Theoretical and computational aspects of nonlinear surface waves

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1 Overview

Waves on the surface of a fluid — or the interface between different fluids — are omnipresent phenomena. In water, ripples driven by surface tension affect remote sensing of underwater obstacles; waves on the surface and the interface between internal layers of water of differing densities affect ocean shipping, coastal morphology, and near-shore navigation; and tsunamis and hurricane-generated waves can cause devastation on a global scale. Hydroelastic waves are generated by man-made floating structures (especially airports) and by Antarctic exploration where heavy equipment travels over roads on floating ice. Hydromagnetic waves arise on the surface of ferrofluids in use in high-technology applications.

From a mathematical viewpoint, the surface-wave equations pose surprisingly deep and subtle challenges for rigorous analysis and numerical simulation. The governing equations are widely accepted and there has been substantial research into their validity and limitations. However, a rigorous theory of their solutions is extremely complex due not only to the fact that the water-wave problem is a classical free-boundary problem (where the problem domain, specifically the water surface, is one of the unknowns), but also because the boundary conditions (and, in some cases, the equations) are strongly nonlinear. The level of difficulty is such that the theory has merely begun to answer the fundamental questions which must be addressed before our understanding can be termed 'adequate'. The workshop was convened in response to recent advances in analysis and scientific computing, and the consequent renewed interest in this classical problem from several different mathematical directions.

The workshop was organised around five main themes related to key mathematical and numerical issues about nonlinear waves arising in a variety of free-surface problems: initial-value problems, coherent structures, stability of coherent structures, model equations and waves in domains with complex geometry. The problems under consideration (water waves, hydroelastic waves, internal waves, etc.) have similar formulations and thus can be addressed by closely related methods. The participants reflected the multidisciplinary nature of these problems: over forty researchers – including mathematical analysts, applied mathematicians, numerical analysts as well as fluid dynamicists, oceanographers and engineers – from twelve different countries took part.

Substantial progress was made in each of the themes (see below for a detailed description of the current state of the art together with the advances reported at the conference). Our main objective – catalysing progress in this classical problem by promoting direct interaction of researchers from different communities, and cross-fertilisation of ideas among them – was achieved, with many discussions taking place during and outside the scheduled talks. A roadmap for making future progress on some of the difficult open problems in nonlinear surface waves was prepared; it includes proposals for intensive research semesters at leading mathematical research institutes, setting up a new interdisciplinary scientific journal on water waves and possibly arranging a follow-up workshop at Casa Matemática Oaxaca in 2019 or 2020.

2 Themes and presentations

The workshop consisted of thirty-five 30-minute presentations, each of which was associated with one of the central themes.

2.1 Initial-value problems

There is now a wealth of local well-posedness results for various versions of the water-wave problem, each relying upon a different formulation (see e.g. [6] and references therein). Recently, global well-posedness results for small waves on water of infinite depth or finite depth with a flat bottom have become available in both two and three dimensions for various combinations of gravity and capillarity; see [2] for a recent overview. These results all exclude the existence of solitary waves with small energy; to go beyond this obstacle (and thus include gravity waves on water of finite depth, for example) is a challenging open problem. In the opposite direction, theories of singularity formation due to wave breaking and overturning have been developed in the last few years. At the same time, a number of accurate and efficient numerical methods have been developed to solve the full Euler equations for irrotational water waves on finite or infinite depth, based on boundary integral equations, conformal mappings and Taylor series expansions. Here the main task consists in evaluating the Dirichlet–Neumann operator associated with the fluid domain. Special effort has also been devoted to designing numerical schemes that preserve important properties of the mathematical system such as energy conservation.

- Walter Craig (McMaster University, Canada) gave the talk 'Hamiltonians and normal forms for water waves', in which he presented a generalization of Zakharov's Hamiltonian formulation for the water-wave problem to overturning wave profiles. He also discussed the question of Birkhoff normal forms for the water-wave equations in the setting of spatially periodic solutions, including the function-space mapping properties of these transformations.

