$ momentum relaxation time from impurities

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

J. Crossno *et al.* arXiv:1509.04713; Science online

Quantum matter without quasiparticles

1. SYK model and AdS_2 metals

2. Quantum-critical metals with translational invariance

3. Breaking translational invariance

Quantum matter without quasiparticles

1. SYK model and AdS_2 metals
2. Quantum-critical metals with translational invariance
3. Breaking translational invariance

Infinite-range model of a strange metal

$$H = \frac{1}{(NM)^{1/2}} \sum_{i,j=1}^{N} \sum_{\alpha,\beta=1}^{M} J_{ij} c_{i\alpha}^{\dagger} c_{i\beta} c_{j\beta}^{\dagger} c_{j\alpha}$$

$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0 \quad , \quad c_{i\alpha} c_{j\beta}^{\dagger} + c_{j\beta}^{\dagger} c_{i\alpha} = \delta_{ij} \delta_{\alpha\beta}$$

$$\frac{1}{M} \sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} = \mathcal{Q}$$

J_{ij} are independent random variables with $\overline{J_{ij}} = 0$ and $\overline{J_{ij}^2} = J^2$
$N \to \infty$ at $M = 2$ yields spin-glass ground state.
$N \to \infty$ and then $M \to \infty$ yields critical strange metal

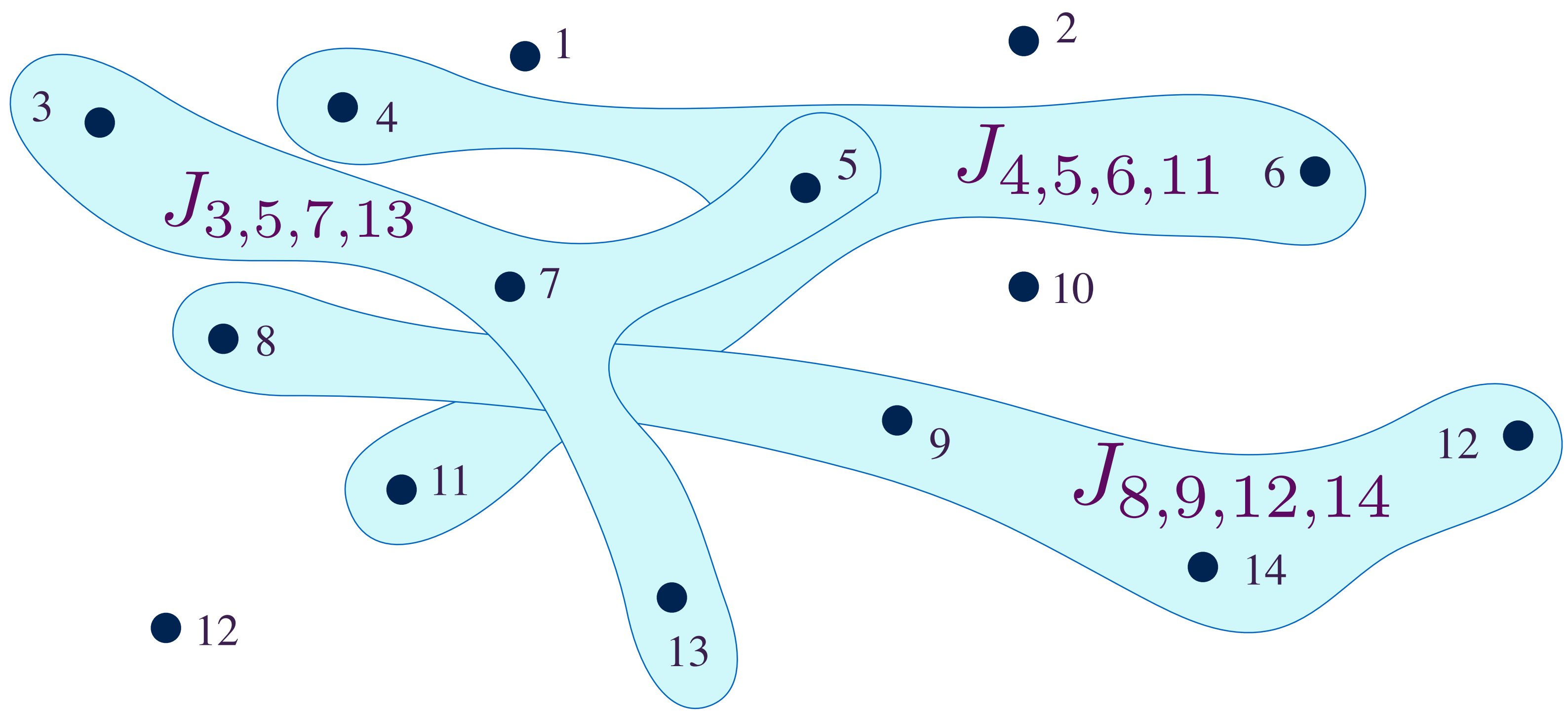
S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

Infinite-range model of a strange metal

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$



$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$

$N \to \infty$ yields same critical strange metal; simpler to study numerically

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

Infinite-range strange metals

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex A.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

Infinite-range strange metals

Local fermion density of states

$$\rho(\omega) = -\mathrm{Im}\, G(\omega) \sim \begin{cases} \omega^{-1/2} & , \omega > 0 \\ e^{-2\pi\mathcal{E}} |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

$\mathcal{E}$ encodes the particle-hole asymmetry

While $\mathcal{E}$ determines the *low* energy spectrum, it is determined by the *total* fermion density $\mathcal{Q}$:

$$\mathcal{Q} = \frac{1}{4}(3 - \tanh(2\pi\mathcal{E})) - \frac{1}{\pi}\tan^{-1}\left(e^{2\pi\mathcal{E}}\right).$$

Analog of the relationship between $\mathcal{Q}$ and k_F in a Fermi liquid.

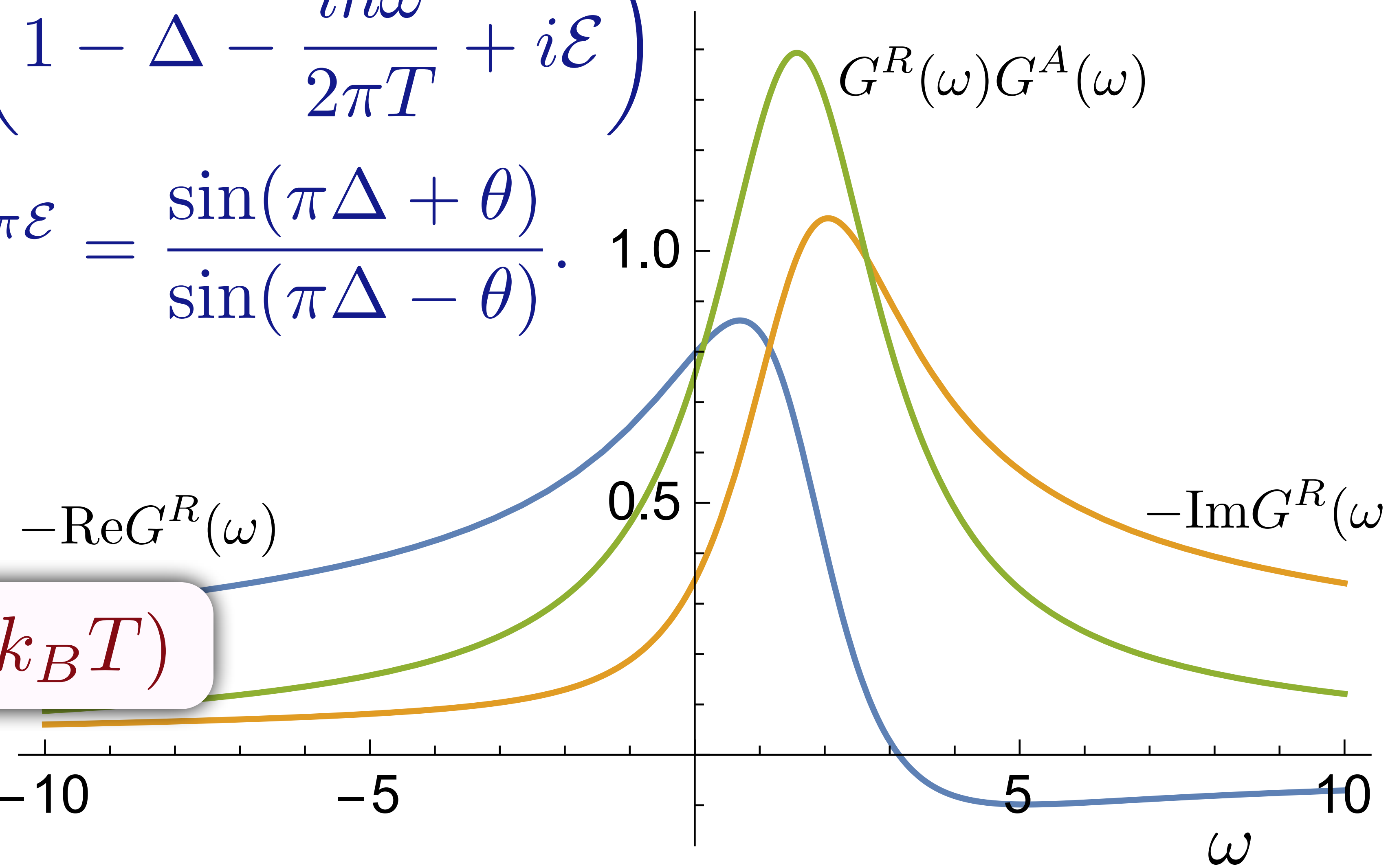
S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

Infinite-range strange metals

At non-zero temperature, T, the Green's function also fully determined by $\mathcal{E}$.

$$G^R(\omega) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma\left(\Delta - \dfrac{i\hbar\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(1 - \Delta - \dfrac{i\hbar\omega}{2\pi T} + i\mathcal{E}\right)}$$

where $\Delta = 1/4$ and $\quad e^{2\pi\mathcal{E}} = \dfrac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}$.



Note $G(\omega) \equiv f(\hbar\omega/k_B T)$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)
A. Georges and O. Parcollet PRB **59**, 5341 (1999)
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

Infinite-range strange metals

The entropy per site, $\mathcal{S}$, has a non-zero limit as $T \to 0$, and can be viewed as each site acquiring the universal boundary entropy of the multichannel Kondo problem.

N. Andrei and C. Destri, PRL **52**, 364 (1984).
A. M. Tsvelick, J. Phys. C **18**, 159 (1985).
I. Affleck and A. W. W. Ludwig, PRL **67**, 161 (1991).
S. Sachdev, C. Buragohain, and M. Vojta, Science **286**, 2479 (1999).

This entropy obeys

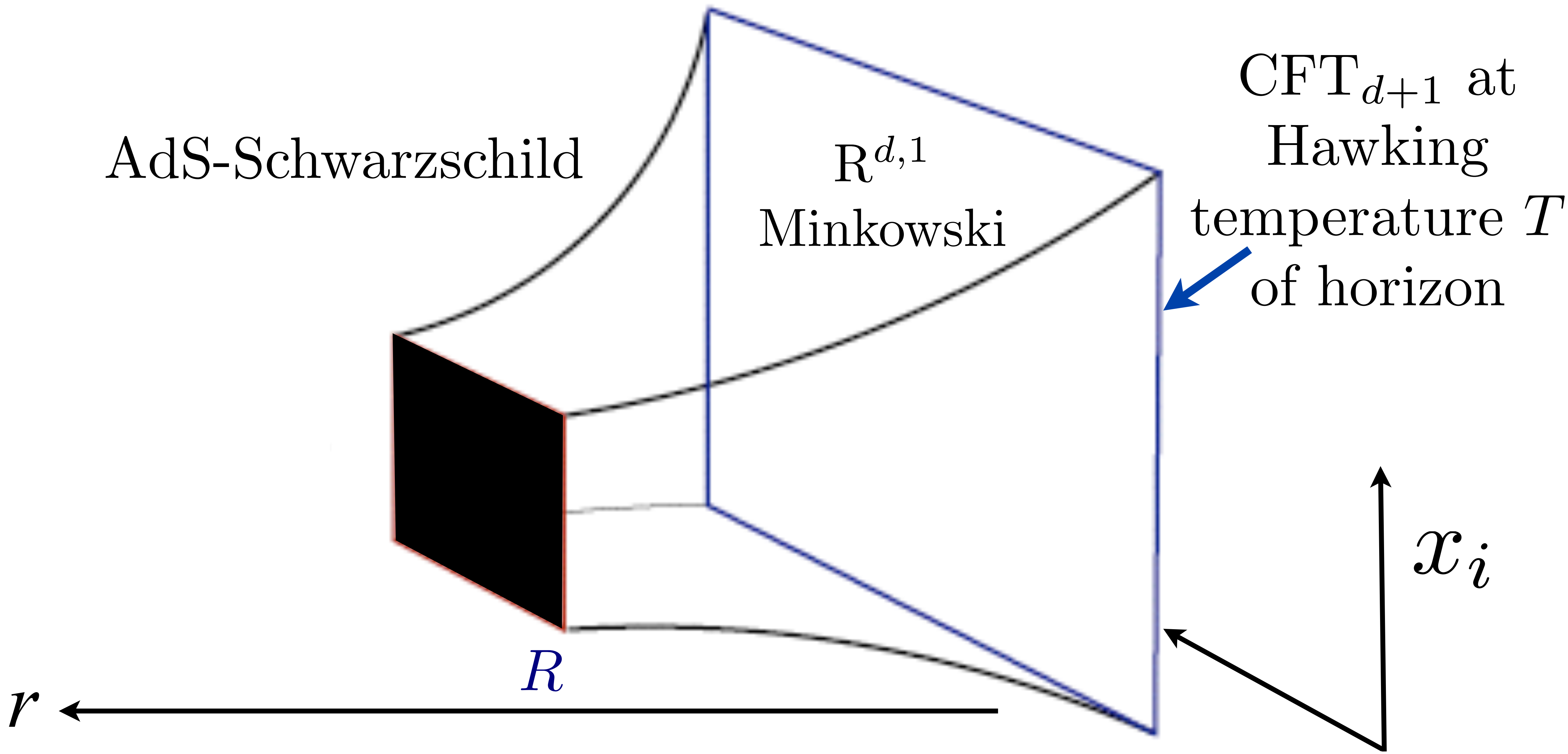
$$\left(\frac{\partial \mathcal{S}}{\partial \mathcal{Q}}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = 2\pi\mathcal{E}$$

Note that $\mathcal{S}$ and $\mathcal{E}$ involve low-lying states, while $\mathcal{Q}$ depends upon *all* states, and details of the UV structure

O. Parcollet, A. Georges, G. Kotliar, and A. Sengupta Phys. Rev. B **58**, 3794 (1998)
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

AdS/CFT correspondence at non-zero temperature

Einstein gravity $\mathcal{S}_E = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2}\right)\right]$

