Faculty of Engeneering and Natural Sciences

Novel manifestations of the noise-aided signal enhancement

(Joint work with Kostal, Lansky, Levakova)

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My research interests:

- Statistical inference for stochastic processes (with application to neuroscience).
- Mathematical modeling of neuronal activity and physiological sytems.

Considered type of data (so far): Extra-cellular recordings.

Statistical methods for:

- **Detecting connectivity** (with Sacerdote and Zucca)
    - (ST) Leaky Integrate and Fire models coupled through copulas: association properties of the Interspike Intervals. J. Physiol., 53 (6): 396–406, 2010.
    - (STZ) Detecting dependencies between spike trains of pairs of neurons through copulas. Brain Res., 1434: 243–256, 2012.

- **(Noisy first-spike) Response latency** (with Ditlevsen and Lansky):
    - (TDL) Identification of noisy response latency. Phys. Rev. E, 86, 021128, 2012.
    - (TDL) Parametric inference of neuronal response latency in presence of a background signal. BioSystems, 112: 249–257, 2013.
    - (LevakovaTDL) A review of the methods for neuronal response latency estimation. BioSystem, 136, 23–34, 2015.

Methods for passage times/Spike times:

- **Single neuron modeling**:
  - (T) Approximation of the first passage time density of a Brownian motion to an exponentially decaying threshold by two-piecewise linear threshold. Application to neuronal spiking activity. Math. Biosci. Eng., 13 (3), 613–629, 2016. (**poster**)

- **Multivariate processes**:
  - (STZ) First passage times of two-dimensional correlated processes: analytical results for the Wiener process and a numerical method for diffusion processes. J. Comput. Appl. Math., 296, 275-292, 2016. (**poster**)

Mathematical modeling of networks:

- (TSJacobsen) Weak convergence of marked point processes generated by crossings of multivariate jump processes. Application to neural network modeling. Physica D, 288: 45–52, 2014.
- (TDMarkussenKyllingsbæk) Gaussian counter models for Visual Identification of Briefly Presented, Mutually Confusable Single Stimuli in Pure Accuracy Tasks. J. Math. Psych., in press, 2017.

More from my institute:
* Ableidinger, Buckwar, Thalhammer. An importance sampling technique in Monte Carlo methods for SDEs with a.s. stable and mean-square unstable equilibrium, J. Comput. Appl. Math., 2017.
* Lima, Buckwar. Numerical Solution of **Stochastic Neural Fields with Delays** (on ArXiv), 2017.
* Lima, Buckwar. Numerical solution of the **neural field equation in the two-dimensional case**, SIAM J. Scient. Comput., 2015.
* Riedler, Buckwar. Laws of large numbers and Langevin approximations for stochastic neural field equations, J. Math. Neurosc., 2013.
* Buckwar, Riedler. An exact **stochastic hybrid model** of excitable membranes including spatio-temporal evolution. J. Math. Biol., 2011.

From **Objectives** of *Brain Dynamics and Statistics: Simulation versus Data (17w5036)*

"Despite a considerable recent body of literature on statistical inference and stochastic modeling in neuroscience, there remain substantial unsolved problems and challenges, some of which we will address during the workshop. Examples are:

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Today's talk: Novel manifestations of the noise-aided signal enhancement.

(The role of noise on the signal decoding accuracy)
(Spontaneous activity and information transmission in single neuron)

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We know that noise

- corrupts signal transmission in linear systems.
+ may have positive effect in non-linear systems.

Positive effects of the noise

- *Stochastic resonance* [McDonnell [1,2], Greenwood, Lindner, Longtin, Ward]:
  - typically systems with a threshold in presence of weak signals [3].
  - suprathreshold signal may be also enhanced by noise in a network of threshold devices [4, 5]

[1] McDonnell and Abbott. PLoS Comput. Biol. 2009.

[2] McDonnell and Ward. Nature Reviews Neuroscience, 2011.

[3] Gammaitoni, Hanggi, Jung, and Marchesoni. Stochastic resonance. Reviews of Modern Physics, 70(1), 223–287, 1998.

[4] Stocks. Suprathreshold stochastic resonance in multilevel threshold systems. Phys. Rev. Lett., 84(11), 2310–2313, 2000.

[5] Stocks. Information transmission in parallel threshold arrays: Suprathreshold stochastic resonance. Phys. Rev. E, 63(4), 041114, 2001.

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- *coherence resonance* [6,7].
- *firing-rate resonance*. [8]

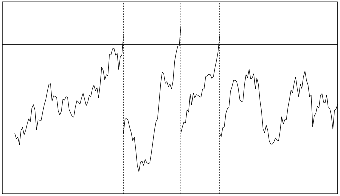
[6] Lindner, Schimansky-Geier and Longtin. Maximizing spike train coherence or incoherence in the Leaky integrate-and-fire model. Phys. Rev. E, 66(3), 031916, 2002.

[7] Kostal, Lansky and Zucca. Netw. Comput. Neural Syst., 2007.

[8] Brunel, Hakim and Richardson. Phys. Rev. E, 2003.

Single Neuronal model: Spontaneous activity

* Stochastic PIF model (Gerstein and Mandelbrot, 1964).



