A dynamical geography of the Gulf of Mexico

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Objectives

Using **only** drifters data (trajectories) in the Gulf of Mexico (GoM):

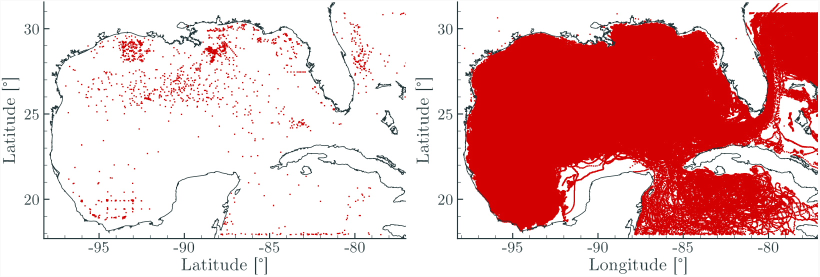
- Subdivide the GoM into regions with similar dynamics
- Predict transport of passive and possibly non-passive tracers

Introduction

Drifters database in the GoM (1994-2016)

Problematics: non uniform data (data per year, type of drifters, time resolution)

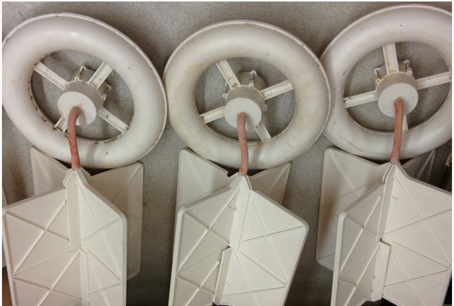
- Left: initial positions
- Right: all trajectories data points



3312 drifters from different sources (LASER & GLAD / CARTHE, GDP / NOAA, BOEM / SCULP, PEMEX / CICESE)

Biodegradable drifters (Novelli et al., 2017, University of Miami)

- 1000 drifters deployed during the LAgrangian Submesoscale ExpeRiment (LASER) in 2016
- Total height of 0.6 m
- GPS precision ± 10 m
- Quarter-hourly acquisition ($\sim$ 3 months)

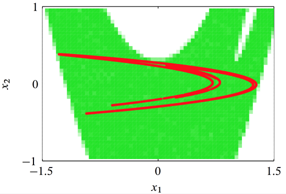


Transfer Operator theory

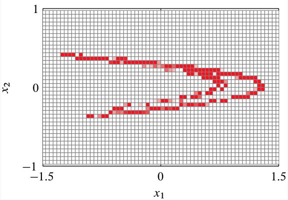
Approximation of an attractor (Dellnitz and Junge, 1997; Froyland, 2001a)

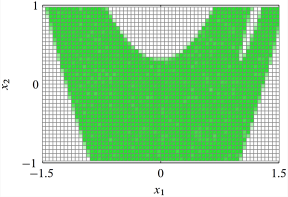
<u>Hénon Map</u>: $(x_1, x_2) \to \left(1 - 1.4x_1^2 + 0.3x_1x_2\right)$

10000 iterations



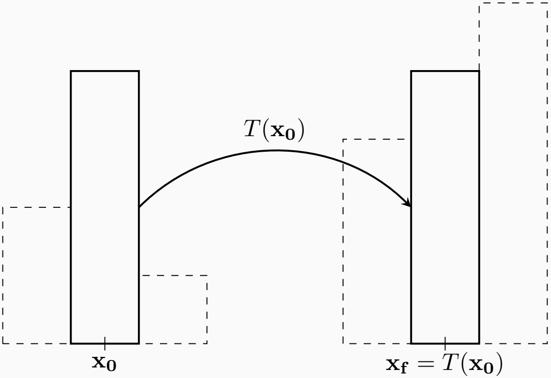
1 iteration





Transfer operator

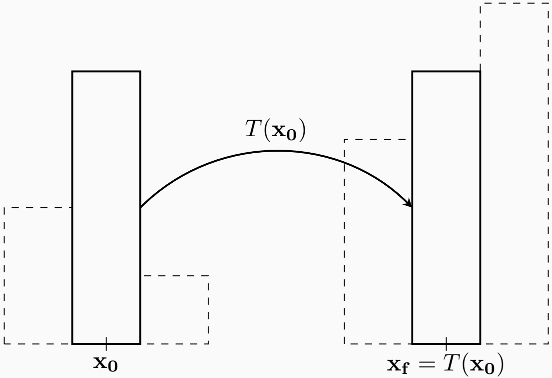
- Domain X
- Map $T : X \circlearrowleft$ acts on a density $f : X \to \mathbb{R}$



