Workshop «Variational Models of Fracture»

A variational approach to gradient plasticity

G. Lancioni

Dipartimento di Ingegneria Civile, Edile e Architettura,
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Joint work with R. March (Rome), G. Del Piero (Ferrara), G. Zitti (Ancona),
T. Yalcinkaya (Ankara), A. Cocks (Oxford)

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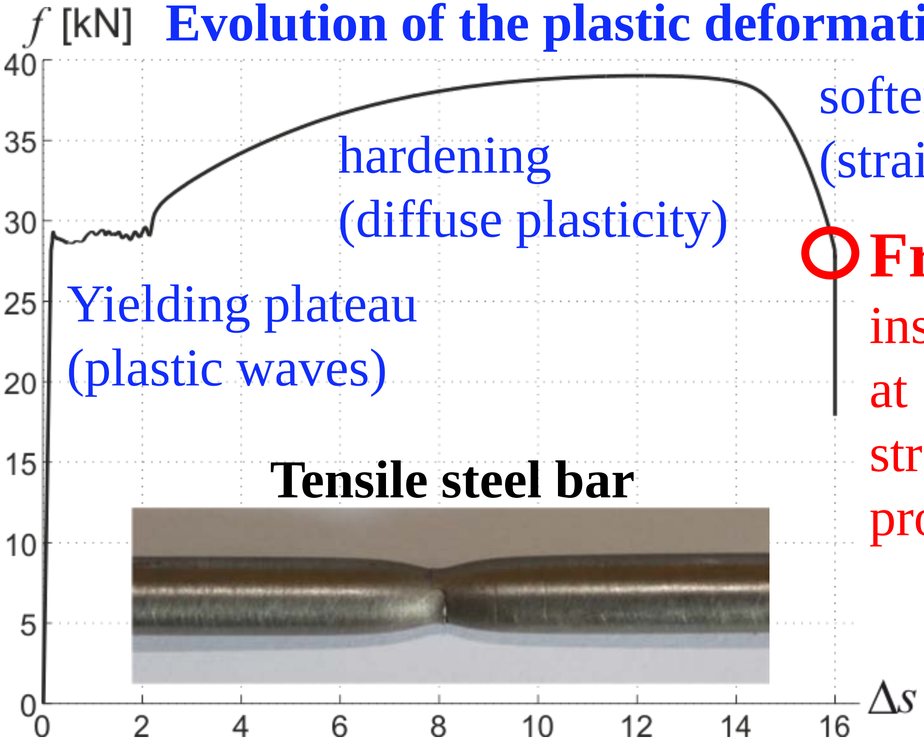
Variational Model

Evolution of the plastic deformation



Fracture: material instability occurring at the end of the strain localization process

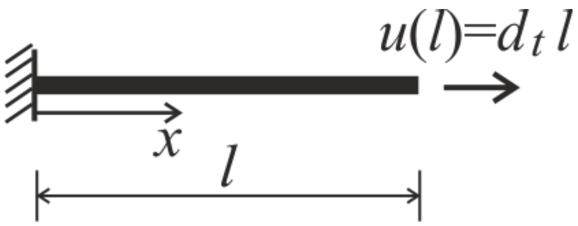
- Rate-independent
- Non-convex totally dissipative plastic energy
- Non-local energy, depending on the plastic strain gradient
- Evolution of the deformation -> Incremental energy minimization

References

- Del Piero, Lancioni, March, JMMS, 2013
- Lancioni, J. Elasticity, 2015

Modeling assumptions

1D Pb.



i. Kinematics $u'(x)=\underbrace{\varepsilon(x)}_{\text{elastic def.}}+\underbrace{\gamma(x)}_{\text{plastic def.}}$

ii. Energy

internal variable $e=e(\gamma)$. In isotropic gradient plasticity $e_t=\int_0^t|\dot{\gamma}_\tau|\,d\tau,\ \dot{e}_t=|\dot{\gamma}_\tau|$

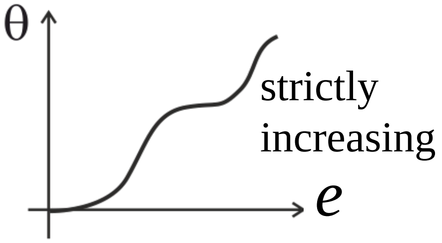
accumulated plastic strain

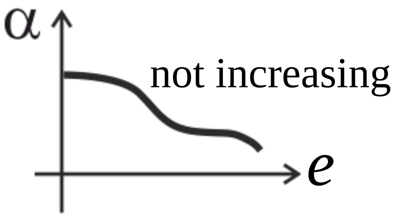
$$E(u,\gamma,e)=\int_0^l(\boxed{\frac{1}{2}E(u'-\gamma)^2}+\boxed{\theta(e)}+\boxed{\frac{1}{2}\alpha(e)e'^2})dx$$

stored elastic energy

dissipative plastic energy

non-local stored plastic energy





iii. Dissipation $\frac{d}{dt}\theta(e)=\theta'(e)\dot{e}\geq 0$

Energy: $$\boxed{E(u,\gamma,e)=\int_0^l \left(\frac{1}{2}E(u'-\gamma)^2+\theta(e)+\frac{1}{2}\alpha(e)e'^2\right)dx}$$

Comparisons with **non-local variational approaches** in literature

Damage energy [Bourdin, Francfort, Marigo, 2000],…

$$E(u,\alpha)=\int_0^l \left(\frac{1}{2}E(\alpha)u'^2+\theta(\alpha)+\frac{1}{2}A\alpha'^2\right)dx$$

Damage and plasticity energy

[Ambrosio, Lemenant, Royer-Carfagni, 2013], [Freddi, Royer-Carfagni, 2014]

$$E(u,\alpha)=\int_0^l \left(\frac{1}{2}E(\alpha)u'^2+\theta(\alpha)+\frac{1}{2}A\alpha'^2+\sigma_0\alpha^2\,|u'|\right)dx$$

yielding stress

[Alessi, Marigo, Vidoli, 2014, 2015]

$$E(u,\alpha,\gamma,e)=\int_0^l \left(\frac{1}{2}E(\alpha)(u'-\gamma)^2+\theta(\alpha)+\frac{1}{2}A\alpha'^2+\sigma_p(\alpha)e\right)dx$$

Damage + von Mises plasticity

Equilibrium

$$\delta E(u,\gamma,e;\delta u,\delta\gamma,|\,\delta\gamma\,|)\geq 0$$

⇩

$$\begin{cases} \sigma'=0, \quad \text{with} \quad \overset{\text{normal stress}}{\sigma} = E(u'-\gamma) \\ \underset{\textbf{Yield condition}}{\boxed{|\,\sigma\,|\leq \sigma^{c}}} \quad \text{with} \quad \underset{\text{Yield limit}}{\sigma^{c}} = \theta'+\frac{1}{2}\alpha' e'^{2}-\frac{d}{dx}(\alpha e') \\ e(0)=e(l)=0, \ \text{or} \ e'(0)\leq 0, \ e'(l)\geq 0 \quad \textbf{b.c.} \end{cases}$$

Flow rule

$$E_{t+\tau}(\dot{u}_t,\dot{\gamma}_t,\dot{e}_t) = E_t + \tau\dot{E}_t(\dot{u}_t,\dot{\gamma}_t,\dot{e}_t),$$

with $\dot{E}_t = \int_0^l (-sign(\dot{\gamma}_t)\sigma_t + \sigma_t^c)\dot{e}\,dx$

Necessary condition for a minimum

$$\delta\dot{E}_t(\dot{u}_t,\dot{\gamma}_t,\dot{e}_t;0,0,\delta e) \geq 0,\ \forall\,\delta e:\ \dot{e}_t + \delta e \geq 0$$

$$\Downarrow$$

$$\boxed{\dot{e}_t \geq 0, \quad -sign(\dot{\gamma}_t)\sigma_t + \sigma_t^c \geq 0, \quad (-sign(\dot{\gamma}_t)\sigma_t + \sigma_t^c)\dot{e}_t = 0}$$

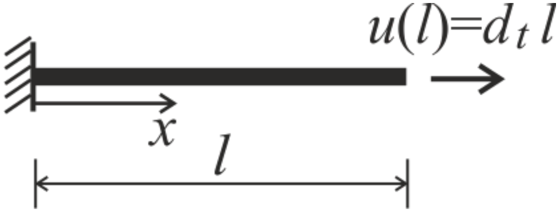
$$\Downarrow$$

$$\dot{\gamma}_t = \frac{\sigma_t}{\sigma_t^c}\dot{e}_t$$ **Flow rule**

Tensile test

Assume $d_t \geq 0$

$\sigma_t \geq 0 \Rightarrow \dot{\gamma} = \dot{e}$



$$\boxed{E(u,\gamma) = \int_0^l \left(\frac{1}{2}E(u'-\gamma)^2 + \theta(\gamma) + \frac{1}{2}\alpha(\gamma)\gamma'^2\right)dx}$$

Dissipation inequality $\dot{\gamma} \geq 0$

Quasi-static evolution

Incremental minimum problem

$$(u_t,\gamma_t)\to(u_{t+\tau},\gamma_{t+\tau}),\qquad \varepsilon_{t+\tau}=u'_{t+\tau}-\gamma_{t+\tau}$$

$$u_{t+\tau}=u_t+\tau\dot{u}_t,$$

$$\gamma_{t+\tau}=\gamma_t+\tau\dot{\gamma}_t$$

$$E(u_{t+\tau},\gamma_{t+\tau})\approx E(u_t,\gamma_t)+\tau\,\dot{E}(u_t,\gamma_t)+\tfrac{1}{2}\tau^2\ddot{E}(u_t,\gamma_t)=$$

$$=E(u_t,\gamma_t)+\tau\,F(u_t,\gamma_t;\dot{u}_t,\dot{\gamma}_t)$$

quadratic functional

$$\boxed{(\dot{u}_t,\dot{\gamma}_t)=\arg\min\{F(u_t,\gamma_t;\dot{u}_t,\dot{\gamma}_t),\ \dot{\gamma}\geq 0,\ \text{b.c.}\}}$$

Constrained quadratic programming pb.

Necessary condition for a minimum

$$\delta F(\dot{u}_t, \dot{\gamma}_t; \delta\dot{u}, \delta\dot{\gamma}) \geq 0$$

$$\dot{\gamma}_t + \delta\gamma \geq 0$$

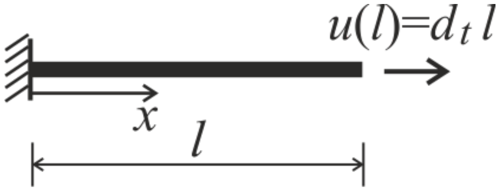
$$\Downarrow$$

Kuhn-Tucker conditions (**flow rule**)

$$\left\{\begin{array}{l} \dot{\sigma}' = 0, \\ \boxed{\dot{\gamma}_t \geq 0, \quad \sigma_t + \tau\dot{\sigma}_t \leq \sigma_t^c + \tau\dot{\sigma}_t^c, \quad \underbrace{[\sigma_t + \tau\dot{\sigma}_t - (\sigma_t^c + \tau\dot{\sigma}_t^c)]\dot{\gamma}_t = 0}_{\textbf{consistency condition}}} \end{array}\right.$$

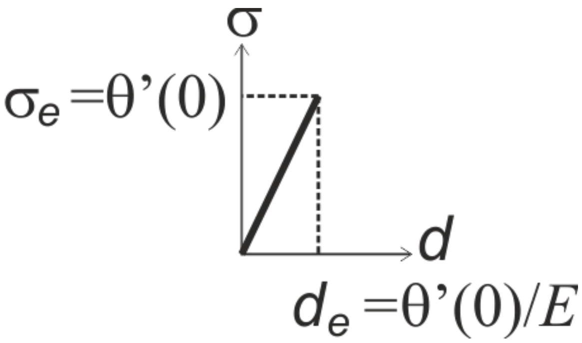
(the yield function maintains
equal to zero when γ grows)

Elastic evolution



1. *Elastic regime*

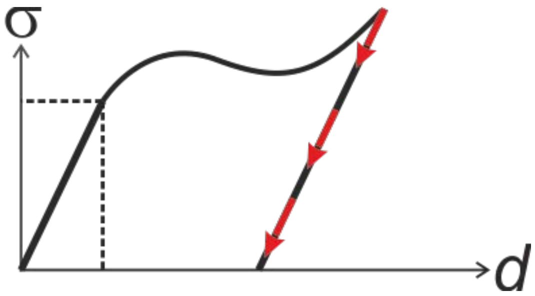
$$0 < d_t \le d_e = \theta'(0)/E$$



$$u_t = xd_t,$$
$$\gamma_t = 0,$$
$$\sigma_t = Ed_t$$

2. *Elastic unloading*

$$\dot{d}_t < 0, \quad d_t \ge 0$$



$$\dot{\gamma}_t = 0,$$
$$\dot{\sigma}_t = E\dot{d}_t < 0$$

Evolution of plastic def. from homogeneous configurations

$$u_t = \text{const}, \quad \gamma_t = \text{const}, \quad \sigma_t = \sigma_t^c,$$

bc: $\dot{\gamma}_t(0) = 0 \qquad \dot{\gamma}_t(l) = 0$

$\dot{u}_t(0) = 0 \qquad \dot{u}_t(l) = \dot{d}_t l$

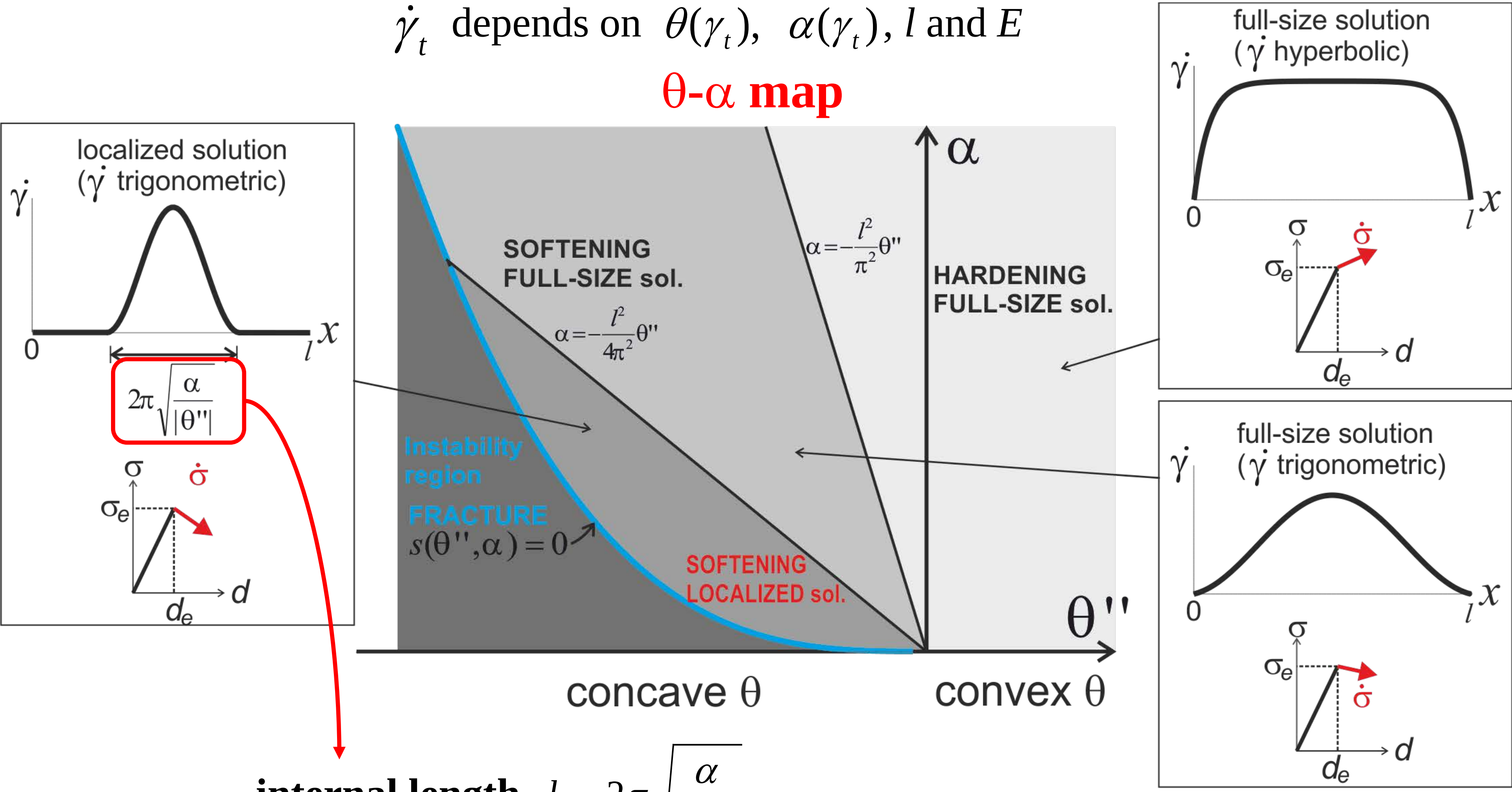
Evolution pb.