* Membrane potential dynamics X at time t:

$$dX(t) = \mu_0 dt + \sigma_0 dW(t), \quad t > 0, \quad X(0) = 0.$$

W: standard BM; $\mu_0 > 0$ drift parameter, $\sigma_0 > 0$ diffusion parameter.

* Spiking mechanism: crossing of constant threshold $B > 0$ with instantaneous reset of X to its resting condition $\Rightarrow$ **renewal point process** with iid ISIs $T \sim IG(B/\mu_0, B^2/\sigma_0^2)$.

Remember

iid ISIs $T \sim IG(B/\mu_0, B^2/\sigma_0^2)$ (stable distribution)

$\Rightarrow \mathbb{E}[T] = \frac{B}{\mu_0}, \qquad \mathrm{Var}(T) = \frac{B\sigma_0^2}{\mu_0^3}$

$\Rightarrow$ If $B = 1$,

$$\mu_0 = \frac{1}{\mathbb{E}[T]} = \text{ firing rate}$$

$\Rightarrow$ μ_0 can be interpreted as spontaneous firing rate.

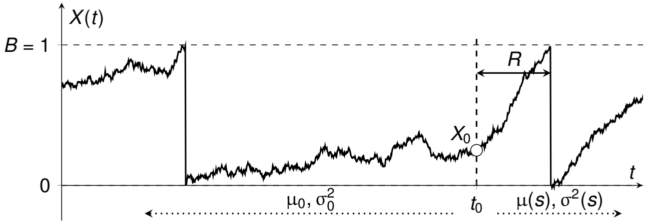
Single Neuronal model: Evoked activity

* Stimulus of unknown intensity level s presented at (given) time t_0.

* The stimulus:

- does NOT change the type of model;
- changes the parameters of the model.

$$(\mu, \sigma^2) \quad = \quad \begin{cases} (\mu_0, \sigma_0^2) & \text{before } t_0 \quad \text{(spontaneous activity)} \\ (\mu(s), \sigma^2(s)) & \text{after } t_0 \quad \text{(evoked activity)} \end{cases}$$



Our goal

Issue: not all stimulus can be decoded with the same accuracy...

Investigate the (optimal, ultimate) stimulus decoding accuracy from available data.

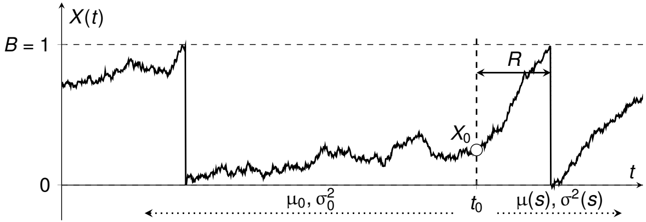
Arising questions:

1. What type of data do we have?
2. How does the stimulus level s enter in $\mu(s)$ and $\sigma^2(s)$?
3. How do we evaluate the decoding accuracy?

* My Hope for today's talk...

Available data: Latency (temporal) coding.

Levakova, Tamborrino, Kostal, Lansky, Neural Comput., 2016.



known: t_0 (time of stimulus onset).

unobserved $X(t), X_0 := X(t_0)$

measured $R(s)$.

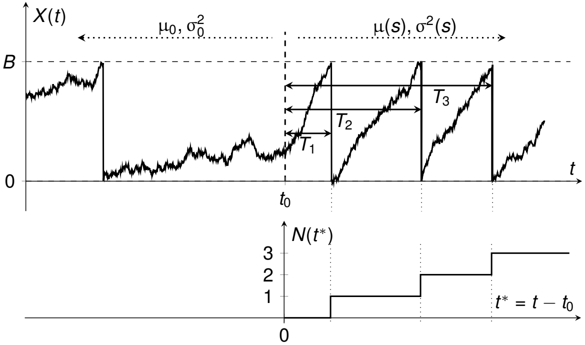
parameter s.

* $R(s)$: random time from the stimulus onset to the first evoked-spike, called *first-spike latency*.

* Pdfs, and first 2 moments of X_0, R available. (Tamborrino, Ditlevsen, Lansky, LDA, 2015).

Available data: Rate coding.

Levakova, Tamborrino, Kostal, Lansky, Phys. Rev. E, 2017.



known t_0.

unobserved $X(t), X_0 := X(t_0)$

observed $n(t^*)$, number of evoked spikes in $[t_0, t_0 + t^*]$ (from $N(t^*)$)

New "parameter" t^* of the model: length of the time window (used by the nervous system).

Goals

Analyze the (optimal, ultimate) stimulus decoding accuracy from:

1. Temporal coding (first-spike latency coding) based on $R(s)$.
2. Rate coding based on the counting process $N(t^*)$.

Two more questions to answer:

1. How does the stimulus level s enter in $\mu(s)$ and $\sigma^2(s)$?
2. How do we evaluate the decoding accuracy?

Transfer function and diffusion coefficient

$$\begin{aligned}
\mu(s) &= \text{spontaneous drift } \mu_0 + \text{ stimulus-driven increment } \Delta\mu(s) \\
&= \mu_0 + \frac{A}{1+e^{-b(s-s_0)}}, \qquad s \in \mathbb{R} \\
\sigma^2(s) &= k\mu(s) + m, \qquad k, m \geq 0,
\end{aligned}$$

with transfer function $\mu(s)$ derived from the Hill function [1] for a stimulus level s expressed on a logarithmic scale, and diffusion parameter $\sigma^2(s)$ linearly dependent on $\mu(s)$ [2].