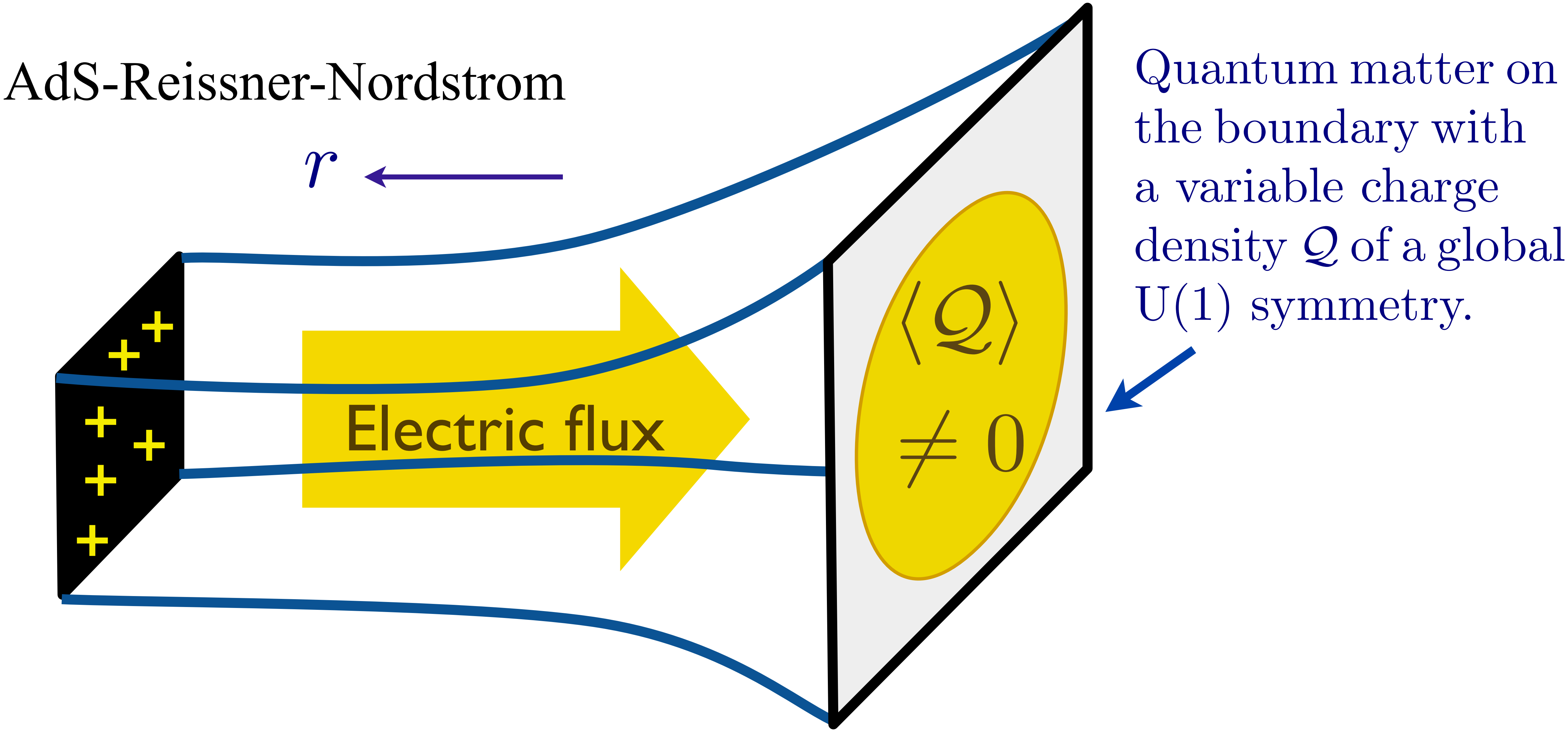


For SU(N) SYM in $d = 3$, $\mathcal{S}_{\mathrm{BH}} = (\pi^2/2)N^2T^3$. But there is (still) no confirmation of this from a field-theory computation on SYM.

S. S. Gubser, I. R. Klebanov, A. W. Peet PRD **54**, 3915 (1996)

Charged black branes

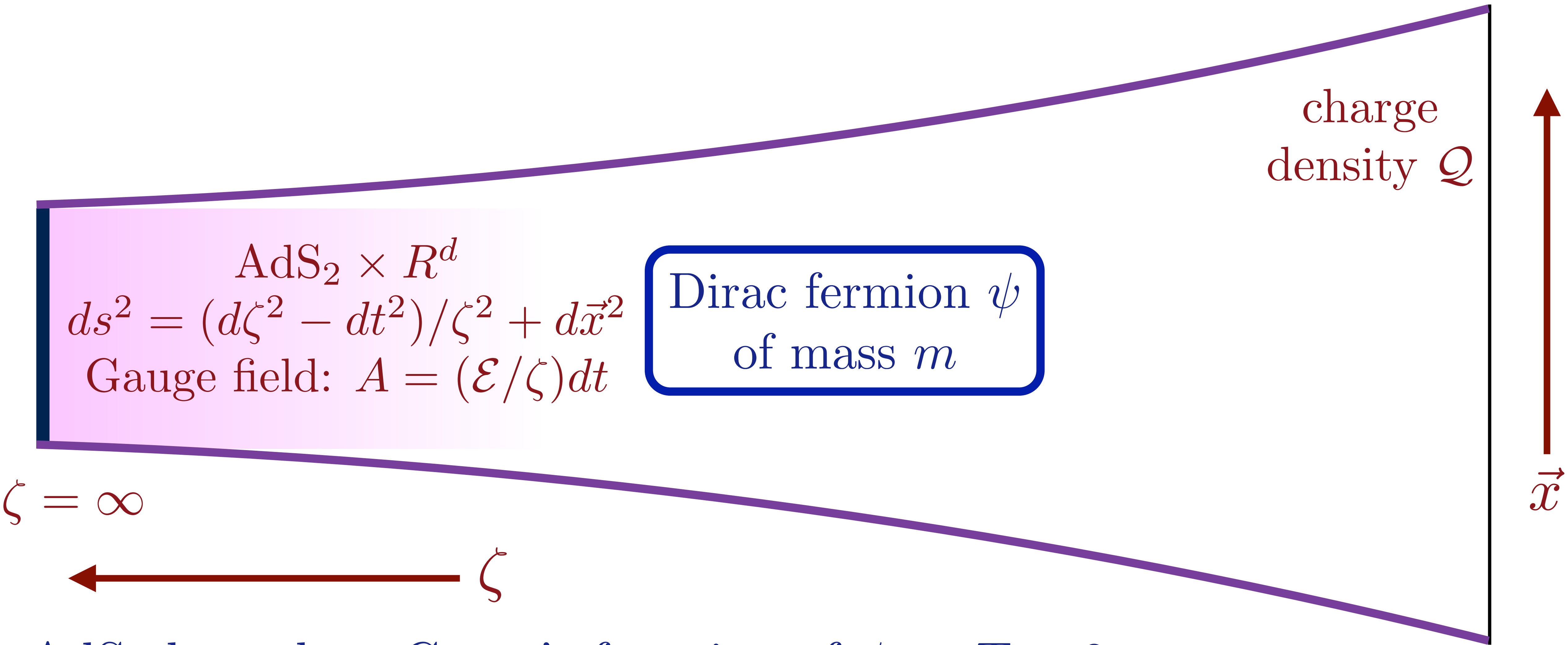
Einstein-Maxwell theory $$\mathcal{S}_{EM} = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2} - \frac{R^2}{g_F^2}F^2\right)\right]$$



Realizes a strange metal: a state with an unbroken global U(1) symmetry with a continuously variable charge density, $\mathcal{Q}$, at $T = 0$ which does not have any quasiparticle excitations.

A. Chamblin, R. Emparan, C.V. Johnson, and R. C. Myers, PRD **60**, 064018 (1999)

Quantum fields on charged black branes



AdS_2 boundary Green's function of ψ at $T = 0$

$$\mathrm{Im}G(\omega) \sim \begin{cases} \omega^{-(1-2\Delta)} & , \omega > 0 \\ e^{-2\pi\mathcal{E}} \, |\omega|^{-(1-2\Delta)} & , \omega < 0. \end{cases}$$

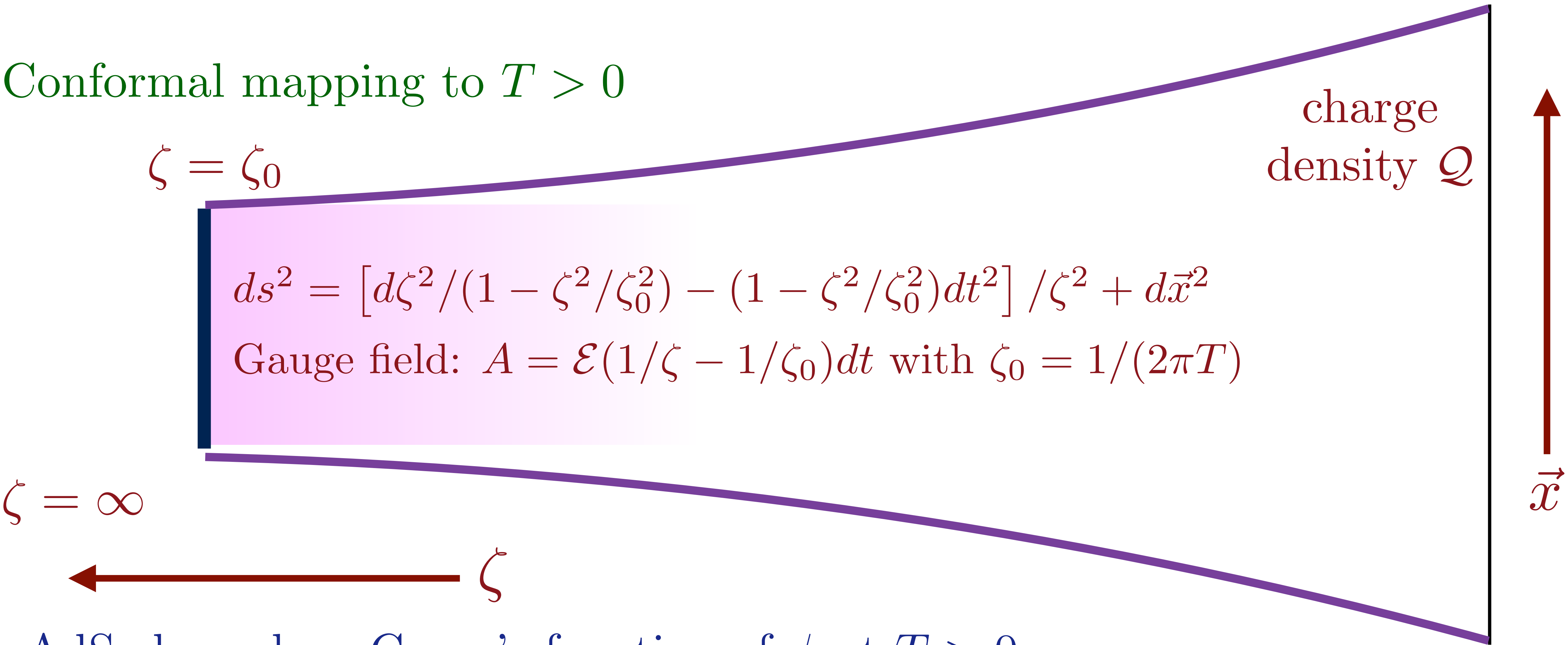
where the fermion scaling dimension Δ is a function of m

$\mathcal{E}$ encodes the particle-hole asymmetry

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, PRD **83**, 125002 (2011)

Quantum fields on charged black branes

Conformal mapping to $T > 0$



AdS_2 boundary Green's function of ψ at $T > 0$ is fully determined by $\mathcal{E}$

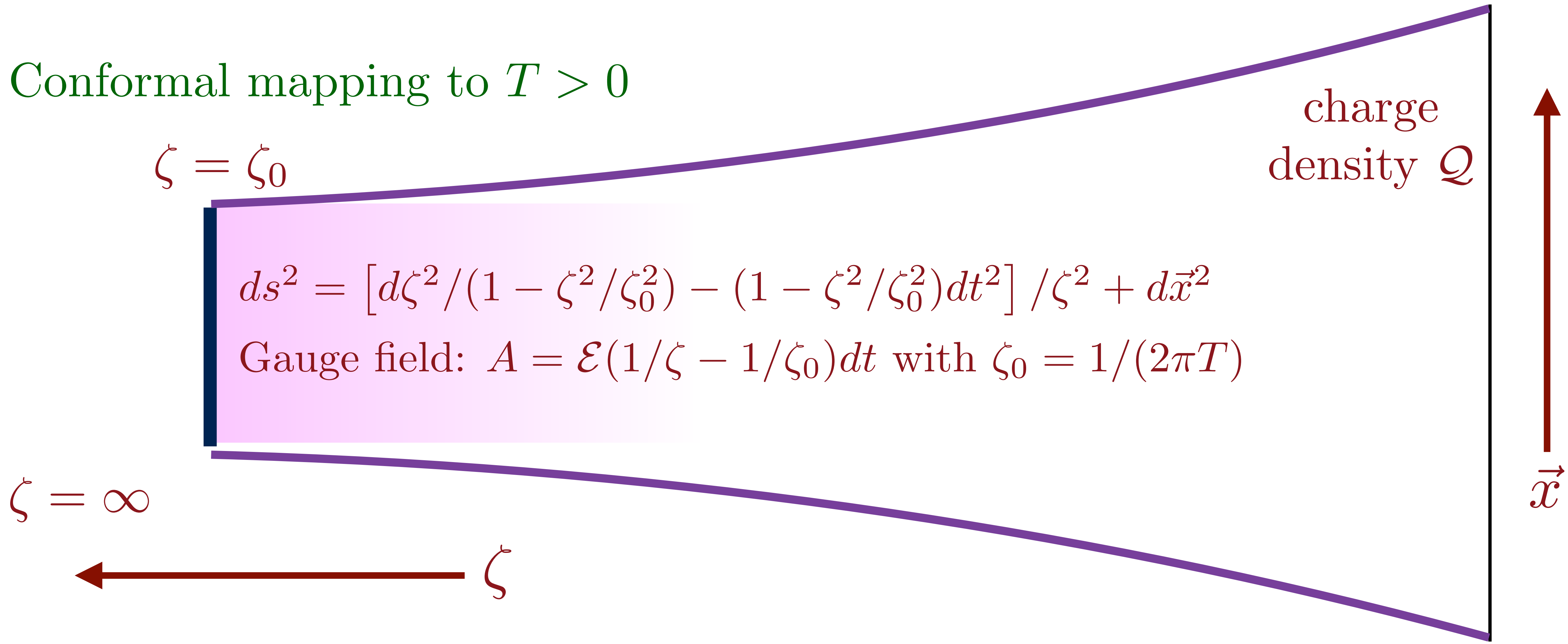
$$G^R(\omega) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma\left(\Delta - \frac{i\hbar\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(1 - \Delta - \frac{i\hbar\omega}{2\pi T} + i\mathcal{E}\right)}$$

where $e^{2\pi\mathcal{E}} = \frac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}$.

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, PRD **83**, 125002 (2011)

General Relativity of charged black branes

Conformal mapping to $T > 0$



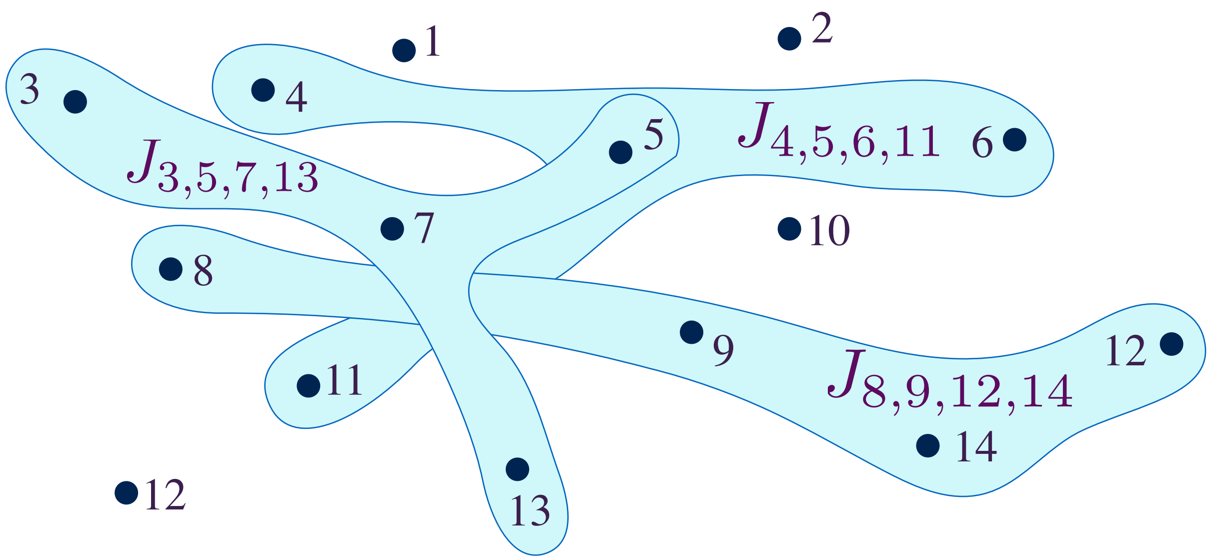
- As $T \to 0$, there is a non-zero Bekenstein-Hawking entropy, $\mathcal{S}_{BH}$.
- Using Gauss's Law, it can be shown that $\mu(T) = -2\pi\mathcal{E}T+$ constant as $T \to 0$.
- Using a thermodynamic Maxwell relation (also obeyed by gravity),

A. Sen
hep-th/0506177
S. Sachdev
PRX 5, 041025 (2015)

$$\left(\frac{\partial \mathcal{S}_{\rm BH}}{\partial \mathcal{Q}}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = 2\pi\mathcal{E}$$

Also obeyed by Wald entropy in higher-derivative gravity.