$$\begin{cases} \dot{\gamma}_t \geq 0, \\ \dot{\sigma}_t \leq \dot{\sigma}_t^c, \ \text{with} \ \dot{\sigma}_t = E(\dot{d}_t - \frac{1}{l}\int_0^l \dot{\gamma}_t dx) \\ [\dot{\sigma}_t - \dot{\sigma}_t^c)]\dot{\gamma}_t = 0 \end{cases} \quad \Rightarrow \quad \boxed{\dot{\gamma}_t}$$

$$\boxed{\dot{\varepsilon}_t} = \dot{d}_t - \frac{1}{l}\int_0^l \dot{\gamma}_t dx$$

$\dot{\gamma}_t$ depends on $\theta(\gamma_t)$, $\alpha(\gamma_t)$, l and E

θ-α map



internal length $l_i = 2\pi\sqrt{\frac{\alpha}{|\theta''|}}$

$$\left.\begin{array}{l} l \le l_i \quad \Rightarrow \quad \text{full size sol.} \\ l > l_i \quad \Rightarrow \quad \text{localized sol.} \end{array}\right\} \text{manifestation of the } \textbf{size-effect}$$

Instability and fracture

Sufficient condition for a minimum:

$\delta F \geq 0$, and $\delta^2 F \geq 0$ for all perturbations for which $\delta F = 0$

$$\delta^2 F(\dot{\gamma}_t, \delta\dot{\gamma}) = \int_0^l (\theta''(\gamma_t)\delta\dot{\gamma}^2 + E\delta\bar{\dot{\gamma}}^2 + \alpha\delta\dot{\gamma}'^2)dx \geq 0,$$ for all $\delta\dot{\gamma}$ in the plastic zone,

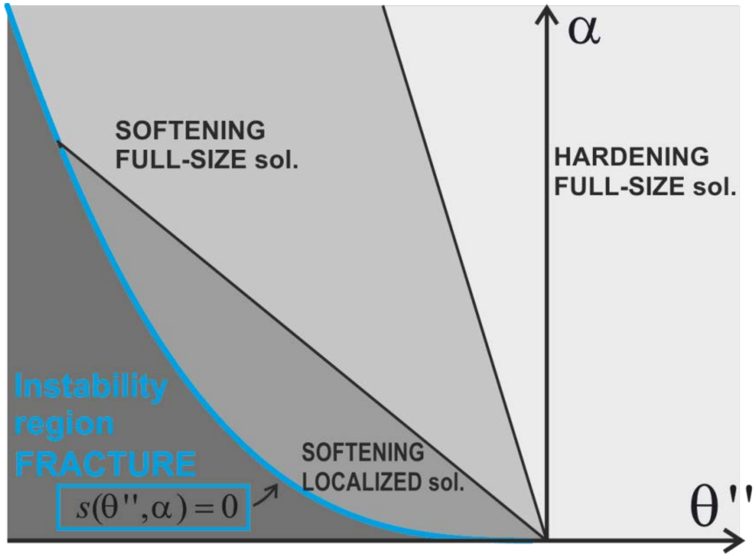
where $\sigma_t = \sigma_t^c$ and $\delta F = 0$

⇕

The smallest eigenvalue ρ_1 of the eigenvalue pb

$$\int_0^l (\theta''(\gamma_t)\delta\dot{\gamma}^2 + E\delta\bar{\dot{\gamma}}^2 + \alpha\delta\dot{\gamma}'^2)dx = \rho\int_0^l \delta\dot{\gamma}^2 dx$$

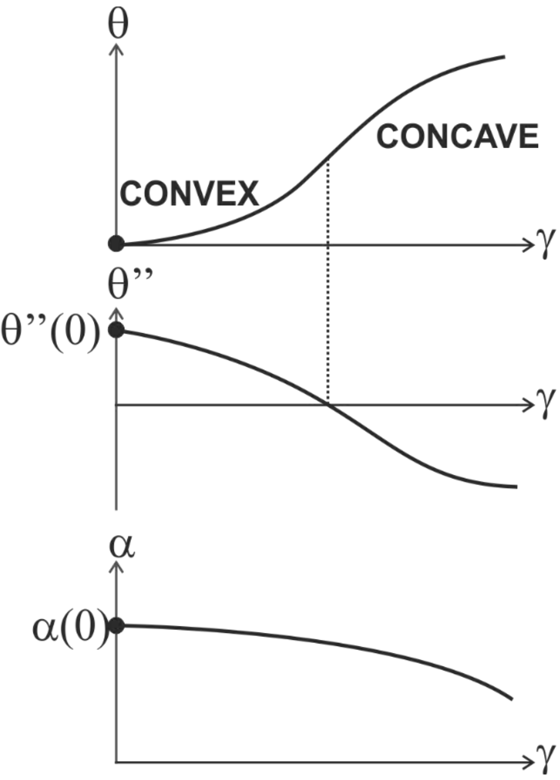
is positive

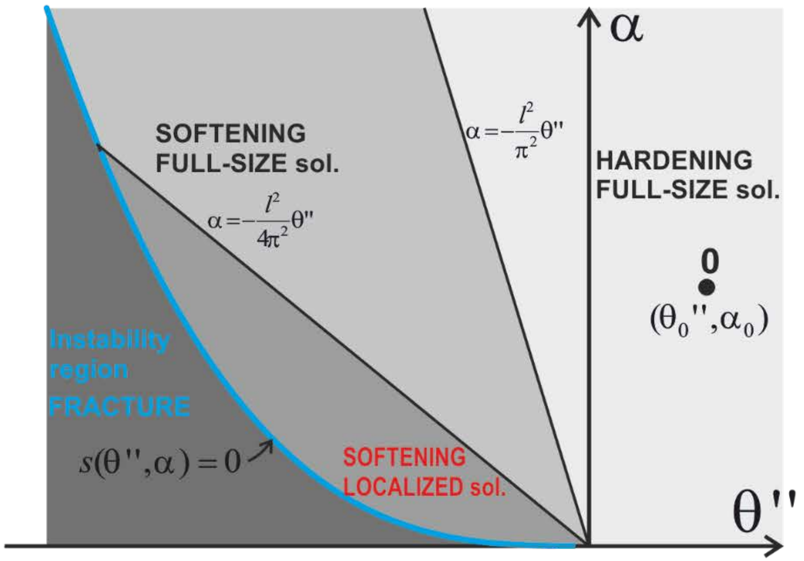


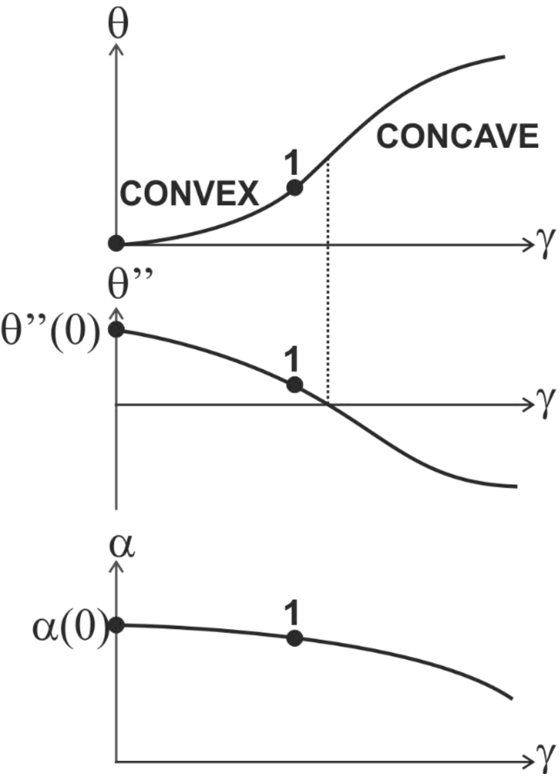
$$\rho_1 \geq 0 \Leftrightarrow \boxed{s(\theta'',\alpha) = \psi(l\sqrt{-\theta''/\alpha}) + \theta''/E \geq 0}$$

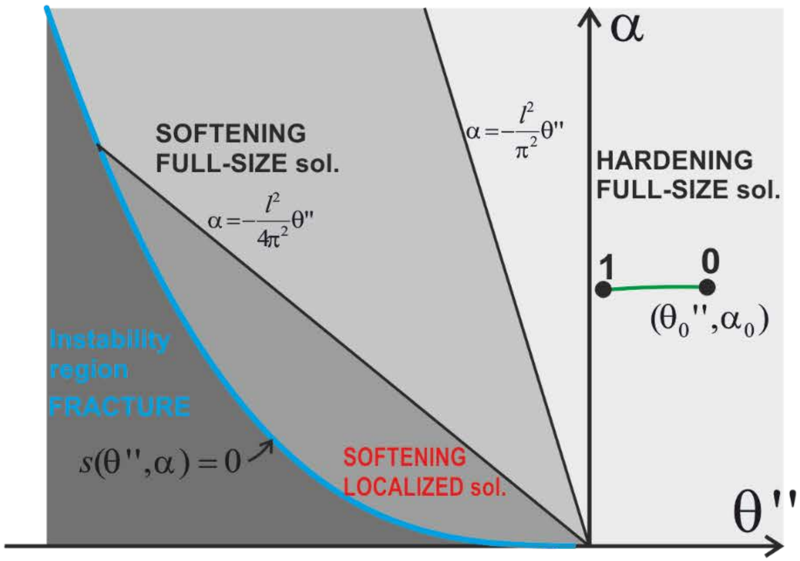
If $s < 0$, F can attain unlimited negative values for perturbations concentrated on intervals of sufficiently small length. ⇒ This situation variationally characterizes ***fractured configurations***.

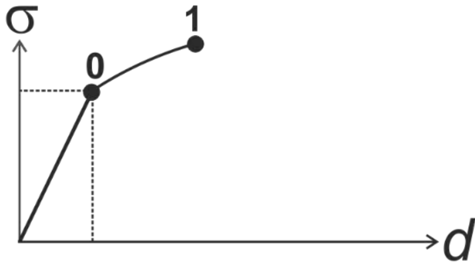
Which shapes to $\theta(\gamma)$ and $\alpha(\gamma)$? ... hints from the analytical solution