$A > 0$: maximum possible increment in $\mu(s)$.
$b > 0$: quantity controlling the steepness of the curve.
s_0: location parameter and $\max_{s\in\mathbb{R}} \partial_s \mu(s)$.

[1] Frank. Biology Direct, 2013.

[2] Tuckwell. Introduction to Theoretical Neurobiology, Vol.2: Nonlinear and Stochastic Theories, 1988.

A bit more on $\sigma^2(s)$

Three considered situations for $\sigma^2(s)$:

1. $\sigma^2(s) = k\mu(s) + m$ (Tuckwell, 1988).
2. $\sigma^2(s) = k\mu(s)$ (balanced excitatory and inhibitory inputs, Miura et al., 2007, Sengupta et al. 2013)
3. $\sigma^2(s) = \sigma_0^2$: no dependence on s.

Similar for $\sigma_0^2 \ldots$

Decoding accuracy analysis: wrong approach

- (μ_0, σ_0^2): spontaneous activity parameters (background noise).
- $(\mu(s), \sigma^2(s))$: evoked activity parameters.

$$\mu(s) = \mu_0 + \frac{A}{1+e^{-b(s-s_0)}}, \qquad \sigma^2(s) = k\mu(s) + m, \quad s \in \mathbb{R} \quad k, m \geq 0,$$

Intuition:

- If s changes $\Rightarrow$ $\mu(s) = \mu_0 + \Delta\mu(s)$ and $\mathbb{E}[R(s)]$ change $\Rightarrow$ Best discrimination s^* of s achieved in the region where $\mu(s)$ changes most rapidly, i.e.

$$\tilde{s}^* = \max_{s\in\mathbb{R}} \partial_s \mathbb{E}[R(s)] = \max_{s\in\mathbb{R}} \partial_s \frac{\mu_0 B + \sigma_0^2}{2\mu_0\mu_s} = s_0 - \frac{1}{b}\log\left(1 + \frac{A}{\mu_0}\right) < s_0.$$

Remark:

- $\partial_s \mathbb{E}[R(s)]$ decreasing in μ_0 increases (for fixed σ_0^2, s).
$\Rightarrow$ **The decoding accuracy of s deteriorates for increasing μ_0.**

Decoding accuracy analysis

* **Remark**: the response to the stimulus is stochastic. The previous criterion ignores the variability of the response.

* **Idea** [1]: view the decoding as a problem of statistical estimation: s parameter with estimator $\hat{s}$.

$$\Rightarrow \mathrm{MSE}(\hat{s}) \stackrel{\text{def}}{=} \mathbb{E}[(\hat{s}-s)^2] \stackrel{\hat{s}\text{ unbiased}}{=} \qquad ,$$

[1] Dayan and Abbott, Theoretical Neuroscience, 2001.
[2] Greenwood, Ward, Wefelmeyer, Statistical analysis of stochastic resonance in a simple setting, Phys. Rev. E, 1999.

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$J(s)$: Fisher information about s carried by $R(s)$ (or $N(t^*)$) in the first-spike latency (or rate coding) approach.

Idea[2] : Maximizing $J(s)$ $\Rightarrow$ minimizing $\mathrm{MSE}(\hat{s})$

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$$\textbf{Idea}[2]: \text{Maximizing } J(s) \quad \Rightarrow \quad \text{minimizing MSE}(\hat{s})$$
$$\Rightarrow \quad \text{improving the decoding accuracy}$$

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[2] Greenwood, Ward, Wefelmeyer, Statistical analysis of stochastic resonance in a simple setting, Phys. Rev. E, 1999.

Goals

Analyze the stimulus decoding accuracy from:

1. Temporal coding (first-spike latency coding) based on $R(s)$.
2. Rate coding based on the counting process $N(t^*)$.

In particular: Investigate $J(s)$ and

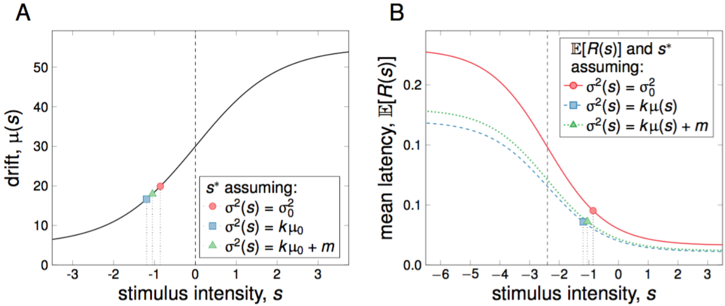
$$s^* = \max_{s \in \mathbb{R}} J(s)$$

and study the roles of

- the spontaneous activity (as μ_0);
- k, m in $\sigma_0^2, \sigma^2(s)$;
- the time window length t^*.

