Transport in Unsteady Flows: from Deterministic Structures to Stochastic Models and Back Again

Sanjeeva Balasuriya (University of Adelaide),
Daan Crommelin (CWI Amsterdam),
Gary Froyland (University of New South Wales),
Adam Monahan (University of Victoria),
Nicholas Ouellette (Stanford University),
Laure Zanna (University of Oxford)

15 January 2017–20 January 2017

1 Background and Rationale for Workshop

Transport in unsteady turbulent flows is an issue of central importance in many geophysical and engineering problems. The problem is challenging from a mathematical perspective because of the broad range of scales of motions present in such flows. Additionally, these flows tend to self-organize into coherent structures, such as small scale filaments and large scale jets. The multiscale and nonlinear interactions across these spatial scales has a profound influence on the transport of energy, enstrophy, momentum, heat, plankton, or pollutants and chemicals in the oceans and atmosphere. In recent years, there has been considerable interest and progress in identifying larger scale time-varying structures (such as mesoscale oceanic eddies, the circumpolar Antarctic vortex, oceanic jets and fluid interfaces) from observational and experimental data using methods based on rigorous mathematics rather than heuristic arguments. Such structures, sometimes called Lagrangian Coherent Structures (LCSs [23, 38]), identify crucial dynamical barriers in the flow and characterize transport and mixing. At the same time, a parallel line of research has been pursued investigating stochastic dynamical representation of transport by small, unresolved scales of flow [16, 36]. This question is complicated by the two-way nature of the interaction: the larger, resolved scales organize the smaller, unresolved ones; but these then feed back on the resolved scales through nonlinear interactions. Processes that have been approached from this stochastic dynamical perspective include transport by mesoscale ocean eddies in ocean models [10, 50, 51, 27], stochastic backscatter of energy in numerical weather prediction models [43], and the mixing of scalar fields or chemical species in engineering devices [40, 14].

The dominant diagnostic methods for determining LCSs typically use velocity data to impute where fluid particles go using *deterministic* advection [23, 6, 17, 18, 42, 38, 32, 21]. The velocity field at each position is assumed to be known precisely for all time, permitting the numerical advection of fluid particles from one time instance to another. For real unsteady flows, however, the full velocity field at future times and across all relevant spatial scales is typically not known. Stochastic modelling approaches have been developed to accommodate the lack of information about flow structures on some scales. To date, however, these approaches have made limited use of the insights regarding transport afforded by LCS for either the development of stochastic parameterizations or the study of how these small-scale transports are modulated by larger, resolved coherent structures.

The presence of unresolved small-scale variability limits the genuinely *predictive* capabilities of deterministic coherent structure identification. However, coherent structures provide a low-dimensional yet meaningful description of the flow dynamics and so developing ways to use them for flow prediction would be tremendously valuable for both geophysical and engineering applications. The natural question that then emerges is how the stochastic dynamics and LCS perspectives can be brought together for a fluid dynamical analysis of the (probabilistic) predictability of transport in unsteady flows. One approach is through finite-time coherent sets for advection-diffusion equations and other stochastic processes [17]. There are also promising tools to approach this problem have recently emerged from interesting new theoretical approaches in random dynamical systems [5, 13], which for example indicate that classical deterministic bifurcation behavior of invariant entities is qualitatively different under stochastic perturbations.

In the past year or so, there has been an emerging tentative dialogue between researchers from the stochastic analysis and the fluid dynamics LCS communities. This workshop was designed to strengthen these nascent interactions and promote their development towards an active, fruitful research dialogue by bringing together researchers with complementary approaches to a topical problem. The hope was to push deterministic flow models towards being more realistic, and stochastic approaches towards being more applicable in fluid dynamics settings. This workshop therefore focussed on the interplay of *stochastic* effects and coherent flow structures on transport in unsteady flows, with a longer-term goal of addressing *predictability*, for which a probabilistic interpretation seems the most appropriate approach. By bringing together participants with expertise in theory, modeling and experiment, the workshop planned to coax theory from dynamical systems, fluid dynamics and stochastic analysis to work together. The presence of experimental and observational researchers, whose insight helped guide models and theory in directions relevant for applications, was an important aspect of the workshop. Several early career researchers in modeling, theory and experiments were also among the invitees, with the goal of ensuring that the impact of this workshop lasts into the future.

The workshop's stated goals were:

1. Assess from experiment and observation the predictive utility of LCS and transport estimates arising from deterministic models of real world systems;

2. Investigate how LCS-based perspectives on transport can inform the development of stochastic models of subgrid-scale transport;

3. Harmonize the deterministic and stochastic perspectives on transport, and discover which objects are analogous to LCS for stochastic models.

