Capturing spike variability in noisy Izhikevich neurons using point process GLMs

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1. A Tale of Two Paradigms

Generalized Linear Models (GLMs) are often simple to formulate and estimate and may therefore aid researchers in designing statistical models to analyze associations in complex data structures, such as spike trains. This model class includes a vast amount of estimation theory and tools, to both fit and assess the model when working with real data. Due to their statistical origin, GLMs are able to capture both structural features, such as association, as well as variability of the input data.

Computational Models (CMs) are designed from a mechanistic perspective, with the goal of describing intrinsic properties of the studied phenomenon, through a deterministic model design of, say, a neuron. Famous examples include classical neuronal models such as Hodgkin-Huxley, FitzHugh-Nagumo, Morris-Lecar and the more recent Izhikevich model. Common to these models is the fact that they often have a meaningful biophysical interpretation.

In this study we compare how well GLMs capture both the variability and the structure from simulations of different types of Izhikevich neurons, injected with noisy stimulus of varying levels. We simulate spike trains ranging from near deterministic to almost complete randomness in spiking behavior. We then fit a simple class of multiplicatively separable history dependent GLMs to these spike trains and show, through a goodness-of-fit analysis, that these models perform optimally in a range of input noise.

2. Simulation Model

The Izhikevich CM is a two-dimensional model capable of simulating various types of observed neuron acitivity. It simulates action potentials v, with a spiking threshold of 30mV, using a secondary refractory variable u

$$\begin{aligned} dv_t &= 0.04v_t^2 + 5v_t + 140 - u_t + I_t \\ du_t &= a(bv_t - u_t), \end{aligned} \quad \Bigg| \quad \text{if } v_t \geq 30 \quad \begin{aligned} v_{t+} &= c \\ u_{t+} &= u_t + d, \end{aligned} \tag{1}$$

where a, b, c, d are parameters chosen to display a certain behavior. I_t is the injected stimulus. The parameters c, d controls the resetting values for v and u whenever $v \geq 30$.
Variability was introduced through the stimulus I_t, where

$$I_t \sim \mathcal{N}(I_0, \sigma^2).$$

Hence, the standard deviation σ becomes the controlling parameter of the noise injected into the simulation. We simulated 6 types of neurons with varying noise levels, $\sigma \in (0, 20]$, ranging from near deterministic to almost completely random spike pattern.

3. GLM Design

We use a discrete time approximation to an orderly continuous time counting process $N(t)$ for a sufficiently small timestep $\Delta t = t_k - t_{k-1}$, such that $P(\Delta N(t) > 1) \approx 0$. Conditional on the past spiking history H_t, the intensity is

$$\lambda(t|H_t) = \lim_{\Delta t \to 0} \frac{P(\text{spike in } [t, t + \Delta t)|H_t)}{\Delta t}.$$

Assuming multiplicatively separability, the log intensity can be modeled as

$$\log(\lambda(t_k|H_{t_k})) = \beta_0 + \sum_{j=1}^{p} \beta_j \sum_{i=1}^{m} y_{k-i} b_{ij}, \tag{2}$$

where b_{ij} are indicator functions dependent on a window width parameter w

$$b_{ij} = \mathbf{1}_{\{(j-1)w+1,\dots,jw\}}(i),$$

such that w controls the number of parameters to estimate. Given spike times $\{s_k\}_{k=1}^{n}$, the conditional log-likelihood with parameter θ,