$s^* \neq \tilde{s}^* \neq s_0$

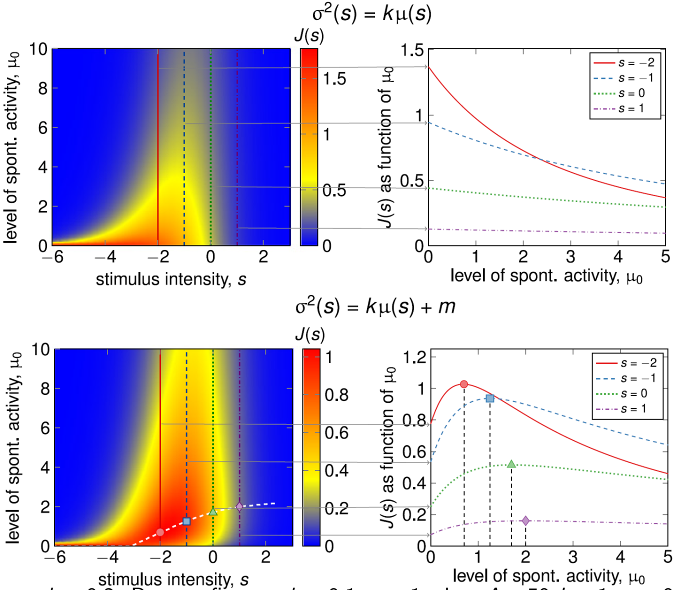
$$s_0 := \max \partial_s \mu(s) > \tilde{s}^* := \max \partial_s \mathbb{E}[R(s)] \neq s^* := \max J(s)$$



* Dashed vertical lines: s_0 (left), $\tilde{s}^*$ (right).

* Parameters: $\mu_0 = 5$, $A = 50$, $b = 1$ and $s_0 = 0$.
Red: $\sigma_0^2 = 4$. Blue: $k = 0.2$. Green: $k = 0.1$ and $m = 1$.

Decoding accuracy from 1st-spike latency coding



Top figures: $k = 0.2$. Bottom figures: $k = 0.1, m = 1$ when $A = 50, b = 1, s_0 = 0$.

Fisher information

	Scenario		
	$\sigma^2(s) = \sigma_0^2$	$\sigma^2(s) = k\mu(s)$	$\sigma^2(s) = k\mu(s) + m$
$J(s)$ wrt μ_0	maximum for $\mu_0 > 0$	decreasing in μ_0	maximum for $\mu_0 > 0$
$J(s)$ wrt other parameters	decreasing in σ_0^2	decreasing in k	decreasing in k decreasing in m

* σ_0^2, k, m always deteriorates the signal decoding accuracy.
* μ_0 may improve the stimulus decoding accuracy!

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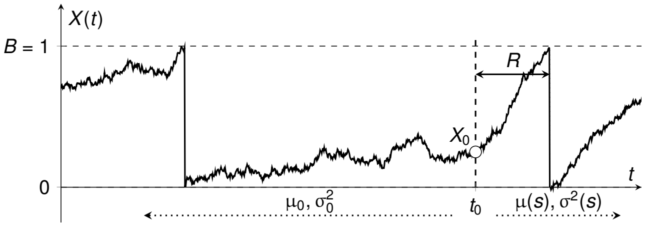
* σ_0^2, k, m always deteriorates the signal decoding accuracy.

* μ_0 may improve the stimulus decoding accuracy! Why?

The reason for the noise-induced enhancement



$$f_{R(s)} = \int_{-\infty}^{B} f_{R(s)|X(t_0)}(r|x) f_{X(t_0)}(x) dx, \qquad X(t_0) = X_0$$

$\overset{Idea}{\Rightarrow}$ study the influence of μ_0 on $R(s)|X(t_0)$ and $X(t_0)$.

For $\sigma^2(s) = k\mu(s) + m$,

$$J_{R|X(t_0)}(s) = \frac{\left[\partial_s \mu(s)\right]^2}{\mu(s)} \frac{k^2\mu(s) + 2(1-x_0)\sigma^2(s)}{2\sigma^4(s)}$$

always decreasing in μ_0 $\forall \mu(s) = \mu_0 + f(s), k \geq 0, m \geq 0$.

The (unobserved) membrane potential $X(t_0)$

* Setting $\alpha = \mu_0/\sigma_0^2$, we get:

$$\begin{aligned} f_{X(t_0)}(x) &= \mathrm{e}^{\alpha(x-|x|)} - \mathrm{e}^{2\alpha(x-1)}, \\ \mathbb{E}[X(t_0)] &= \frac{1}{2} - \frac{1}{2\alpha}, \\ \mathrm{Var}[X(t_0)] &= \frac{1}{12} + \frac{1}{4\alpha^2}, \end{aligned}$$