2 Workshop Structure

Given the diverse nature of the research areas of the participants, we designed this workshop in a very specific way to elicit interactions between the participants, in order to work towards the goals of the workshop. Thus, we structured the workshop around several distinct components:

1. Perspectives: Eleven participants, selected because of their broad knowledge in their subject area, their dissemination skills, and their perceived potential to reach beyond their area, were invited to present 'Perspective' talks on the first and part of the second days of the workshop. These talks were limited to 15 minutes, with 15 minutes discussion thereafter. The Perspectives were intended to introduce certain areas of research to participants from other areas, and the presenters were asked specifically to pose questions and issues that would spill over into related research areas. This highly challenging task, limited as it was to 15 minutes, was taken up with avidity by all our presenters. Illuminating and thought-provoking Perspectives were presented by Stephen Griffies (ocean mesoscales), Michael Ghil (variability in the ocean and atmosphere), Adam Monahan (atmospheric transport), Judith Berner (prediction), Jonathan Lilly (ocean observations and turbulence), Cecile Penland (limitations of models), Guido Boffetta (bio-organisms under turbulence), Nicholas Ouellette (extracting coherence from experimental data), Gary Froyland (set-oriented techniques for transport and coherence), Oliver Junge (computational methods for coherence using transfer operators), and Jason Frank (numerical discretization impacts on preserving coherence). Several presenters identified well-defined issues or potential links to other areas, which led to rich discussions, often involving many of the participants.

2. ***Issue identification***: The Perspectives and subsequent discussions brought into focus many issues for future discussion, and potential multidisciplinary collaborations between the participants. A listing of these issues was maintained, and on day two of the workshop, participants were asked to vote on which of these issues they would like to focus. By a process of elimination, several subject areas were identified. After much discussion, issues were amalgamated into three specific areas:

   1. Lagrangian Coherent Structures: What Good Are They?
   2. Stochastic Parameterization.
   3. Prediction.

   Thus, the issues to be discussed in detail *emerged* organically from the structure of the workshop.

3. ***Problem-solving groups***: Significant time was devoted to addressing specific issues under the three groups that were identified above. The time allocated was roughly 1.5 hours on the second day, 5 hours on the fourth day, and 1 hour on the fifth day. Outlines of the discussions from each of these groups appears in later sections of this report. On the whole, the discussions in these groups was lively and constructive, with the participants taking considerable effort to reach across disciplinary barriers. The organizers wish to thank Shane Keating and Jonathan Lilly in particular for help in coordinating the discussions, and setting up online resources to help document them. The discussions in general helped in bridging the terminology divide, in helping understand why certain issues were considered important or difficult in various research areas, in seeking overlapping interests, and in formulating issues that could lead to future collaborations.

4. ***Standard talks***: The workshop also had ongoing talks that were of the more standard variety. In keeping with the goals of the workshop, efforts were made to give as much exposure as possible to early-career researchers to present their research. One way in which this was achieved was by limiting standard talks also to 15 minutes. This resulted in well-thought out and interesting talks from both junior and senior participants, which also elicited much discussion. Talks were presented by Cecilia González-Tokman, Hussein Aluie, Alexis Tantet, Thomas Peacock, Valerio Lucarini, Jeroen Wouters, Kathrin Padberg-Gehle, Jacques Vanneste, Daniel Karrasch, Philippe Miron, Nikki Vercauteren, Ryan Abernathey, Irina Rypina, Georg Gottwald, Péter Koltai and Shane Keating. The talks cut across many of the research areas represented in the workshop, and included aspects of ergodic theory, fluid experiments, turbulence approaches, transfer operators in the presence of noise, coral reef connectivity, multistability in climate, subgrid-scale parameterization for finite time-scale separation, coherence obtained from trajectories, dispersion in large deviation regimes, coherence as thought of in terms of a heat equation, Gulf of Mexico geometry analysis from float data, atmospheric boundary layer flow structures, Eulerian eddy fluxes and Lagrangian eddies, encounter number as a new diagnostic of mixing, statistical consistency of numerical integrators, space-time characterization of coherence, and stochastic turbulent modeling based on satellite observations of the ocean.

5. ***Poster session***: A poster session was held in the evening of the second day. In contrast possibly with many other workshops or conferences, the majority of the poster presenters were *senior* participants. Posters were presented by Michael Allshouse, Sanjeeva Balasuriya, Daan Crommelin, Amber Holdsworth, Douglas Kelley, Björn Schmalfuß and Marek Stastna.