- Mihaela Ifrim (UC Berkeley, USA) gave the talk 'Constant vorticity water waves', in which she considered the incompressible, infinite depth water-wave equations in two space dimensions, with gravity and constant vorticity but without surface tension. She described how to prove local well-posedness for large data, as well as cubic lifespan bounds for small data solutions, by expressing the problem in position-velocity potential holomorphic coordinates.

- Paul Milewski (University of Bath, UK) gave the talk 'Wave collapse for ripples: dynamics of CG waves', in which he discussed capillary-gravity wave packets that are described asymptotically by the focussing 2D NLS equation. This equation has solutions that blow up in finite time. In his talk, Milewski numerically explored the dynamics of blowup initial data under the full Euler equations.

- Daniel Tataru (UC Berkeley, USA) discussed 'Long time solutions for finite bottom gravity waves'. He gave an overview of recent work with Benjamin Harrop-Griffiths and Mihaela Ifrim concerning enhanced lifespan bounds for small data gravity waves in the presence of a flat bottom. This work is based on two key ideas: (i) the use of holomorphic coordinates; (ii) the modified energy method, a quasilinear alternative to normal forms.

- Sijue Wu (University of Michigan, USA) gave the talk 'On the motion of a self-gravitating incompressible fluid with free boundary', reporting joint work with Lydia Bieri, Shuang Miao and Sohrab Shahshahani. She considered the motion of the interface separating a vacuum from an inviscid, incompressible, and irrotational fluid subject to self-gravitational force and neglecting surface tension in two space dimensions. The main result was a long-time existence result for small perturbations of an equilibrium state. The proof is based on a nonlinear transformation of the unknown function and a coordinate change which eliminates quadratic nonlinear terms.

2.2 Coherent structures

Recent advances in this area include the development – from a base of almost zero knowledge – of existence theories for travelling waves with vorticity, standing waves (waves which are periodic in space and time) and three-dimensional surface waves in the context of the water-wave problem (see e.g. [1, 3, 4, 5, 9]). Various types of coherent structures in two and three dimensions have been computed in the last few years using boundary integral equations and spectral methods (see e.g. [10, 8, 7]).

- Mark Groves (Universität des Saarlandes, Germany) presented the talk 'Fully localised solitary gravity-capillary water waves', which is joint work with Boris Buffoni and Erik Wahlén. He considered the classical gravity-capillary water-wave problem in its usual formulation as a three-dimensional free-boundary problem for the Euler equations for a perfect fluid. The existence of fully localised solitary waves has been predicted on the basis of simpler model equations, namely the Kadomtsev-Petviashvili (KP) equation in the case of strong surface tension and the Davey-Stewartson (DS) system in the case of weak surface tension. The talk confirmed the existence of such waves as solutions to the full water-wave problem and gave rigorous justification for the use of the model equations.

- Philippe Guyenne (University of Delaware, USA) gave the talk 'Nonlinear waves in ice sheets' concerning the mathematical modelling and numerical simulation of waves in ice sheets as occurring, e.g., in polar regions. A three-dimensional Hamiltonian formulation for ice sheets deforming on top of an ideal fluid of arbitrary depth was presented and nonlinear wave solutions were examined. In certain asymptotic regimes, analytical solutions were derived and compared with fully nonlinear solutions obtained numerically by a pseudospectral method.

- David Henry (University College Cork, Ireland) gave the talk 'Nonlinear water waves and wave-current interactions', in which he examined the nonlinear water waves and wave-current interactions, which may be prescribed by Gerstner-like exact and explicit solutions to the geophysical $\beta-$plane equations in the equatorial region. In particular, he presented recent work which highlights the role played by the typically-neglected centripetal force terms in describing such physical processes.

