Nonlinear Quantum Bifurcations in Semiconductor Superlattices

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Outline

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Nonlinear Coherent Electron States

- In GaAs/AlGaAs quantum wells (QWs), the Coulomb self-interaction of the two-dimensional electron gas (2DEG) leads to the emergence of experimentally observable nonlinear effects:
  - the *depolarization shift* of the intersubband absorption peak
  - modeled considering the interplay between the one-electron density matrix and
  - mean-field (Hartree) potential
  - dipole-coupled optical driving field
  - and thermal dissipation rates (depolarization and decorrelation)
- The nonlinear effects are most pronounced for high carrier densities and driving field amplitudes

Outline

Nonperturbative theory of intersubband dynamics in quantum wells

- Galdrikian, Batista, and Birnir et al. (GBB) extended the theory of Ando et al. [2]
  - By self-consistent numerical computation of the time-dependent coherent electronic states [8, 7, 4, 3]
  - Nonlinearities were enhanced in slightly *assymetric* quantum wells, containing one or more 'step' features in the confinement potential (Figure 1)
- One such step, nonlinear phenomena such as bistability, period-doubling bifurcations, and period-doubling cascades to chaos were observed in the simulations
- Two or more steps resulted in a Hopf bifurcation of the periodic orbit, giving quasi-periodic orbits on a torus, followed by a period-doubling cascade of the associated invariant tori

The two-step Quantum Well

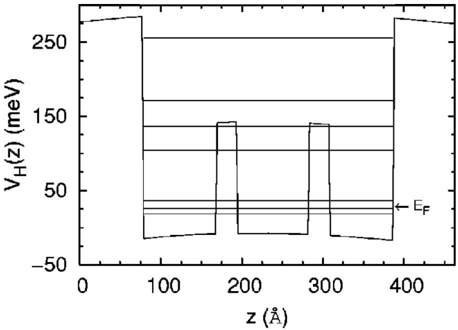


Figure: The stationary self-consistent potential of the assymetric GaAs-AlGaAs quantum well from Batista et al. [4]. The eigenstates' energy levels are indicated by the horizontal lines.

Outline

GaAs/AlGaAs superlattices (SLs)

- Dynamics are similarly dominated by the interplay between the electronic populations of each site and the self-consistent potential
- Well-described by the resonant sequential tunneling (RST) model of Bonilla et al [5]
- In the RST model, the electronic states are assumed to be localized at each site of the SL, while the electronic states in the GBB model simultaneously occupy multiple regions between the steps
- Intersubband dynamics at each site of the RST model are not resolved in time; instead, they are treated according to the Chapman-Enskog method [6]

Quantum Wells layered together in a Superlattice

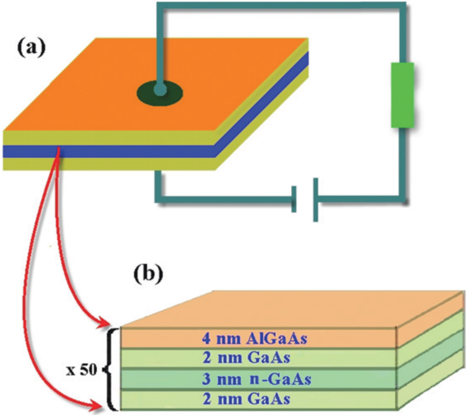


Figure: A mesa-shaped SL of 1.2 *mm* (square) width and 1.5 μm thickness. (a) shows a dc voltage-biased SL consisting of two contact regions of about 0.5 μm thickness each and 50 periods, formed by the two semiconductor layers of different bandgaps (b).

Nonlinear Bifurcations in a Superlattice

- We study the usual sequential tunneling model of electron transport in a weakly coupled n-doped SL, under a (DC) voltage bias and search for nonlinear bifurcations of coherent electron states, parametrized by DC voltage V.
- The model is adapted from M. Alvaro, M. Carretero, and L. Bonilla, [1].
- Let the electric field, in well i, be F_i, where $i = 1, ..., N (N \leq 50)$ is the number of SL periods, and $J_{i\to i+1}$ and $J(t)$ is the tunneling current density from well i to well $i+1$, and the total current density, respectively.