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$



$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

Local fermion density of states

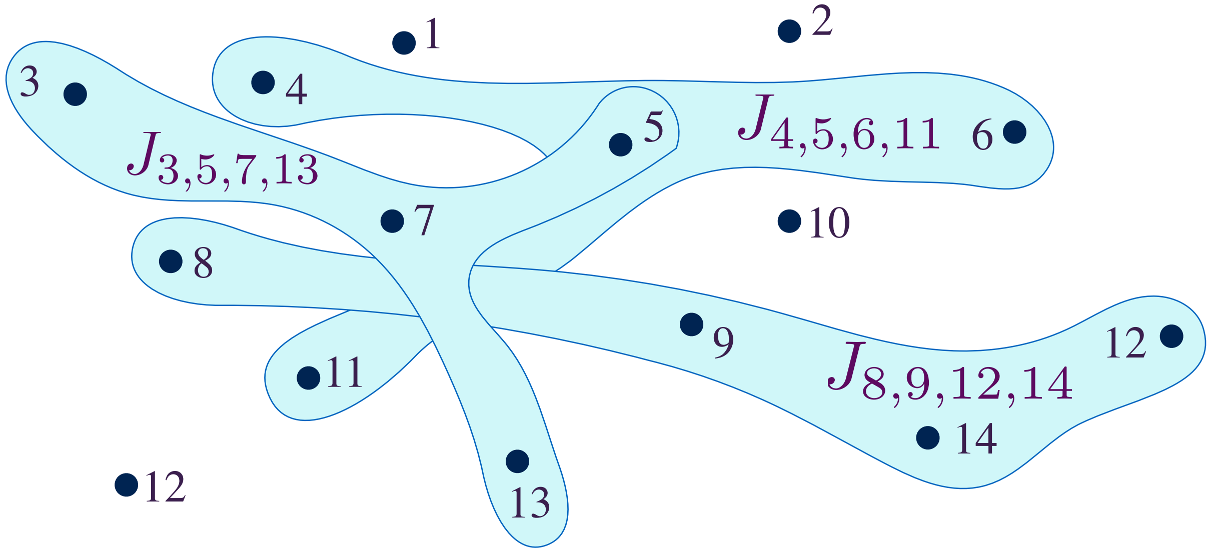
$$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \omega > 0 \\ e^{-2\pi\mathcal{E}} \, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature entropy density, $\mathcal{S}$, obeys

$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$



$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

Local fermion density of states

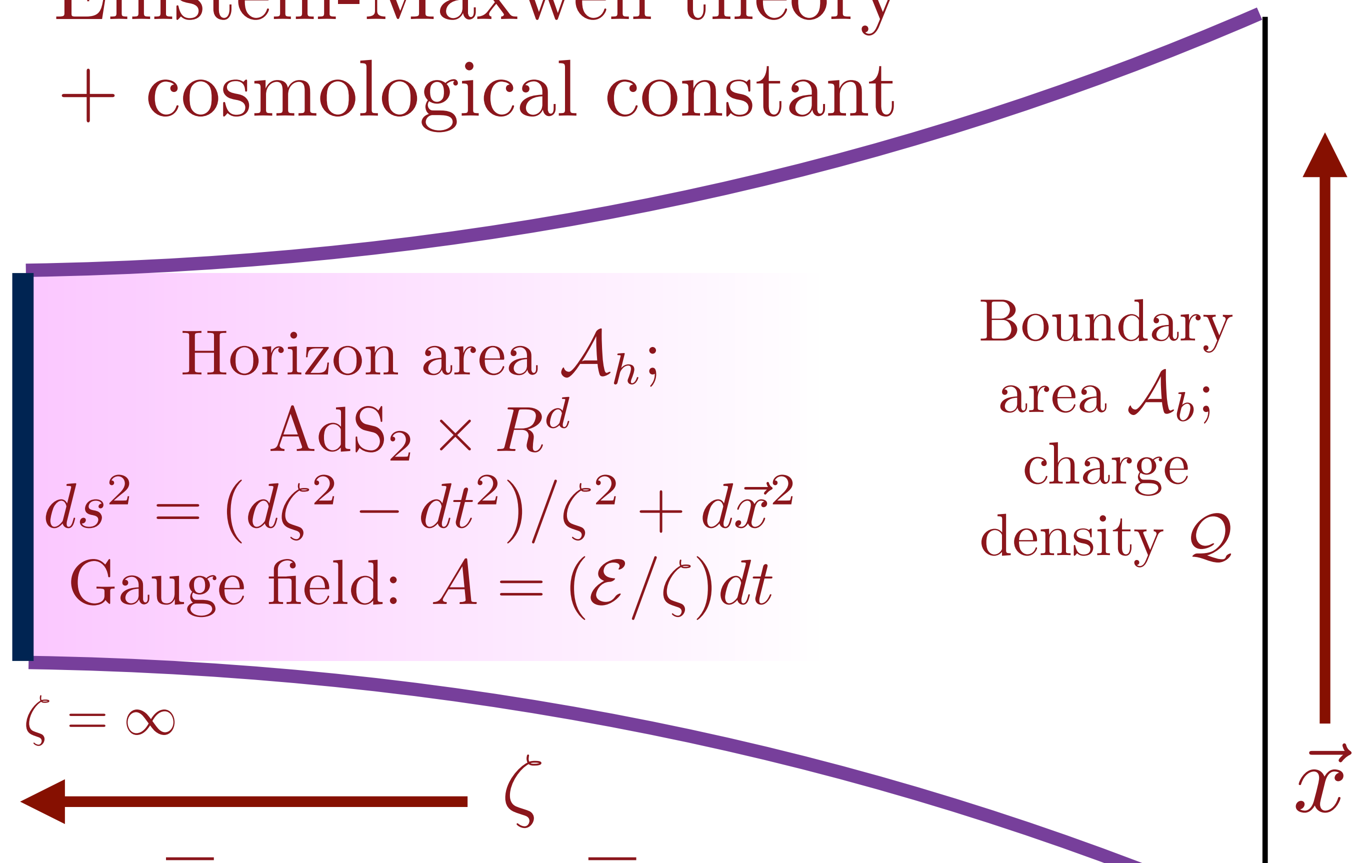
$$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \ \omega > 0 \\ e^{-2\pi\mathcal{E}}\, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature entropy density, $\mathcal{S}$, obeys

$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

Einstein-Maxwell theory + cosmological constant



$$\mathcal{L} = \overline{\psi}\Gamma^\alpha D_\alpha \psi + m\overline{\psi}\psi$$

Local fermion density of states

$$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \ \omega > 0 \\ e^{-2\pi\mathcal{E}}\, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

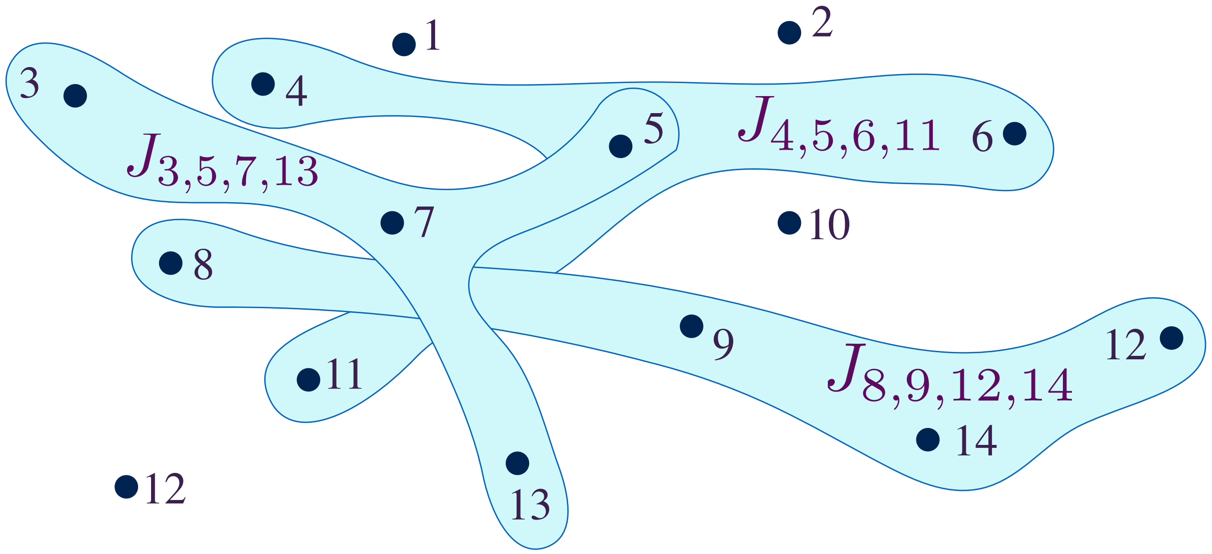
'Equation of state' relating $\mathcal{E}$ and $\mathcal{Q}$ depends upon the geometry of spacetime far from the AdS_2

Black hole thermodynamics (classical general relativity) yields

$$\frac{\partial \mathcal{S}_{\rm BH}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

A. Sen hep-th/0506177; S. Sachdev PRX **5**, 041025 (2015)

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$



$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

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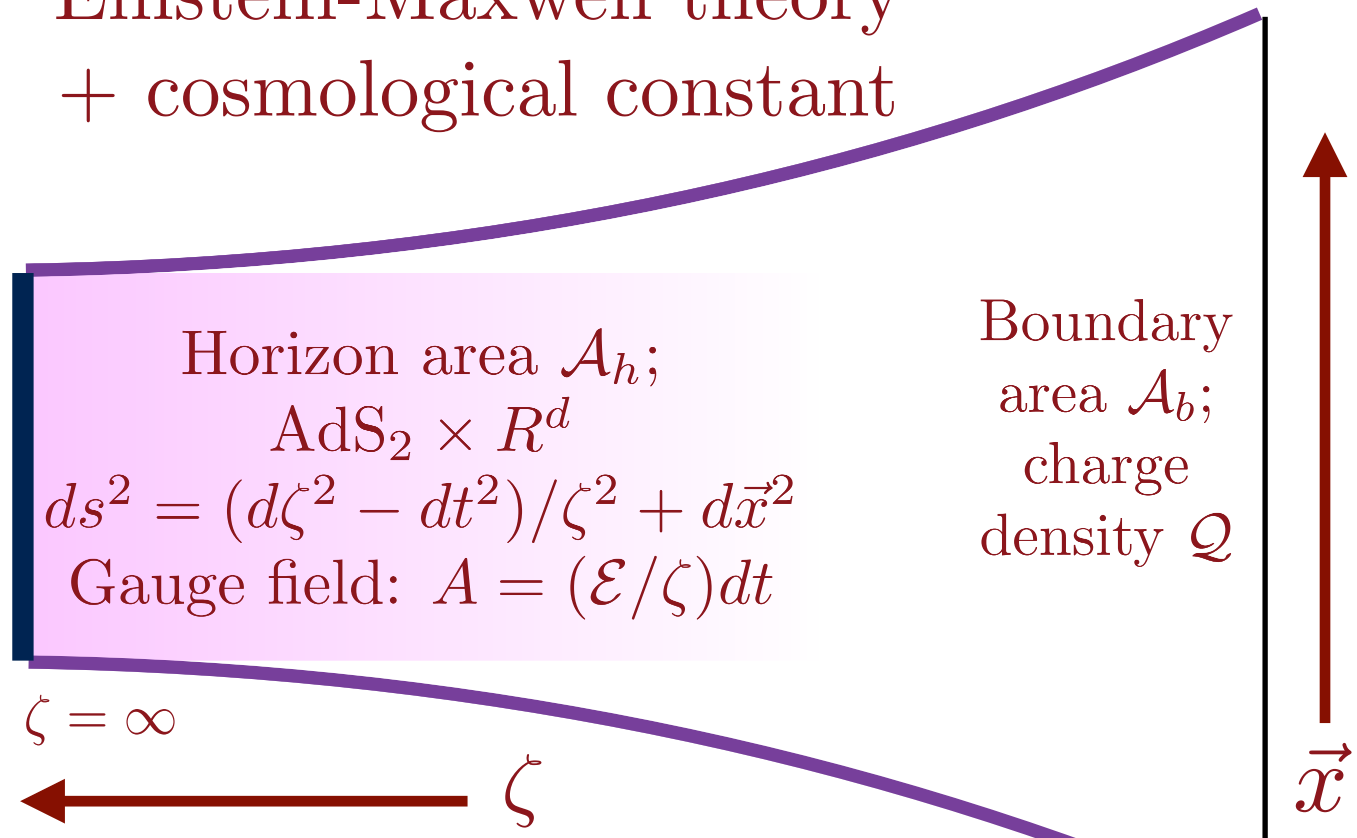

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Evidence for AdS_2 gravity dual of H