- Olga Trichtchenko (University College London, UK) presented the talk 'Computing three-dimensional flexural gravity water waves', in which she discussed an efficient and accurate method for computing solutions to Euler's equations for water waves underneath an ice sheet in three dimensions (joint work with Emilian Parau, Jean-Marc Vanden-Broeck and Paul Milewski). The equations were solved via a numerically implemented boundary integral-equation method and some high performance computing techniques were utilized. Trichtchenko gave details of the current methods and compared solutions for different models of the ice sheet.

- Jean-Marc Vanden-Broeck (University College London, UK) presented 'A numerical investigation of non-symmetric nonlinear water waves'. He considered nonlinear waves travelling at a constant velocity at the surface of a fluid of finite depth, assuming the fluid to be incompressible and inviscid and the flow to be irrotational. Gravity and surface tension were taken into account and both two and three dimensional waves were studied. Classical solutions usually assume that the waves are symmetric. Vanden-Broeck showed that there are in addition an infinite number of branches of non-symmetric waves which include periodic waves, solitary waves and generalised solitary waves.

- Samuel Walsh (University of Missouri, USA) gave the talk 'Existence and qualitative theory of stratified solitary water waves', in which he reported some recent results concerning two-dimensional gravity solitary water waves with heterogeneous density obtained in collaboration with Robin Ming Chen and Miles Wheeler. The fluid domain is assumed be bounded below by an impenetrable flat ocean bed, while the interface between the water and vacuum above is a free boundary. Their main main existence result states that, for any smooth choice of upstream velocity and streamline density function, there exists a path connected set of such solutions that includes large-amplitude surface waves. Indeed, this solution set can be continued up to (but does not include) an 'extreme wave' that possess a stagnation point. Walsh also discussed a number of results characterizing the qualitative features of solitary stratified waves.

- Miles Wheeler (Courant Institute, USA) discussed 'Global bifurcation of rotating vortex patches', (joint work with Zineb Hassainia and Nader Masmoudi), in which continuous curves of rotating vortex patch solutions to the two-dimensional Euler equations are constructed. These curves are large in that, as the parameter tends to infinity, the minimum value on the boundary of the relative angular fluid velocity becomes arbitrarily small.

2.3 Stability of coherent structures

One crucial question facing all developments, both analytical and numerical, in the theory of surface waves is that of stability: Once one has found a solution of interest (e.g., a travelling or standing wave) will it be observable in the laboratory setting? Will it be found in the open ocean? There are now a range of rigorous mathematical linear instability results for two-dimensional periodic wave trains and solitary waves on deep water (Benjamin-Feir, superharmonic instabilities) and two-dimensional waves under three-dimensional perturbations. Conditional stability results for solitary waves are also available ('conditional' since they hold only over the unknown, possibly small, existence times of solutions).

- Mariana Hărăguş (Université de Franche-Comté, France) gave the talk 'Counting unstable eigenvalues in Hamiltonian spectral problems via commuting operators'. She presented a general counting result for the unstable eigenvalues of linear operators of the form JL, in which J and L are respectively skew- and self-adjoint operators. Under the assumption that there exists a self-adjoint operator K such that JL and JK commute, the result states that the number of unstable eigenvalues of JL is bounded by the number of nonpositive eigenvalues of K. As an application, Hărăguş discussed the transverse stability of one-dimensional periodic traveling waves in the classical KP-II (Kadomtsev–Petviashvili) equation.

- Christian Klein (Institut de Mathématiques de Bourgogne, France) discussed 'Multidomain spectral methods for Schrödinger equations'. A multidomain spectral method with compactified exterior domains combined with stable second and fourth order time integrators was presented for Schrödinger equations. The numerical approach allows high precision numerical studies of solutions on the whole real line or in higher dimensions. The method was compared with transparent boundary conditions and perfectly matched layers. The code can deal with asymptotically non vanishing solutions such as the Peregrine breather (currently being discussed as a model for rogue waves). It was shown that the Peregrine breather can be numerically propagated with essentially machine precision, and the localized perturbations of this solution were discussed.