3 Group on "Lagrangian Coherent Structures: What Good Are They?"

As the central topic of the workshop, this group was the largest of the three and included many participants from the oceanographic community and the mathematics/dynamics community. Several participants from the atmospheric science community also attended some of these sessions. Because of the open question asked of the group ("What good are Lagrangian coherent structures"), the first two sessions were mainly wide-ranging discussions about which questions are of scientific significance.

These sessions were also crucially an open and highly informative learning and knowledge-transfer environment, where the mathematicians/dynamicists were able to communicate directly with mathematically savvy oceanographers. In one direction, this served to explain the features and limitations of current mathematical and computational approaches for analysing dynamics with coherent structures, for example, sampling issues [28]. In the other direction, among several aspects, this provided an explanation of key aspects of ocean models, how the physics of fluid flow might better connect with the relevant mathematics, and an appreciation of the important, unknown aspects of the lifecycle of coherent structures. Several participants remarked that these open discussion sessions provided a truly unique opportunity to engage with different communities in a very collegial atmosphere, and were in many ways the highlight and most valuable part of the workshop.

The question "What good are coherent structures?" was accompanied by the related questions "What is the role of coherent structures in dynamics?" and "What sort of transport is useful in ocean and atmospheric science? (at which scales, of what quantities, etc...)." Physical manifestations of coherent structures include hurricanes, western boundary currents, and jets. Eddies/vortices at all scales are another type of coherent structure occurring frequently in both the atmosphere [45] and the oceans [48]. Many aspects related to the mixing and transport associated with all of these coherent entities were highlighted in the discussions [28, 44, 45, 2, 48]. In addition to the usual impacts on transport in the ocean, these features also play a role in coral reef connectivity and search and rescue efforts. Submesoscale coherent vortices are very important to fisheries. There is a hypothesis within the fisheries community, for example, that they act as "nurseries" for larval fish for periods of time that are relevant for larval development; that is, the time it takes for a fish larva to grow large enough to swim. Thus, it would be interesting to know how much exchange occurs across barriers. Coherent structures may also be useful for dimension reduction, as touched on by the "Predictability" group, and identifying regions across which it would make sense to compute fluxes more generally. The dimension reduction in this case might, for example, be in extracting "modes" corresponding to each coherent structure.

There was a substantial discussion concerning coherent structures and the physical dynamics that must be obeyed in the oceanographic and atmospheric contexts. One could consider coherent structures as emergent behavior, and it was noted that they arise in "maximal entropy" solutions. There is for example a strong sense in the atmospheric community that coherent structures emerge due to baroclinic instabilities, and their geometry is associated with wavenumbers that are preferentially chosen by the dynamics. Most current work with coherent structures does not take into account such dynamics, but instead works directly with a given velocity field. Is it possible to incorporate the governing dynamics into standard coherent structure analysis? This broad theme was identified as requiring additional investigation, and an area in which there could be further interaction between the (mathematical) coherent structures analysts, and the (physical law abiding) geophysical scientists.

Another aspect that arose in the discussions was that coherent structure identification could also be used as a criterion of "fitness" of global circulation models. The quality of global circulation models could be assessed by the coherent structures they give rise to, and comparing these to observations.

This was strong interest from the oceanographic community in identifying coherent structures directly from observations, and it was noted that recent methods have been developed to do this, particularly in the case where the data is sparse, scattered, and possibly incomplete [3, 20, 22, 8, 19, 41]. Over the past few years, the coherent structure community has gradually been developing tools that are targeted for explicitly this situation, and it is hoped that an ongoing development—consistent with the limitations of the observational data—will continue to occur. The dialogue established at this workshop is expected to help in this endeavor, as the needs of the practitioners are becoming more apparent to the theorists, and the theoretical limitations of conclusions more obvious to the geophysicists.

In addition to coherent structures acting as efficient transporters of heat, salt, nutrients, etc., there is also a modeling interest in better understanding their formation, interaction with surrounding fluid and each other, and death. A brief discussion on this issue did not lead to any obvious conclusions, but could potentially be an aspect for future investigation.

The group was conscious of having to formulate some concrete problems for work in the immediate future. The following were identified as having the potential for such work, possibly among the participants of the workshop:

- Downscale GCM (Global Climate Model) model/drifter data and quantify sensitivity and uncertainty in diagnosis of coherent structures.
- Look at the dynamical role or physical significance of coherent structures in simple (but realistic)

turbulence models, e.g., the recent data set [1] generated from the quasigeostrophic model, and others [44, 2, 39].