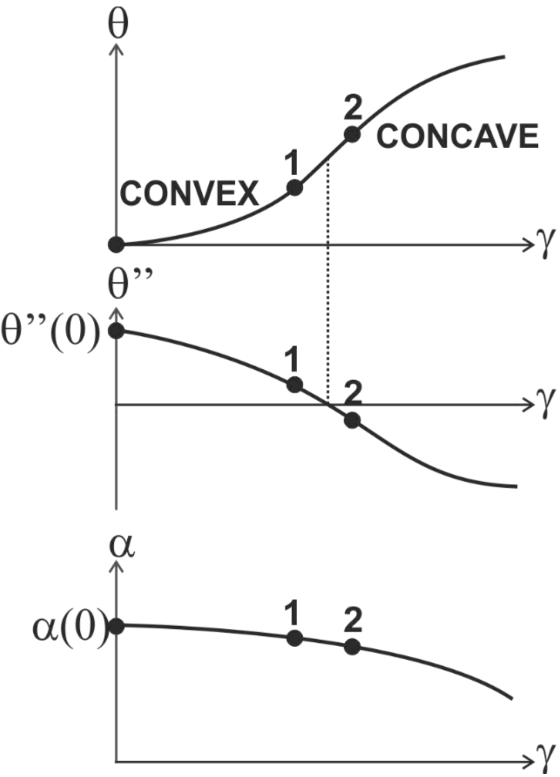


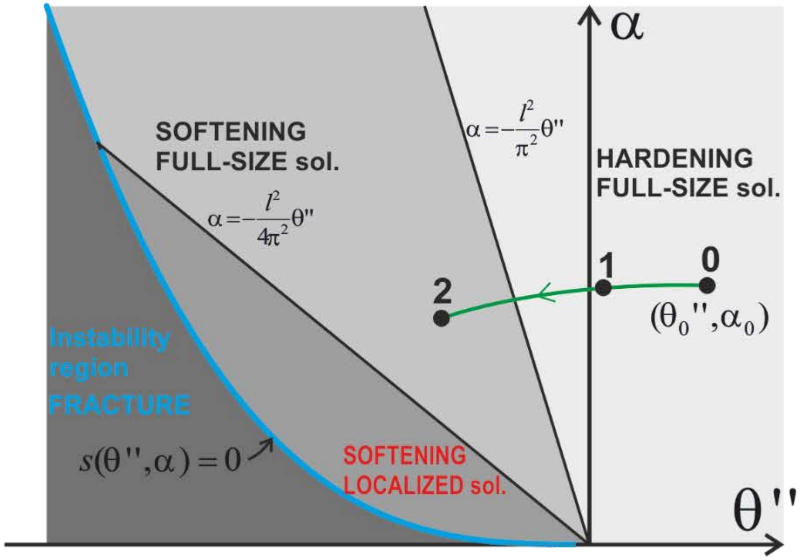


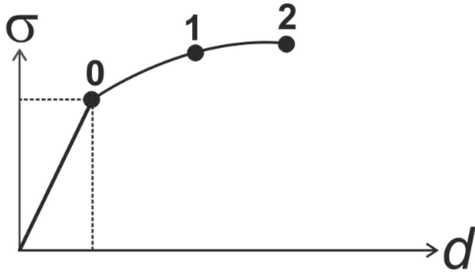




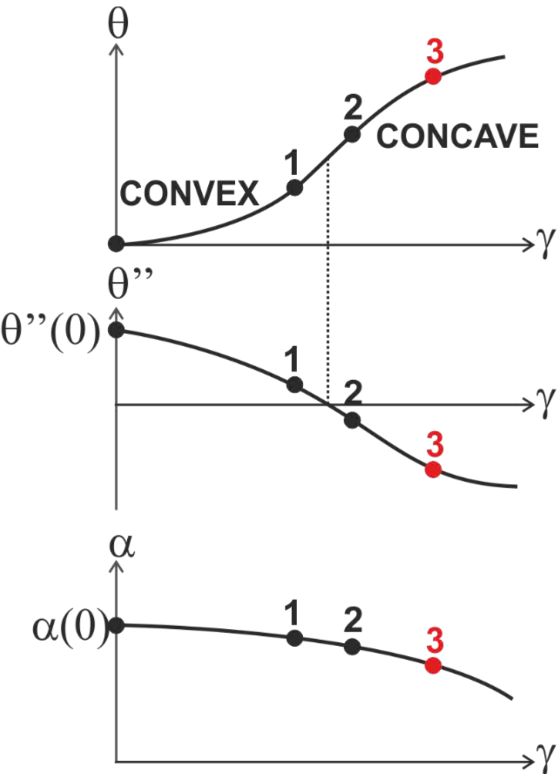
Stress-hardening
Full size solution

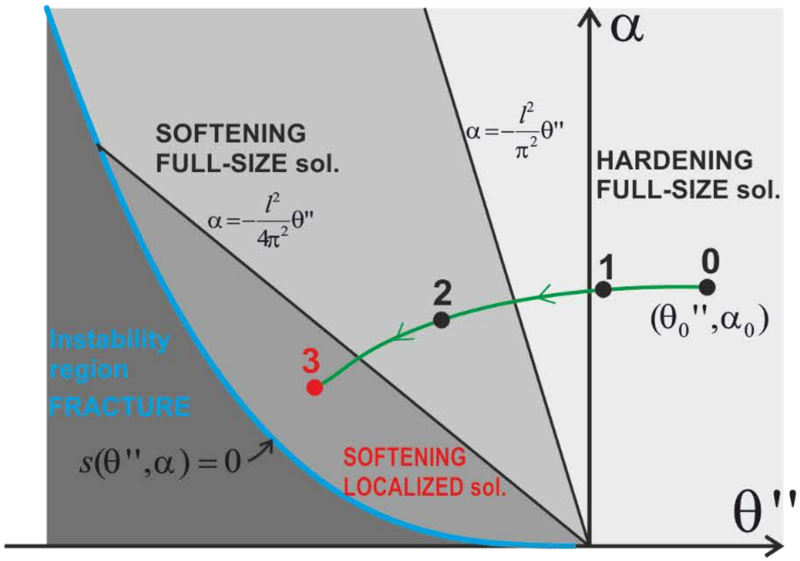


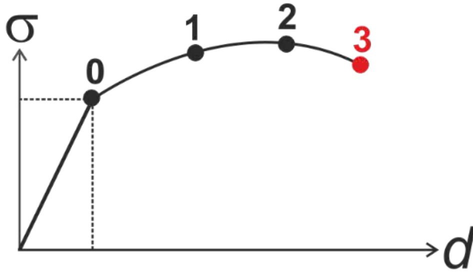




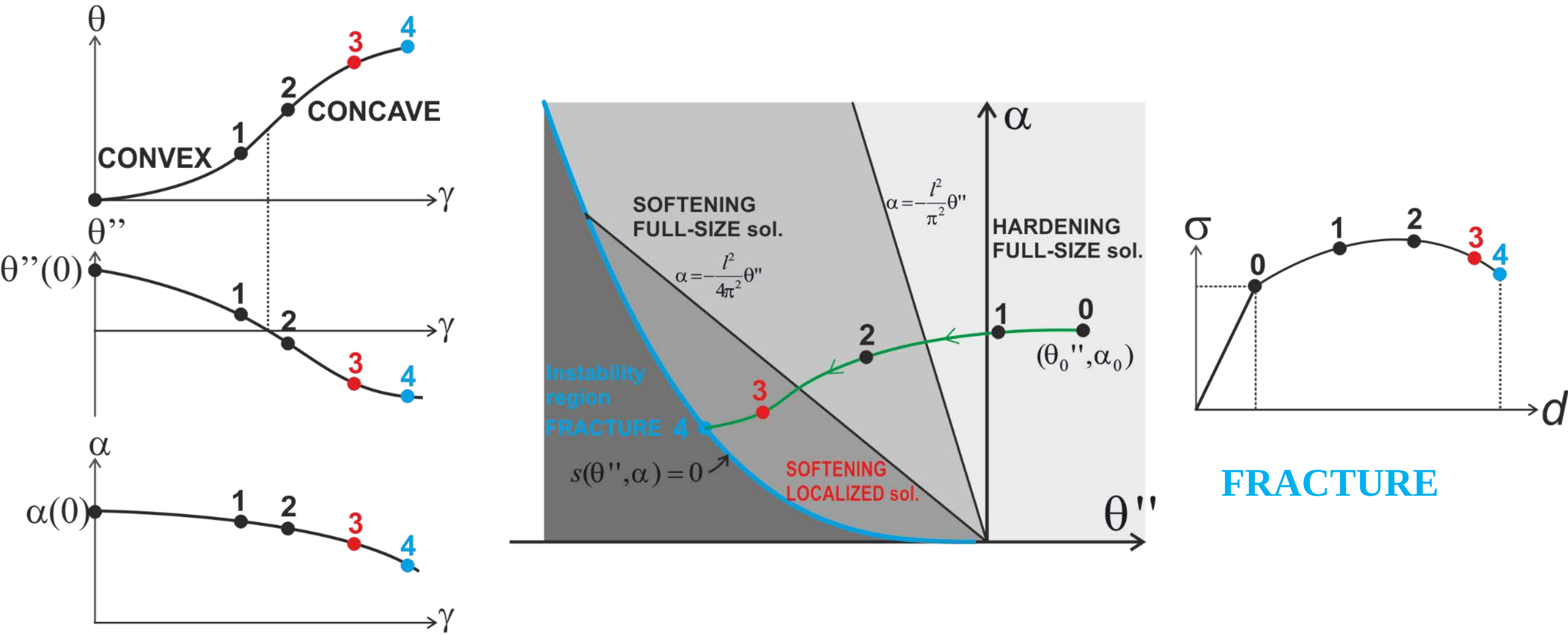
Stress-softening
Full size solution



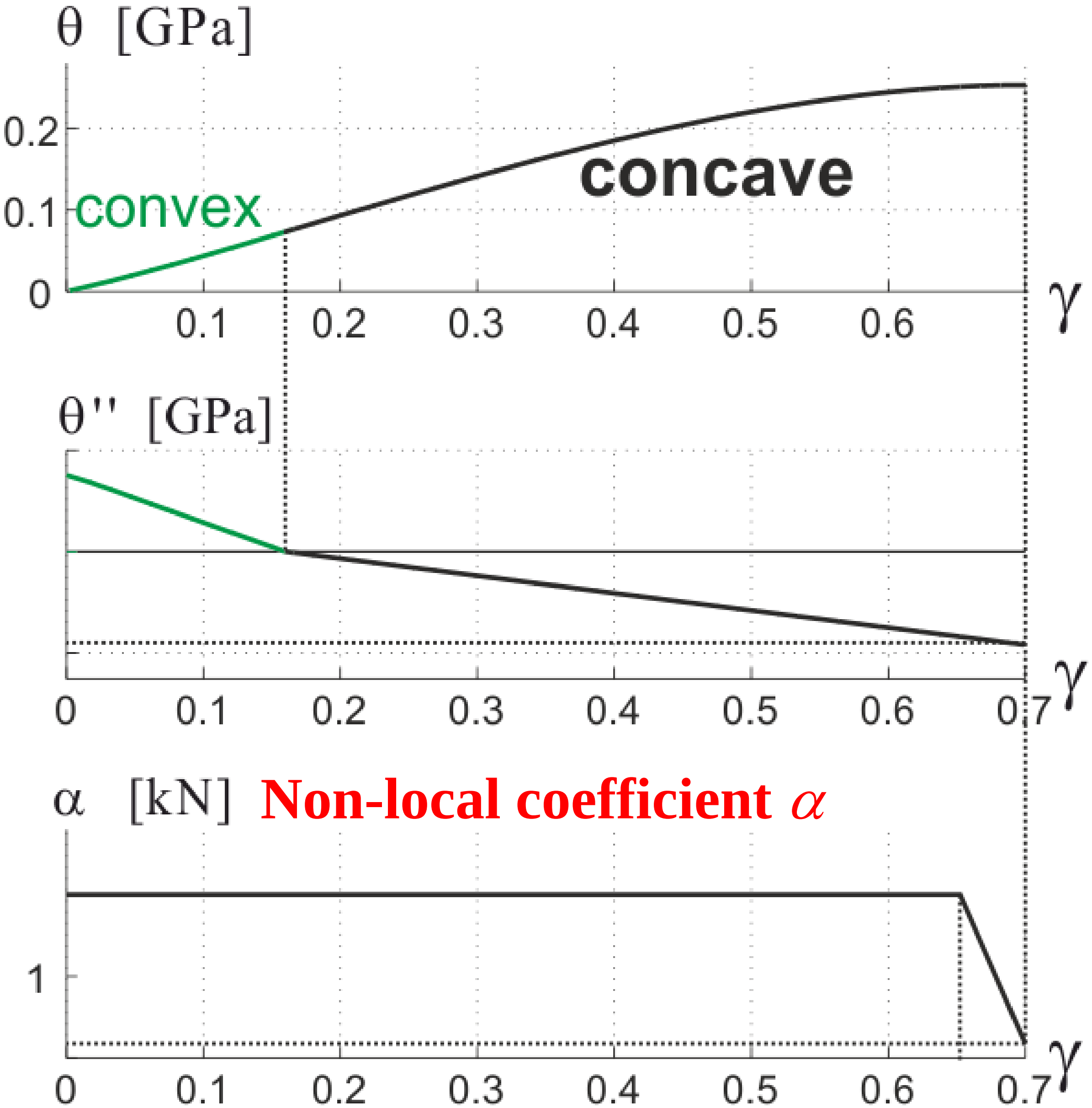




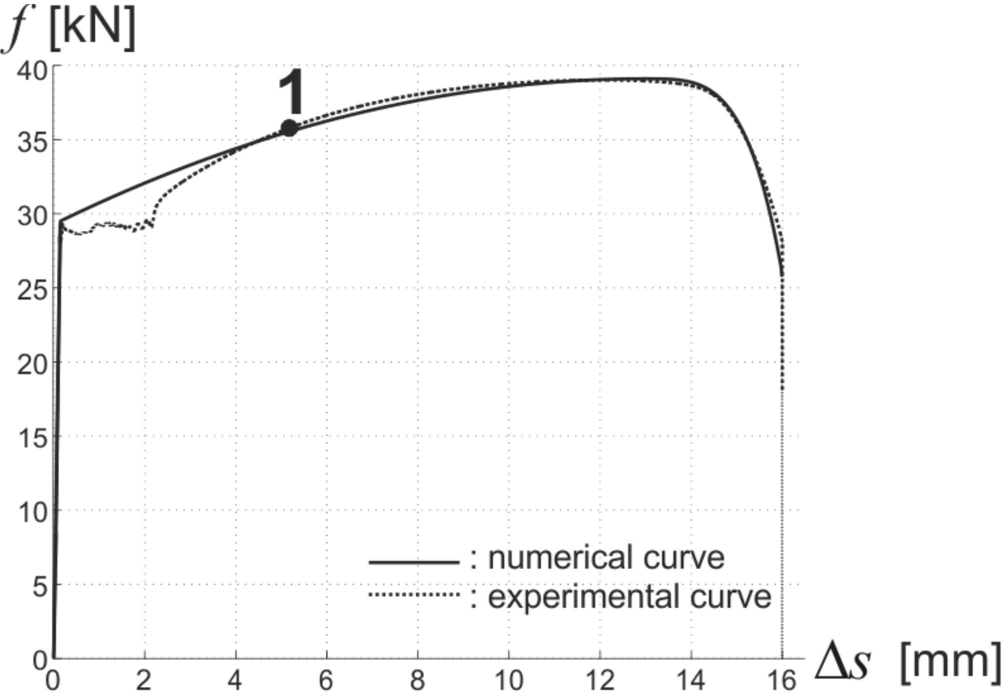
Stress-softening
Localized solution


θ
CONVEX
CONCAVE
1
2
3
4
γ
θ''
θ''(0)
1
2
3
4
γ
α
α(0)
1
2
3
4
γ
SOFTENING FULL-SIZE sol.
$\alpha=-\frac{l^2}{\pi^2}\theta''$
$\alpha=-\frac{l^2}{4\pi^2}\theta''$
HARDENING FULL-SIZE sol.
α
1
0
(θ_0'',α_0)
2
3
Instability region
FRACTURE 4
$s(\theta'',\alpha)=0$
SOFTENING LOCALIZED sol.
θ''
σ
0
1
2
3
4
d
FRACTURE