- Alexander Korotkevich (University of New Mexico, USA and Landau Institute, Russia) gave the talk 'Circular instability of a standing surface wave: numerical simulation and wave-tank experiments', in which he showed that a standing wave is unstable if four-wave nonlinear processes are considered. This is joint work with Sergei Lukaschuk. Korotkevich compared numerical simulations of unstable weakly nonlinear standing waves on the surface of a deep fluid in the framework of the primordial dynamical equations and in a laboratory wave-tank experiment. Direct measurements of the spatial Fourier spectrum confirm the existence of the instability in real life conditions for gravity-capillary surface waves.

- Emilian Părău (University of East Anglia, UK) presented the talk 'Stability of capillary waves on fluid sheets'. The talk was based on joint work with Mark Blyth in which the linear stability of finite-amplitude capillary waves on inviscid sheets of fluid is investigated; superharmonic and subharmonic perturbations are considered and a conformal mapping technique is used. The instability results are also checked by time integration of the fully nonlinear unsteady equations.

- Erik Wahlén (Lund University) gave the talk 'Variational existence and stability theory for hydroelastic solitary waves' (joint work with Mark Groves and Benedikt Hewer). He presented an existence and stability theory for solitary waves at the interface between a thin ice sheet and an ideal fluid, which is based on minimising the total energy subject to the constraint of fixed total horizontal momentum. The ice sheet is modelled using the Cosserat theory of hyperelastic shells. Since the energy functional is quadratic in the highest derivatives, stronger results are obtained than in the case of capillary-gravity waves.

2.4 Model equations

An important direction of inquiry in the general field of nonlinear waves is the development and application of simplified model equations. In asymptotic regimes where small parameters can be defined, many such models can be derived at various orders of approximation. Their interest lies in the fact that they are usually more tractable analytically and numerically than the full equations. As a consequence, they have been a popular tool in the engineering community where they have been used with varying degrees of success. The well-posedness as well as rigorous justification of model equations are fundamental mathematical questions that have recently led to a surge of activity (see e.g. [6]). Such analysis is crucial in determining their precise range of validity. Their numerical simulation also requires sophisticated numerical methods that are suited to their mathematical structure. In addition, some models may exhibit blow-up which calls for careful analysis and computation.

- David Ambrose (Drexel University, USA) gave the talk 'Sufficiently strong dispersion removes ill-posedness of truncated series models of water waves' (joint work with Shunlian Liu). Previous joint work with Jerry Bona, David Nicholls and Michael Siegel, demonstrated that truncated series models of gravity water waves exhibit ill-posedness. Ambrose explained that the addition of sufficiently strong dispersion makes such a system well-posed. Physically, this strong dispersion can be relevant, for instance, for hydroelastic waves. The proof uses techniques of paradifferential calculus.
- Jerry Bona (University of Illinois at Chicago, USA) discussed 'Higher-order Hamiltonian models for water waves'. He introduced higher-order models for unidirectional propagation of long-crested water waves and sketched their analysis.
- Gabriele Bruell (NTNU, Norway) presented the talk 'On symmetry and decay of traveling wave solutions to the Whitham equation'. The Whitham equation is a nonlocal, nonlinear dispersive wave equation introduced by G. B. Whitham as an alternative to the Korteweg-de Vries equation, describing the wave motion at the surface on shallow water. Bruell showed that any supercritical solitary waves decays exponentially, is symmetric and has exactly one crest. She also presented a result stating that, conversely, any classical, symmetric solution constitutes a traveling wave. The latter result holds true for a large class of partial differential equations sharing a certain structure.
- John Carter (Seattle University, USA) discussed 'Frequency downshifting in a viscous fluid'. Frequency downshift, i.e. a shift in the spectral peak to a lower frequency, in a train of nearly monochromatic gravity waves was first reported by Lake *et al.* (1977). Even though it is generally agreed upon that frequency downshifting (FD) is related to the Benjamin-Feir instability and many physical phenomena (including wave breaking and wind) have been proposed as mechanisms for FD, its precise cause remains an open question. Dias *et al.* (2008) added a viscous correction to the Euler equations and derived the dissipative NLS equation (DNLS). In his talk, Carter introduced a higher-order generalization of the DNLS equation, which he called the viscous Dysthe equation. He outlined the derivation of this new equation and presented many of its properties. He established that it predicts FD in both the spectral mean and spectral peak senses. Finally, he demonstrated that predictions obtained from the viscous Dysthe equation accurately model data from experiments in which frequency downshift occurred.
- Christopher Curtis (San Diego State University, USA) discussed 'Surface and interfacial waves over currents and point vortices'. The computation of surface and interfacial waves is a central problem in fluid mechanics. While much has been done, the effect of vorticity on surface and internal wave propagation is still poorly understood. Curtis first looked at shallow-water propagation in density stratified fluids with piecewise linear shear profiles. He showed that by allowing for jumps in the