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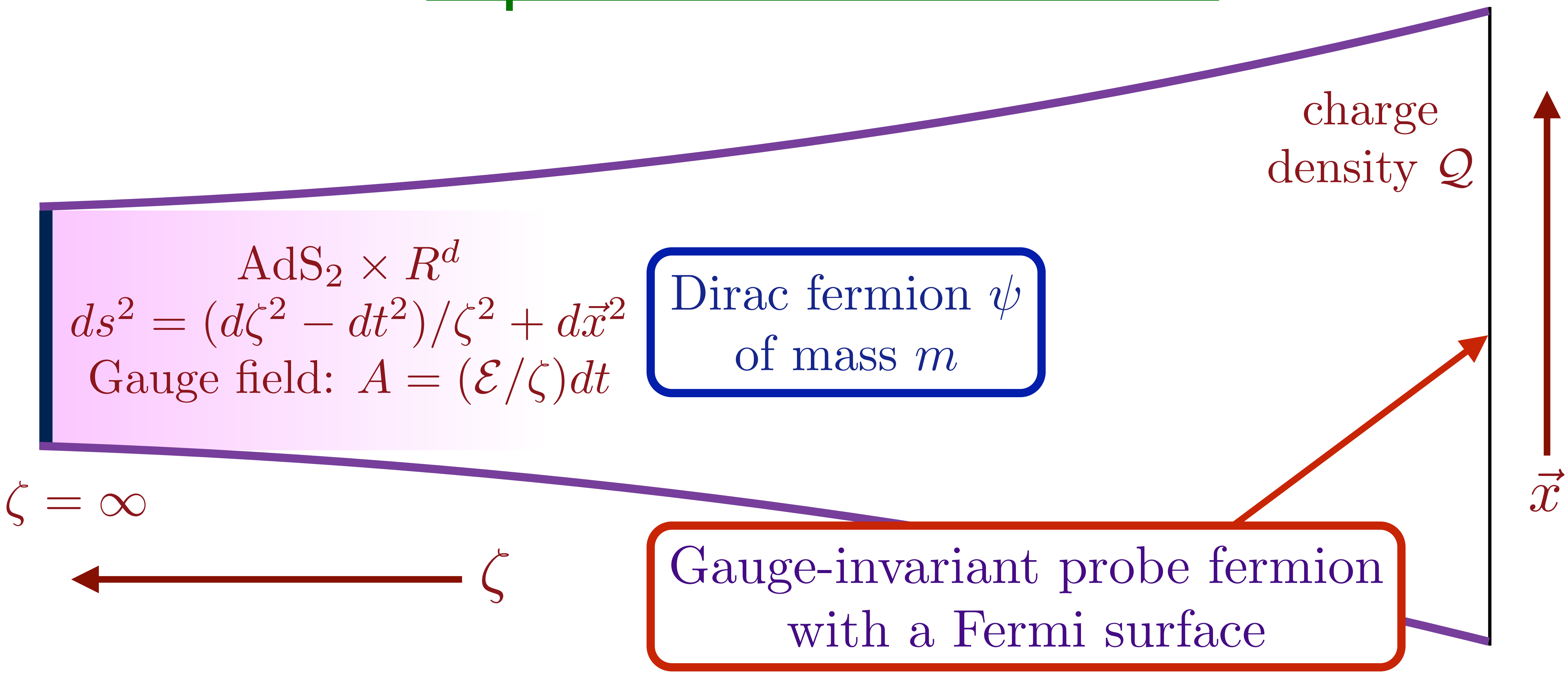
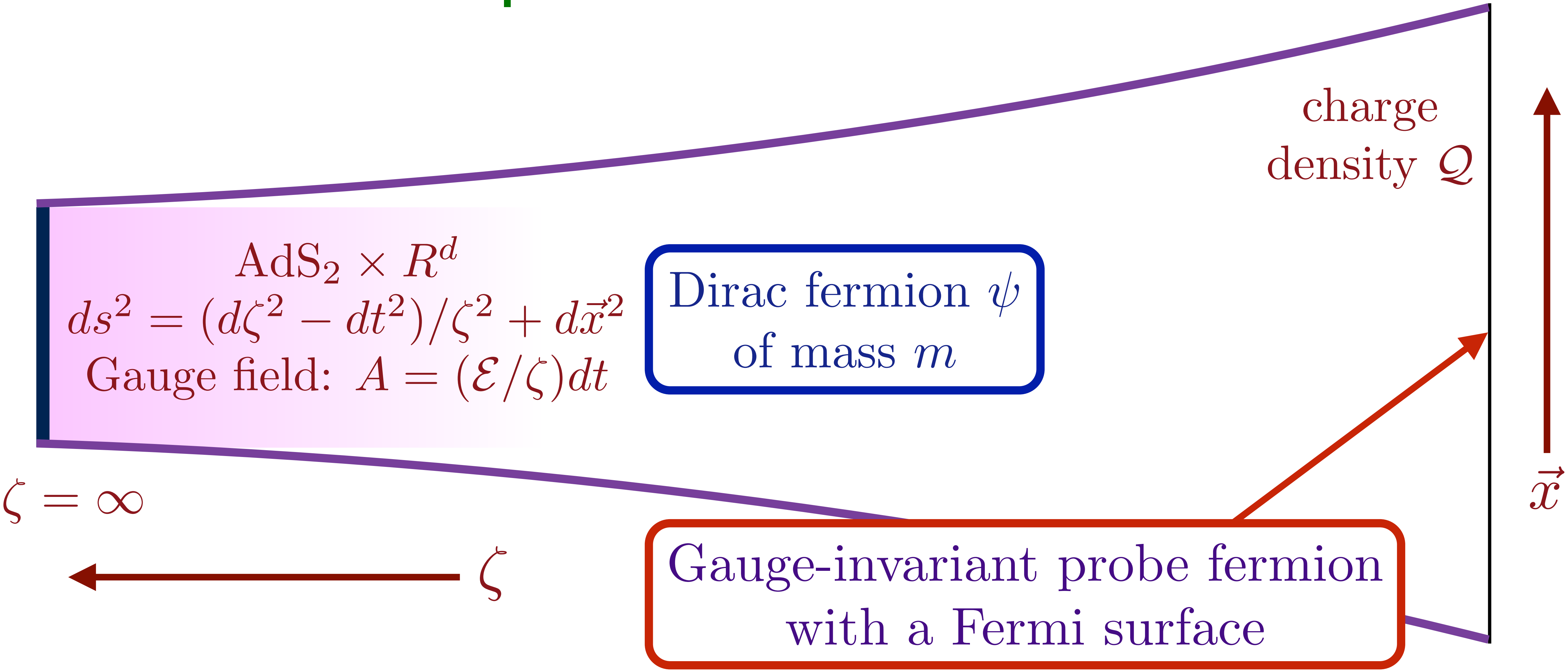
S. Sachdev, PRX 5, 041025 (2015)

Implications of SYK model



T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, PRD **83**, 125002 (2011)

Implications of SYK model



Charge density of Fermi surface

- Fermi surface has $\mathcal{O}(1)$ charge density, and the $\mathcal{O}(N^2)$ charge is "behind the horizon".

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, PRD **83**, 125002 (2011)

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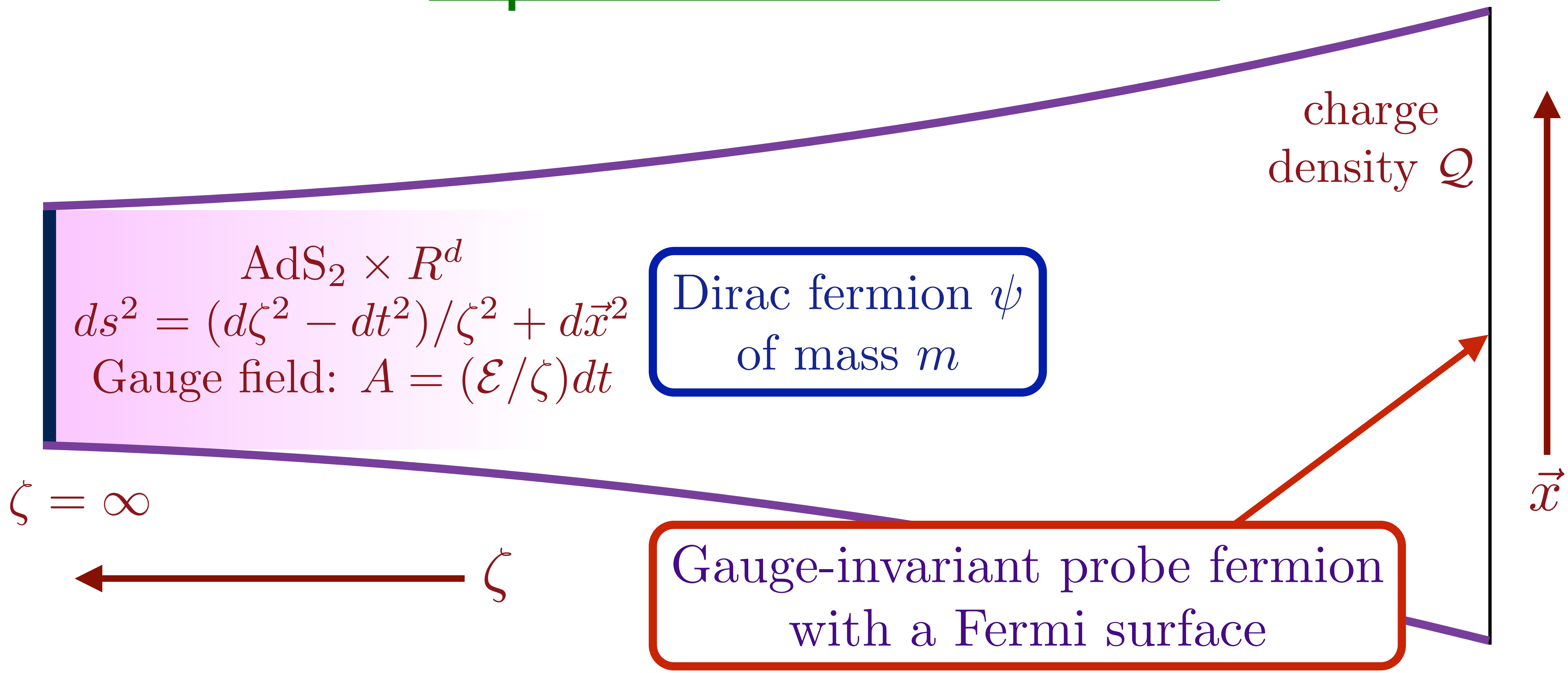

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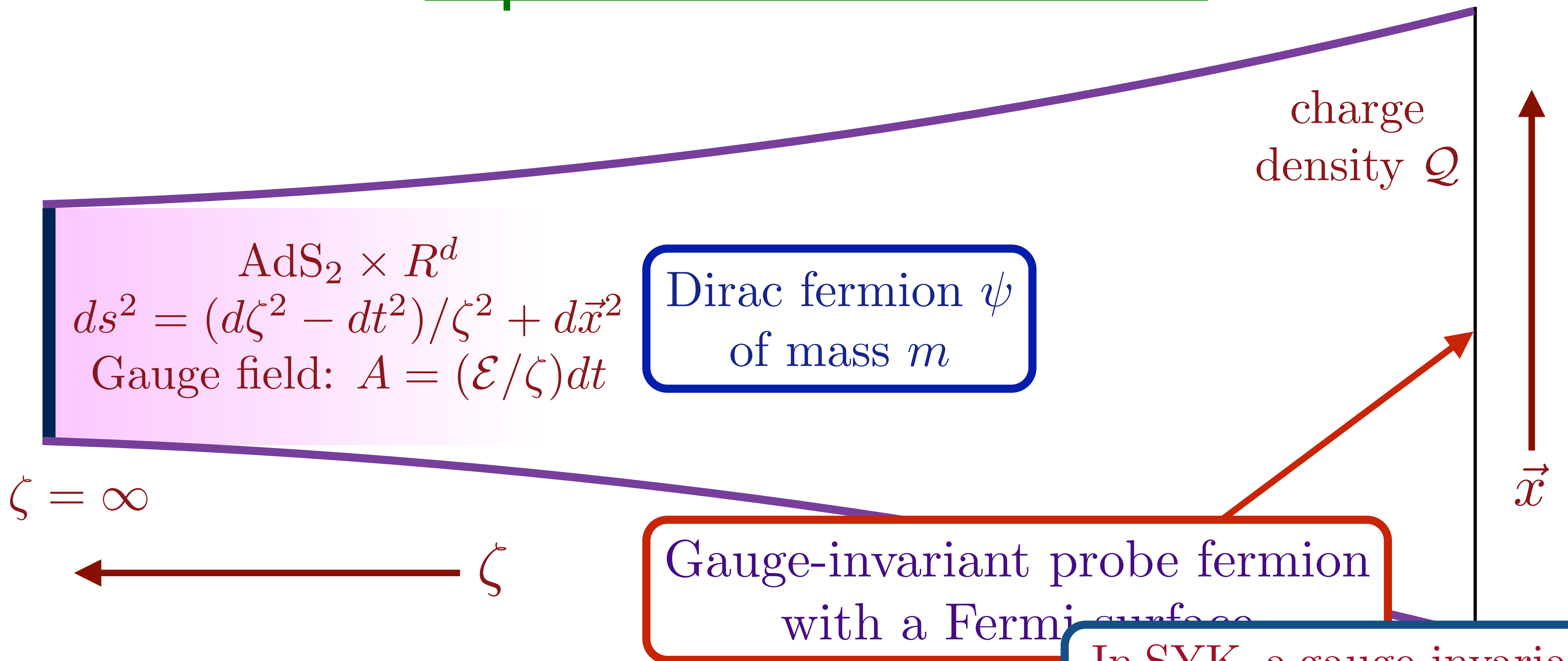
- Fermi surface has $\mathcal{O}(N^2)$ charge density, because of almost Bose-condensed scalar in boundary theory

  O. DeWolfe, S. S. Gubser, and C. Rosen, PRL **108**, 251601 (2012)

Implications of SYK model



In SYK, a gauge-invariant UV fermion carries all the charge density, and renormalizes to a gauge-charged IR fermion with same correlators as ψ.

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  T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, PRD 8

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Implications of SYK model

A bound on quantum chaos:

- The "Lyapunov exponent" for chaos, λ_L, is given by out-of-time-order correlators, and for quantum systems near equilibrium, it obeys the bound $\lambda_L \leq 2\pi k_B T/\hbar$.

J. Maldacena, S. H. Shenker and D. Stanford, arXiv:1503.01409

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S. H. Shenker and D. Stanford, arXiv:1306.0622

- The bound is also saturated by the SYK model

A. Kitaev, unpublished
J. Polchinski and V. Rosenhaus, arXiv:1601.06768

Quantum matter without quasiparticles

1. SYK model and AdS_2 metals
2. Quantum-critical metals with translational invariance
3. Breaking translational invariance

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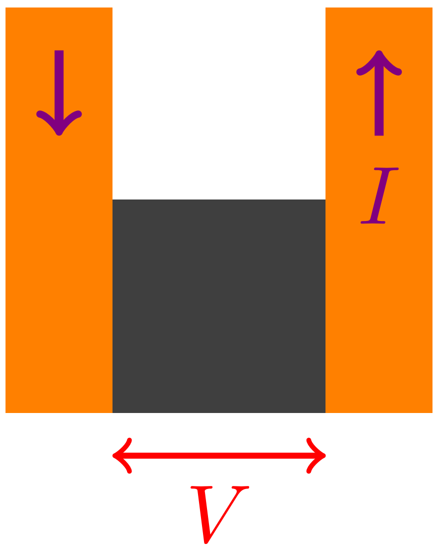

$$V = IR \qquad R \sim \frac{1}{\sigma}$$

- more generally, measure thermoelectric transport:

$$\begin{pmatrix} \delta J_i \\ \delta Q_i \end{pmatrix} = \begin{pmatrix} \sigma_{ij} & \alpha_{ij} \\ T\bar{\alpha}_{ij} & \bar{\kappa}_{ij} \end{pmatrix} \begin{pmatrix} \delta E_j \\ -\partial_j \delta T \equiv T\delta\zeta_j \end{pmatrix}.$$

- σ = easy experiment; related to QFT correlators:

$$\sigma_{ij}(\omega) = \frac{\mathrm{i}}{\omega}\langle J_i(-\omega) J_j(\omega)\rangle, \quad \text{etc.}$$

Thermoelectric transport coefficients

Transport has two components: a "momentum drag" term, and a "quantum critical" term.

$$\sigma = \frac{\mathcal{Q}^2}{\mathcal{M}} \left(\frac{1}{-i\omega} \right) + \sigma_Q(\omega)$$

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with entropy density $\mathcal{S}$, $\mathcal{Q} \equiv \chi_{J_x, P_x}$, and $\mathcal{M} \equiv \chi_{P_x, P_x}$.

The thermal conductivity $\kappa = \bar{\kappa} - \frac{T\alpha^2}{\sigma}$

As $\omega \to 0$, $\kappa = \bar{\kappa}_Q - 2 \left(\frac{T\mathcal{S}}{\mathcal{Q}} \right) \alpha_Q + \left(\frac{T\mathcal{S}^2}{\mathcal{Q}^2} \right) \sigma_Q$

Obtained in hydrodynamics, memory functions, and holography

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

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In relativistic theories (apart from T and non-zero μ),

$$T\alpha_Q(\omega) = -\mu\sigma_Q(\omega), \qquad T\bar{\kappa}_Q(\omega) = \mu^2\sigma_Q(\omega),$$

and there is only one independent transport co-efficient (σ_Q).
Also $\mathcal{M} = T\mathcal{S} + \mu\mathcal{Q} = \mathcal{H}$ the enthalpy density, and $\mathcal{Q} = n$ the electron density.