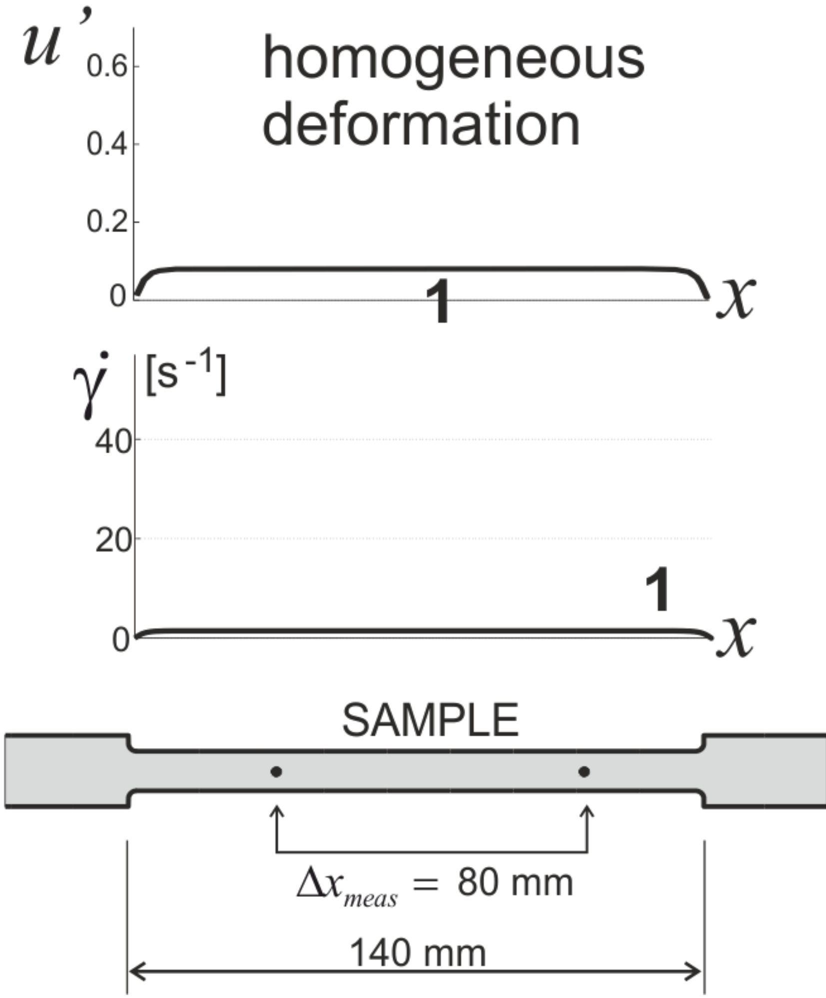
Plastic energy θ, piecewise cubic

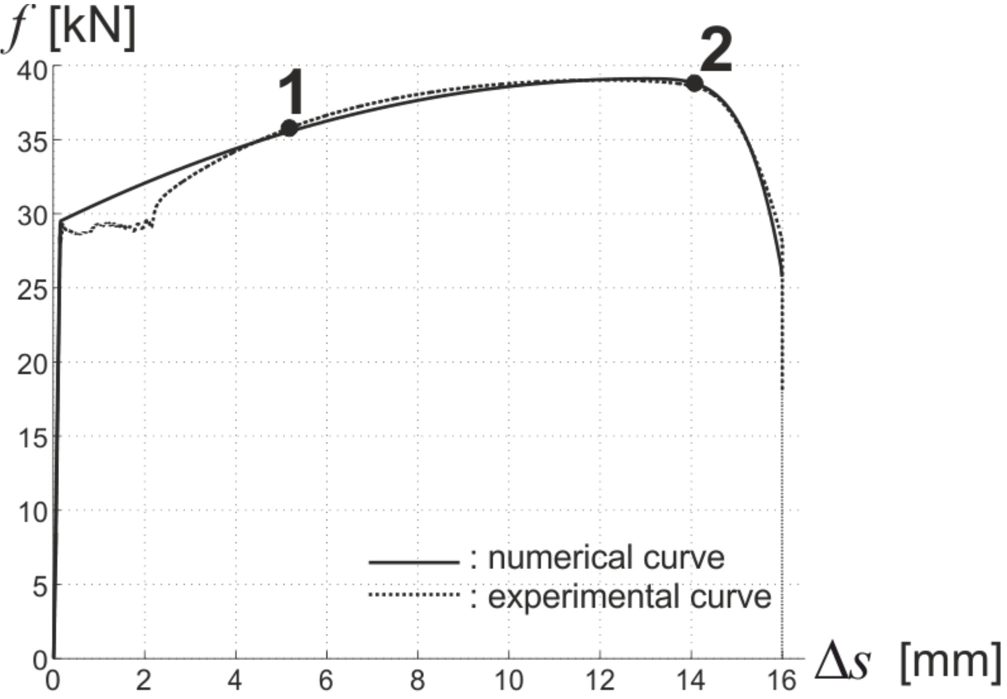


Non-local coefficient α

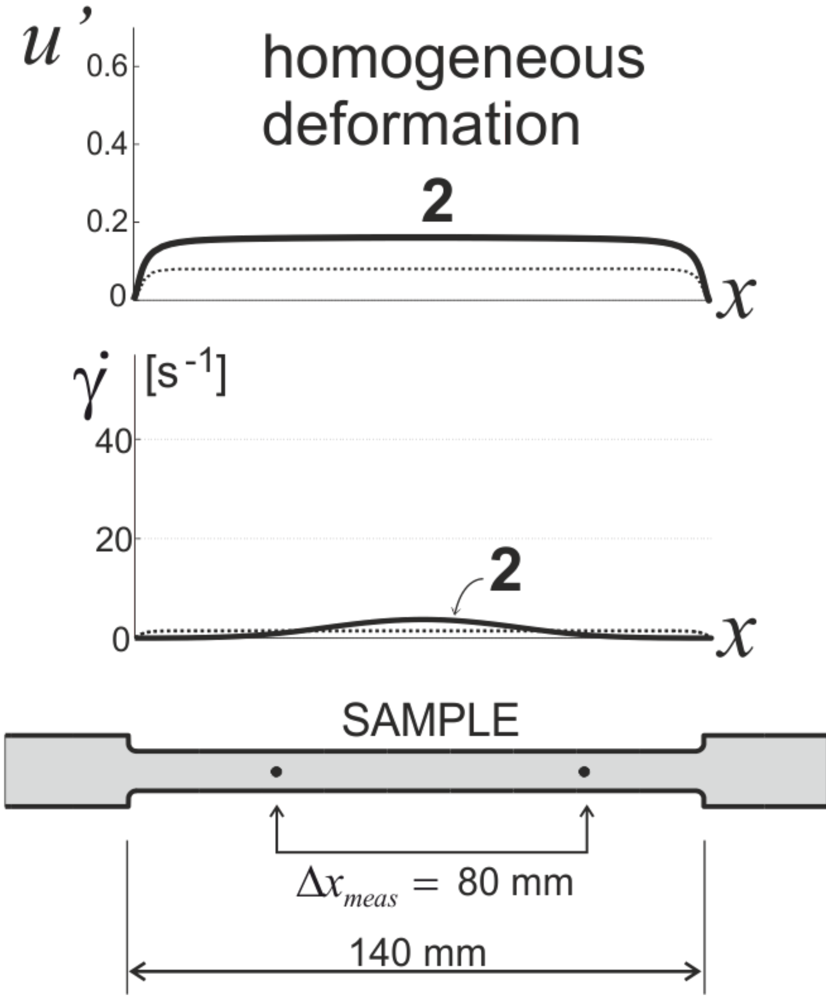


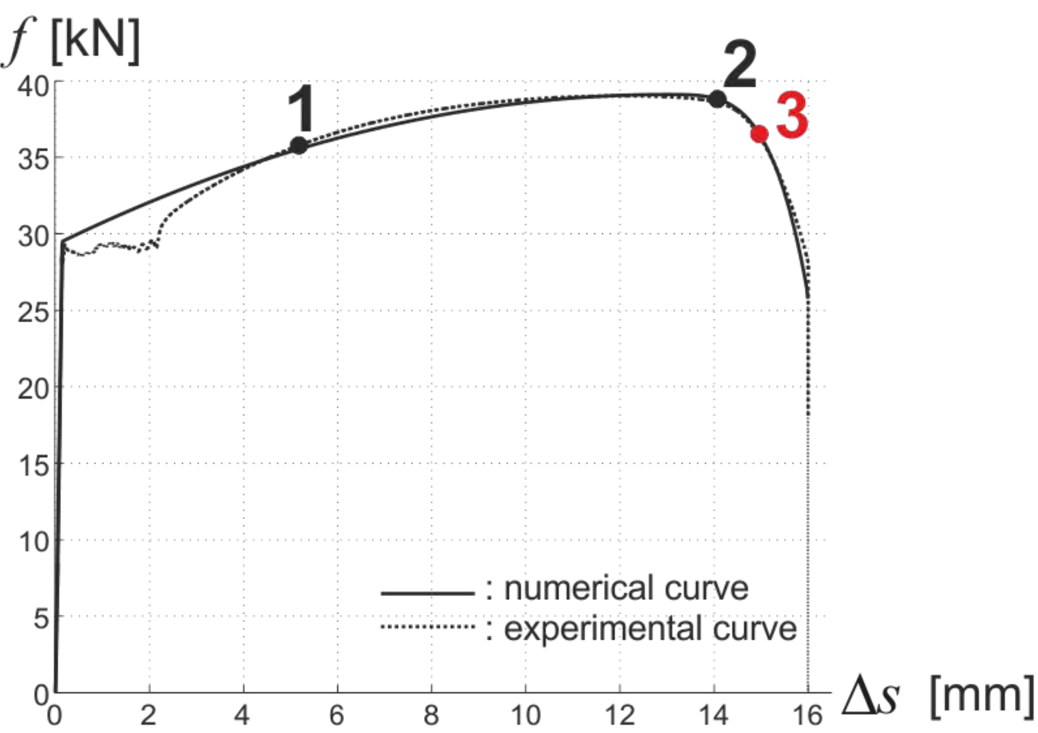
Lancioni, J. Elasticity, 2015



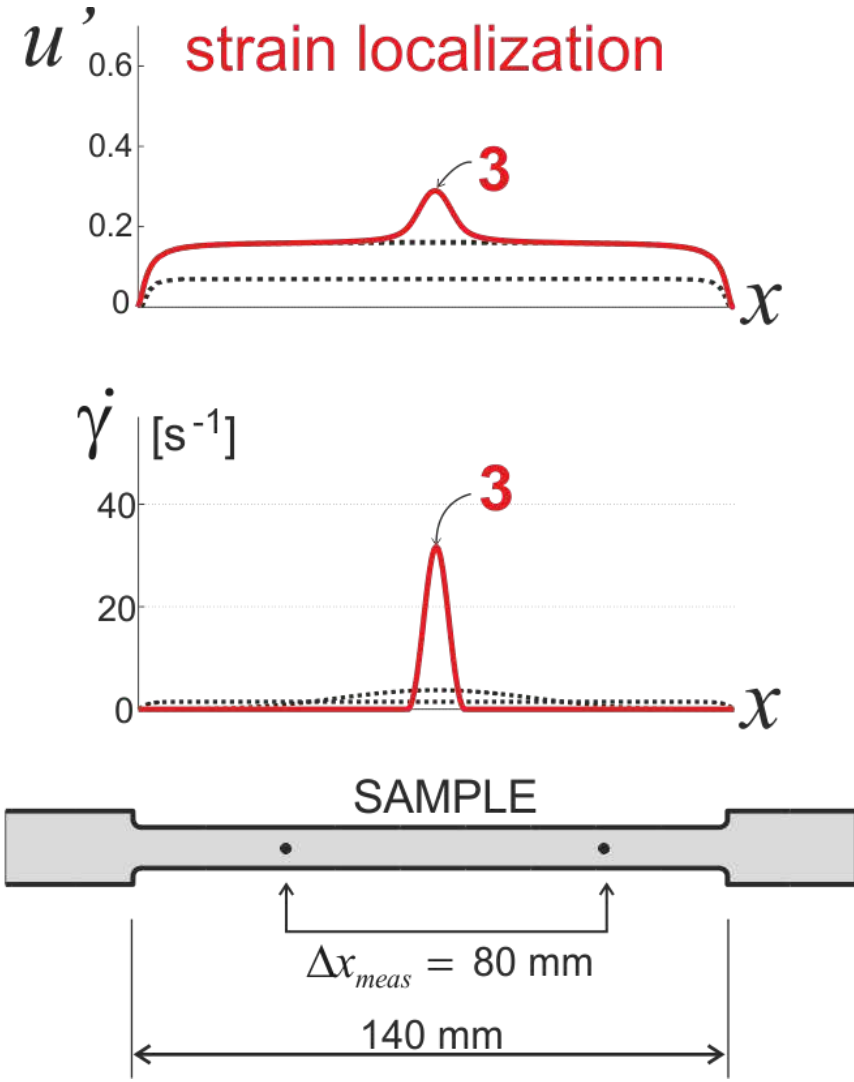


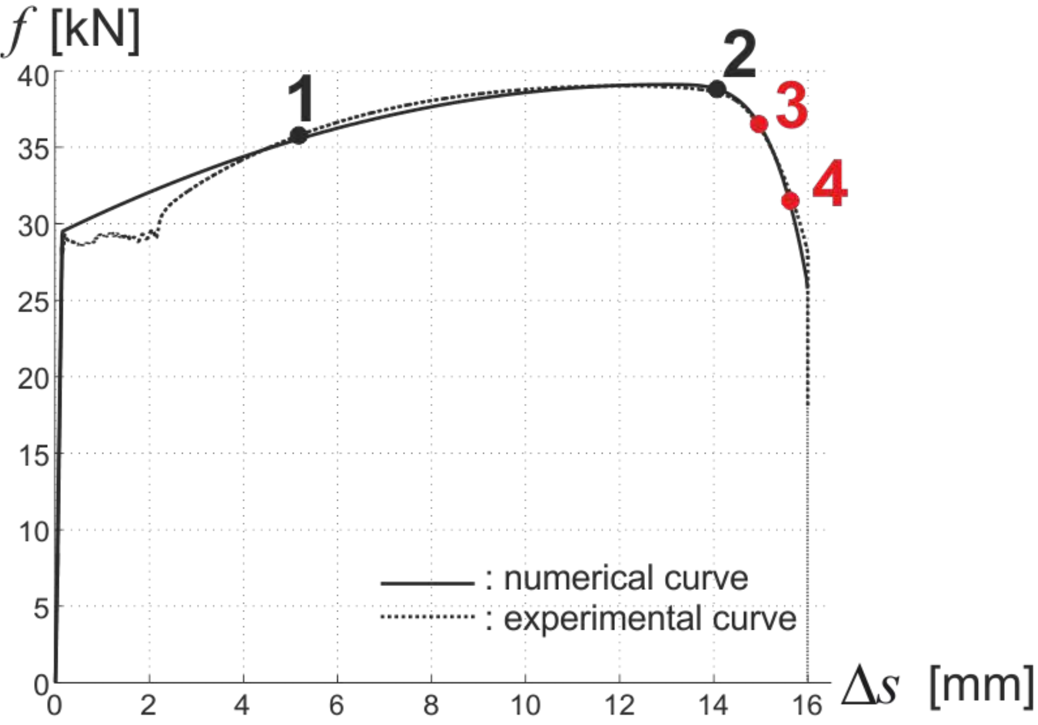
Lancioni, J. Elasticity, 2015



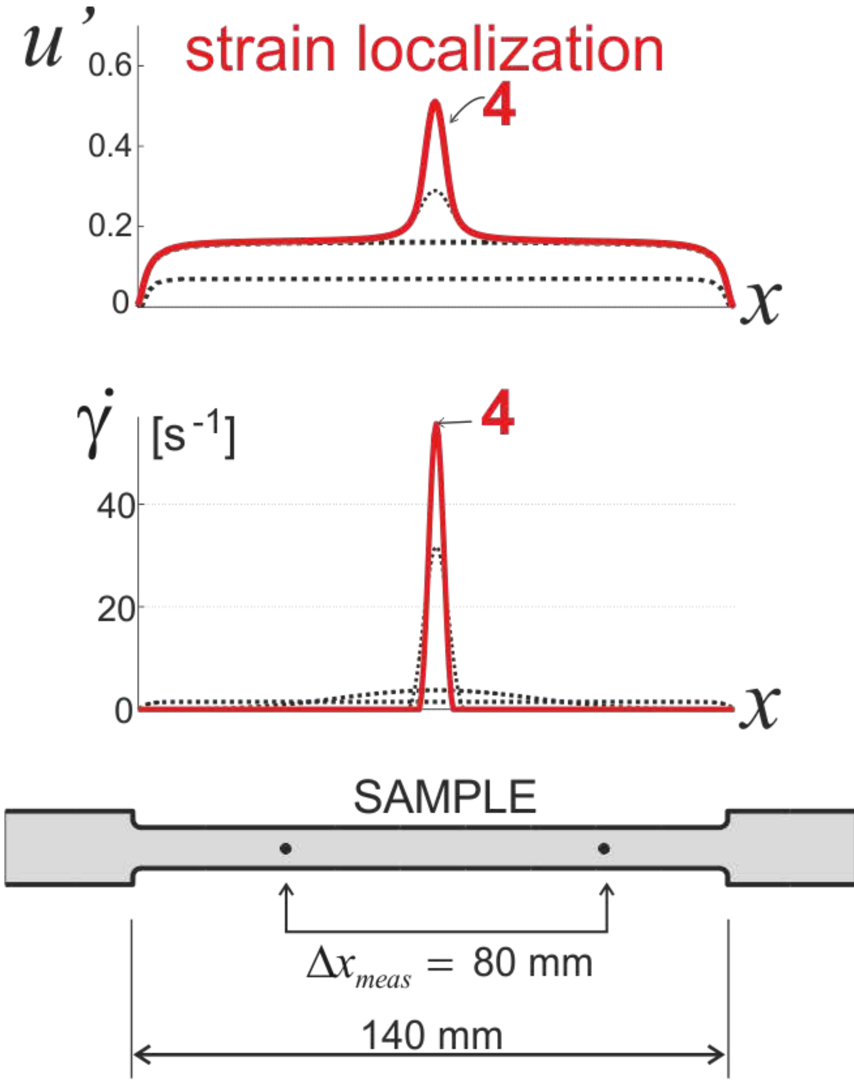


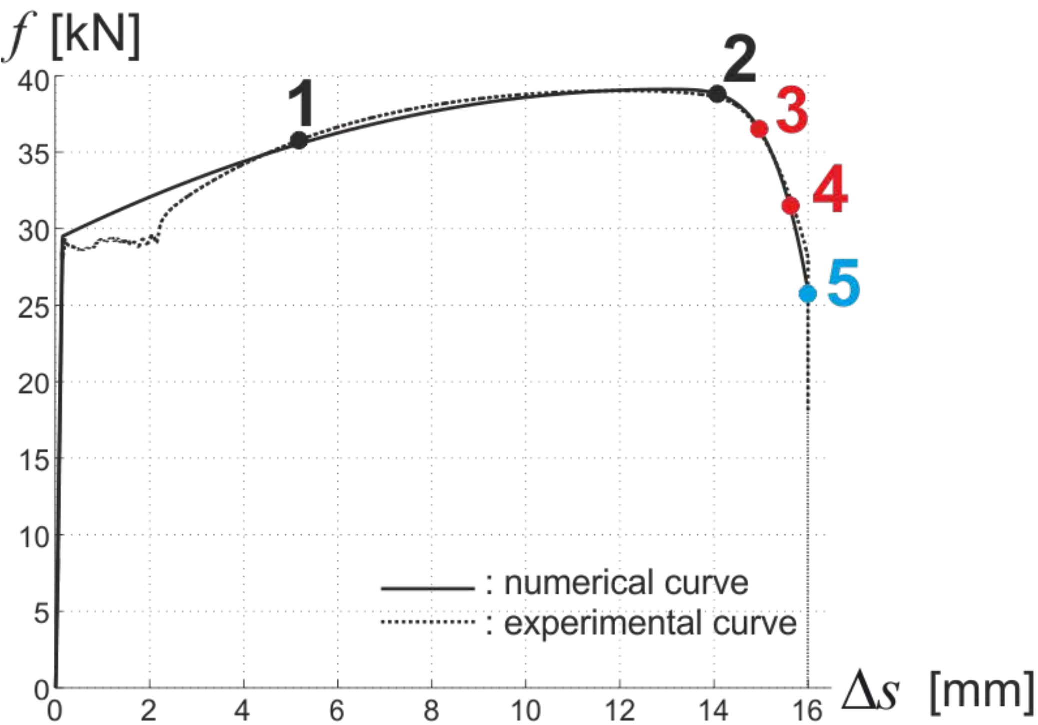
Lancioni, J. Elasticity, 2015



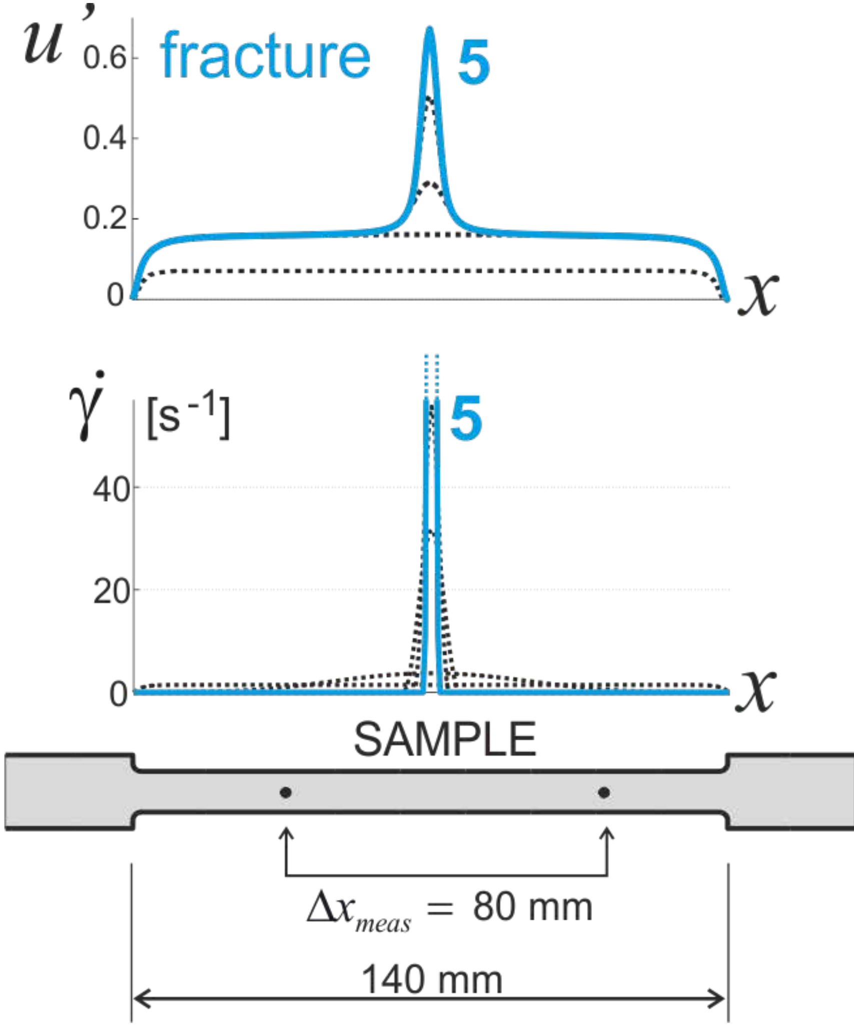


Lancioni, J. Elasticity, 2015





Lancioni, J. Elasticity, 2015



Constitutive parameters setting [Lancioni, J. Elast., 2015]