shear across the interface, strong nonlinear responses can be generated, resulting in phenomena such as dispersive shock waves. Thus depth varying currents could play a larger role in interface dynamics than is currently understood. Second, he studied the problem of collections of irrotational point vortices underneath a free fluid surface. He presented a derivation of a model and numerical scheme which allows for arbitrary numbers of vortices in a shallow-water limit and argued that this might give some hint as to how underwater eddies can generate free surface waves.

- Mats Ehrnström (NTNU, Norway) gave the talk 'On waves of greatest height in fully dispersive equations', in which he discussed the existence of large-amplitude periodic traveling waves to the nonlocal Whitham equation. In particular, he discussed the existence of a highest, $C^{1/2}$-cusped, traveling wave solution, which is obtained as a limiting case at the end of the main bifurcation branch of P-periodic traveling wave solutions. He proved that this regularity is optimal. Given that the Euler equations admits a highest wave that is not cusped, but Lipschitz continuous, it is an interesting question whether a bidirectional Whitham equation, which carries the full two-way dispersion relation from the Euler equations, could encompass a Lipschitz wave as well. In his talk, Ehrnström showed however that the highest wave for the bidirectional Whitham equation is not Lipschitz. He characterized its behaviour near the wave crest. Finally, he outlined the first steps towards a more general theory. The talk comprised joint work with Erik Wahlén, Mathew Johnson and Kyle Claassen.

- Anna Geyer (University of Vienna, Austria) presented the talk 'On periodic traveling waves of the Camassa-Holm equation', in which she considered the wave length λ of smooth periodic travelling wave solutions of the Camassa-Holm equation. The set of these solutions can be parametrized using the wave height a. Her main result establishes monotonicity properties of the map $a \mapsto \lambda(a)$ i.e., the wave length as a function of the wave height. She explained how to obtain the explicit bifurcation values, in terms of the parameters associated with the equation, which distinguish between the two possible qualitative behaviours of $\lambda(a)$, namely monotonicity and unimodality. The key point is to relate $\lambda(a)$ to the period function of a planar differential system with a quadratic-like first integral, and to apply a criterion which bounds the number of critical periods for this type of system.

- Henrik Kalisch (University of Bergen) gave the talk 'On existence and uniqueness of singular solutions for systems of conservation laws', in which he discussed existence and admissibility of delta-shock solutions for hyperbolic systems of conservation laws. One of the systems discussed is fully nonlinear, and does not admit a classical Lax-admissible solution to certain Riemann problems. By introducing complex-valued corrections in the framework of the weak asymptotic method, he showed that a compressive delta-shock wave solution resolves such Riemann problems. By letting the approximation parameter tend to zero, the corrections become real valued and the resulting distributions fit into a generalized concept of singular solutions. In this framework, it can be shown that every 2×2 system of conservation laws admits delta-shock solutions. As an example, he showed that the combination of discontinuous free-surface solutions and bottom step transitions naturally leads to singular solutions featuring Dirac delta distributions in the context of shallow-water flows.