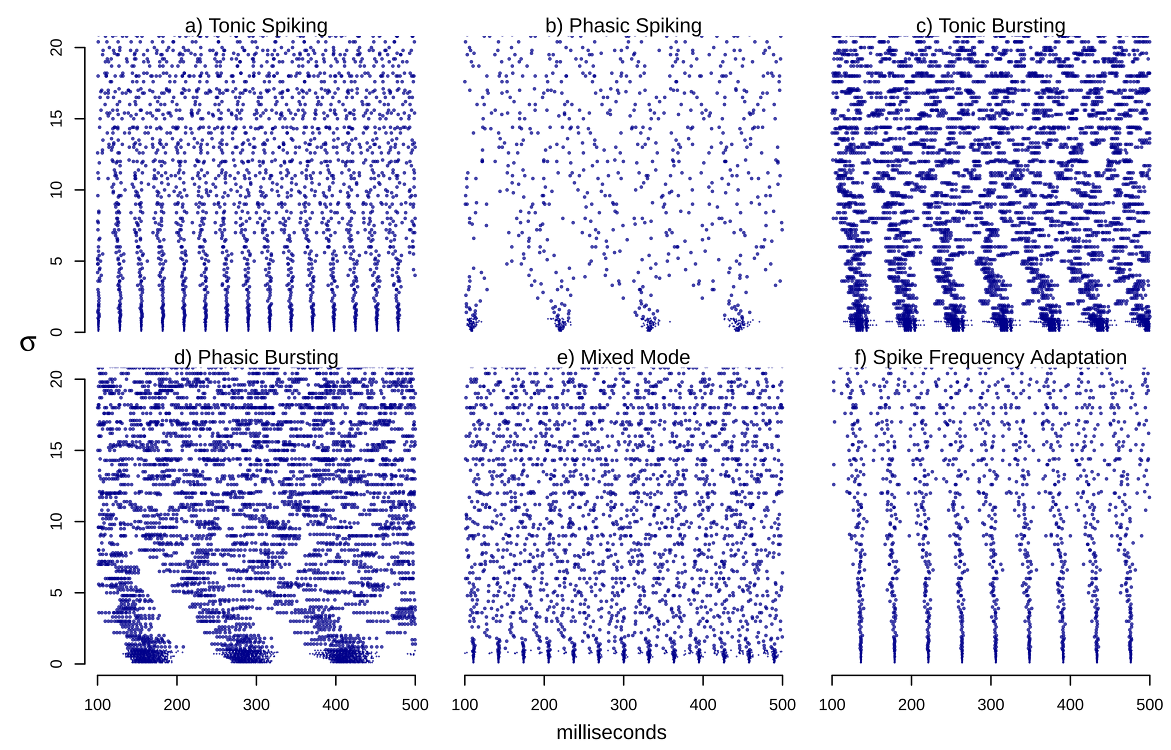
$$\log L(s; \theta) = \int_0^T \log \lambda(t|H_t, \theta) dN(t) - \int_0^T \lambda(t|H_t, \theta) dt \tag{3}$$

can be approximated as a Poisson distribution with a history dependent intensity in small time bins of size Δt, and the GLM framework to fit models can thus be exploited.

Maximization of the log-likelihood (3) was performed as a penalized L1 regression (LASSO) using the `R` package `glmnet`, due to convergence issues for near deterministic spike trains.

4. Simulated Spike Trains

The figure below presents spike trains simulated using an Euler-Maruyama scheme with timestep $\Delta t = 0.1$ms for 6 types of neurons and varying noise level $\sigma \in (0, 20]$. Notice that the sensitivity of the regular spike patterns to σ varies between types, with Spike Frequency Adaption diffusing slowly into random behavior, where as Mixed Mode diffuse almost instantaneously.