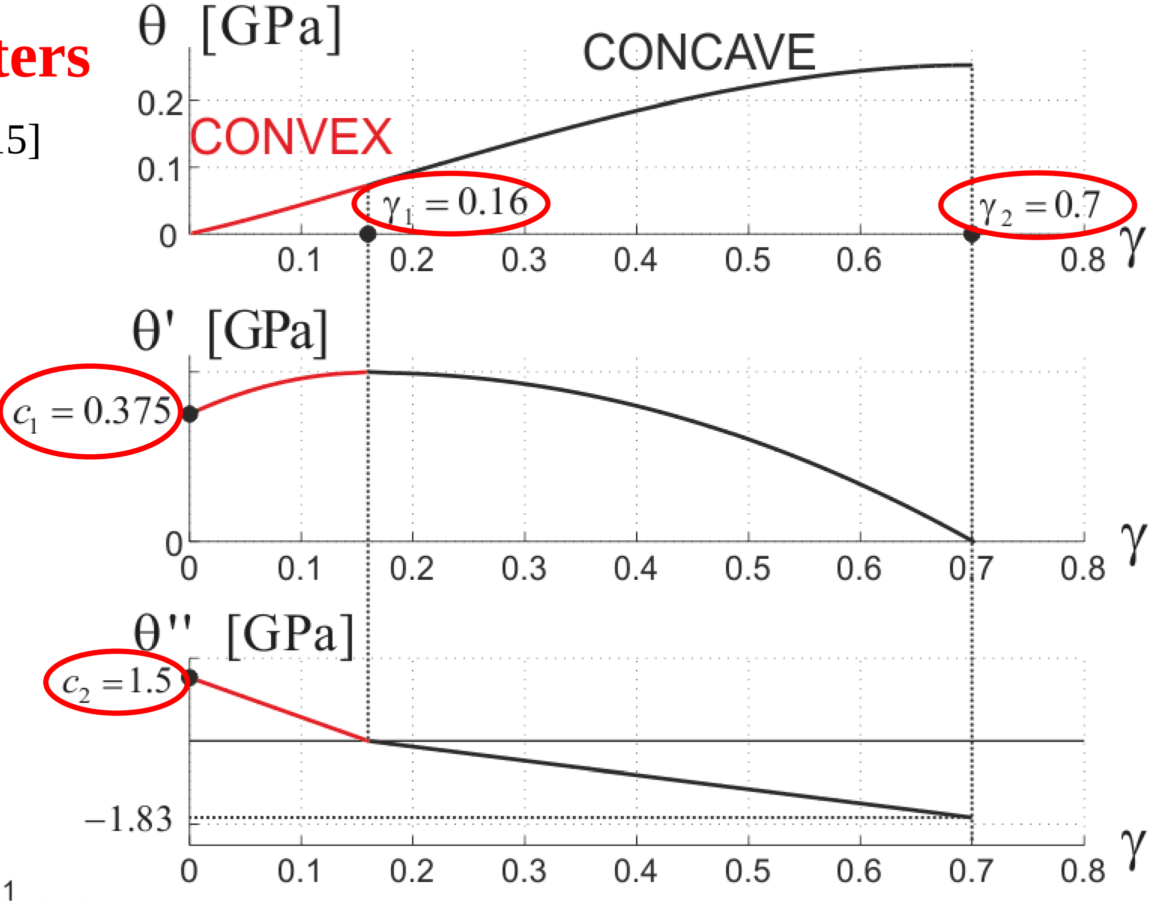
E=210 kN/mm^2,

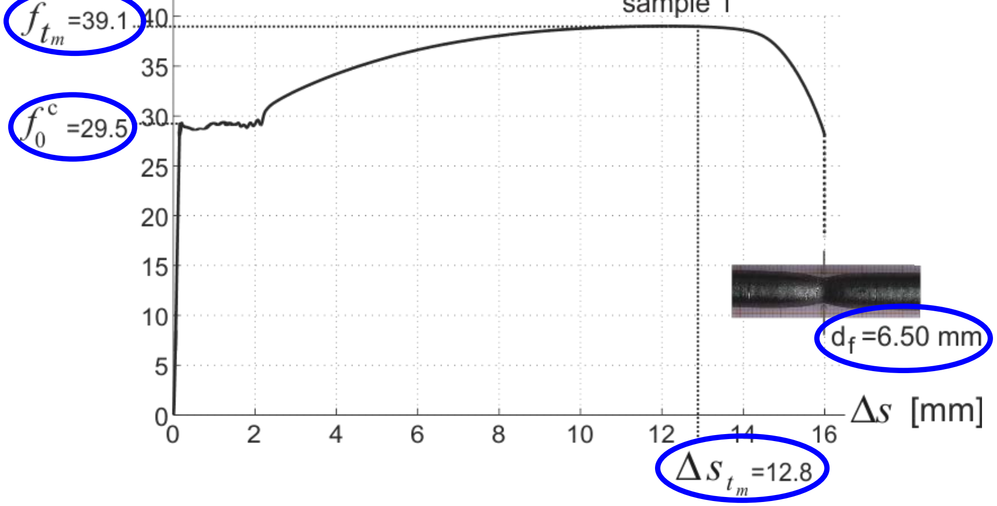
$$c_1 = \frac{f_0^c}{A}$$

$$\gamma_1 = \frac{\Delta s_{t_m}}{\Delta x} - \frac{f_{t_m}}{EA}$$

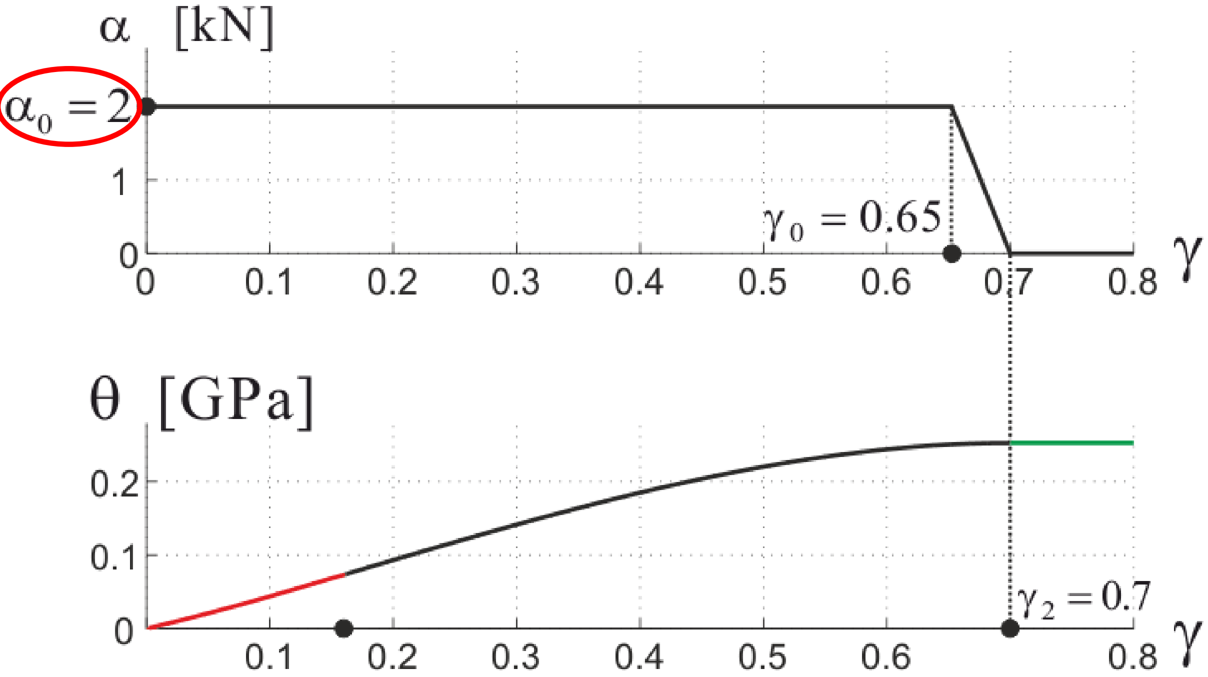
$$c_2 = \frac{2}{A\gamma_1}\left(f_{t_m} - f_0^c\right)$$