- Consider the lifecycle of coherent structures, the role of geometry in interacting / merging structures [4, 49], and the role of "entrained periphery" of the coherent core.
- Work on theoretical and applied approaches to the issue of incorporating stochasticity into deterministic coherent structure viewpoints. (There is current progress in this direction [7] as a result of discussions at the workshop, with several follow-up analyses currently in preparation.)

4 Group on "Stochastic Parameterization"

The discussion sessions on stochastic parameterization were attended by participants from the atmospheric science community, mathematicians involved in stochastic modeling, and oceanographers.

Initially, much of the discussion focused on past experiences and results from stochastic parameterizations for atmospheric processes. This was perhaps not surprising, as stochastic methods for parameterizing atmospheric processes (for example convection) have been under development for over 15 years [37, 11], whereas exploration of these methods for ocean modeling is a more recent development [39, 29, 51, 27, 50]. However, later on topics more specific to oceanography were also addressed in the discussions.

A marked difference in what is expected from stochastic parameterization in ocean and atmosphere modeling is that in current atmospheric GCMs, most of the spectrum of eddies are explicitly resolved, whereas in ocean GCMs, they are not. This is directly related to the difference in Rossby radius between atmosphere and ocean. In the atmosphere, it is on the order of 1000 km, a scale that is well resolved in current atmospheric GCMs. As a consequence, atmospheric eddies exist primarily on synoptic scales and these are well resolved. By contrast, the oceanic Rossby radius becomes as small as $\mathcal{O}(10 \text{ km})$ at high latitudes, so that oceanic eddies are not well resolved in most ocean GCMs. For ocean modeling, an important driver for the interest in stochastic parameterization comes from the intent to use these as a way of representing mesoscale eddies. These can be seen as part of the adiabatic processes of ocean flow, hence it is of interest that parameterizations do as little as possible to disturb the energy-conserving nature of the adiabatic processes. In the atmosphere, on the other hand, much of the research on stochastic parameterization is targeting the representation of diabatic processes (notably, convection).

A few of the issues and questions that were brought up during the discussions were:

- Are there guidelines for knowing when a stochastic parameterization will be useful or effective in the atmosphere/ocean/GCMs? Before engaging in a search for the optimal method or approach for stochastic parameterization, one would like to be able to assess in what situations and under what conditions a stochastic parameterization can be useful at all. Two important aspects were discussed in this context: (i) Is there temporal scale separation between the processes that are resolved and the processes that must be parameterized? This is to be distinguished from the question of whether or not there is spatial scale separation. (ii) If the resolved scales of a system receive a large amount of random "kicks" (perturbations) from the unresolved scales, so that the system is approaching (but has not reached) a kind of thermodynamic limit, a stochastic parameterization of these kicks can be appropriate.
- It was discussed that there are different goals that researchers have in mind for stochastic parameterizations. Representing sub-gridscale coherent structures and partially resolved coherent structures is one such goal. A relevant distinction here relating to coherent structures and transport is between transport *of* coherent structures and transport *by* coherent structures. Other aims that were stated in the discussion were (i) capturing non-diffusive behavior (neglected in traditional diffusive eddy parameterizations), (ii) increasing variability on small scales, and (iii) improving the numerical stability of models.
- An issue that was raised several times during discussions is the absence of broadly agreed-on performance measures for stochastic parameterization. Should there be a "skill-score" for parameterizations,

similar in spirit to the skill scores that are used to evaluate performance of numerical weather prediction models? This was considered useful, while it was also acknowledged that the different aims and expectations that researchers have for including stochastic parameterizations in their models make it difficult to identify performance measures that will be widely regarded as useful and important.

- Two further topics addressed in the discussion were (i) methodology and best-practices, and the advantages and disadvantages of methodologies based on first principles versus more empirical approaches; and (ii) the usefulness of intermediate-complexity models for developing and testing stochastic parameterizations methods. This is a class of models that is no longer state of the art. However, testing methods in these models can be very useful as an intermediate step between highly idealized models on the one hand and state of the art models on the other.