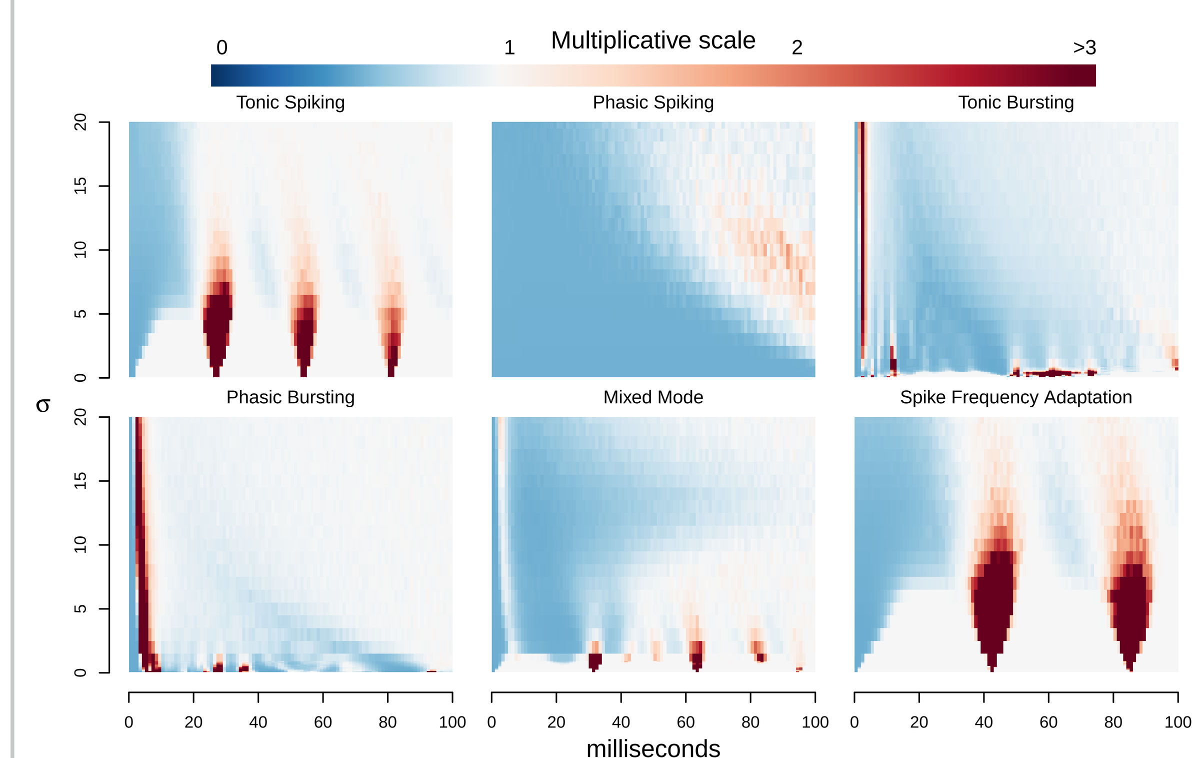
Is is evident that bursting is present even at high noise levels, whereas inter-burst periods and interspike intervals (ISIs) for regular spiking neurons becomes almost completely random in highly noisy regimes. As seen to the right in Section 5, the estimated models also capture bursting, even for highly noisy spike trains, whereas regular spike activity is more difficult to extract from noisy data.

5. Esimated Filters

The estimated exponential 100ms filters $\vec{F}$, with

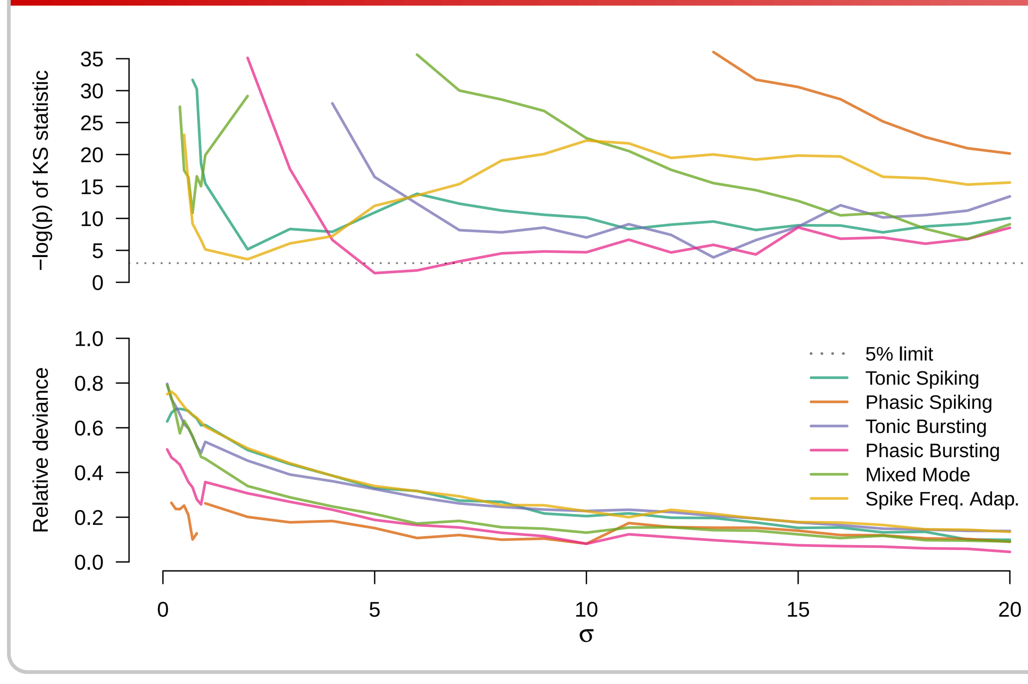
$$\vec{F}_i = \exp\left(\sum_{j=1}^{p} \beta_j b_{ij}\right), i = 1, \dots, 1000 \tag{4}$$