Superlattice Equations

$$\varepsilon \frac{dF_i}{dt} + J_{i\to i+1} = J(t), \tag{1}$$

$$J^-_{i\to i+1} = \frac{en_i}{l} v^{(f)}(F_i) - J^-_{i\to i+1}(F_i, n_{i+1}, T), \tag{2}$$

$$n_i = N_D + \frac{\varepsilon}{e}(F_i - F_{i-1}), \tag{3}$$

$$J^-_{i\to i+1}(F_i, n_{i+1}, T) = \frac{em^* k_B T}{\pi \hbar^2 l} v^{(f)}(F_i) \ln \left[1 + e^{-\frac{eF_i l}{k_b T}} \left(e^{\frac{\pi \hbar^2 n_{i+1}}{m^* k_b T}} - 1 \right) \right] \tag{4}$$

$$J_{0\to 1} = \sigma_0 F_0, \; J_{N\to N+1} = \sigma_0 \frac{n_N}{N_D} F_N, \tag{5}$$

$$\sum_{i=1}^{N} F_i = \frac{V}{l}. \tag{6}$$

Outline

The Bifurcation Diagram

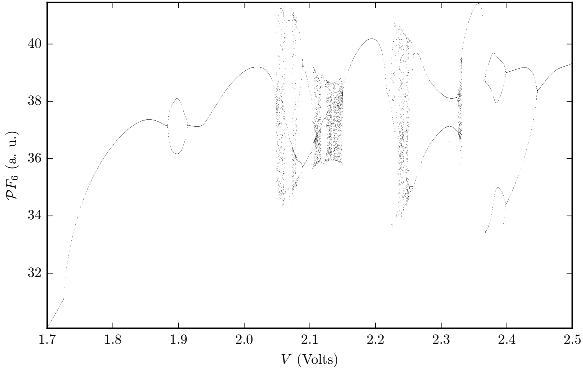
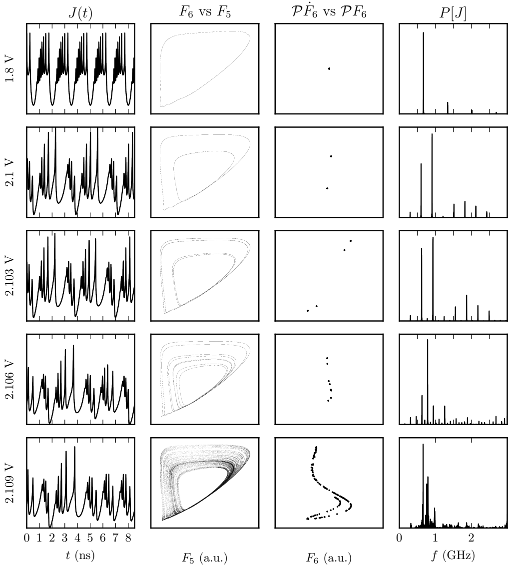


Figure: The bifurcation diagram for $N = 10$ and $V_{barr} = 600$ meV. The Hopf bifurcation from the steady state occurs at the kink above 1.7V. The period-doubling cascade occurs in the shaded region above 2.1V.

The Bifurcations



Outline

Bifurcations and Devices

- We simulate SLs that can be used as sources, period halvers and squeezers, random sequence generators and frequency mixers, at room temperature
  - Hopf bifurcation: The first bifurcation is the Hopf bifurcation from stationary state to periodic orbit. This electron state is a *source*; it will emit radiation at the frequency $\omega = 2\pi/T$, where T is the fundamental period, and at its superharmonics.
  - Period doubling bifurcation: The second bifurcation that we find in the SL is the period-doubling bifurcation. Now the periodic orbit has period twice that of the original periodic orbit, so the frequency is halved: $\omega \to \omega/2$. Can then be used to compress information into a desirable frequency range or to squeeze out of it undesirable noise [9].

A Strange Attractor

- The period doubling cascade leads to a strange attractor:
  - Period doubling cascade: The period doubling of the periodic orbit continues into a period-doubling cascade to a strange attractor. Based upon the emergence of a broad peak between the two strongest harmonics, we conclude that the invariant manifold is a strange attractor.
  - A practical application of the dynamics in this regime is ultrafast generation of random number sequences [12]. This has many applications in areas such as secure communication and data storage, stochastic modeling, and Monte Carlo simulations.

The Design of the Superlattice

- The design (b) that effectively quenches the background noise is taken from Huang et al. [11, 10].

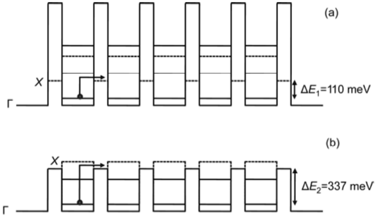


Figure: Two superlattices, respectively $GaAs/AlAs$ (a) and (b) $GaAs/Al_{0.45}Ga_{0.55}As$.

Conclusions

- This work demonstrates that semiconductor heterostructures, consisting of a superlattice of weakly-coupled quantum wells under a DC voltage bias, are nonlinear quantum systems
- The coherent electronic dynamics in the superlattice bifurcate with increased voltage.
- In our simulations, we find a Hopf bifurcation, a period-doubling of the resulting periodic orbit and a period-doubling cascade to a strange attractor
- These bifurcations are observable at room temperature. These enable the design of a number of devices operating in the GHz range.

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