- Vera Hur (University of Illinois at Urbana-Champaign, USA) discussed 'Wave breaking and modulational instability in full-dispersion shallow water models'. In the 1960s, Benjamin and Feir, and Whitham, discovered that a Stokes wave would be unstable to long wavelength perturbations, provided that (the carrier wave number) $\times$ (the undisturbed water depth) $> 1.363...$. In the 1990s, Bridges and Mielke studied the corresponding spectral instability in a rigorous manner. But it leaves some important issues open, such as the spectrum away from the origin. The governing equations of the water wave problem are complicated, and one may resort to simple approximate models to gain insights. Hur began by discussing Whitham's shallow water equation and the wave breaking conjecture, and then moved on to the modulational instability index for small-amplitude periodic traveling waves, the effects of surface tension and constant vorticity. She also discussed higher order corrections as well as extensions to bidirectional propagation and two-dimensional surfaces. The talk was based on joint work with Jared Bronski, Mathew Johnson, Ashish Pandey and Leeds Tao.

- Dag Nilsson (Lund University, Sweden) discussed 'Solitary waves of a class of Green-Naghdi type systems' (work in in progress with Erik Wahlén and Vincent Duchêne). He considered a class of nonlocal Green-Naghdi type systems and presented an existence result for solitary wave solutions. The solutions are constructed as constrained minimizers of a certain scalar functional. A key component of the proof is the use of the concentration compactness principle.

- Daniel Ratliff (University of Surrey, UK) discussed 'The emergence of higher order dispersion from periodic waves'. Following the method of Bridges (2013), he demonstrated how one may derive PDEs with fifth order dispersion from periodic waves (and, in general, relative equilibria). Many of the coefficients of the emerging nonlinear approximations are directly related to the system's conservation laws, and those of the dispersive terms are tied to a Jordan chain analysis. Applications of the theory were also discussed.

- Steve Shkoller (UC Davis, USA) discussed 'Nonuniqueness of weak solutions to the SQG equation'. This is joint work with Tristan Buckmaster and Vlad Vicol, in which it is proved that weak solutions of the inviscid surface quasi-geostrophic (SQG) equation are not unique, thereby answering an open problem posed by De Lellis and Szekelihidi. Moreover, he showed that weak solutions of the dissipative SQG equation are not unique, even if the fractional dissipation is stronger than the square root of the Laplacian.

2.5 Waves in domains with complex geometry

In view of real-world applications, it is crucial to consider wave problems in domains with complex geometry or complex boundary conditions, e.g. surface waves propagating over topography near the shore or generated by a wavemaker in a laboratory basin. The modelling of surface-wave propagation under wind forcing and bottom friction also poses many theoretical challenges. Other examples in fluid mechanics include hydroelastic waves in ice of variable thickness and hydromagnetic waves under the influence of a multidirectional magnetic field. These effects add considerably to the technical difficulties of the problem and also require special treatment in the numerical schemes.

- Harry Bingham (Technical University of Denmark) discussed 'Stable, high-order finite difference methods for nonlinear wave-structure interaction in a moving reference frame' which are used for solving potential flow approximations of nonlinear surface waves interacting with marine structures. Of special interest is the loading and wave-induced response of sailing ships. The work builds on the numerical solution strategy described in Bingham and Zhang (2007); in contrast to a simple upwinding strategy, the approach is found to give accurate and stable solutions for all combinations of wave celerity and ship forward speed. An immersed boundary method based on weighted least-squares difference approximations was developed to introduce the ship geometry into the numerical solution. Challenges with respect to tracking the body-free surface intersection line, and treating wave-breaking in a rational way were raised for discussion.