During the week, a subgroup formed to discuss stochastic parameterization issues specifically regarding the atmospheric boundary layer (ABL). The primary question of interest was the simulation of the observed multiple regimes of the stably stratified ABL (SBL) and transitions between them [31, 33, 46, 47]. Under conditions of stable stratification, the ABL is observed to exist in two states: the weakly stable boundary layer (WSBL) characterized by strong flow, a weak temperature inversion, and sustained turbulence; and the very stable boundary layer (VSBL) characterized by weak flow, a strong temperature inversion, and weak and intermittent turbulence. These features are quite shallow (typically within a few tens of metres of the surface) and are therefore poorly resolved by standard regional and global models of the atmosphere. Transitions from the WSBL to the VSBL typically occur shortly after sunset during periods of strong radiative surface cooling; this process is expected to be captured reasonably well by models. In contrast, the VSBL to WSBL transition is not well understood. It appears to be related to a collection of intermittent turbulent processes of diverse origin (collectively denoted as submesoscale, many of which processes are coherent structures) that has been described as a ”stochastic mix” [31]. Some of these processes, such as instabilities associated with the resolved shear, may be simulated by regional and global models. Others, such as density currents and breaking gravity waves, will not be. Representation of the SBL has been a persistent bias of weather and climate models, with consequent biases in simulation and prediction of near-surface temperature and wind fields of interest in applications ranging from agriculture to renewable energy.

The subgroup first discussed the problem in broad terms, considering which phenomena physically relevant to SBL dynamics can potentially be captured by existing parameterizations and which likely cannot. Initial attention was focused on the results of [9], which showed that even in high-resolution (1/3 km horizontal) mesoscale models such as Weather Research and Forecasting (WRF), parameterized eddy diffusion strongly suppresses submesoscale motion. Attention then turned to the fact that even those processes that can in principle be captured require finer than standard vertical resolution. The following research plan was developed: targeted case studies will be carried out using WRF for the time period and location corresponding to the well-instrumented CASES-99 field campaign. The initial simulations will vary vertical resolution, horizontal resolution, and the strength of parameterized diffusion to assess the effect of these factors on the simulation of SBL regimes and transitions between them. Having assessed this deterministic “baseline” of simulation quality, a new state-dependent stochastic representation of submesoscale vertical turbulent transport will be introduced. The starting point for this parameterization will be the statistical analyses of [47] and the proof-of-concept stochastic parameterization of [24]. By driving the local WRF simulation with the large-scale forecast ensemble produced for that time, we will also be able to assess how the factors of increased resolution and stochastic physics improve ensemble spread in boundary layer quantities.

Some ideas were discussed for the next-generation of ocean parameterizations. Stochastic instead of deterministic methods were regarded as an interesting and promising approach, however other promising ideas were considered as well. Anisotropic diffusion is drawing quite some interest in the oceanography community [15]. Another promising approach is to adopt methods from non-Newtonian fluid mechanics [39, 4, 25].

5 Group on “Prediction”

For much of their history of study, there has been a recognized link between coherent structures and prediction in complex flows. By construction, coherent structures capture parts of the flow that maintain their structure for long times relative to the rest of the field. Thus, their dynamics may be in some sense "simpler" than the rest of the flow, which could potentially be modeled in a purely stochastic way [35]. Lagrangian structures, like those that were the focus of this workshop, typically contain much more information and are much more stringently coherent than their Eulerian counterparts, since they explicitly account for fluid advection. Thus, when they were first introduced, there was significant excitement about using them as new predictive tools. Unfortunately, Lagrangian structures have not yet lived up to their promise in this regard, in part because methods for computing them typically require knowledge of the future evolution of the flow. To discuss whether this and other issues can be surmounted or whether Lagrangian structures will only ever be useful as a post-hoc analysis tool, one of the working groups at this workshop focused on potential strategies for using Lagrangian structures for flow prediction.

One of the first issues the group faced is the question of what precisely is meant by 'prediction'; that is, what do we want to predict, and over what time horizon? In many cases of practical interest, one wants to know where some transported quantity will go; in the Deepwater Horizon oil spill, for example, the most important quantity to be predicted is the future distribution of the oil. But in some other situations, such as in predicting the evolution of a turbulent flow, one wants to know about the evolution of the flow properties themselves. It is possible that different strategies may have to be employed to handle these different kinds of predictions. It was also pointed out that, given the finite-time and aperiodic nature of the flows of interest, prediction is never going to be exact or mathematically rigorous, in part because the coherent structures themselves do not last forever in these kinds of flows. The best we may be able to do is to assign likelihoods to possible future scenarios based on the current coherent structures.