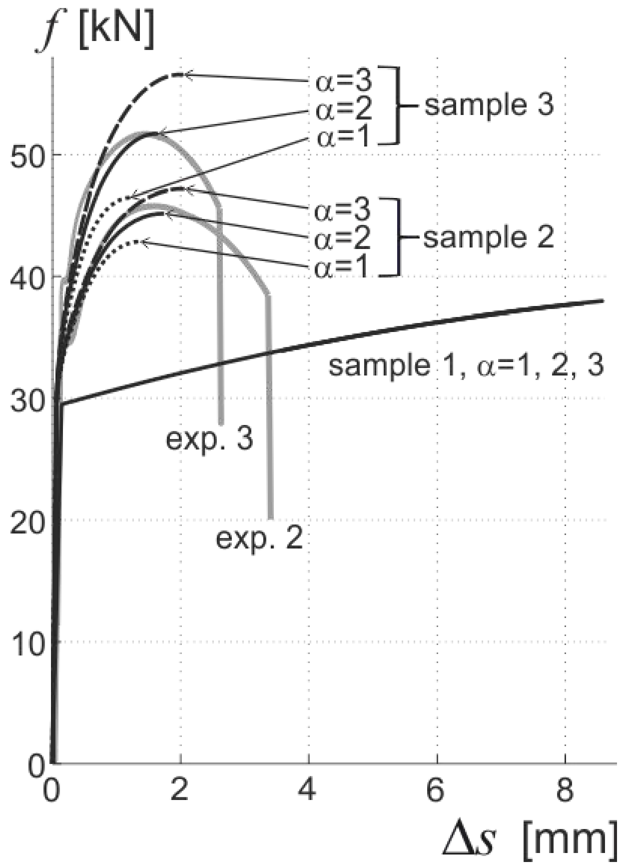
$$\gamma_2 = 2\frac{d_0 - d_f}{d_0}$$

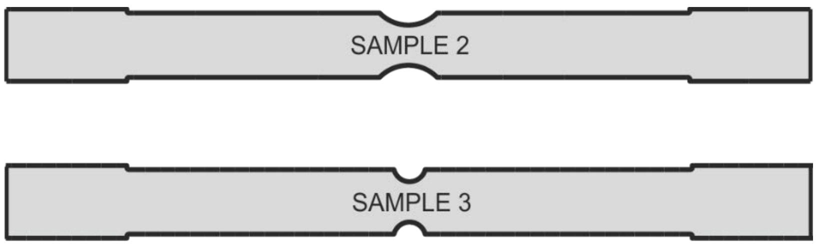




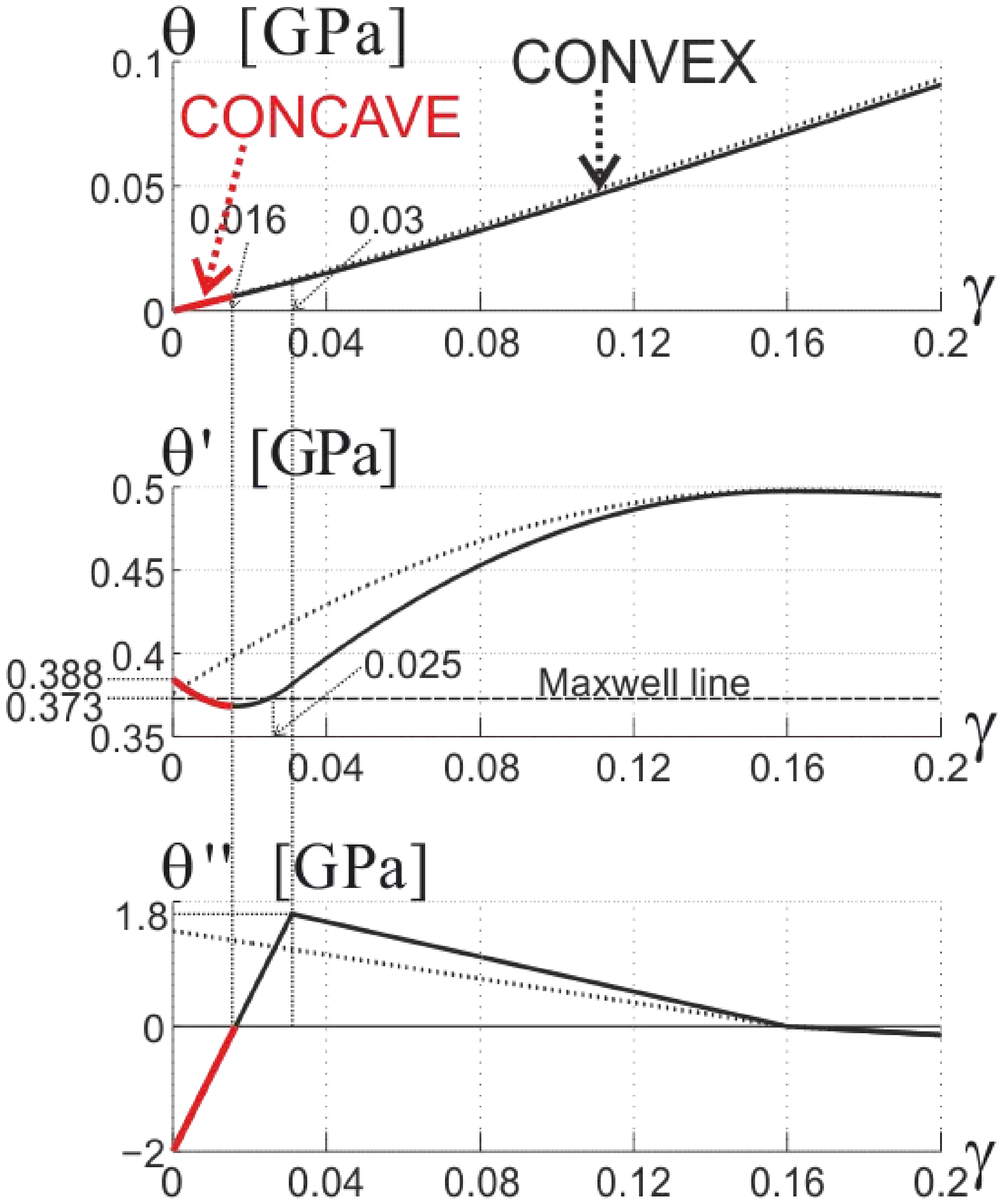
α

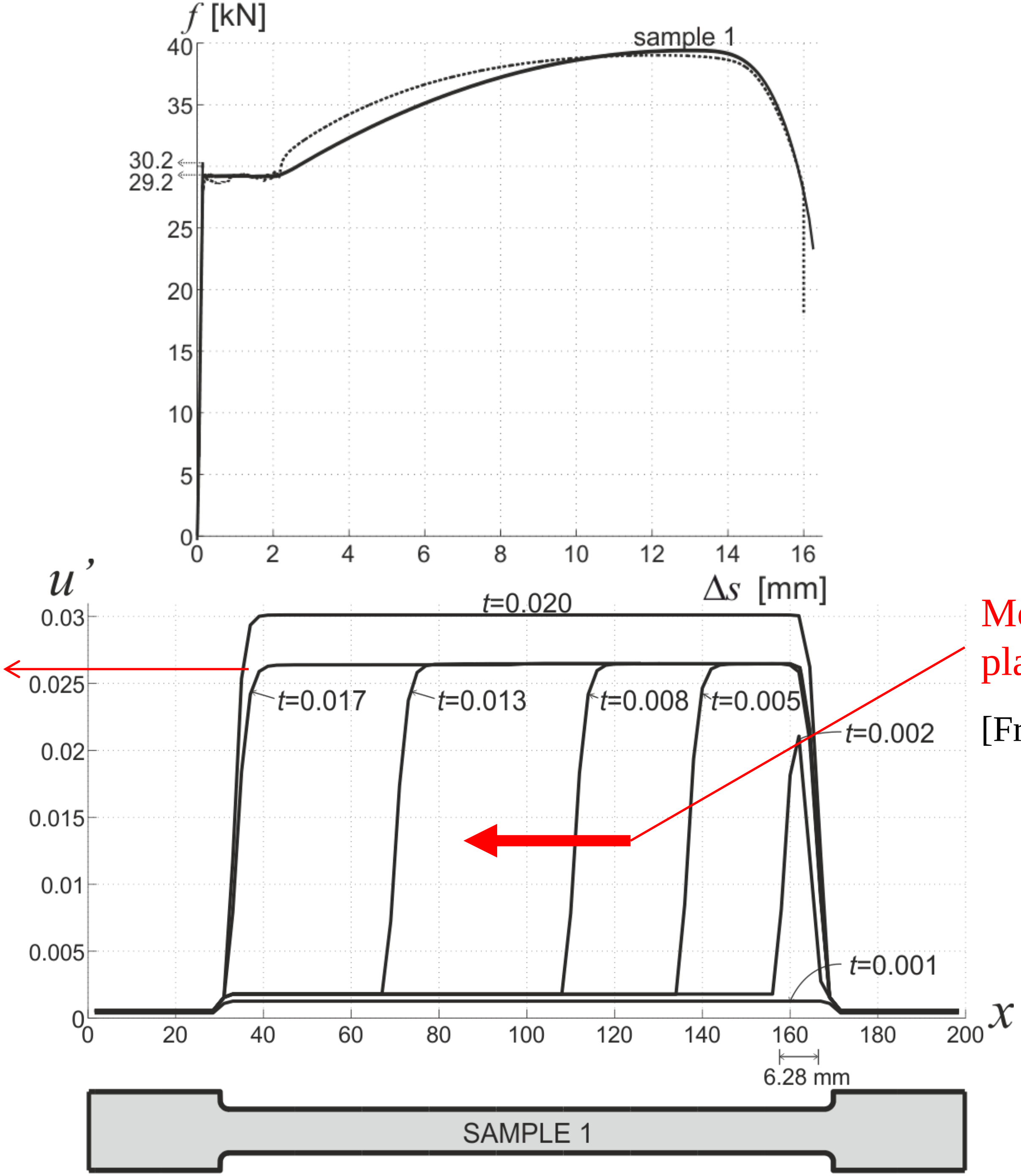

α [kN]
$\alpha_0 = 2$
1
0
$\gamma_0 = 0.65$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
γ
θ [GPa]
0.2
0.1
0
$\gamma_2 = 0.7$
0.1
0.2
0.3
0.4
0.5
0.6
0.8
γ


f [kN]
α=3
α=2
α=1
sample 3
α=3
α=2
α=1
sample 2
sample 1, α=1, 2, 3
exp. 3
exp. 2
50
40
30
20
10
0
0
2
4
6
8
Δs [mm]


SAMPLE 2
SAMPLE 3

θ **concave**-convex-concave

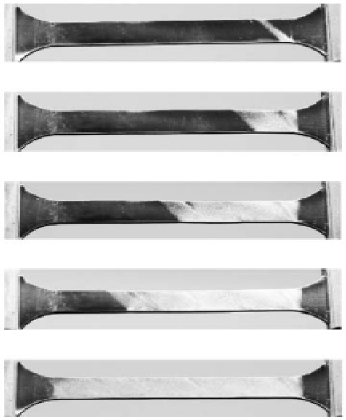




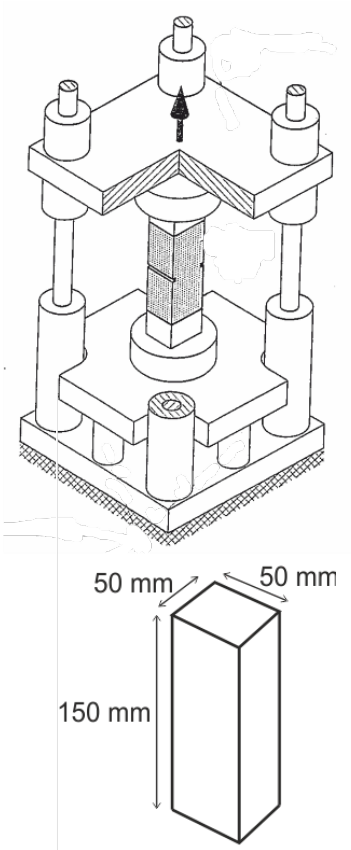
Maxwell line

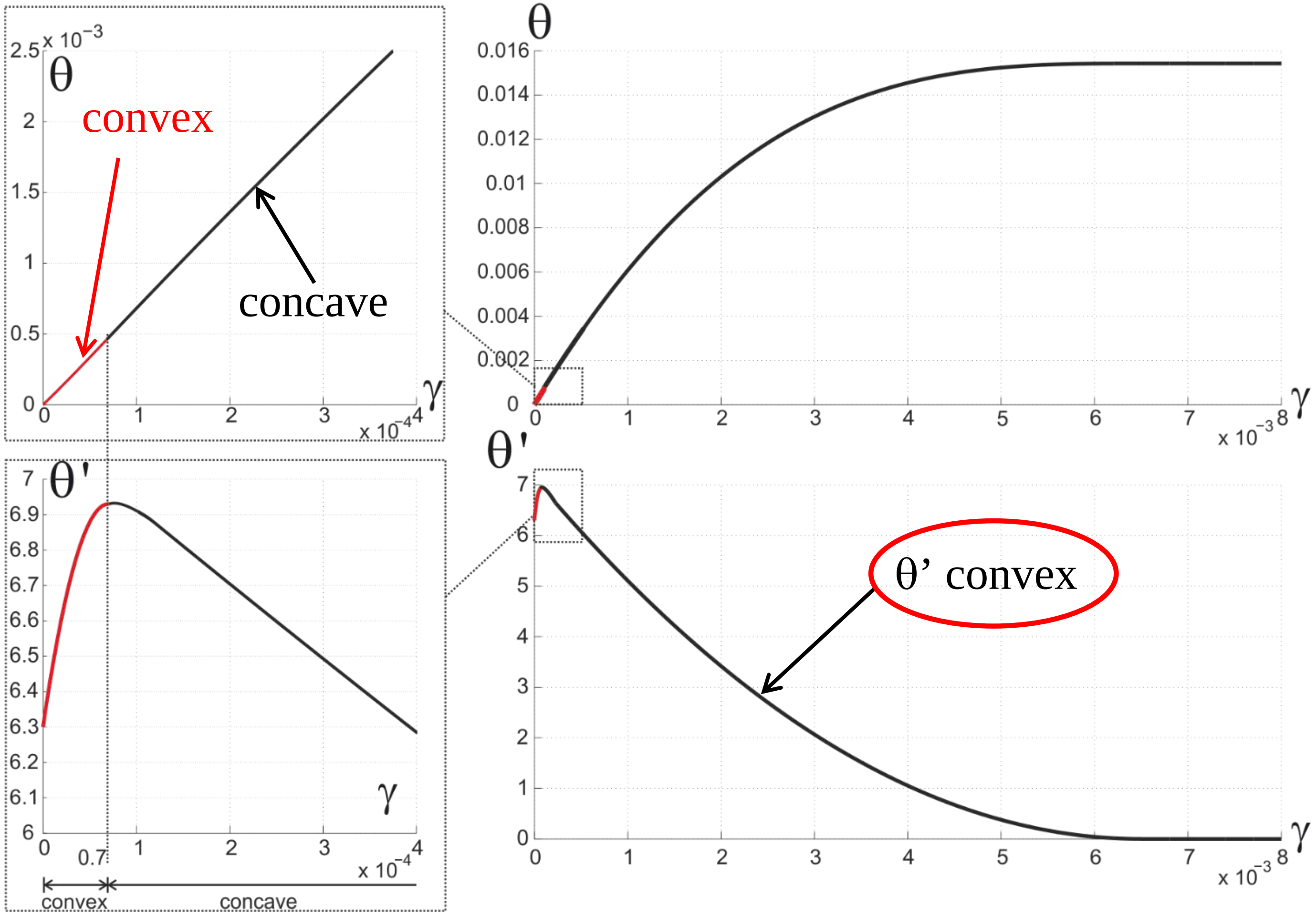
McReynolds' slow plastic wave (1948)

[Froli, Royer-Carfagni, 1999]



Tensile response of a concrete specimen



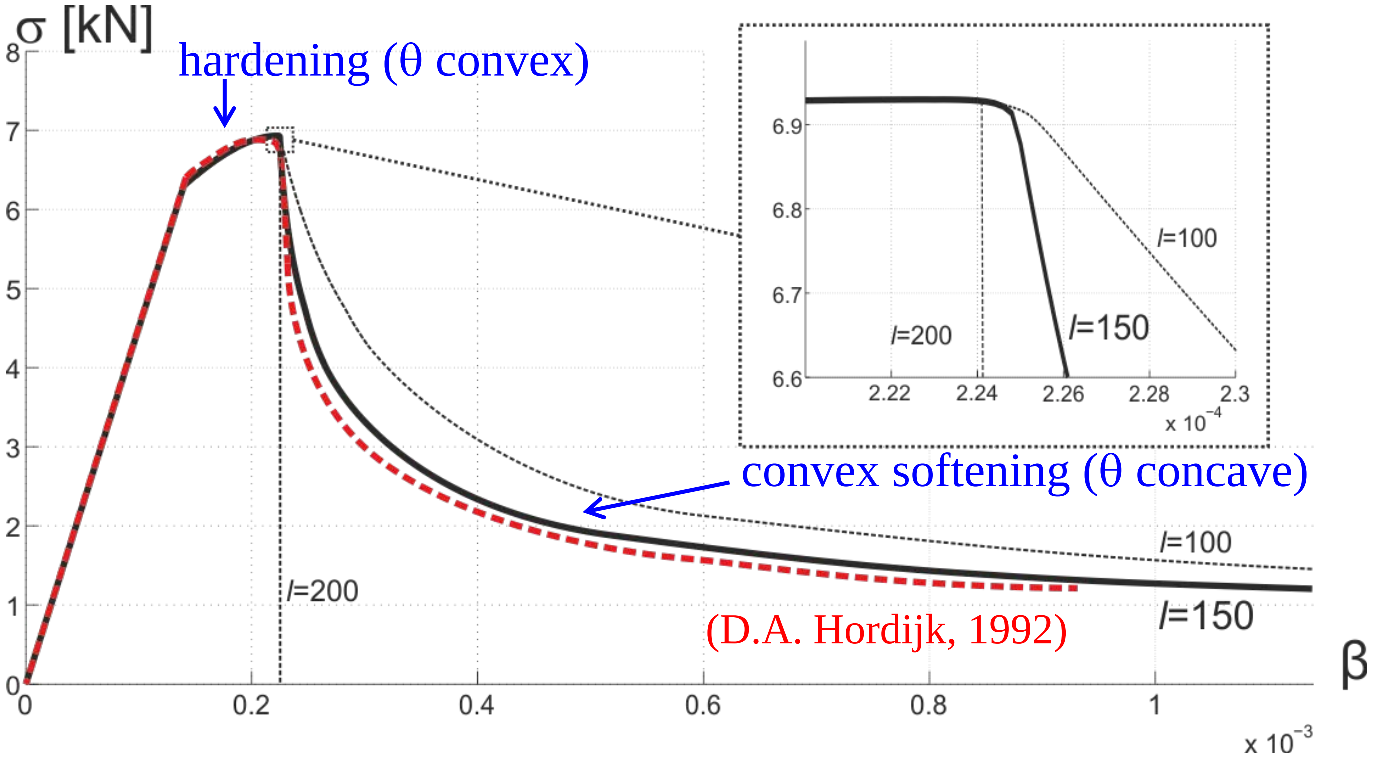


E=18 kN/mm^2,

A=50*50 mm^2,

$\theta'(0)$=6.9 kN (yielding force),

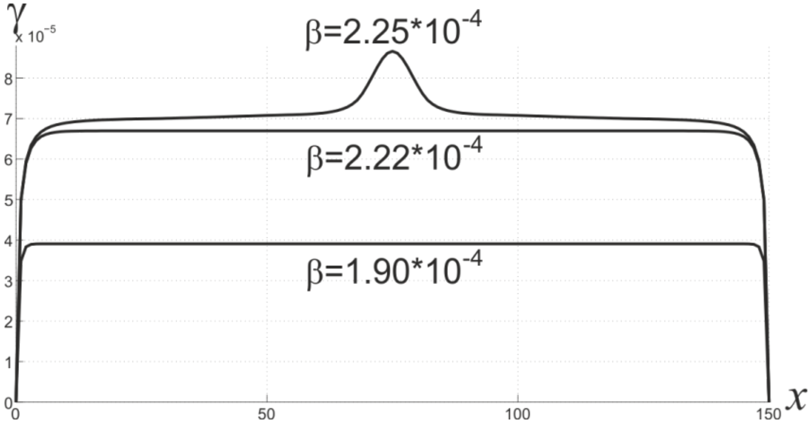
α=3500 kN mm^2



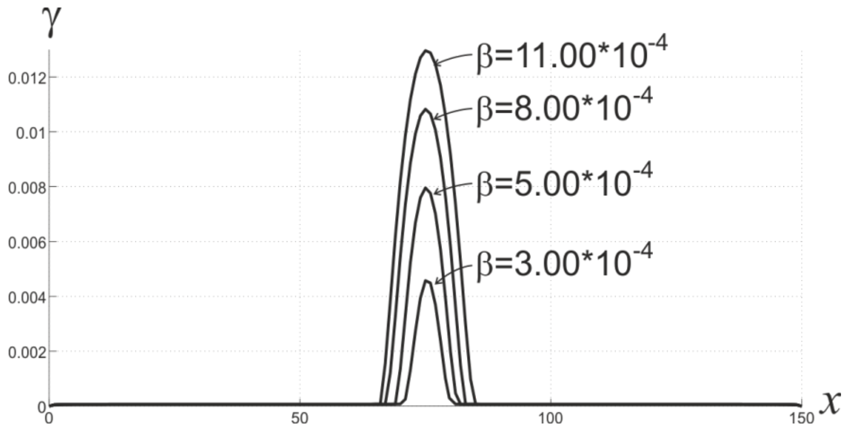
hardening (θ convex)

convex softening (θ concave)

(D.A. Hordijk, 1992)



full-size inelastic deformation (θ convex)



Localization and enlargement of the plastic zone (θ concave)

Multi-dimensional extensions

1D rate-dependent plasticity model

[Yalcinkaya, Brekelmans, Geers, JMPS, 2011]

Virtual work principle, dissipation inequality;

Nonconvex plastic potential;

Non-local gradient energy term.

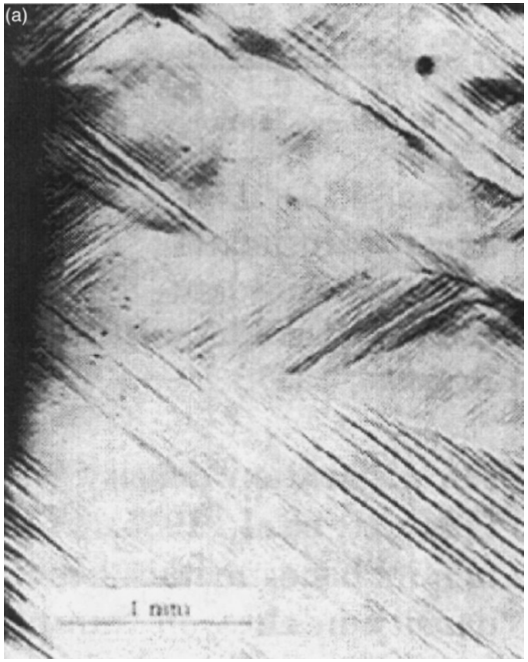
Plastic deformation partially recoverable and partially dissipated through a viscous micro-stress

See [Lancioni, Yalcinkaya, Cocks, *Proc. R. Soc. A*, 2015] for models comparison.