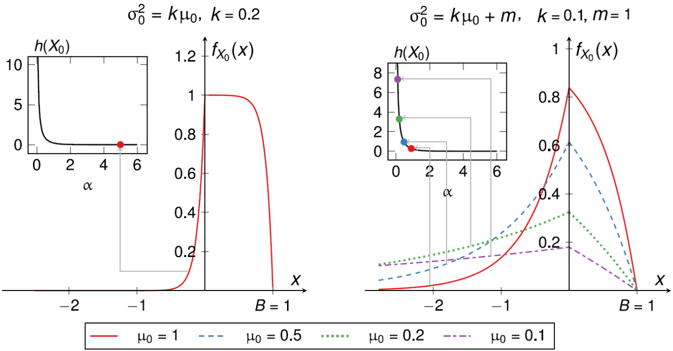
Differential entropy (measure of the randomness) of $X(t_0)$:

$$h(X(t_0)) = -\int_{-\infty}^{1} f_{X(t_0)}(x) \log f_{X(t_0)}(x)\,\mathrm{d}x = \frac{\pi^2 - 6\mathrm{Li}_2\left(\mathrm{e}^{-2\alpha}\right)}{12\alpha},$$

with $\mathrm{Li}_2(x) = \int_x^0 \log(1-t)/t\,\mathrm{d}t$ dilogarithm function.

How are $f_{X(t_0)}$ and $h(X(t_0))$ behaving wrt μ_0?

Differential entropy h of $X(t_0)$



$h(X(t_0))$ decreasing in $\alpha \Rightarrow$ decreasing in μ_0 (if there).

If $\sigma_0^2 = k\mu_0 \Rightarrow \alpha = 1/k \Rightarrow$ no effect on $X(t_0), h(X(t_0))$!

$\Rightarrow$ Influence of the spontaneous activity on the *stabilization* of the membrane potential in the absence of stimulation.

Main results on the temporal coding scenario

- spontaneous activity μ_0 may enhance the signal in a model as simple as the Brownian motion.
- The shown phenomenon does not result from a subthreshold signal (as for stochastic resonance).
- The optimal level μ_0^* is approximately zero for weak stimuli, increases with increasing s and gradually saturates.
- **Key factor**: noise-induced stabilization of the membrane potential in the stimulation-free regime (τ, μ may play similar role for OU)

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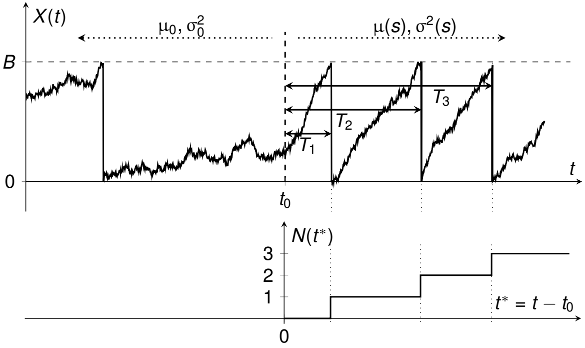
Rate-coding scenario?

Rate coding scenario.



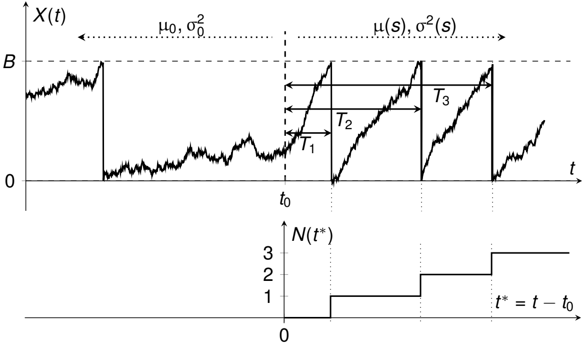
known t_0.

unobserved $X(t), X_0 := X(t_0)$

observed $n(t^*)$, number of evoked spikes in $[t_0, t_0 + t^*]$ (from $N(t^*)$)

Crucial new "parameter" t^* of the model: length of the time window (used by the nervous system).

Two possible scenarios...



- The observation time window starts at $t_0 \Rightarrow X_0 = X(t_0)$ random and unobserved (but $f_{X_0}, \mathbb{E}[X_0], \mathrm{Var}(X_0)$ available).
- The observation time window starts with an evoked spike $\Rightarrow X_0$ known and $X_0 = 0$.

Remark: We assume $B = 1 \Rightarrow \mu_0, \mu(s)$ are spontaneous and evoked firing rates!

Fisher information based on $N(t^*)$

1. Determine T_n.
2. Determine $p_{N(t^*)}(n) := \mathbb{P}(N(t^*) = n; s)$.
3. Determine $J_{N(t^*)}(s) = \sum_{n=0}^{\infty} \frac{\partial_s p_{N(t^*)}(n)^2}{p_{N(t^*)}(n)}$.

$$f_{T_n}(t) \begin{cases} \text{pdf of } IG\left(\frac{nB}{\mu(s)}, \frac{n^2B^2}{\sigma^2(s)}\right) & \text{if } X_0 = 0 \\ = \int_{-\infty}^{B} f_{T_n|X_0}(t|x) f_{X_0}(x)dx \ \text{(computed)} & \text{if } X_0 \text{ random} \end{cases}$$