To do so, the group recognized that an important gap in our current understanding of coherent structures is that we do not know much about their life cycle. That is, given the locations and some properties of coherent structures now, and potentially at times in the past, we have little sense of how they will evolve in the future. Part of the reason for this lack of understanding is that current methods for locating Lagrangian structures have been developed simply by considering the structure of the Lagrangian evolution equation $\dot{\mathbf{x}} = \mathbf{u}$, where $\mathbf{x}$ is position, the dot signifies the time-derivative along a Lagrangian trajectory, and $\mathbf{u}$ is the velocity field. Typically, the velocity field is assumed to be given, and no characteristics of its dynamics are assumed. In reality, however, the evolution of the velocity field, and therefore the coherent structures themselves, is constrained by physics: its dynamics must satisfy conservation of mass, momentum, and energy, the laws of thermodynamics, and the like. Often, the velocity field will be a solution of the Navier–Stokes equations, which bring their own complications; but in some cases, such as in data-assimilated large-scale ocean models, it will come from a somewhat simpler system. Thus, one potential path forward for making coherent structures more predictive is to learn how to constrain their possible future evolution by the properties of the velocity field that they arise from.

A second strategy the group discussed for making coherent structures more predictive is to model their evolution by an ordinary differential equation. The (approximate) flow evolution could then be captured by a finite (and hopefully small) set of such ODEs, one per structure, that are coupled, but that would be easier to solve than the full Navier-Stokes equations. This idea is reminiscent of how the utility of coherent structures was originally conceived of by the turbulence community [35]. Developing such an ODE model, however, may be quite challenging, since Lagrangian structures are potentially complex, spatially extended objects. It may thus be necessary to simplify the structures themselves to be able to model them appropriately. It was noted that this approach is now being taken in the context of Koopman operators, by approximating them in terms of only a few eigenmodes. Something similar may be possible to do with transfer operators and almost-invariant sets, and then their evolution may potentially be modeled by an ODE (which may potentially have to be stochastic to compensate for the information removed by the spectral truncation). Much work remains to be done to make these ideas viable in practice for the unsteady flows of interest in many applications, since these eigenmode decompositions have been only been studied so far for steady or periodic cases. But they do represent a promising strategy going forward.

The final topic discussed by the group was what appropriate test cases are for developing strategies for prediction using coherent structures. Clearly, steady or time-periodic flows are not sufficient; in those cases, prediction is anomalously easy, since the time dependence is simple. Thus, some of the classic test cases for studying Lagrangian structures, such as the double gyre, may not be appropriate for looking at their connection to prediction. Looking to observational data from, for example, the ocean is likely not the best choice either, due to inherent finite spatiotemporal resolution and numerous physical processes that will affect the evolution of structures outside of pure fluid advection. The group suggested that an appropriate case of intermediate difficulty might be a well controlled laboratory experiment or direct numerical simulation of Navier–Stokes, where the flow can be forced to be statistically stationary and will have known length, time, and velocity scales.

Thus, although much work remains to be done, the discussion group on prediction via coherent structures was able to identify concrete paths forward that should be explored in future research.

6 Outcomes and the Future

The topic of this workshop was highly multidisciplinary, and its structure was designed to initiate dialogues between researchers from many areas. The general sentiment from participants and organizers was that this purpose had been achieved to a large degree. The highly collegial atmosphere in which all discussions were held led to a greater understanding of each others' research areas, and bodes well for future interactions between the participants.

The groups discussed many different ways in which statistical models, coherent structures, and applications in oceanography, atmospheric sciences and turbulence overlapped. Many nascent intersections were identified, as have been detailed in the separate reports from each of the groups. (These will not be separately enumerated here.) It is expected that some of these will coalesce into coherent transdisciplinary research projects in the near future.

Additionally, there were many areas/topics that saw lively debates but that did not necessarily result in a well-defined identification of a problem or an approach. It is hoped that the seeds planted at this workshop will in the medium term result in the crystallization of these into cogent research questions and/or grant proposals. A number of the participants discussed initial plans to organize a follow-up meeting within a couple of years.

References

[1] R. Abernathey, C.B. Rocha, F.J. Poulin, M. Jansen and J. Penn, PYQG: Python Quasigeostrophic Model (2016). pyqg: v0.2.0 [Data set]. *Zenodo. http://doi.org/10.5281/zenodo.50569.*

[2] R. P. Abernathey and P. Cessi, Topographic enhancement of eddy efficiency in baroclinic equilibration. *J. Phys. Oceanogr.* **44** (2014) 2107-2126.

[3] M. Allshouse and J.-L. Thiffeault, Detecting coherent structures using braids, *Physica D* **241** (2012) 95-105.

[4] J. Anstey and L. Zanna, A deformation-based parametrization of ocean mesoscale eddy Reynolds stresses, *Ocean Modelling* **112** (2017) 99-111.

[5] L. Arnold, *Random Dynamical Systems*, Springer-Verlag, NY, 1998.

[6] S. Balasuriya, *Barriers and transport in unsteady flows: A Melnikov approach*, Series on Mathematical Modeling and Computation, SIAM Press, Philadelphia, 2016.