are presented below for each neuron type and varying noise σ. These filters correspond to the modulation of the baseline $\lambda_0 = \exp(\beta_0)$ at lags $i = 1, \dots, 1000$, which corresponds to a 100ms lagged history.



The filters approach delta functions for $\sigma \to 0$ and vanishes for $\sigma \to 20$, besides a refractory period after spiking, which is present along all noise levels. Bursting is captured even at the highest noise levels.

6. Goodness-Of-Fit



The plot show $-\log(p\text{-values})$ of KS statistics for rescaled spiketimes, based on the estimated models and the relative deviance for the 6 neuron types as functions of σ.

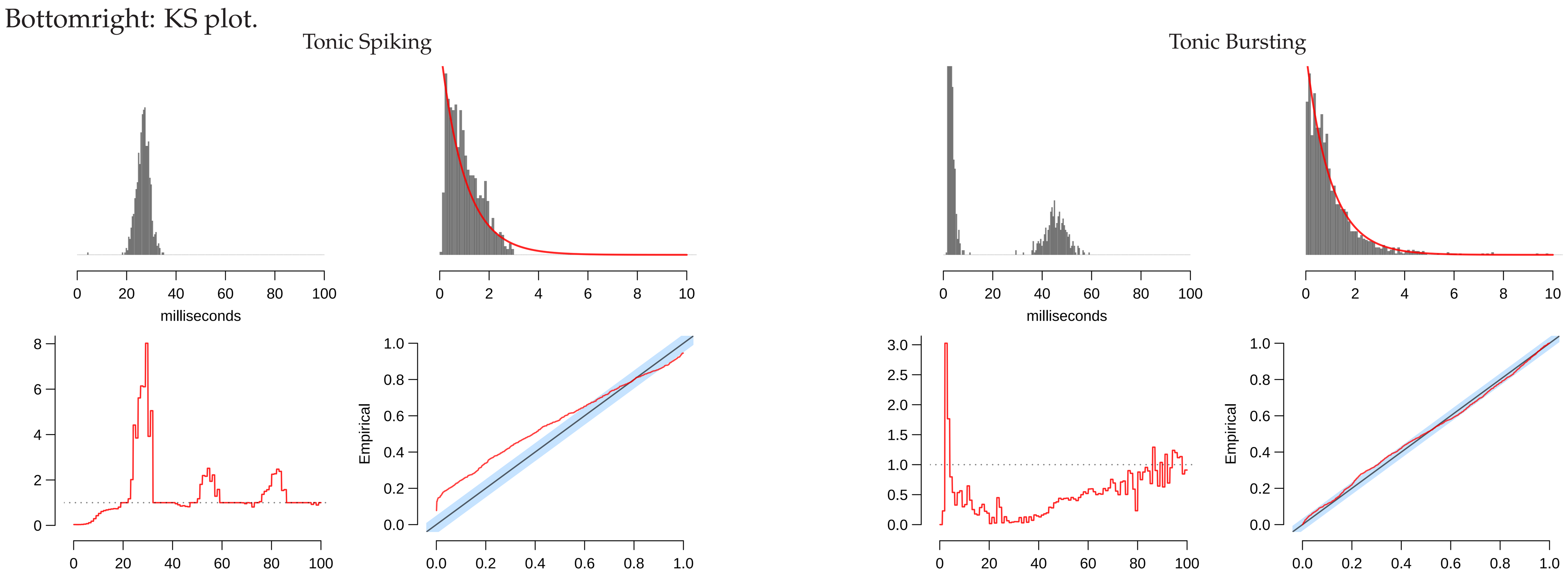
The $-\log$ of the p-values decay towards the 5% limit when σ increases, where as relative deviances indicate that as the noise level increases, the estimated GLMs approach the null Poisson, which does not capture any intrinsic neuron features, but only the baseline firing rate λ_0.