For $n = 0$:

$$p_{N(t^*)}(0) = \mathbb{P}(T_1 \geq t^*).$$

For $n \geq 1$:

$$p_{N(t^*)}(n) = \begin{cases} \int_{-\infty}^{B} p_{N(t^*)|X_0}(n|x) f_{X_0}(x)dx \ \text{(computed)} \\ \\ \end{cases}$$

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For $n \geq 1$:

$$p_{N(t^*)}(n) = \begin{cases} \int_{-\infty}^{B} p_{N(t^*)|X_0}(n|x) f_{X_0}(x) dx \text{ (computed)} \\ \mathbb{P}(T_n \leq t^* < T_{n+1}) = \mathbb{P}(T_n \leq t^*) - \mathbb{P}(T_{n+1} < t^*) \end{cases}$$

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1. Determine T_n.
2. Determine $p_{N(t^*)}(n) := \mathbb{P}(N(t^*) = n; s)$.
3. Determine $J_{N(t^*)}(s) = \sum_{n=0}^{\infty} \frac{\partial_s p_{N(t^*)}(n)^2}{p_{N(t^*)}(n)}$.

$$f_{T_n}(t) \begin{cases} \text{pdf of } IG\left(\frac{nB}{\mu(s)}, \frac{n^2B^2}{\sigma^2(s)}\right) & \text{if } X_0 = 0 \\ = \int_{-\infty}^{B} f_{T_n|X_0}(t|x) f_{X_0}(x) dx \ \text{(computed)} & \text{if } X_0 \text{ random} \end{cases}$$

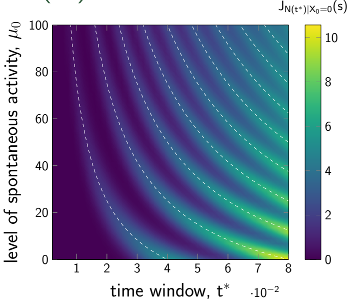
For $n = 0$:

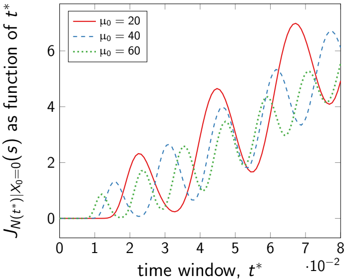
$$p_{N(t^*)}(0) = \mathbb{P}(T_1 \geq t^*).$$

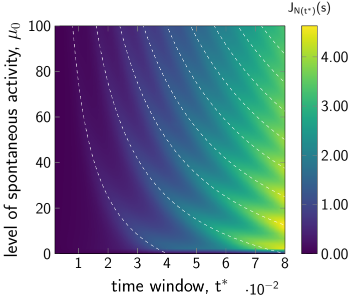
For $n \geq 1$:

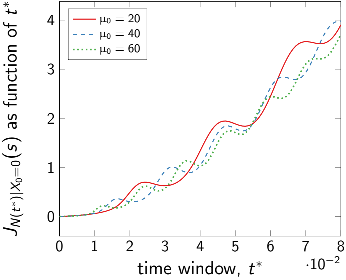
$$p_{N(t^*)}(n) = \begin{cases} \int_{-\infty}^{B} p_{N(t^*)|X_0}(n|x) f_{X_0}(x) dx \ \text{(computed)} \\ \mathbb{P}(T_n \leq t^* < T_{n+1}) = \mathbb{P}(T_n \leq t^*) - \mathbb{P}(T_{n+1} < t^*) \\ \int_0^{t^*} f_{T_n}(\tau) \mathbb{P}(T \geq t^* - \tau) d\tau \end{cases}$$

$J_{N(t^*)}(s=0)$ wrt the time window length t^*

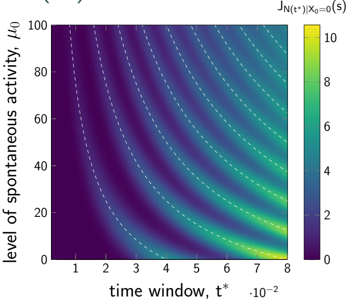


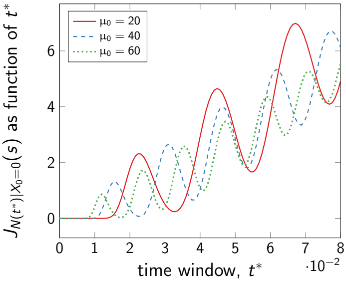






$J_{N(t^*)}(s=0)$ wrt the time window length t^*





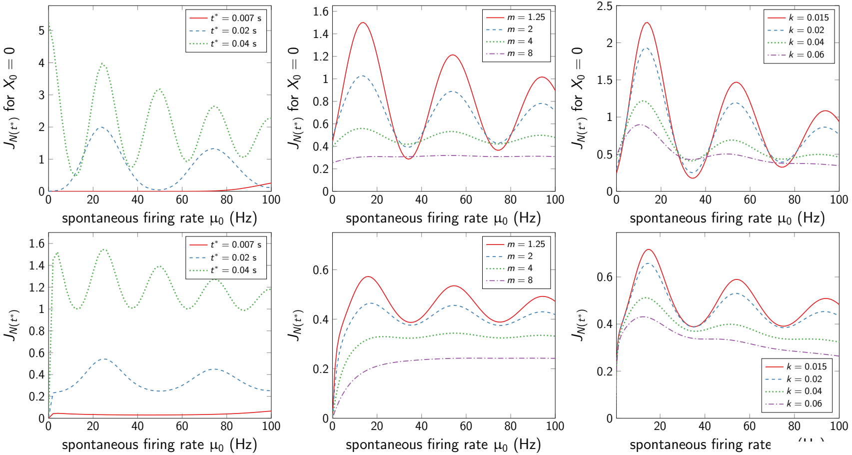
* **Longer time window may not necessarily improve the decoding accuracy.**