[7] S. Balasuriya, Stochastic uncertainty of advected curves in finite-time unsteady flows, *Phys. Rev. E* (2017), submitted.

[8] R. Banisch and P. Koltai, Understanding the geometry of transport: Diffusion maps for Lagrangian trajectory data unravel coherent sets, *Chaos* **27** (2017), 035804.

[9] D. Belušić and I. Güttler, Can mesoscale models reproduce meandering motions, *Q. J. R. Meterol. Soc.* **136** (2010), 553-565.

[10] P.S. Berloff, Random-forcing model of the mesoscale oceanic eddies. *J. Fluid Mech.* **529** (2005), 71–95.

[11] J. Berner et al., Stochastic parameterization: towards a new view of weather and climate models. *Bull. Amer. Meteorol. Soc.*, **98** (2017), 565-588.

[12] A. E. BozorgMagham, S. D. Ross, and D. G. Schmale III, Real-time prediction of atmospheric Lagrangian coherent structures based on forecast data: An application and error analysis, *Physica D* **258** (2013), 47-60.

[13] M.D. Chekroun, E. Simonnet and M. Ghil, Stochastic climate dynamics: Random attractors and time-dependent measures. *Physica D* **240** (2011), 1685-1700.

[14] P. Dimotakis, Turbulent mixing, *Annu. Rev. Fluid Mech.* **37** (2005), 329–356.

[15] B. Fox-Kemper, R. Lumpkin, and F. O. Bryan, Lateral transport in the ocean interior. In *Ocean Circulation and Climate: A 21st century perspective (G. Siedler, S. M. Griffies, J. Gould, and J. A. Church, Eds.)*, International Geophysics Series, **103**, pages 185-209. Academic Press, 2013.

[16] J.S. Frederiksen and A.G. Davies, Eddy viscosity and stochastic backscatter parameterizations on the sphere for atmospheric circulation models. *J. Atmos. Sci.* **54** (1997), 2475–2492.

[17] G. Froyland, An analytic framework for identifying finite-time coherent sets in time-dependent dynamical systems, *Physica D* **250** (2013), 1–19.

[18] G. Froyland, Dynamic isoperimetry and the geometry of Lagrangian coherent structures, *Nonlinearity* **28** (2015), 3587.

[19] G. Froyland and O. Junge, Robust FEM-based extraction of finite-time coherent sets in using scattered, sparse, and incomplete trajectories. *Arxiv* 2017.

[20] G. Froyland and K. Padberg-Gehle. A rough-and-ready cluster-based approach for extracting finite-time coherent sets from sparse and incomplete trajectory data. *Chaos* **25** (2015), 087406.

[21] A. Hadjighasem, M. Farazmand, D. Blazevski, G. Froyland and G. Haller, A critical comparison of Lagrangian methods for coherent structure detection, *Chaos* (2017), in press.

[22] A. Hadjighasem, D. Karrasch, H. Teramoto, and G. Haller, Spectral clustering approach to Lagrangian vortex detection, *Phys Rev E* **93** (2016), 063107.

[23] G. Haller, Lagrangian Coherent Structures, *Annual Review of Fluid Mechanics* **47** (2015), 137–162.

[24] Y. He, A.H. Monahan and N.A. McFarlane, The influence of bounary layer processes on the diurnal variation of the climatological near-surface wind speed probability distribution over land, *J. Climate* **25** (2012), 6441-6458.

[25] D. D. Holm and B. A. Wingate, Baroclinic instabilities of the two-layer quasigeostrophic alpha model. *J. Phys. Oceanogr.* **35** (2005), 12871296.

[26] A.A.M. Holtslag, G. Svensson, P. Baas, S. Basu, B. Beare, A.C.M. Beljaars, F.C. Bosveld, J. Cuxart, J. Lindvall, G.J. Steeneveld, M. Tjernstrom and B.J.J. van de Wiel, Stable atmospheric boundary layers and diurnal cycles - challenges for weather and climate models, *Bull. Amer. Meteor. Soc.* **11** (2013), 1691-1706.

[27] S. Juricke, T. N. Palmer, and L. Zanna. Stochastic perturbations to sub-grid scale ocean mixing: Impacts on low frequency variability. *Journal of Climate* **doi=10.1175/JCLI-D-16-0539.1** (2017).

[28] S. Keating, K. Smith, and P. Kramer, Diagnosing lateral mixing in the upper ocean with virtual tracers: spatial and temporal resolution dependence. *J. Phys. Oceanogr.* **41** (2011), 1512-1534.