Then $$\kappa = \sigma_Q \left(\frac{\mathcal{H}^2}{T\mathcal{Q}^2} \right)$$

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In non-relativistic theories, we expect

$\mathcal{Q} \sim$ constant, and $\mathcal{M} \sim$ constant

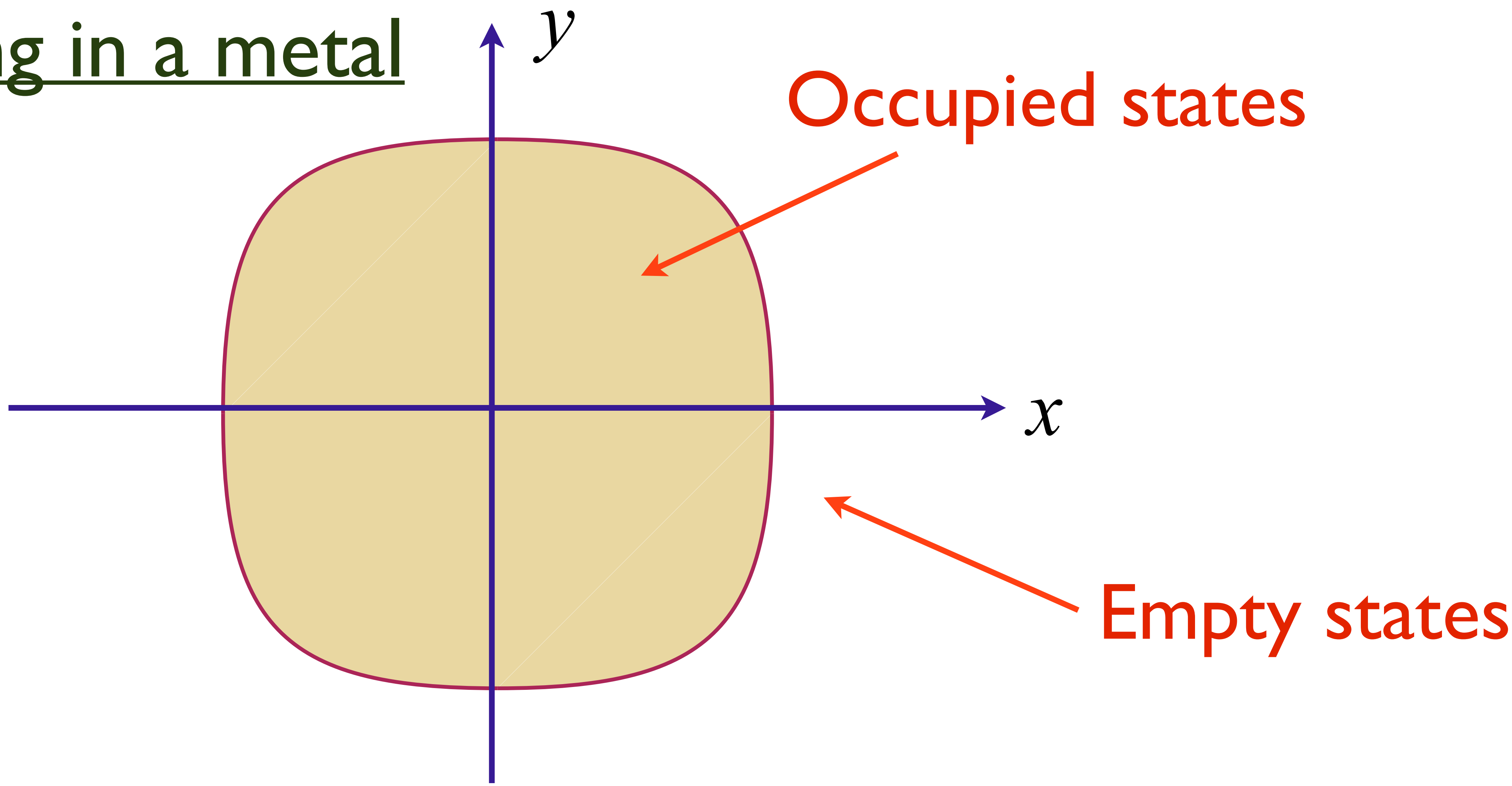
2. Quantum-critical metals with translational invariance

A. Quantum criticality of Ising-nematic ordering in a metal

B. Fermi surface+antiferromagnetism

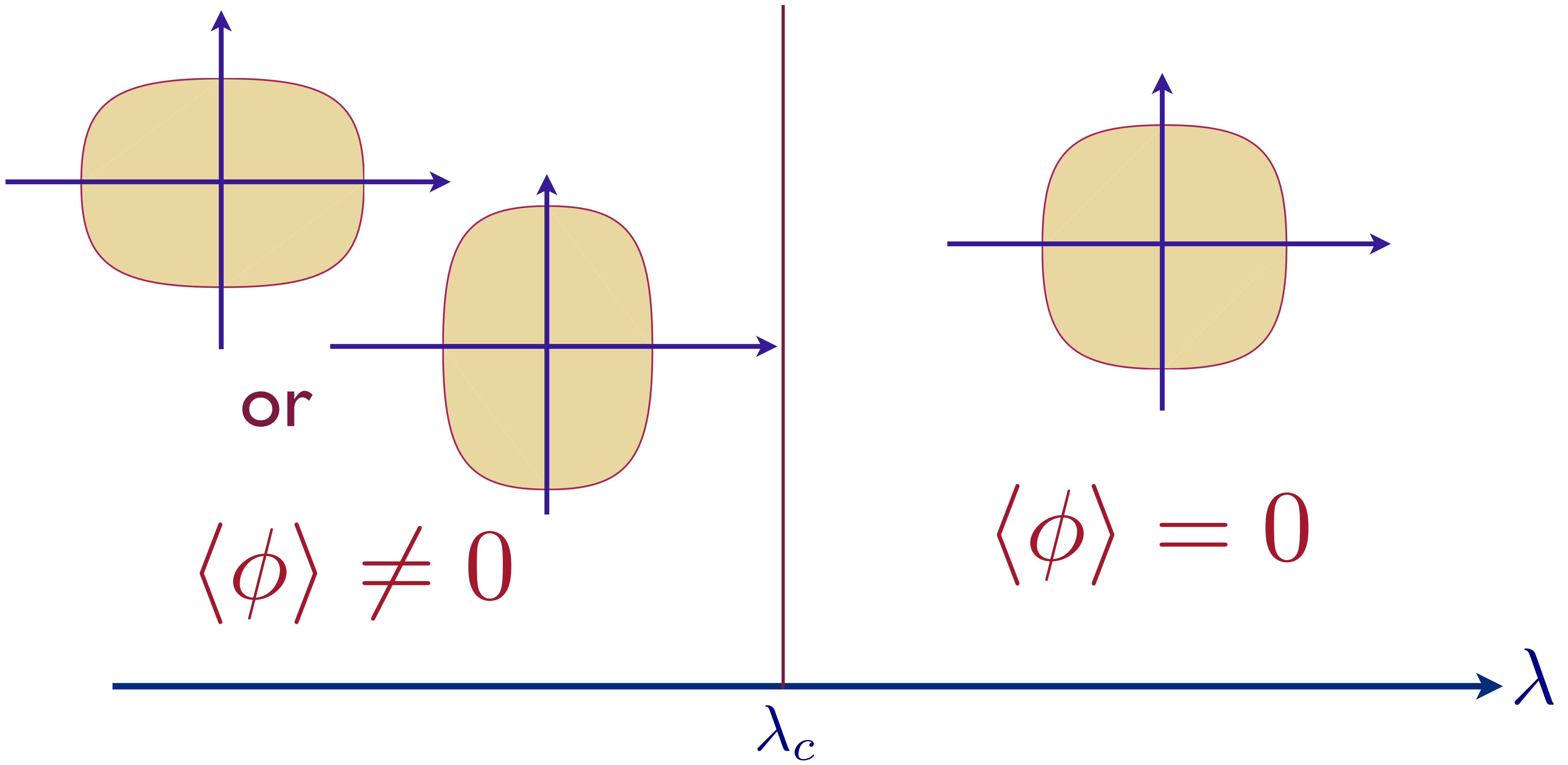
C. Holography: charged black branes

A. Quantum criticality of Ising-nematic ordering in a metal



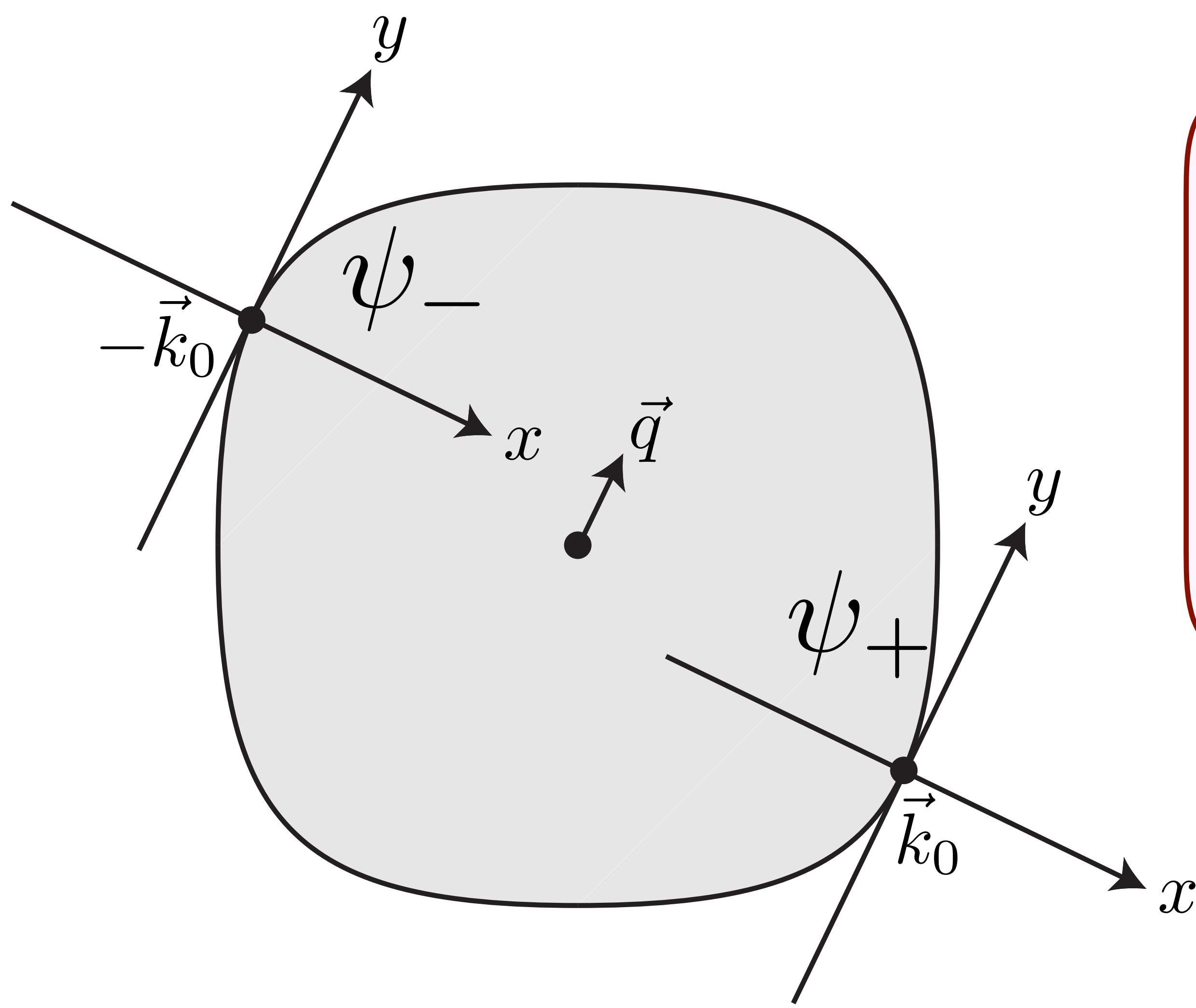
A metal with a Fermi surface
with full square lattice symmetry

A. Quantum criticality of Ising-nematic ordering in a metal



Pomeranchuk instability as a function of coupling λ

A. Quantum criticality of Ising-nematic ordering in a metal

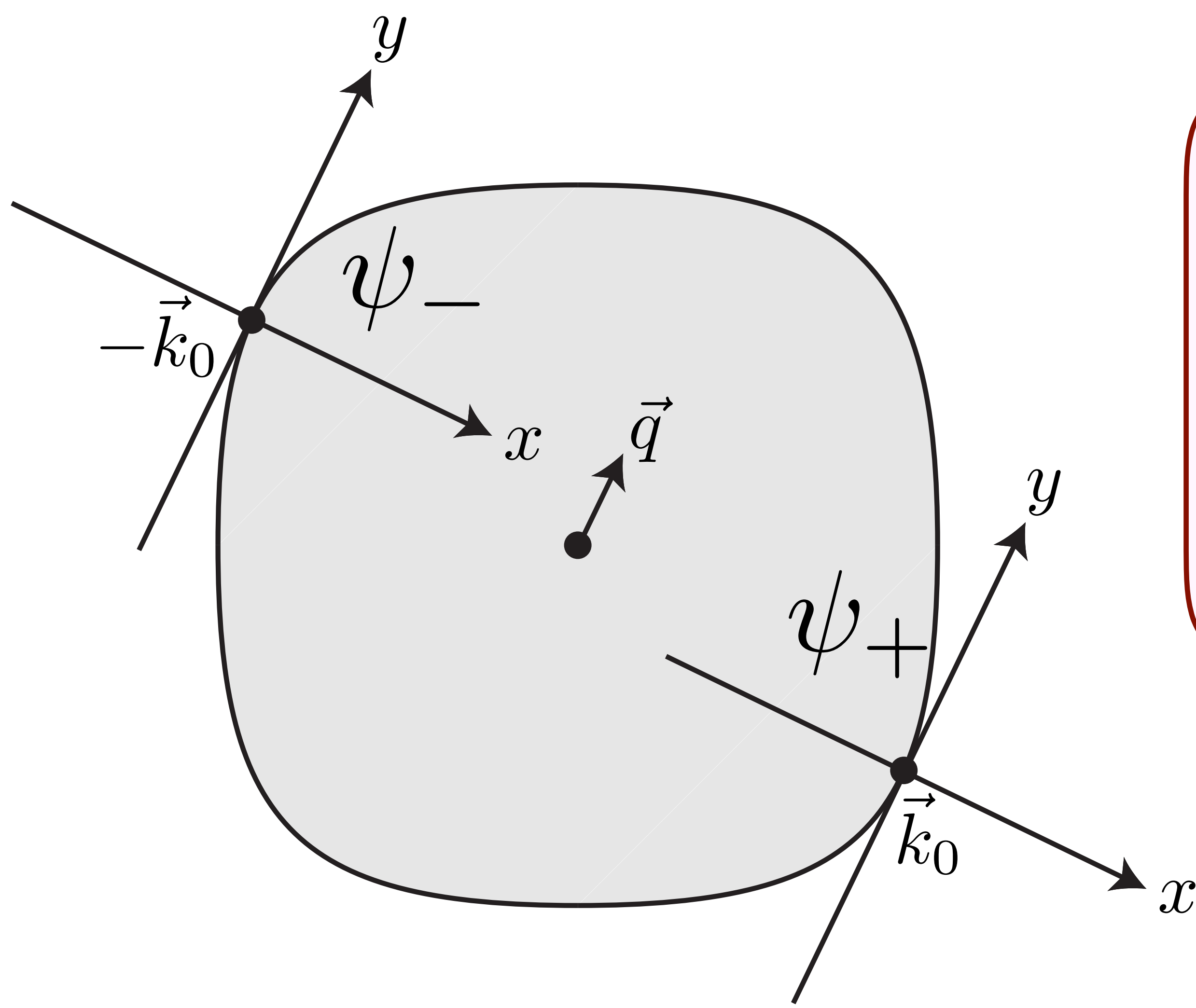


Field theory also applies (with small changes) to a Fermi surface coupled to an abelian or non-abelian gauge field.

$$\mathcal{L}[\psi_\pm, \phi] =$$
$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$
$$-\phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

A. Quantum criticality of Ising-nematic ordering in a metal



Field theory also applies (with small changes) to a Fermi surface coupled to an abelian or non-abelian gauge field.

Using the dimensional regularized $\epsilon = 5/2 - d$ expansion ($d = 2$ is spatial dimension) introduced by D. Dalidovich and Sung-Sik Lee (PRB **88**, 245106 (2013)), we find

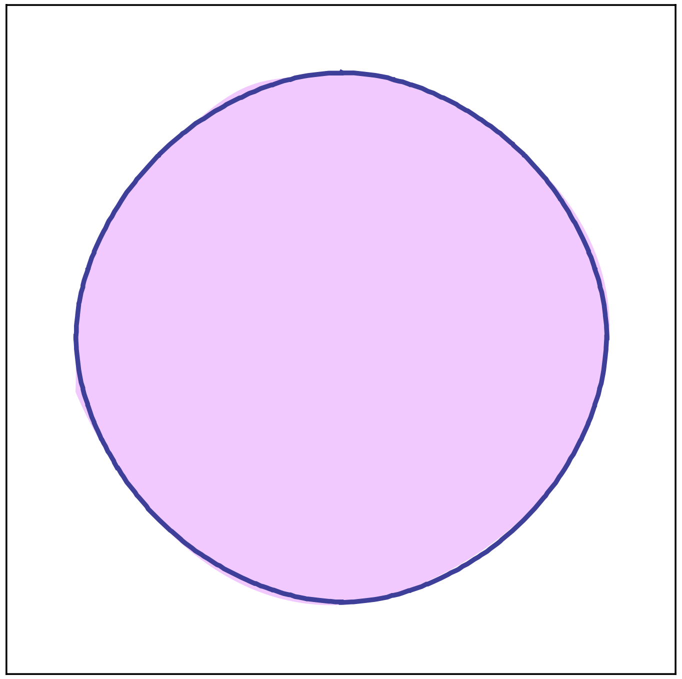
$$\mathcal{S} \sim T^{(d-\theta)/z}$$
$$\sigma_Q \sim T^{(d-2-\theta)/z}\Phi(\omega/T)$$

with $z = 3/2$, $\theta = 1$, and $\Phi(\omega/T)$ a universal scaling function.
θ is the violation of hyperscaling exponent.