Extension to 2D single crystal plasticity

[Yalcinkaya, Brekelmans, Geers, Int. J. Solids Struct., 2012]

… joint work with Gianluca Zitti (PhD at Univpm)



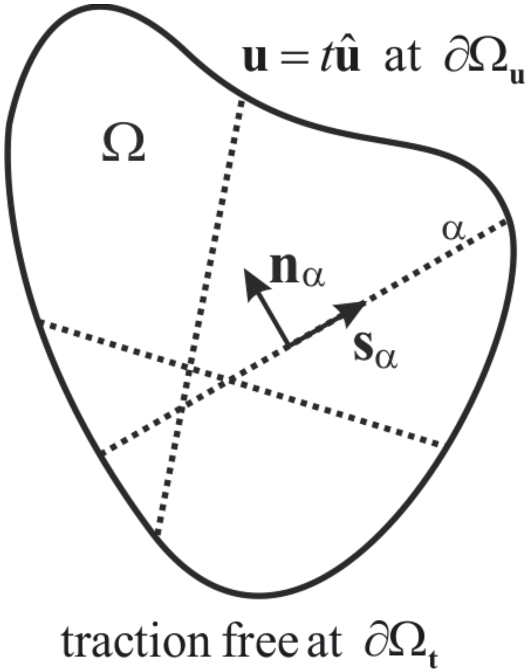
Plastic single-slip domains
[Saimoto, 1963]

Deformation

$$\underbrace{sym\nabla\mathbf{u}(x)}_{\substack{\text{total}\\ \text{def.}}} = \underbrace{\mathbf{E}^{e}(x)}_{\substack{\text{elastic}\\ \text{def.}}} + \underbrace{\mathbf{E}^{p}(x)}_{\substack{\text{plastic}\\ \text{def.}}}$$

$$\mathbf{E}^{p}(x) = \sum_{\alpha} \underset{\substack{\downarrow\\ \text{plastic}\\ \text{slip}}}{\gamma_{\alpha}}\, sym\overbrace{(\underset{\substack{\downarrow\\ \text{slip}\\ \text{direction}}}{\mathbf{s}_{\alpha}} \otimes \underset{\substack{\searrow\\ \text{slip-plane}\\ \text{normal}}}{\mathbf{n}_{\alpha}})}^{\text{Schmid tensor}}$$

Single crystal



Energy

$$E(\mathbf{u},\gamma_\alpha)=\int_\Omega\Big(\underbrace{\psi_e(\mathbf{E}^e)}_{\text{Elastic energy}}+\underbrace{\theta(|\gamma_\alpha|)}_{\text{Plastic energy}}+\underbrace{\psi_{\nabla\gamma}(\nabla\gamma_\alpha)}_{\text{Non-local energy}}\Big)dx$$

Free energy density (stored)

$$\psi(\mathbf{u},\gamma_\alpha)=\psi_e(\mathbf{E}^e)+\psi_{\nabla\gamma}(\nabla\gamma_\alpha)$$

$$\psi_e(\mathbf{E}^e)=\frac{1}{2}C[\mathbf{E}^e]\cdot\mathbf{E}^e,\qquad \psi_{\nabla\gamma}(\nabla\gamma_\alpha)=\frac{1}{2}\sum_\alpha\mathbf{A}_\alpha[\nabla\gamma_\alpha]\cdot\nabla\gamma_\alpha$$

$$\Big\downarrow$$

$$\mathbf{A}_\alpha=A_{s\alpha}\mathbf{s}_\alpha\otimes\mathbf{s}_\alpha+A_{n\alpha}\mathbf{n}_\alpha\otimes\mathbf{n}_\alpha.$$

Dissipative plastic energy

$$\frac{d}{dt}\theta(|\gamma_\alpha|)=\sum_\alpha\operatorname{sign}(\gamma_\alpha)\frac{d\theta(|\gamma_\alpha|)}{d|\gamma_\alpha|}\dot{\gamma}_\alpha\geq 0$$

Suppose that $\theta(|\gamma_\alpha|)$ is strictly increasing in each variable $|\gamma_\alpha|$,

the **dissipation condition** reduces to $\boxed{\operatorname{sign}(\gamma_\alpha)\dot{\gamma}_\alpha\geq 0.}$

Equilibrium

$$\delta E(\mathbf{u},\gamma_\alpha,\delta\mathbf{u},\delta\gamma_\alpha)\geq 0,\qquad \operatorname{sign}(\gamma_\alpha)\overset{\text{perturbation}}{\overset{\swarrow}{\delta\gamma_\alpha}}\geq 0$$

$$\Downarrow$$

$$\boxed{\operatorname{div}\mathbf{T}=0,\ \text{ with } \mathbf{T}=C[\mathbf{E}^e]}$$

Macroscopic balance equation

$$\boxed{\underbrace{|\mathbf{Tn}_\alpha\cdot\mathbf{t}_\alpha|}_{\text{resolved shear}}\leq\underbrace{\pi_\alpha-\operatorname{sign}(\gamma_\alpha)\operatorname{div}\boldsymbol{\xi}_\alpha}_{\text{yield limit}}}$$

Yield condition

$$\text{with}\quad \underset{\substack{\Big\downarrow\\ \text{microscopic stress power-conjugated to } \dot{\gamma}_\alpha}}{\pi_\alpha}=\frac{d\theta(|\gamma_\alpha|)}{d|\gamma_\alpha|}\quad\text{and}\quad \underset{\substack{\Big\downarrow\\ \text{microscopic stress power-conjugated to } \nabla\dot{\gamma}_\alpha}}{\boldsymbol{\xi}_\alpha}=\frac{d\psi_{\nabla\gamma}(\nabla\gamma_\alpha)}{d\nabla\gamma_\alpha}$$

Evolution Pb. ⇨ Incremental energy minimization

$$(\mathbf{u}_t , \gamma_{\alpha,t}) \rightarrow \begin{cases} \mathbf{u}_{t+\tau} = \mathbf{u}_t + \tau \boxed{\dot{\mathbf{u}}_t} \\ \gamma_{\alpha,t+\tau} = \gamma_{\alpha,t} + \tau \boxed{\dot{\gamma}_{\alpha,t}} \end{cases}$$

Unknowns

$$E_{t+\tau}(\dot{\mathbf{u}}, \dot{\gamma}_\alpha) \approx E_t + \tau \dot{E}_t(\dot{\mathbf{u}}, \dot{\gamma}_\alpha) + \tfrac{1}{2}\tau^2 \ddot{E}_t(\dot{\mathbf{u}}, \dot{\gamma}_\alpha) = E_t + \tau\, J_t(\dot{\mathbf{u}}, \dot{\gamma}_\alpha)$$

$$\boxed{(\dot{\mathbf{u}}_t , \dot{\gamma}_{\alpha,t}) = \arg\min\left\{ J_t(\dot{\mathbf{u}}, \dot{\gamma}_\alpha),\ \mathrm{sign}(\gamma_\alpha)\dot{\gamma}_\alpha \geq 0, \mathrm{b.c.} \right\}}$$

Constrained quadratic programming pb.

Necessary condition for a minimum $\delta J_t(\dot{\mathbf{u}}, \dot{\gamma}_\alpha ; \delta\dot{\mathbf{u}}, \delta\dot{\gamma}_\alpha) \geq 0, \quad \dot{\gamma}_\alpha + \delta\dot{\gamma}_\alpha \geq 0$

⇩

$$\boxed{\mathrm{div}\dot{\mathbf{T}} = 0}$$

Balance of the macroscopic stress evolution

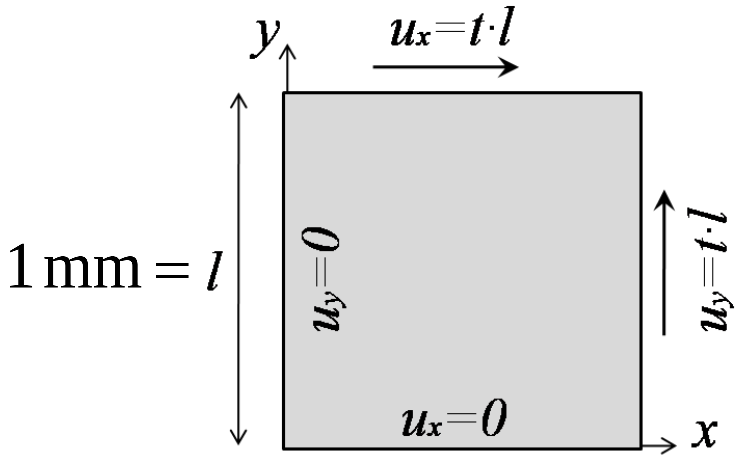
Kuhn-Tucker conditions (**flow rule**)

$$\boxed{\begin{array}{l} \mathrm{sign}(\gamma_{\alpha t})\dot{\gamma}_\alpha \geq 0, \quad |(\mathbf{T}+\tau\dot{\mathbf{T}})\mathbf{n}_\alpha \cdot \mathbf{t}_\alpha| \leq (\pi_\alpha + \tau\dot{\pi}_\alpha) - \mathrm{sign}(\gamma_{\alpha t})\mathrm{div}(\boldsymbol{\xi}_\alpha + \dot{\boldsymbol{\xi}}_\alpha) \\ \underline{\left((\pi_\alpha + \tau\dot{\pi}_\alpha) - \mathrm{sign}(\gamma_{\alpha t})\mathrm{div}(\boldsymbol{\xi}_\alpha + \dot{\boldsymbol{\xi}}_\alpha) - |(\mathbf{T}+\tau\dot{\mathbf{T}})\mathbf{n}_\alpha \cdot \mathbf{t}_\alpha|\right)\dot{\gamma}_\alpha = 0} \end{array}}$$

consistency condition

(the yield function maintains equal to zero when γ grows)

Numerical results – plane pure shear test



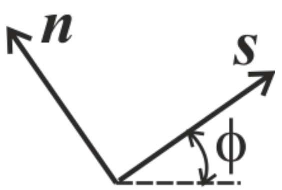
Periodic b.c.