[29] D. Kondrashov and P. Berloff, Stochastic modeling of decadal variability in ocean gyres. *Geophysical Research Letters*, **42** (2015), 1543-1553.

[30] F. Lekien, C. Coulliette, A. J. Mariano, E. H. Ryan, L. K. Shay, G. Haller, and J. Marsden, Pollution release tied to invariant manifolds: A case study for the coast of Florida, *Physica D* **210** (2005), 1-20.

[31] L. Mahrt, Stably stratified atmospheric boundary layers, *Annu. Rev. Fluid Mech.* **46** (2014), 23-45.

[32] I. Mezić, S.Loire, V. Fonoberov and P. Hogan, A new mixing diagnostic and Gulf oil spill movement, *Science* **330** (2010), 486–489.

[33] A.H. Monahan, T. Rees, Y. He and N. McFarlane, Multiple regimes of wind, stratification, and turbulence in the stable boundary layer, *J. Atmos. Sci.* **72** (2015), 3178-3198.

[34] M. J. Olascoaga and G. Haller, Forecasting sudden changes in environmental pollution patterns, *Proceedings of the National Academy of Sciences of the USA* **109** (2012), 4738-4743.

[35] N. T. Ouellette, On the dynamical role of coherent structures in turbulence, *Comptes Rendus Physique* **13** (2012), 866-877.

[36] T.N. Palmer, A nonlocal dynamical perspective on model error: a proposal for nonlocal stochastic-dynamic parametrization in weather and climate prediction models. *Q. J. R. Meteorol. Soc.* **127** (2001), 279–304.

[37] T. Palmer and P. Williams (Eds.), *Stochastic physics and climate modelling*, Cambridge University Press, 2010.

[38] T. Peacock and J. Dabiri, Introduction to focus issue: Lagrangian coherent structures, *Chaos* **20** (2010), 017501.

[39] P. Porta Mana and L. Zanna, Toward a stochastic parameterization of ocean mesoscale eddies, *Ocean Modelling*, **79** (2014), 1-20.

[40] B. Sawford, Turbulent relative dispersion, *Annu. Rev. Fluid Mech.* **33** (2001), 289–317.

[41] K. Schlueter-Kuck and J. Dabiri, Coherent structure coloring: identification of coherent structures from sparse data using graph theory, *Journal of Fluid Mechanics* **811** (2017), 468-486.

[42] S.C. Shadden, Lagrangian Coherent Structures. In: *Transport and Mixing in Laminar Flows: From Microfluidics to Oceanic Currents (R. Grigoriev, ed)*, Wiley-VCH, 2011.

[43] G. J. Shutts, A kinetic energy backscatter algorithm for use in ensemble prediction systems. *Q.J.R. Meteorol. Soc.* **131**, (2005), 3079–3102.

[44] A. Thompson and W. Young, Scaling baroclinic eddy fluxes: Vortices and energy balance. *J. Phys. Oceanogr.* **36** (2006), 720-738.

[45] C.D. Thorncroft, B.J. Hoskins and M.E. McIntyre, Two paradigms of baroclinic-wave life-cycle behaviour. *Quart. J. R. Meteorol. Soc.* **119** (1993), 17-55.

[46] I.G.S. van Hooijdonk, J.M.M. Donda, H.J.H. Clercx, F.C. Bosveld and B.J.H. van de Wiel, Shear capacity as a prognostic for nocturnal boundary layer regimes, *J. Atmos. Sci.* **72** (2015), 1518-1532.

[47] N. Vercauteren, L. Mahrt and R. Klein, Investigation of interactions between scales of motion in the stable boundary layer, *Q.J.R. Meterol. Soc.* **142** (2016), 2424-2433.

[48] Y. Wang, F. J. Beron-Vera and M. J. Olascoaga. The life cycle of a coherent Lagrangian Agulhas ring, *J. Geophys. Res. Oceans* **121** (2016) 3944-3954.

[49] S. Waterman and J. M. Lilly, Geometric decomposition of eddy feedbacks in barotropic systems, *J. Phys. Oceanogr.* **45** (2015), 1009–1024.

[50] P. D. Williams, N. J. Howe, J. M. Gregory, R. S. Smith, and M. M. Joshi, Improved climate simulations through a stochastic parameterization of ocean eddies, *J. Climate* **29** (2016), 8763–8781.

[51] L. Zanna, P. Porta Mana, J. Anstey, T. David and T. Bolton. Scale-aware deterministic and stochastic parametrizations of eddy-mean flow interaction. *Ocean Modelling* **111** (2017), 66-80.