These results indicate that the multiplicatively separable GLM perform optimally in a range of noise, in terms of describing structure and variability, when there is sufficient variability in the data, but with enough structure present to capture intrinsic neuron properties.

7. Tonic Bursting and Spiking Neurons

Below we examine Tonic Spiking and Bursting neurons to contrast regular spiking activity and bursting periods at $\sigma = 5$.

The two plots below each display, Topleft: ISI histogram; Topright: rescaled spiketimes overlaid with an Exp(1) distribution; Bottomleft: estimated filter $\vec{F}$; Bottomright: KS plot.



For Tonic Spiking the ISI histogram shows regular spiking patterns at $\approx$ 27ms. The filter below show peaks at multiples of the ISI mean, $q \cdot 27$ms, for $q = 1, 2, 3$, respectively. The rescaled spike times and the KS plot both show, that the model tends to overestimate smaller ISIs and slightly underestimate longer ISIs.

The Tonic Bursting ISI histogram is bi-modal and the estimated filter shows a peak corresponding to bursting, but does not show positive modulation (> 1) around the inter-burst interval mean $\approx$ 45ms. Looking at the rescaled spike times and the KS plot, this model seems to fit somewhat better than that of the Tonic Spiking neuron. However, this is due to the fact that bursting is captured very well by the model, whereas the regularity between bursts is not, as evident from the filter, and that bursts account for nearly 75-80% of the spike train.

8. Model Extensions

A possible next step is to further investigate history dependent models restricted to the previous spike only, e.g. renewal models, where

$$\log(\lambda(t|H_t)) = \beta_0 + g(s_t^*, t),$$

and $g(\cdot)$ is a function the last observed spike $s_t^* = \max_j\{s_j | s_j < t\}$, and time t, for observed spike times $\{s_j\}_{j=1}^{n}$.
Another extension to the Tonic Spiking & Bursting analysis, is to define a State Space Model, where the state variable X_t is an indicator of whether the neuron is bursting at time t or not

$$\log(\lambda(t|H_t, X_t)) = \beta_0 + \mathbf{1}_{\{X_t=1\}} \lambda^{\text{burst}}(t|H_t) \quad + \mathbf{1}_{\{X_t=0\}} \lambda^{\text{regular}}(t|H_t).$$

This can possibly improve the fit to bi-modal ISI histograms for bursting neurons and it may also serve as a link for interpreting GLM parameters with those of the Izhikevich model (1) that control bursting behavior.

9. The Bottom Line

We have shown that multiplicatively separable history dependent GLMs, perform optimally in a range of injected constant, but noisy stimulus, for Izhikevich neurons, in terms of capturing variability and structural properties of neurons.

Furthermore, our analysis show, that while these GLMs capture in-bursting quite well, the model has more difficulty in accounting for the inter-burst patterns. Future extensions to the simple models considered here may be able to capture this property better and therefore lead to further insight between Izhikevich parameters and interpreting GLMs.

References

Izhikevich EM (2003) Simple model of spiking neurons. Trans. Neur. Netw. 14(6):1569–1572

Truccolo W, Eden UT, Fellows MR, Donoghue JP, Brown EN (2005) A point process framework for relating neural spiking activity to spiking history, neural ensemble, and extrinsic covariate effects. Journal of Neurophysiology 93(2):1074–1089

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