A. Eberlein, S. Sachdev et al., to appear

B. Fermi surface+antiferromagnetism

Metal with "large"
Fermi surface

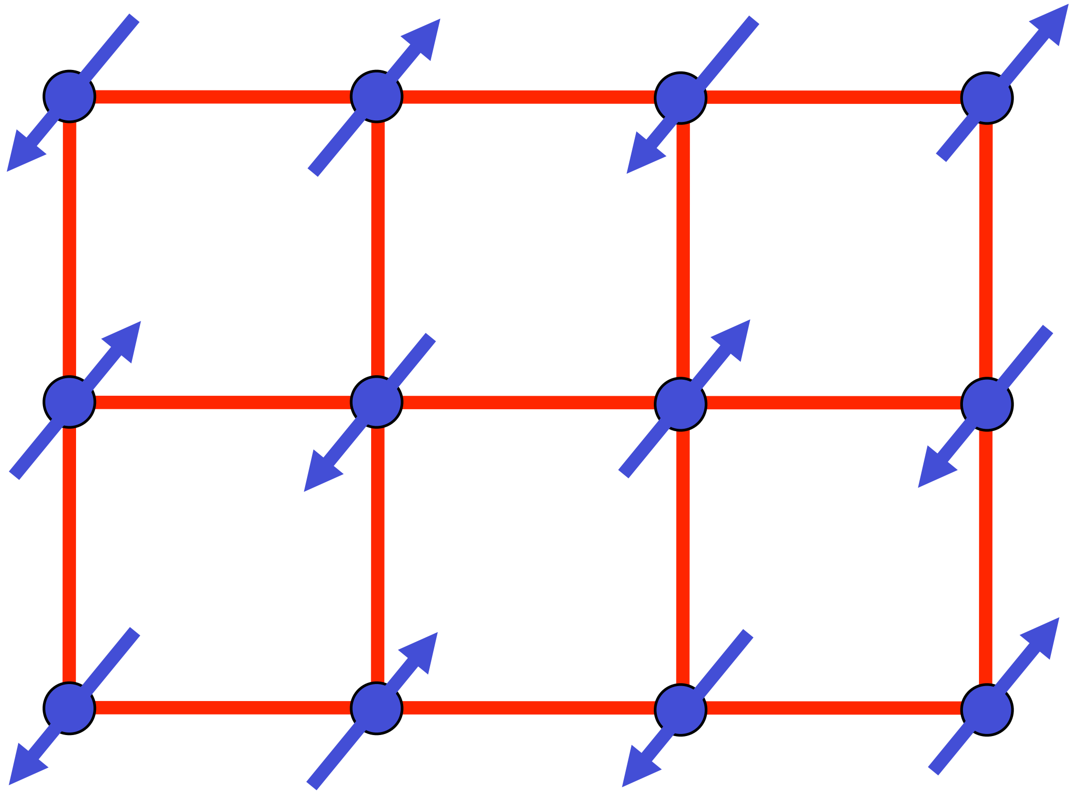


+

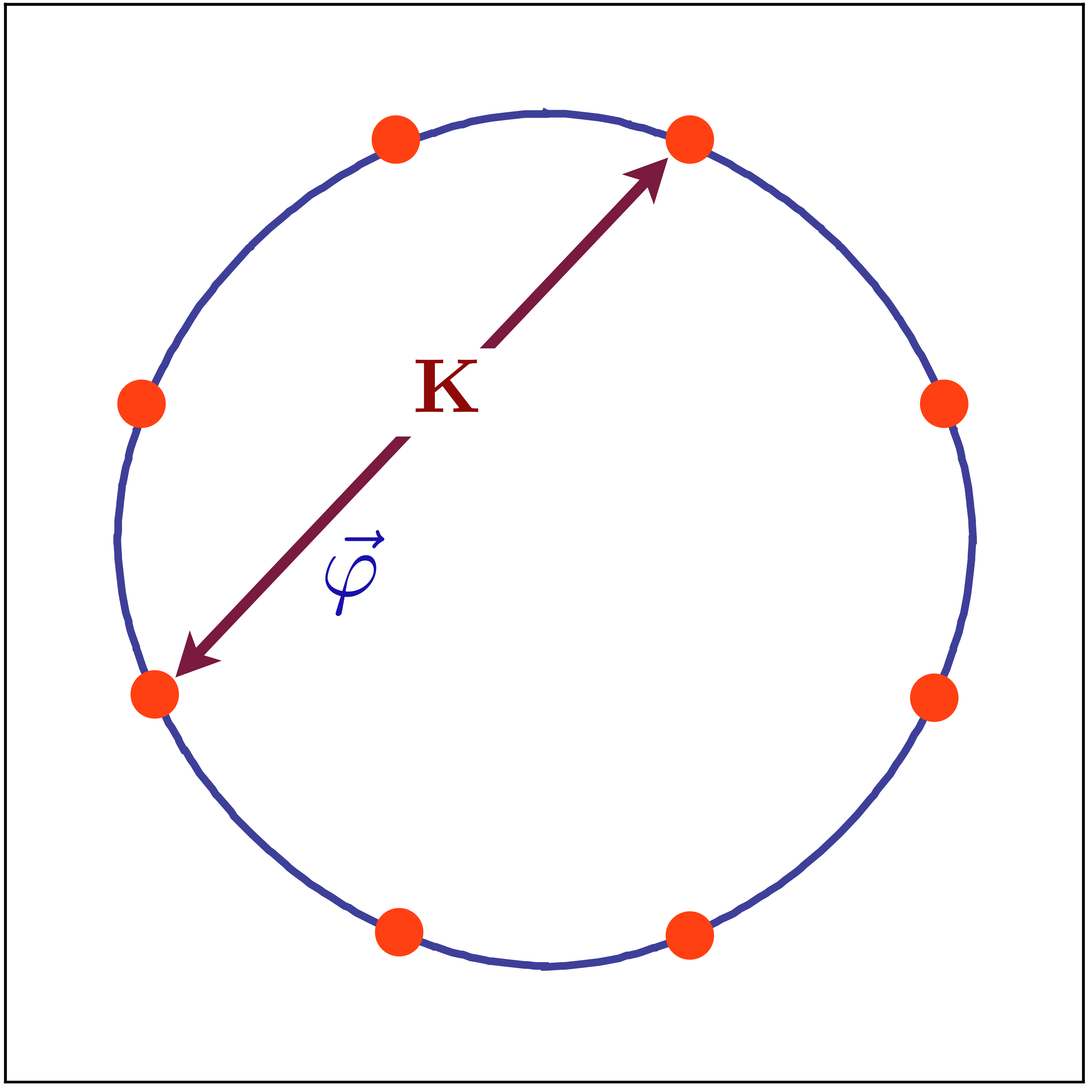
The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.



B. Fermi surface+antiferromagnetism



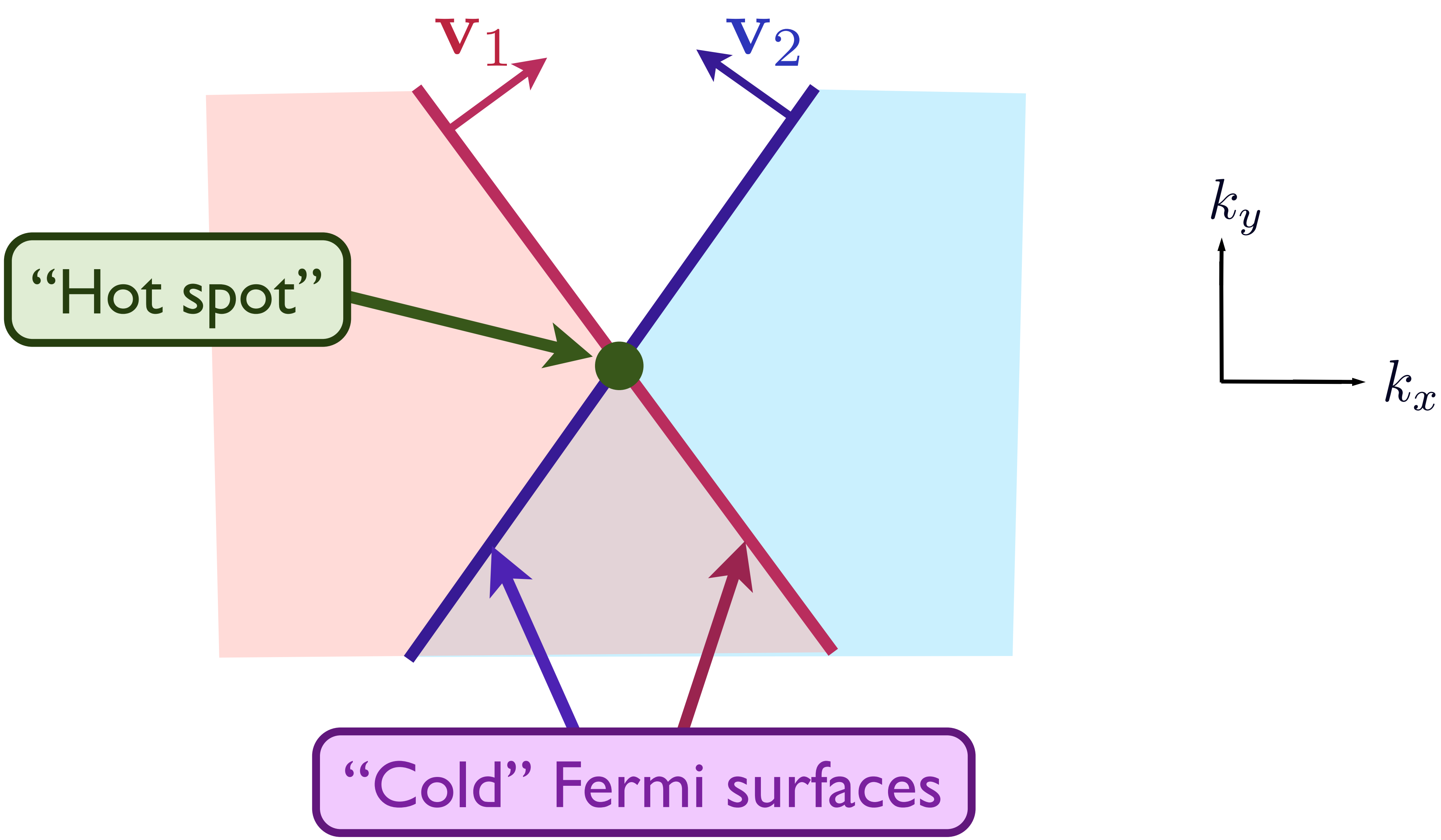
Hot spots in a single band model

$$\mathcal{L}_f = \psi^\dagger_{1\alpha}\left(\partial_\tau - i\mathbf{v}_1 \cdot \boldsymbol{\nabla}_r\right)\psi_{1\alpha} + \psi^\dagger_{2\alpha}\left(\partial_\tau - i\mathbf{v}_2 \cdot \boldsymbol{\nabla}_r\right)\psi_{2\alpha}$$

Order parameter: $$\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{1}{2}\left(\partial_\tau\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$$

"Yukawa" coupling: $$\mathcal{L}_c = -\lambda\vec{\varphi}\cdot\left(\psi^\dagger_{1\alpha}\vec{\sigma}_{\alpha\beta}\psi_{2\beta} + \psi^\dagger_{2\alpha}\vec{\sigma}_{\alpha\beta}\psi_{1\beta}\right)$$

Ar. Abanov and A.V. Chubukov, PRL **93**, 255702 (2004).