Transfer operator

For all the points $\in X$, if the map T is area preserving, the end result can be obtained from the Perron-Frobenius operator ($\mathcal{P}$):

$$\mathcal{P}f(\mathbf{x}) = f \circ T^{-1}(\mathbf{x}).$$



Transition matrix (Froyland, 2001b)

- Probabilistic approach known as Ulam's method (Ulam, 1960)
- Subdivision of the domain in boxes $(B_1, \cdots, B_n)$
- From short-time trajectories, we approximate the probability to go from box i to box j:

$$\mathcal{P}_{ij} \approx \frac{\#\{p : p \in B_i \text{ and } T(p) \in B_j\}}{\#\{p \in B_i\}}$$

by counting the number of particles (drifters) in B_i that are mapped into B_j.

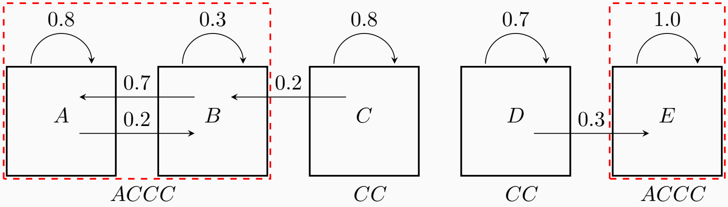
Markov Chain and eigenvectors method (Froyland et al., 2014)

- $\mathcal{P}$ defines a Markov Chain of the dynamics
- Using initial density $f_0 \rightarrow$ future distribution

$$f_1 = f_0\mathcal{P}$$
$$f_N = f_0\mathcal{P}^N$$

- left eigenvectors ($\lambda L = L\mathcal{P}$)
  - for $\lambda = 1$: invariant distribution
  - for $\lambda \approx 1$: almost invariant distribution
- right eigenvector highlights the basin of attraction

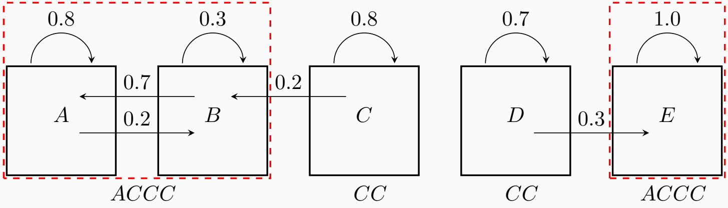
Example with a simple 5 states problem



$\mathcal{P} =$

	A	B	C	D	E
A	0.8	0.2	0	0	0
B	0.7	0.3	0	0	0
C	0	0.2	0.8	0	0
D	0	0	0	0.7	0.3
E	0	0	0	0	1.0

Example with a simple 5 states problem



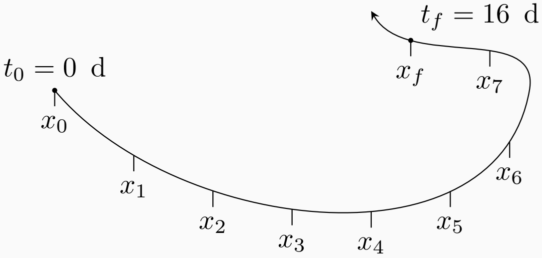
$$L_1^T = \begin{pmatrix} 0.83 \\ 0.55 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad R_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad L_2^T = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad R_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

- L_1: A, B are attractors
- L_2: E is another attractor
- R_1: A, B, C basin of attr.
- R_2: D, E basin of attr.

Algorithm on real data!

Hypothesis

All drifters trajectory start at the same time (Autonomous system) and we construct the transition matrix by looking where drifters end up 2 days later (bins size, data).