* **Possible beneficial compensation between t^* and μ_0!**

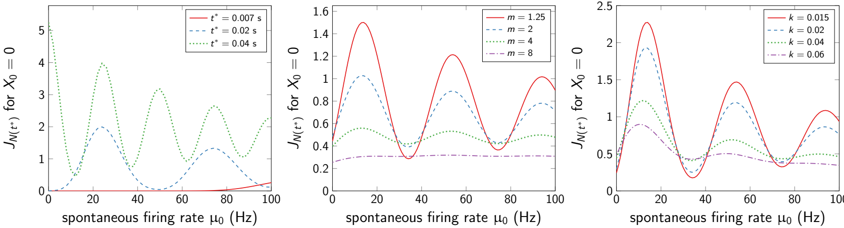
* White dashed lines: points maximizing $J(s)$, with $t^* \approx \frac{nB}{\mu(s)}$ (Why?)

* What is the reason for this oscillatory behavior of the Fisher information?

$J_{N(t^*)}(s)$ wrt the presynaptic spontaneous activity μ_0



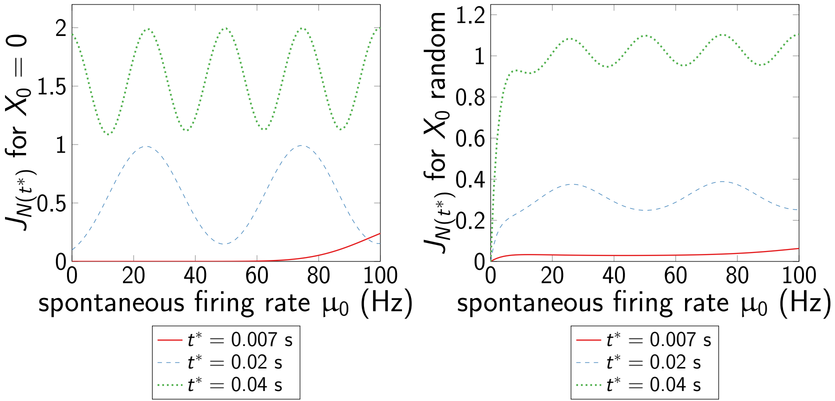
$J_{N(t^*)}(s)$ wrt the presynaptic spontaneous activity μ_0



Presynaptic spontaneous activity may improve the decoding accuracy!

* If k, m are large, the amplitudes of the oscillation decrease.
* Overall tendency of $J_{N(t^*)}$ is decreasing wrt μ_0 but...

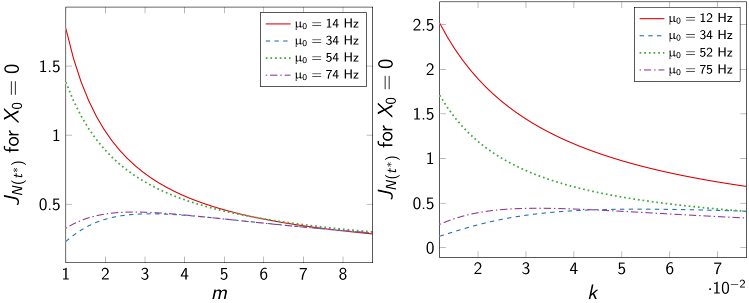
Particular case: $\sigma^2(s) = \sigma_0^2 = 2$ (i.e. $k = 0$)



* (Ignoring local extremes): mild overall increasing tendency of $J_{N(t^*)}$ wrt μ_0, suggesting that the decoding accuracy improves with increasing spontaneous firing rate μ_0! Why?

$J_{N(t^*)}(s)$ wrt $\sigma^2(s)$

* If X_0 is random, $J_{N(t^*)}(s)$ is always decreasing in k and m.
* If $X_0 = 0$:



Increasing $\sigma^2(s)$ improves/deteriorates the decoding accuracy if

$$\mu_0 \approx \frac{2n(n+1)B}{(2n+1)t^*} - \frac{A}{1+e^{-b(s-s_0)}} \quad vs \quad \mu_0 \approx \frac{nB}{t^*} - \frac{A}{1+e^{-b(s-s_0)}}, \qquad n \in \mathbb{N}$$

$\Rightarrow$ The decoding accuracy might be improved by increasing the fluctuation of the membrane potential ($\approx$ stoch. reson.)