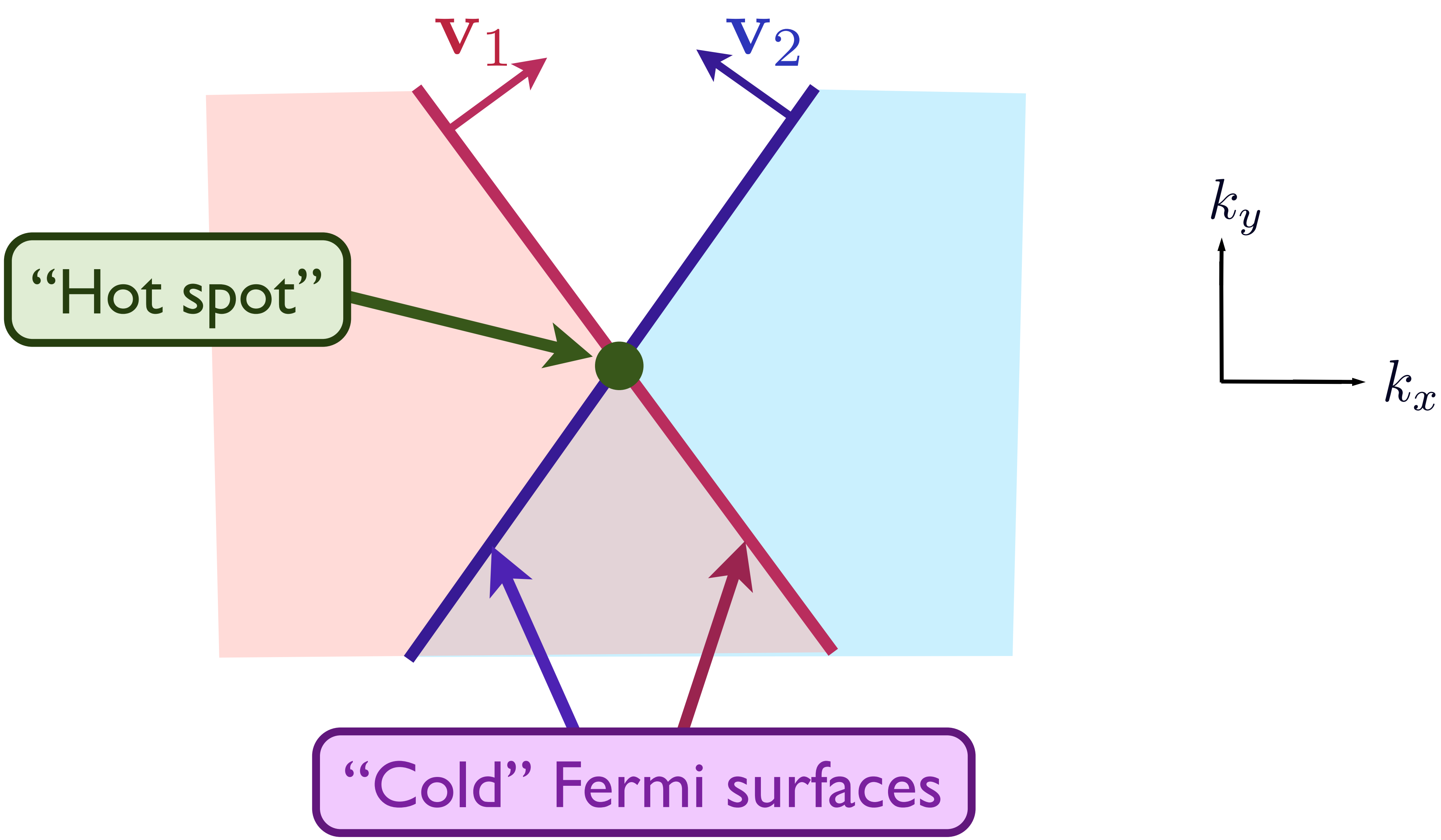


B. Fermi surface+antiferromagnetism

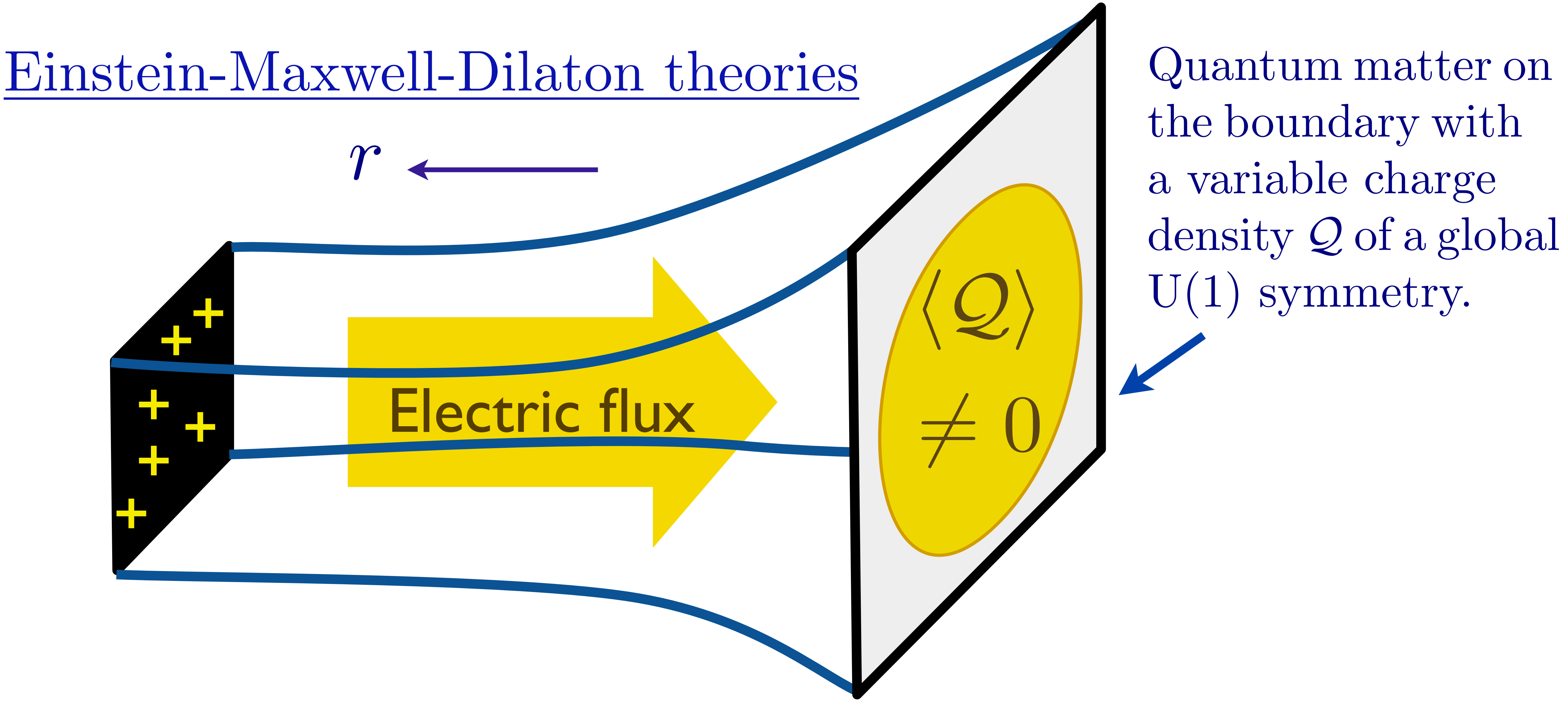
We find $$\mathcal{S}_{\text{singular}} \sim T^{(d-\theta)/z}$$
$$\sigma_Q \sim T^{(d-2-\theta)/z}\Phi(\omega/T)$$

with $\theta = 0$ *i.e.* no violation of hyperscaling.

A. A. Patel, P. Strack, and S. Sachdev, PRB **92**, 165105 (2015)



C. Holography: charged black branes



C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, JHEP **1201**, 94 (2012).
L. Huijse, S. Sachdev, B. Swingle, Phys. Rev. B **85**, 035121 (2012)

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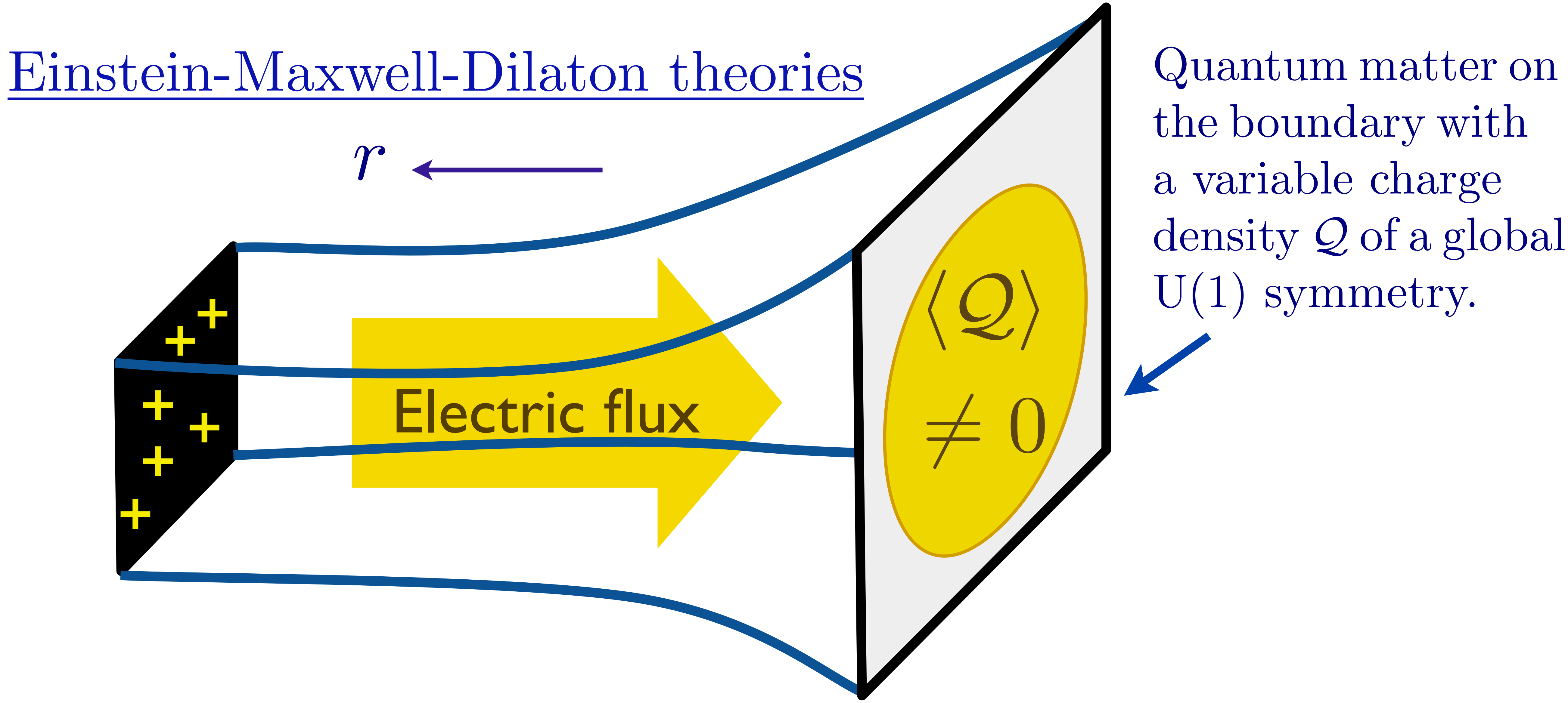

Hyperscaling violating metric in the IR with $z \geq 1 + \theta/d$

Ising-nematic critical theory saturates lower bound on z.

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right) \quad \text{at } T=0$$

C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, JHEP **1201**, 94 (2012).
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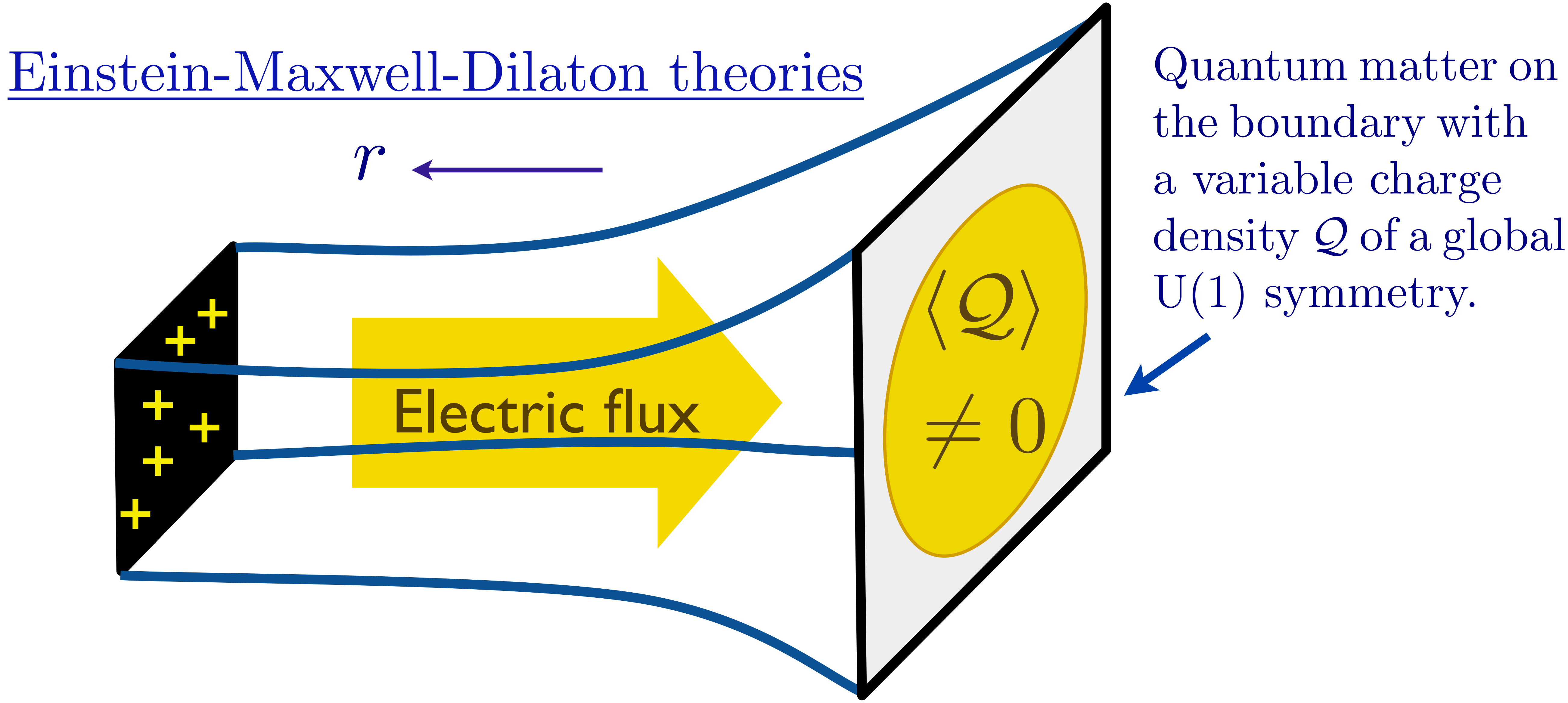

EMD theory yields $\mathcal{S} \sim T^{(d-\theta)/z}$

$$\sigma_Q \sim T^{(d+2\Phi-2-\theta)/z}\Phi(\omega/T)$$

with "current anomalous dimension" $\Phi = z$.

Ising-nematic critical theory has $\Phi = 0$.

B. Gouteraux, arXiv:1308.2084; S. Hartnoll and A. Karch, arXiv:1501.03165

Quantum matter without quasiparticles

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Translational symmetry breaking

Momentum relaxation by an external source h coupling to the operator $\mathcal{O}$

$$H = H_0 - \int d^d x \, h(x) \, \mathcal{O}(x).$$

Leads to an additional term in equations of motion:

$$\partial_\mu T^{\mu i} = \ldots - \frac{T^{it}}{\tau_{\rm imp}} + \ldots$$

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

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Memory functions and holography yield the same expression for τ_{imp}:

$$\frac{\mathcal{M}}{\tau_{\text{imp}}} = \lim_{\omega \to 0} \int d^d q\, |h(q)|^2 q_x^2 \frac{\text{Im}\left(G^{\text{R}}_{\mathcal{O}\mathcal{O}}(q,\omega)\right)_{H_0}}{\omega} + \ldots$$

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

A. Lucas, JHEP **03** (2015) 071

A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

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If $\overline{h(x)h(x')} = h_0^2 \delta^d(x - x')$,
and $\dim[\mathcal{O}] = \Delta$ in the translationally invariant theory, then

$$\frac{\mathcal{M}}{\tau_{\text{imp}}} \sim h_0^2 T^{2(1+\Delta-z)/z}$$

A. Lucas, S. Sachdev, and K. Schalm, PRD **89**, 066018 (2014)

Thermoelectricity with translational symmetry

Transport has two components: a "momentum drag" term, and a "quantum critical" term.

$$\sigma = \frac{\mathcal{Q}^2}{\mathcal{M}} \left(\frac{1}{-i\omega} \right) + \sigma_Q(\omega)$$

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with entropy density $\mathcal{S}$, $\mathcal{Q} \equiv \chi_{J_x,P_x}$, and $\mathcal{M} \equiv \chi_{P_x,P_x}$.

$$\text{The thermal conductivity } \kappa = \bar{\kappa} - \frac{T\alpha^2}{\sigma}$$

$$\text{As } \omega \to 0,\ \kappa = \bar{\kappa}_Q - 2\left(\frac{T\mathcal{S}}{\mathcal{Q}}\right)\alpha_Q + \left(\frac{T\mathcal{S}^2}{\mathcal{Q}^2}\right)\sigma_Q$$

Obtained in hydrodynamics, memory functions, and holography

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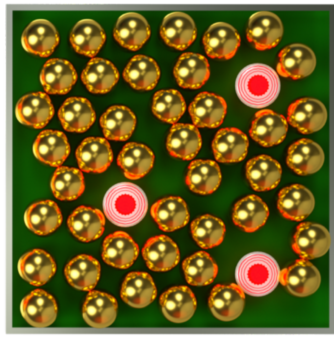

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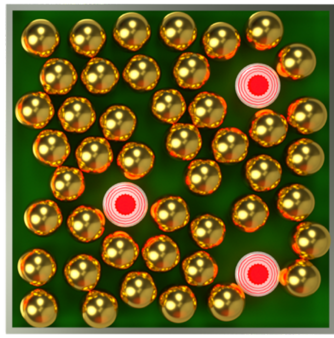
Transport has two components: a "momentum drag" term, and a "quantum critical" term.

$$\sigma = \frac{\mathcal{Q}^2}{\mathcal{M}}\left(\frac{1}{-i\omega + 1/\tau_{\rm imp}}\right) + \sigma_Q(\omega)$$

$$\alpha = \frac{\mathcal{S}\mathcal{Q}}{\mathcal{M}}\left(\frac{1}{-i\omega + 1/\tau_{\rm imp}}\right) + \alpha_Q(\omega)$$

$$\bar{\kappa} = \frac{T\mathcal{S}^2}{\mathcal{M}}\left(\frac{1}{-i\omega + 1/\tau_{\rm imp}}\right) + \bar{\kappa}_Q(\omega)$$



with entropy density $\mathcal{S}$, $\mathcal{Q} \equiv \chi_{J_x,P_x}$, and $\mathcal{M} \equiv \chi_{P_x,P_x}$.