Algorithm

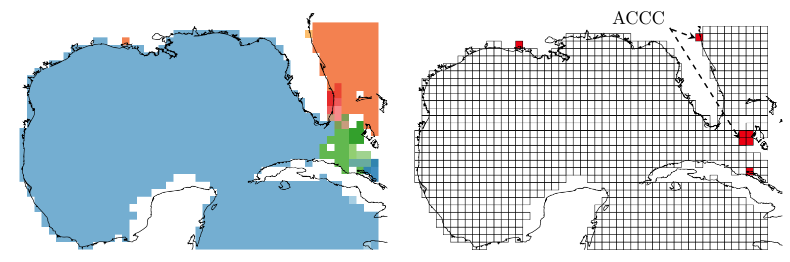
1. Split the domain (GoM) into square boxes
2. For each trajectory segment:
   - find bins i where x_0 is located and store the segment ID in the vector B_i
   - identify bins j where x_f is located and store the segment ID in the vector B_j
3. Calculate the transition matrix $\mathcal{P}_{ij}$ using vectors B_i and B_j
4. Calculate eigenvalues and eigenvectors of $\mathcal{P}$

Let’s look at the results !

Strongly connected components (Tarjan algorithm)

The GoM is almost completely covered by a single CC.

- Left: communicating classes (CC) of the GoM
- Right: the closed communicating classes (CCC) in red with attractive closed communicating classes ($ACCC$)



Limiting distribution from a uniform density

Existence of a westward mean flow: similar to results presented by Sturges (2016)

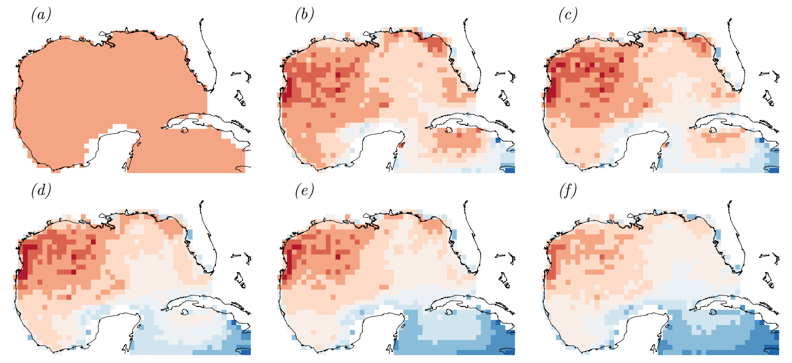
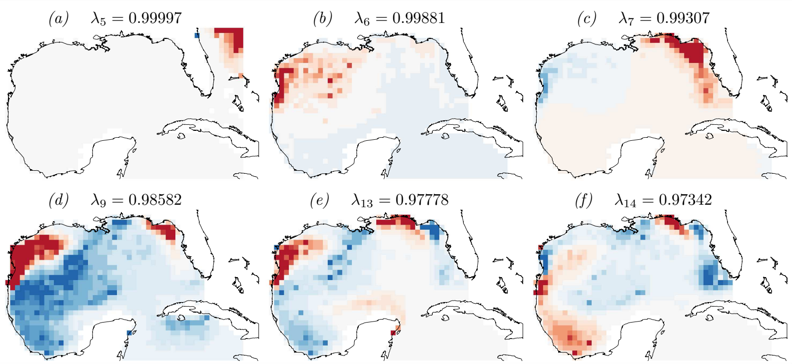
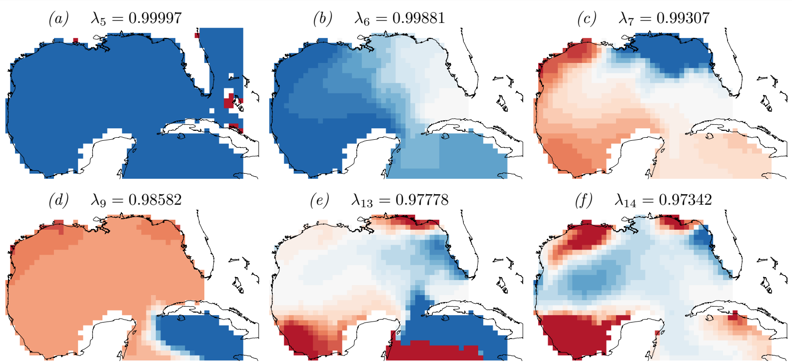


Figure 1: *(a)* 0 d, *(b)* 100 d, *(c)* 200 d, *(d)* 500 d, *(e)* 1000 d, *(f)* 2000 d.