$$u_x(l,y) = u_x(0,y);\ u_y(x,l) = u_y(x,0);$$

$$\gamma(x,l) = \gamma(x,0);\ \gamma(l,y) = \gamma(0,y);$$

Single slip system

$$\mathbf{E}^p(x) = \gamma\, sym(\mathbf{s} \otimes \mathbf{n})$$

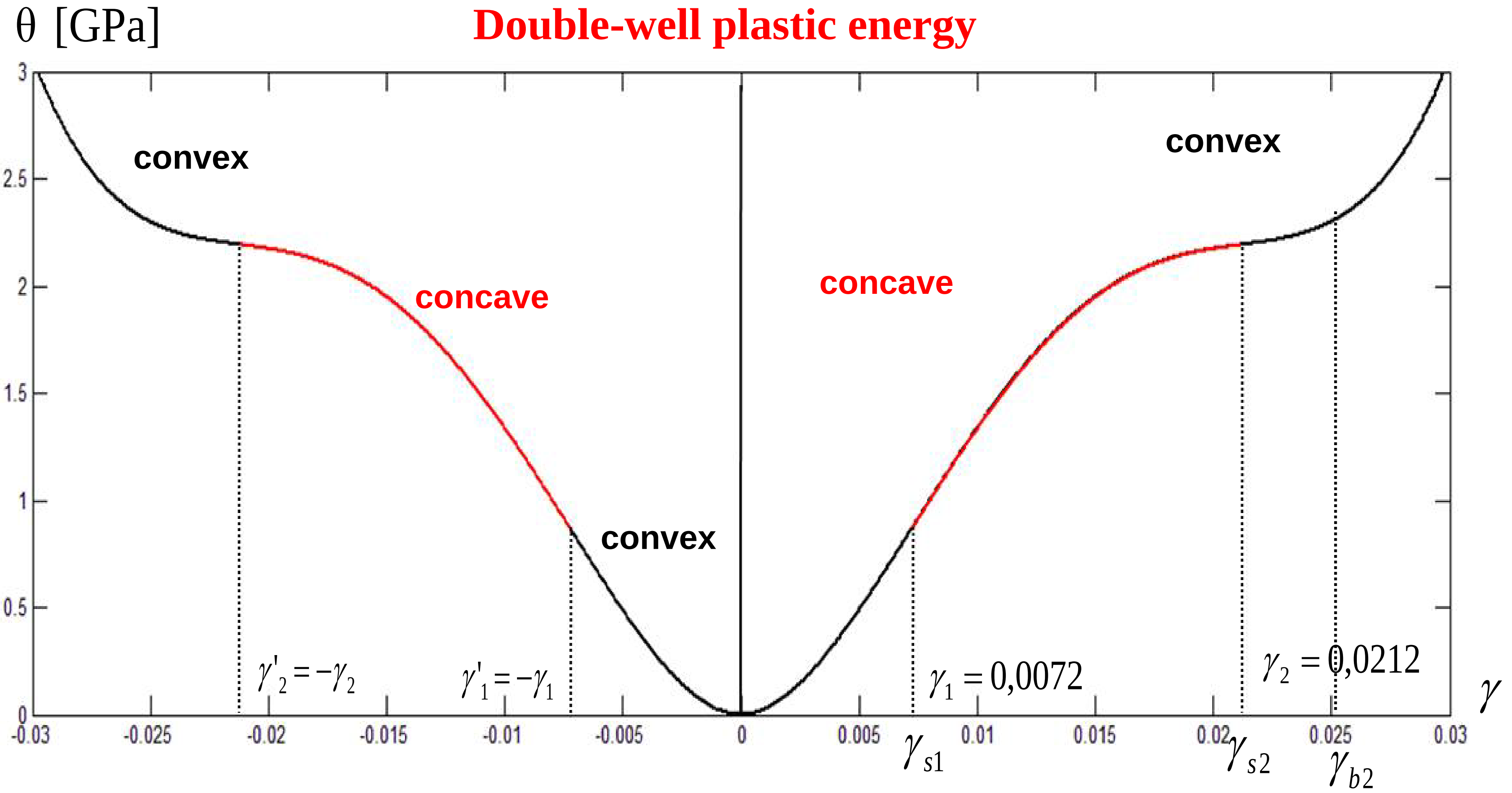


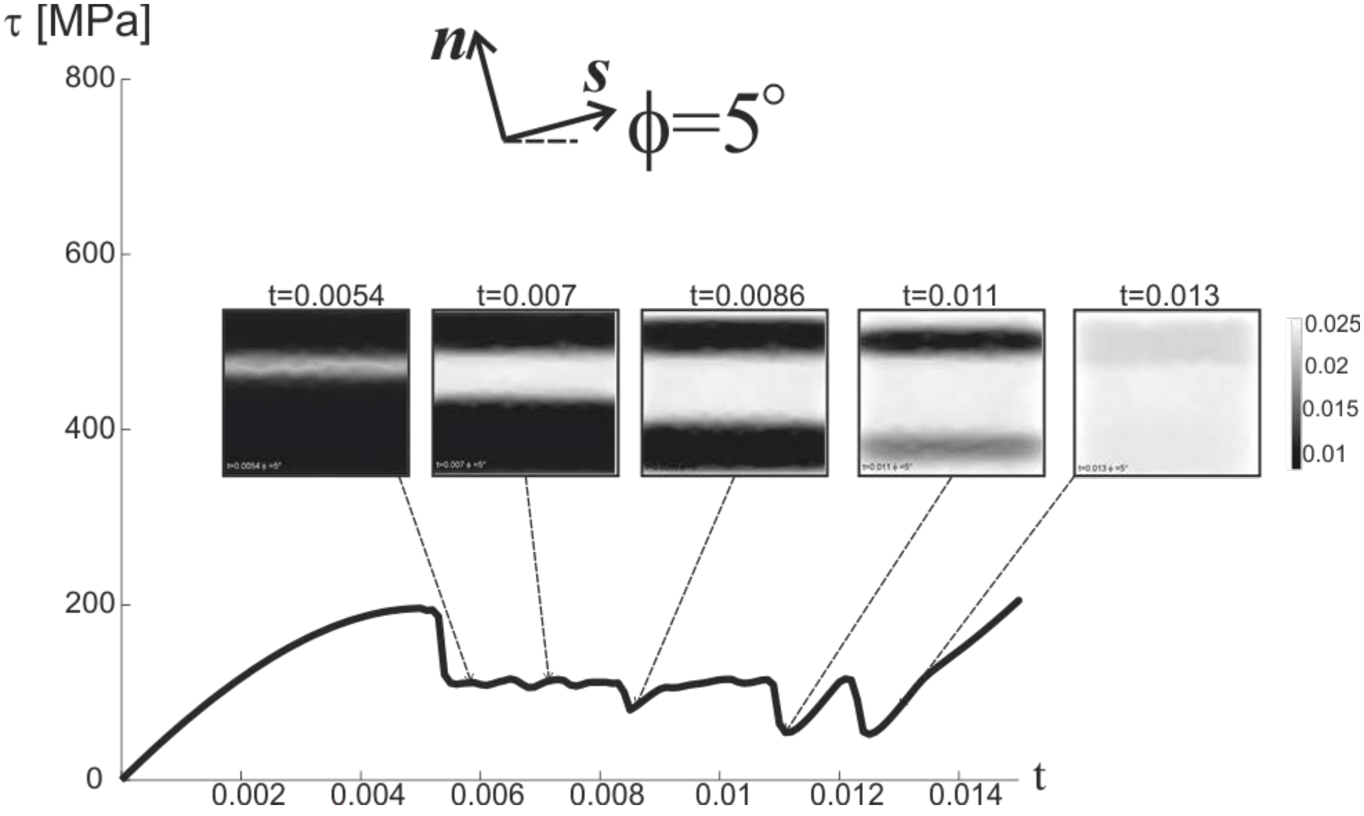
Orientations: $\phi = 5°; 15°; 30°$

$E = 210\,\mathrm{GPa},\ \nu = 0{,}33,$

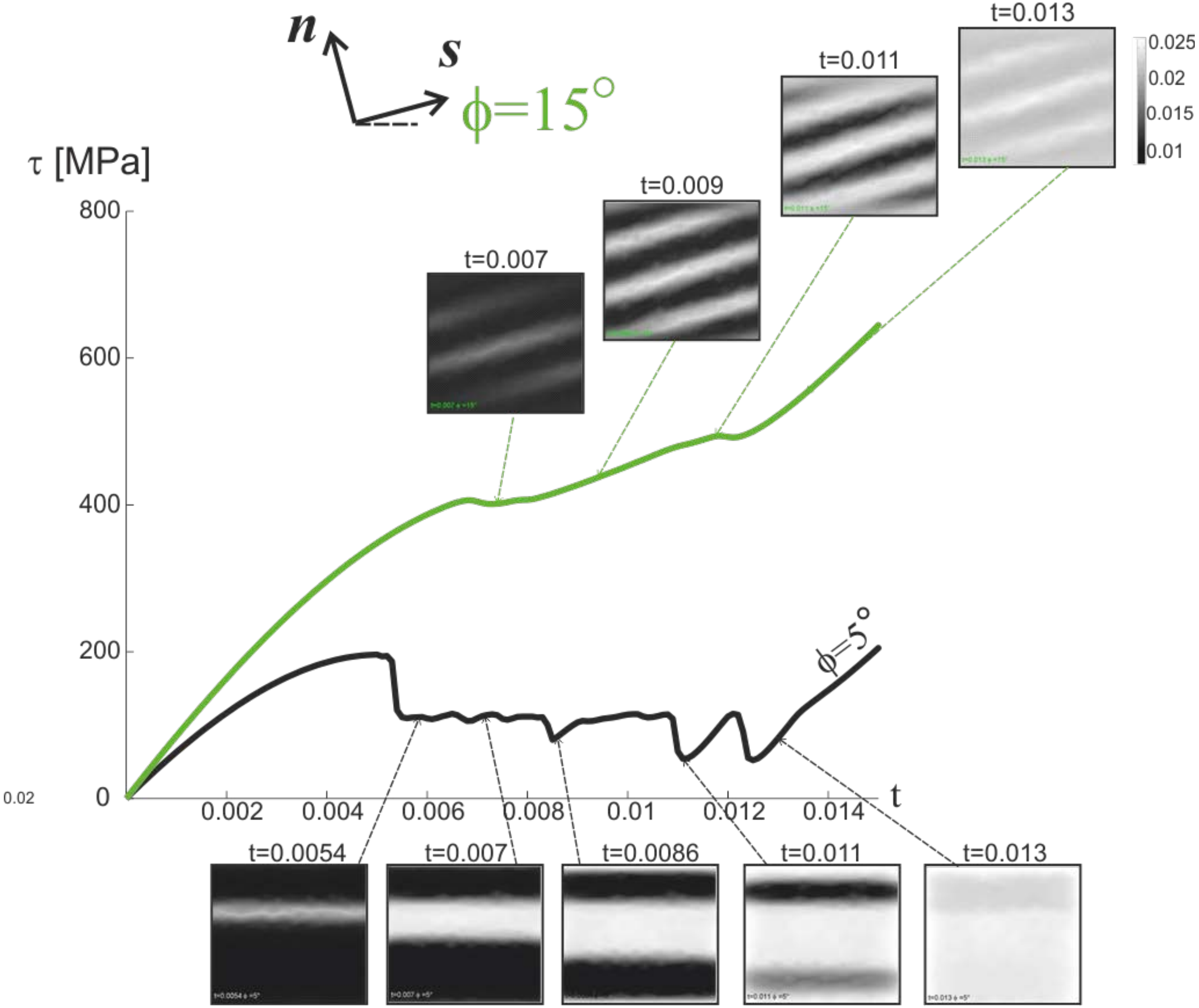
$A_s = 52{,}5\,\mathrm{kN},\ A_n = 10{,}5\,\mathrm{kN}$

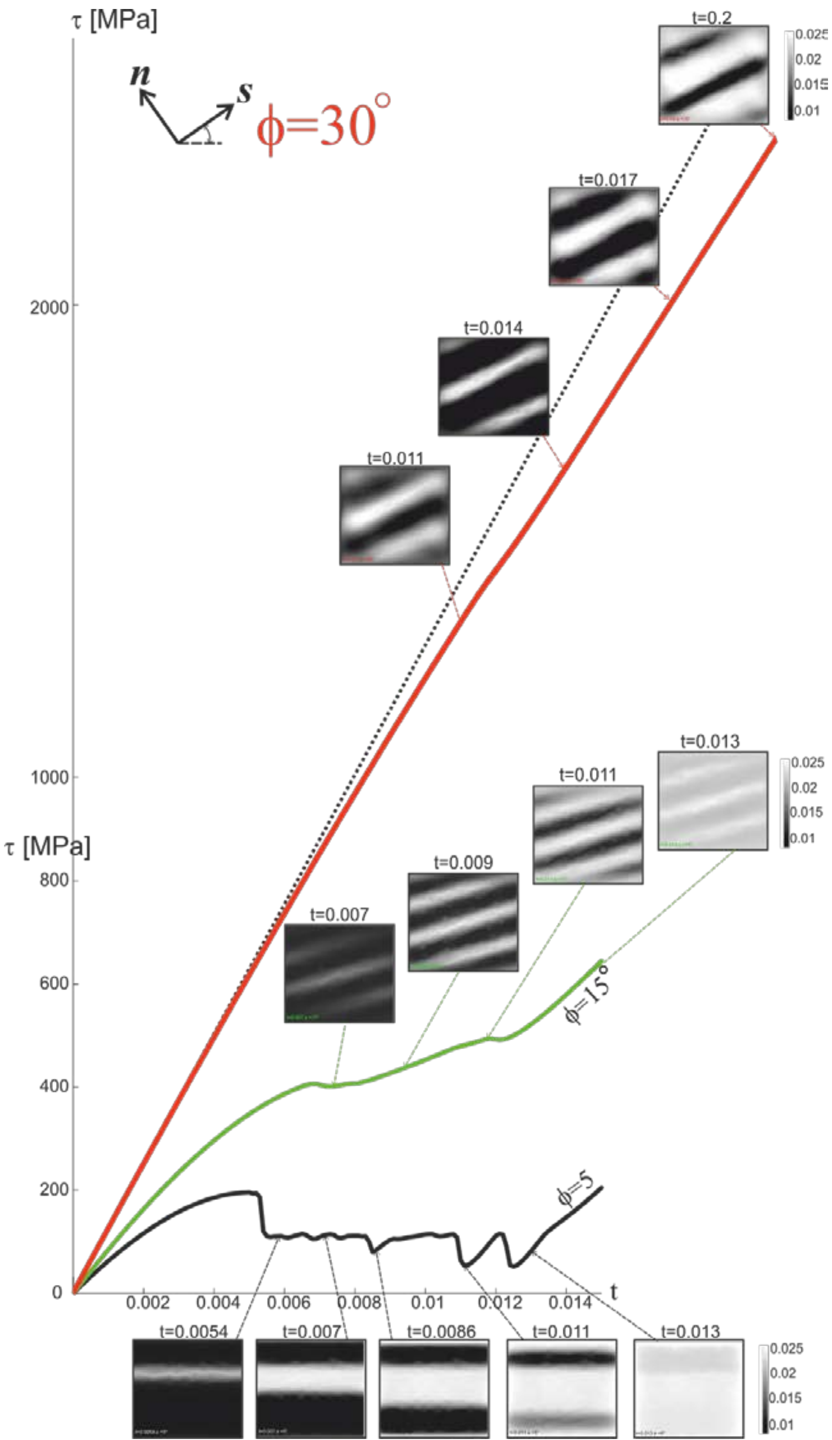
Double-well plastic energy





τ [MPa]
800
600
400
200
0
n
s
ϕ=5°
t=0.0054
t=0.007
t=0.0086
t=0.011
t=0.013
0.025
0.02
0.015
0.01
0.002
0.004
0.006
0.008
0.01
0.012
0.014
t


n
s
ϕ=15°
τ [MPa]
800
600
400
200
0
0.02
0.002
0.004
0.006
0.008
0.01
0.012
0.014
t
ϕ=5°
t=0.007
t=0.009
t=0.011
t=0.013
0.025
0.02
0.015
0.01
t=0.0054
t=0.007
t=0.0086
t=0.011
t=0.013


τ [MPa]
n
s
ϕ=30°
t=0.2
0.025
0.02
0.015
0.01
t=0.017
2000
t=0.014
t=0.011
1000
τ [MPa]
800
600
400
200
0
t=0.013
0.025
0.02
0.015
0.01
t=0.011
t=0.009
t=0.007
ϕ=15°
ϕ=5
0.002
0.004
0.006
0.008
0.01
0.012
0.014
t
t=0.0054
t=0.007
t=0.0086
t=0.011
t=0.013
0.025
0.02
0.015
0.01

Conclusions

The proposed model represents a *variational approach to* ***softening gradient plasticity*** (Aifantis-type model). Advantages:

i. the laws of classical plasticity are variationally deduced (and not given a priori);

ii. clear dependence of the response on the *shape of the plastic energy* $\theta(\gamma)$:

$\theta(\gamma)$ convex -> stress-hardening, diffuse plasticity

$\theta(\gamma)$ concave -> stress-softening, $\theta''(\gamma)$ decreasing -> strain localization

$\theta''(\gamma)$ increasing -> localization zone enlargement

$\theta(\gamma)$ double-wells -> plastic wave propagation

Ductile failure is described as a *bulk process* of progressive strain localization, which concludes with a final *material instability*, variationally interpreting *fracture*.

Physical motivation: *process zone*, where strains localize, and only at the very end they coalesce in fracture surfaces.

⇨ The model presents as an *alternative to classical cohesive fracture theories*, which concentrate inelasticity on surfaces.

Perpectives

1. Extension to **multi-dimension**.
   Crystal plasticity: multiple slip systems
2. Find correlations between the covexity-concavity properties of θ and its derivatives and the microstructure of real materials.
   Crystal plasticity: non-convex energy proposed by Ortiz-Repetto (1999), accounting for latent hardening

Conclusions

Rate-Independent model based on **incremental energy minimization**;

Non-convex dissipative plastic energy ⇒
- Irreversibility of plastic def.
- non-convexity leads to localization

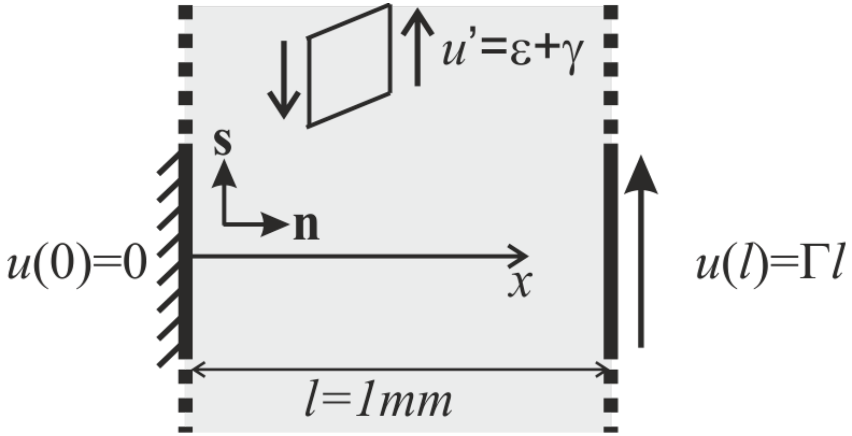
Non-local energy ⇒
- internal length scale (it makes possible to simulate phenomena at different scales)
- stabilizing effect (ductile failure; no brittle fracture)

Perspectives

- Simulations with multiple slip systems and plastic energy funtions of different shapes;
- Find correlations between the covexity-concavity properties of θ and its derivatives and the microstructure of real materials -> non-convex energy proposed by Ortiz-Repetto (1999), accounting for latent hardening

Numerical results

i. Slip patterning in an infinite long strip (1D Pb)

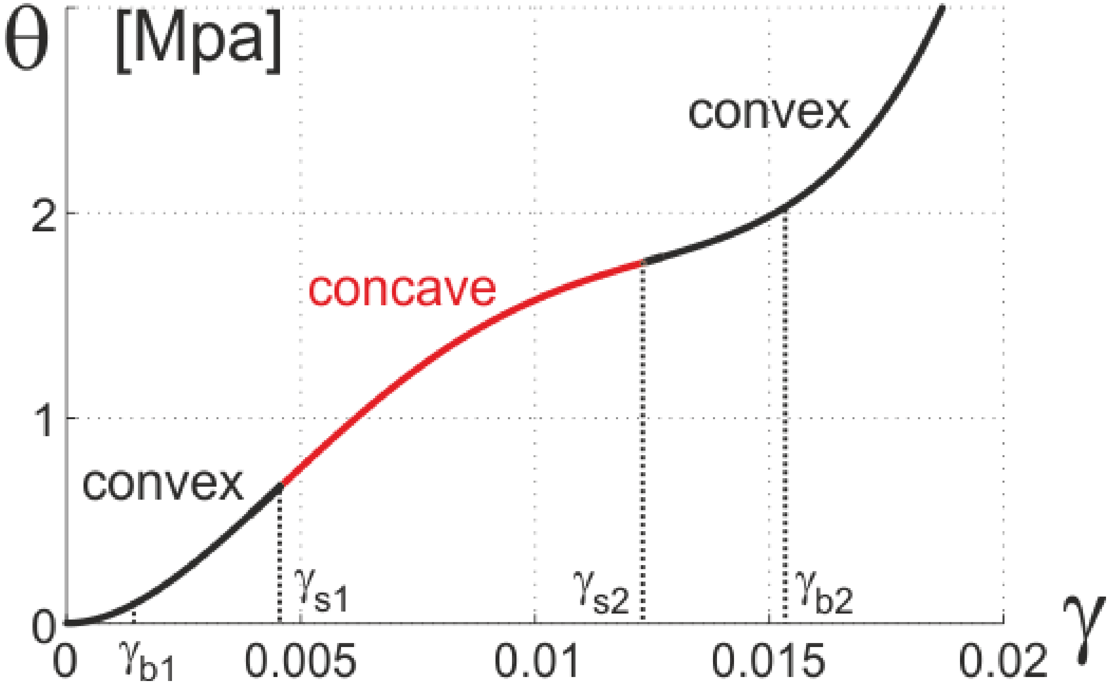


Single slip system

$$\mathbf{E}^{p}(x) = \gamma\, sym(\mathbf{s} \otimes \mathbf{n})$$

E = 210 GPa, ν = 0.33,

A_n = 147.29 N



Soft boundary conditions $\gamma'(0)=0$, $\gamma'(l)=0$