In a relativistic theory (at non-zero T and $\mathcal{Q}$),

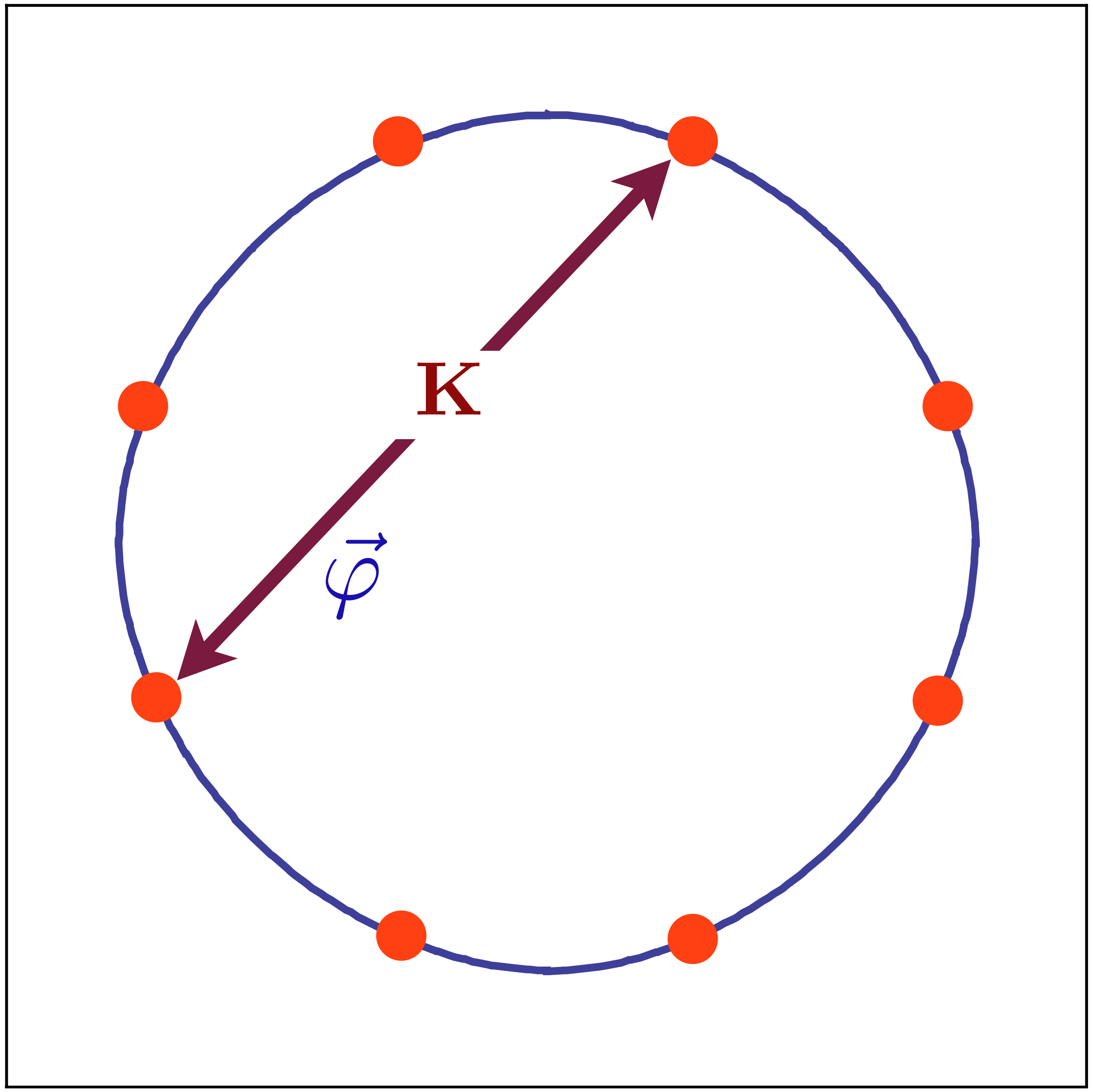
$$\kappa = \sigma_Q\left(\frac{\mathcal{H}^2}{T\mathcal{Q}^2}\right)\frac{1}{(1 + (\mathcal{H}\sigma_Q/\mathcal{Q}^2)(-i\omega + 1/\tau_{\rm imp}))}$$

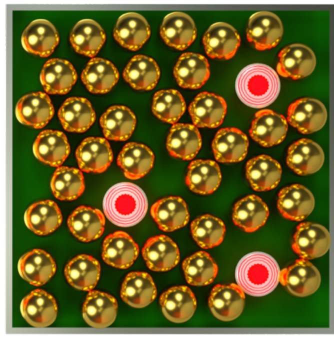
S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

Transport near SDW critical point

- Assume excitations around the full Fermi surface locally thermalize via interactions with excitations of the SDW boson $\vec{\varphi}$. These interactions conserve a (suitably defined) total momentum.

S.A. Hartnoll, D. M. Hofman, M.A. Metlitski and S. Sachdev, PRB **84**, 125115 (2011)





A.A. Patel and S. Sachdev, PRB **90**, 165146 (2014)

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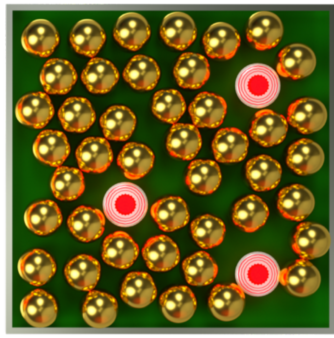
S.A. Hartnoll, D. M. Hofman, M.A. Metlitski and S. Sachdev, PRB **84**, 125115 (2011)

- Momentum relaxation occurs via disorder perturbations which change the local position of the quantum critical point.



$$
\begin{aligned}
H &= H_0 - \int d^d x \, h(x) \, \vec{\varphi}^2(x) \\
\overline{h(x) h(x')} &= h_0^2 \, \delta^d(x - x')
\end{aligned}
$$

A.A. Patel and S. Sachdev, PRB **90**, 165146 (2014)

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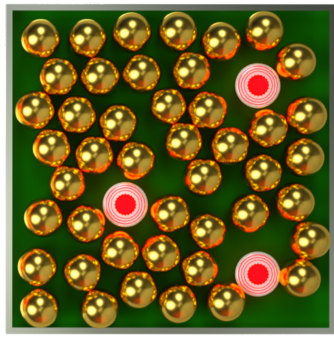
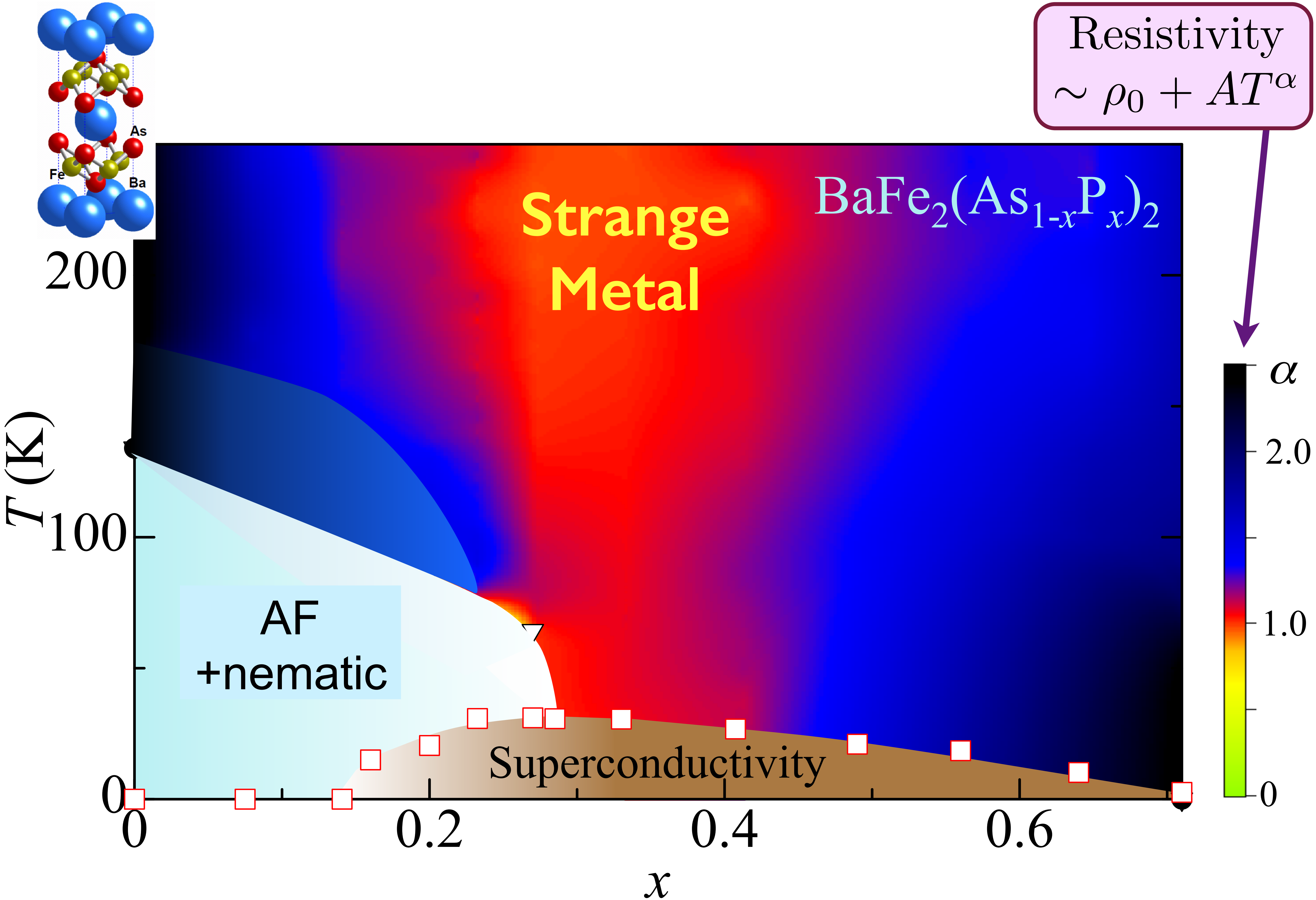


$$\begin{aligned} H &= H_0 - \int d^d x\, h(x)\, \vec{\varphi}^2(x) \\ \overline{h(x)h(x')} &= h_0^2\, \delta^d(x - x') \end{aligned}$$

- Memory function methods yield

$$\begin{aligned} \frac{1}{\tau_{\rm imp}} &\sim \lim_{\omega \to 0} h_0^2 \int d^2 q\, q_x^2 \frac{\mathrm{Im}\left(G^{\rm R}_{\varphi_\alpha^2, \varphi_\alpha^2}(q,\omega)\right)_{H_0}}{\omega} \\ &\sim h_0^2\, T \quad \text{(up to logarithms)} \end{aligned}$$

A.A. Patel and S. Sachdev, PRB **90**, 165146 (2014)



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Stronger disorder

- In the conductivity formula

$$\sigma = \frac{\mathcal{Q}^2}{\mathcal{M}} \left(\frac{1}{-i\omega + 1/\tau_{\text{imp}}} \right) + \sigma_Q(\omega),$$

when including the σ_Q contribution, we should also include the correction to the Drude weight $\mathcal{Q}^2/\mathcal{M}$ at order $1/\tau_{\text{imp}}$.

- From solvable holographic models, it appears the equation

$$\partial_\mu T^{\mu i} = \ldots - \frac{T^{it}}{\tau_{\text{imp}}} + \ldots$$

should be replaced by

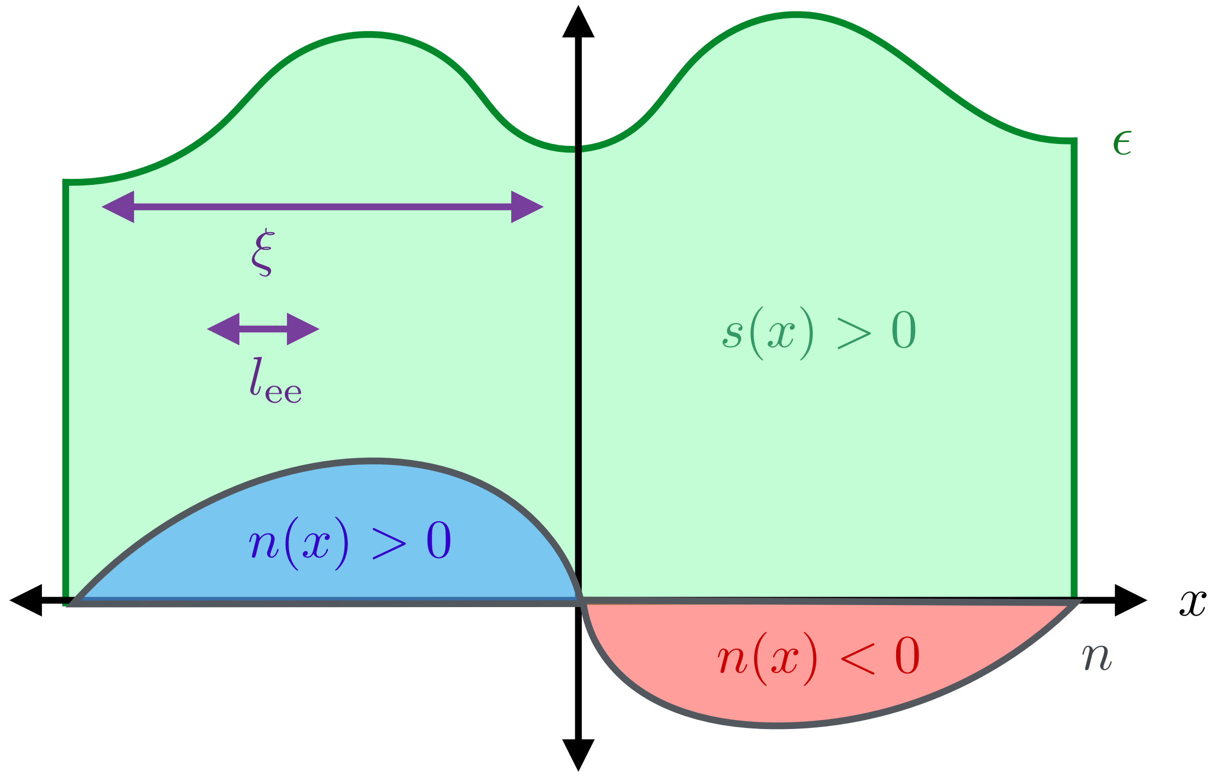
$$\partial_\mu T^{\mu i} = \ldots - \frac{J^i_{\mathcal{S}}}{\tau_{\text{imp}}} + \ldots$$

where $J^i_{\mathcal{S}}$ is the heat current.

R. A. Davison and B. Gouteraux, arXiv:1411.1062

M. Blake, arXiv:1505.06992

Stronger disorder



Note
$n \equiv \mathcal{Q}$

Figure 3: A cartoon of a nearly quantum critical fluid where our hydrodynamic description of transport is sensible. The local chemical potential $\mu(\mathbf{x})$ always obeys $|\mu| \ll k_{\rm B}T$, and so the entropy density $s/k_{\rm B}$ is much larger than the charge density $|n|$; both electrons and holes are everywhere excited, and the energy density ϵ does not fluctuate as much relative to the mean. Near charge neutrality the local charge density flips sign repeatedly. The correlation length of disorder ξ is much larger than $l_{\rm ee}$, the electron-electron interaction length.

Numerically solve the hydrodynamic equations in the presence of a x-dependent chemical potential. The thermoelectric transport properties will then depend upon the value of the shear viscosity, η.

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, PRB **93**, 075426 (2016)

Stronger disorder

Hovering Black Holes from Charged Defects

Gary T. Horowitz,[1,*] Nabil Iqbal,[2,†] Jorge E. Santos,[3,‡] and Benson Way[3,§]

We construct the holographic dual of an electrically charged, localised defect in a conformal field theory at strong coupling, by applying a spatially dependent chemical potential. We find that the IR behaviour of the spacetime depends on the spatial falloff of the potential. Moreover, for sufficiently localized defects with large amplitude, we find that a new gravitational phenomenon occurs: a spherical extremal charged black hole nucleates in the bulk: a hovering black hole.

Analog of many-body localization ?

arXiv:1412.1830

Quantum matter without quasiparticles:

- No quasiparticle excitations
- Shortest possible "collision time", or more precisely, fastest possible local equilibration time $\sim \dfrac{\hbar}{k_B T}$
- Continuously variable density, $\mathcal{Q}$
(conformal field theories are usually at fixed density, $\mathcal{Q} = 0$)
- Theory built from hydrodynamics/holography
/memory-functions/strong-coupled-field-theory
- Exciting experimental realization in graphene.

Open problems:

- Measurement of hydrodynamic flow in strange metal of cuprates, pnictides...
- Difference between $\Phi = z$ in holographic EMD theory, and $\Phi = 0$ in Fermi surface coupled to gauge fields.
- Computation of α_Q, $\bar{\kappa}_Q$, and viscosity η in field theories of Fermi surfaces coupled to gauge fields and order parameters.
- Holography of theories without underlying relativistic Hamiltonian *i.e.* which are not relativistic field theories perturbed only by a chemical potential. These are expected to have three independent thermoelectric co-efficients: σ_Q, $\bar{\kappa}_Q$, and α_Q.
- Systematic understanding of sub-leading terms in theories with weak momentum dissipation.
- Strong disorder effects....