Top left eigenvectors

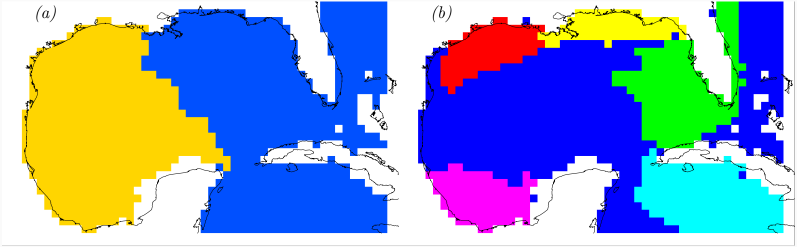


Top right eigenvectors



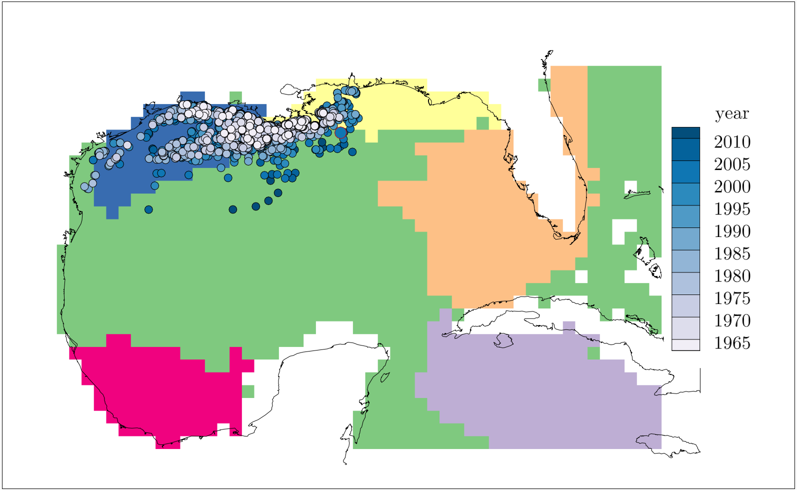
Dynamical geography

- **left:** the main separation of the GoM
- **right:** the five coastal basins of attraction



Oil rigs location

Distance of oil rigs from the coast is increasing in recent years..



Conclusion

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- Identified **almost limiting distributions** and corresponding **basins of attraction** from the inspection of the eigenvectors
- Supported by independent observations of a westward mean flow (Sturges, 2016)
- Ability to push-forward a density to "predict" the dispersion (e.g. after an oil spill or to plan a drifters experiment)

Thank you!

Open questions:

- What is the influence of the drifters' type on the transition matrix ?
- Will it be possible to perform a seasonal (or maybe monthly) evaluation of the transition matrix ?
- How does it compare to high number of artificial drifters or simply density advection using a numerical velocity field?
- How to "easily" extract the different sets ?

Coming soon: Miron et al. (2017): A dynamical geography of the Gulf of Mexico

References I

M. Dellnitz and O. Junge. Almost invariant sets in chua's circuit. *International Journal of Bifurcation and Chaos*, 7(11):2475–2485, 1997.

G. Froyland. Extracting dynamical behaviour via markov models. In A. Mees, editor, *Nonlinear Dynamics and Statistics: Proceedings of the Newton Institute, Cambridge, 1998*, pages 283–324. Birkhauser, 2001a.

G. Froyland. Extracting dynamical behavior via markov models. In *Nonlinear dynamics and statistics*, pages 281–321. Springer, 2001b.

References II

G. Froyland, R. M. Stuart, and E. van Sebille. How well-connected is the surface of the global ocean? *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 24(3):033126, 2014.

P. Miron, F. J. Beron-Vera, M. J. Olascoaga, P. Pérez-Brunius, J. Sheinbaum, and G. Froyland. A dynamical geography of the gulf of mexico. *Preprint*, 2017.

G. Novelli, C. M. Guigand, C. Cousin, E. H. Ryan, N. J. M. Laxague, H. B. K. Dai, Hanjing, and T. M. Özgökmen. A biodegradable surface drifter for ocean sampling on a massive scale: Design, calibration and application. *Preprint*, 2017.

W. Sturges. The mean upper-layer flow in the central gulf of mexico by a new method. *Journal of Physical Oceanography*, 46(10): 2915–2924, 2016.

References III

S. M. Ulam. *A collection of mathematical problems*, volume 8. Interscience Publishers, 1960.