- Onno Bokhove (University of Leeds, UK) gave the talk 'Variational coupling of nonlinear water wave and ship dynamics: continuum and finite element modelling'. He reported on the mathematical and numerical modelling of (non)linear ship motion in (non)linear water waves, and derived a coupled model for the wave-ship dynamics following a variational methodology, in order to ensure zero numerical damping which is important for wave propagation. The final system of evolution equations comprises the classical water-wave equations for incompressible and irrotational waves, and a set of equations describing the dynamics of the ship. The novelty in this model is in the presence of a physical restriction on the water height under the ship, which is enforced through an inequality constraint via a Lagrange multiplier. The model is solved numerically using a variational (dis)continuous Galerkin finite element method with special, new and robust time integration methods. Bokhove showed numerical results for the dynamics of the coupled system in a hierarchy of increasing complexity: linear water-wave and linear ship dynamics, and potentially also fully coupled (non)linear water-wave and nonlinear ship dynamics.

- John Grue (University of Oslo, Norway) presented 'Ship generated tsunamis: linearity vs. nonlinearity'. The motivation for the talk was recent observations of very long waves running ahead of ships in the Oslofjord in Norway. The waves are triggered when the new, very large and relatively fast cruise ferries run across significant depth changes in the shallow fjord. An asymptotic linear analysis expressing the upstream wave field in terms of a pressure impulse at the depth change is complemented by fully dispersive calculations of the upstream waves. At very shallow positions the local ship speed becomes critical where the effect of nonlinearity is analyzed. A wave length of 700 m, wave height at the shore of 1 m, average depth of the fjord of 35 m, depth change similar to the average depth, ship length of 200 m, moving at subcritical speed, are typical characteristics.

- David Lannes (Université de Bordeaux, France) gave a talk entitled 'On the dynamics of floating structures'. He derived some equations describing the interaction of a floating solid structure and the surface of a perfect fluid. This is a double free boundary problem since in addition to the water-wave problem (determining the free boundary of the fluid region), one has to find the evolution of the contact line between the solid and the surface of the water. The so-called floating body problem has been studied so far as a three-dimensional problem. Lannes' first goal was to reduce it to a two-dimensional problem that takes the form of a coupled compressible-incompressible system. It was also shown that the hydrodynamic forces acting on the solid can be partly cast into the form of an added mass-inertia matrix, which turns out to be affected by the dispersive terms in the equations.

- Katie Oliveras (Seattle University, USA) discussed 'Relationships between pressure, bathymetry, and wave-height' (joint work with Vishal Vasan and Daniel Ferguson). A new method was proposed to relate the pressure at the bottom of a fluid, the shape of the bathymetry, and the surface elevation of a wave for steady flow or traveling waves. Given a measurement of any one of these physical quantities (pressure, bathymetry, or surface elevation), a numerical representation of the other two quantities is obtained via a nonlocal nonlinear equation derived from the Euler formulation of the water-wave problem without approximation. From this new equation, a variety of different asymptotic formulas were derived. The nonlocal equation and the asymptotic formulas were compared with both numerical data and physical experiments.

- Rosa Vargas-Magana (Universidad Nacional Autónoma de México) talked about 'A Whitham-Boussinesq long-wave model for variable topography' (joint work with Panayotis Panayotaros). They studied the propagation of water waves in a channel of variable depth using the long-wave asymptotic regime, using the Hamiltonian formulation of the problem in which the non-local Dirichlet-Neumann (DN) operator appears explicitly. They performed an ad-hoc modification of these terms using a pseudo differential operator (PDO) associated with the bottom topography. A Whitham-Boussinesq model for bidirectional wave propagation in shallow water that involves a PDO and which consider explicitly the expression for the depth profile was proposed in the talk. The model generalizes the Boussinesq system, as it includes the exact dispersion relation in the case of constant depth. An accurate and efficient numerical method was developed to compute this PDO and some results for the normal modes and eigenfrequencies of the linearized problem for families of different topographies were presented.

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