We still have to understand the oscillatory behavior of the FI wrt t^* and μ_0 and the reason for these particular values

$$\frac{nB}{t^*}, \qquad \frac{2n(n+1)}{(2n+1)t^*}, \qquad n \in \mathbb{N}.$$

We still have to understand the oscillatory behavior of the FI wrt t^* and μ_0 and the reason for these particular values

$$\frac{nB}{t^*}, \qquad \frac{2n(n+1)}{(2n+1)t^*}, \qquad n \in \mathbb{N}.$$

Obs: $\frac{nB}{t^*}$: deterministic nth FPT when $\sigma^2(s) = \sigma_0^2 = 0, X_0 = 0$.

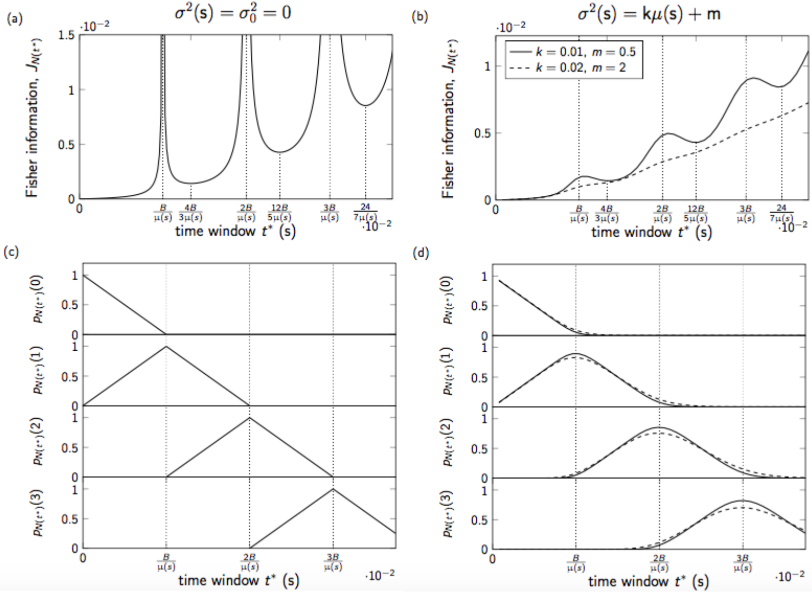
* Idea: study the deterministic system $\sigma^2(s) = \sigma_0^2 = 0$. For X_0 random,

- Explicit expression for $\mathbb{P}(N(t^*) = n)$.
- For $n \in \mathbb{N}$,

$$J_{N(t^*)}(s) = \frac{(\mu'(s)t^*)^2}{[(n+1)B - \mu(t)t^*](-nB + \mu(s)t^*)}, \qquad t^* \in \left(\frac{nB}{\mu(s)}, \frac{(n+1)B}{\mu(s)}\right). \tag{1}$$

- $J_{N(t^*)}(s) \to \infty$ if $t^* = nB/\mu(s)$.
- Minimum of $J_{N(t^*)}(s)$ for $t^*_{\min} = \frac{2n(n+1)B}{(2n+1)\mu(s)}$.

Comparison with the limit case $\sigma_0^2 = \sigma^2(s) = 0$



Main results on the rate-coding scenario

In a setting as simple as the PIF model,

- Longer time window may not necessarily improve the stimulus decoding accuracy.
- Presynaptic spontaneous activity, i.e. μ_0, may improve the decoding accuracy.
- Possible beneficial compensation between shorter time-window and higher presynaptic spontaneous activity, and vice versa.
- If the time window begins with a spike, the decoding accuracy might be improved by increasing the fluctuation of the membrane potential. ($\approx$ Stochastic resonance)

Key factor on t^*: discrete nature of the count of spikes.
Further remark:

- The form of $\mu(s)$ may play a role only on the FI wrt μ_0, but not wrt t^*, k, m (when we fix μ_0, s)!

A last example of noise

A last example of noise

Me: I live in Austria.

A last example of noise

Me: I live in Austria.
RP: Ah, cool, Australia.

A last example of noise

Me: I live in Austria.
RP: Ah, cool, Australia.
Me: Nope, Austria, not Australia.

A last example of noise

Me: I live in Austria.
RP: Ah, cool, Australia.
Me: Nope, Austria, not Australia.

A last example of noise enhancement

Me: I live in Austria.
RP: Ah, cool, Australia.
Me: Nope, Austria, not Australia.

Escaped kangaroo on the run in Austria

The Local
news.austria@thelocal.com

7 July 2016
10:13 CEST+02:00

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Rene Bacher

Police in Styria are being given the run-around by a kangaroo that escaped its enclosure two weeks ago and refuses to go back in captivity.

Tamborrino, Ditlevsen, Lansky. Parameter inference from hitting times for perturbed Brownian motion. *Lifetime Data Analysis*, 21 (3), 331–352, 2015.

Levakova, Tamborrino, Kostal and Lansky. Presynaptic spontaneous activity enhances the accuracy of latency coding. *Neural Comput.*, 28 (10), 2162–2180, 2016.

Levakova, Tamborrino, Kostal and Lansky. Accuracy of rate coding: when shorter time window and higher spontaneous activity may help. *Phys. Rev. E*, 95, 022310, 2017.

Thank you for your attention!