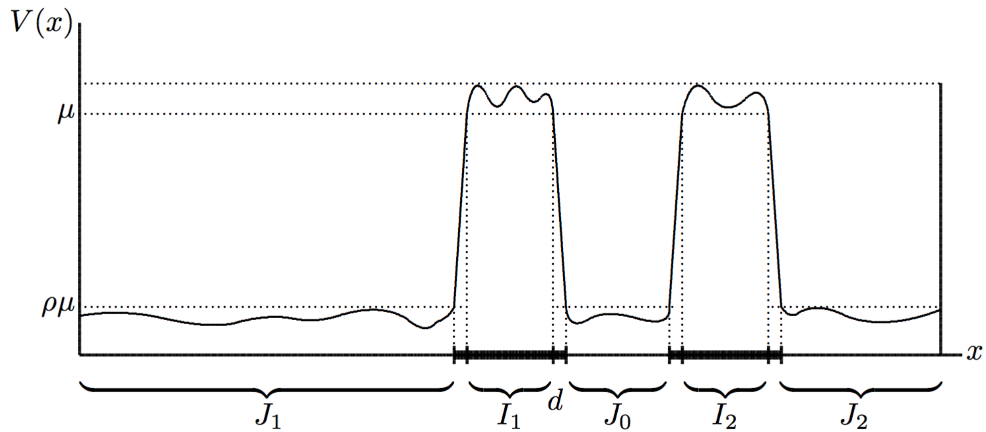
QUESTION 2: NO TOUCHDOWN AT POINTS WHERE V IS *SMALL* ?

Typical situation: **M-shaped** (or 2-bump) functions $V(x)$



Theorem. [Esteve-S., 2017] *Assume* $n = 1$, $f(u) = (1-u)^{-p}$, $p > 0$, $d > 0$,

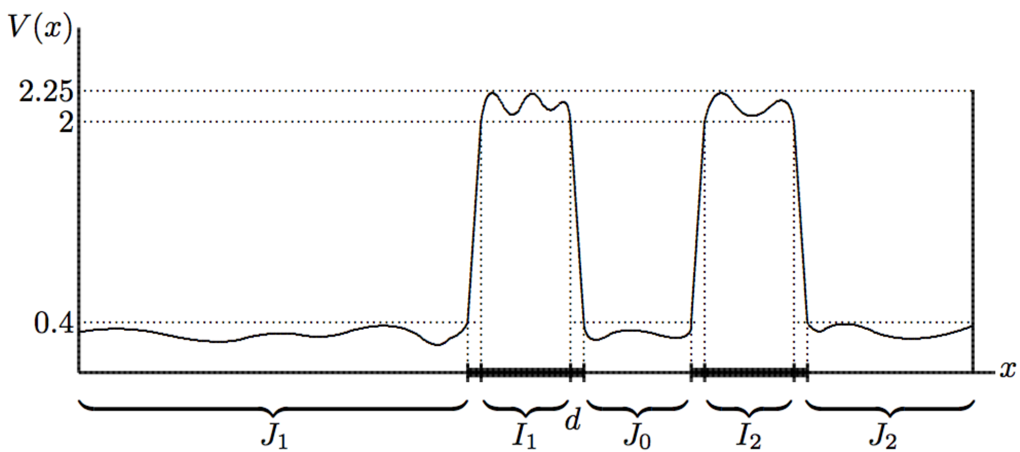
$$V \geq \mu > 0 \quad \text{on } I_1 \cup I_2,$$

$$V \leq \rho\mu \quad \text{on } J_0 \cup J_1 \cup J_2,$$

where $\rho \in (0,1)$ *is the solution of an* **explicit 3-parameter optimization (sup-inf) problem.** *Then no touchdown occurs on* $J_0 \cup J_1 \cup J_2$.

MEMS $(p=2)$, $|I_1|=|I_2|=2$, $|J_1|,|J_2|\geq 4$

μ	$\|V\|_\infty$	d	ρ
1	1.1	0.1	**0.2138**
1.25	1.3	0.1	**0.2166**
2	*2.25*	*0.1*	***0.2005***
2	2.25	0.05	**0.2300**
3	3.5	0.01	**0.2523**
4	4.1	0.05	**0.2289**
4	4.1	0.01	**0.2650**



$d=0.1$

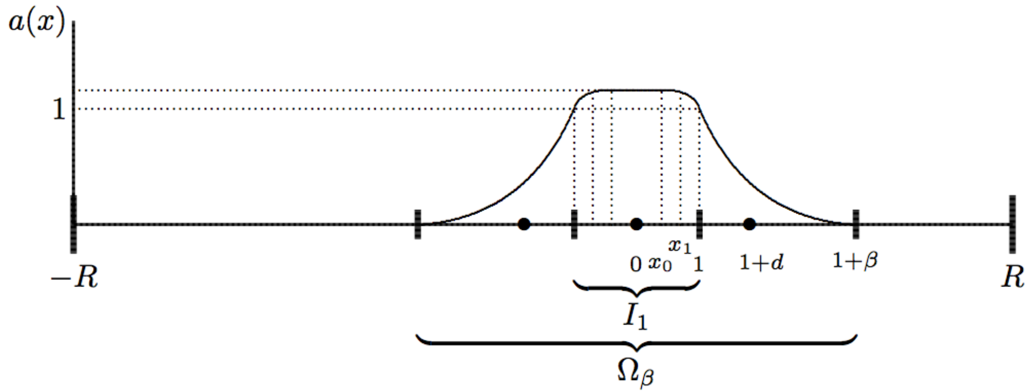
IDEA: PARAMETRIZED VERSION OF THE AUXILIARY FUNCTION J

3 parameters: K, β, τ

$$J = u_t - \varepsilon a(x) g(u) \qquad \text{in } (\tau, T) \times \Omega_\beta$$

$$g(u) = (1-u)^{-p} + K(1-u)$$

$$a(x) = a_{K,\beta}(x) \text{ solution of suitable piecewise (optimal) ODE } \longleftrightarrow \mathcal{L}J \leq 0$$



$$a_{\beta,K}(x) = \begin{cases} D_1, & x \in [0, x_0], \\ D_1 \cos^\alpha(c_1 x + \theta_1), & x \in [x_0, x_1], \\ D_2 \cos^{p+1}(c_2 x + \theta_2), & x \in [x_1, 1], \\ \left(\dfrac{1+\beta-x}{\beta}\right)^{p+1}, & x \in [1, 1+\beta] \end{cases}$$

OPTIMIZATION PROBLEM

$$\boxed{\rho = \sup_{(K,\beta,\tau)\in\mathcal{A}} \left[\phi(K,\beta,\tau;d,R) \inf_{x\in\Omega_\beta} \frac{(G_\tau * \chi_{I_1})(x)}{a_{\beta,K}(x)} \right]}$$

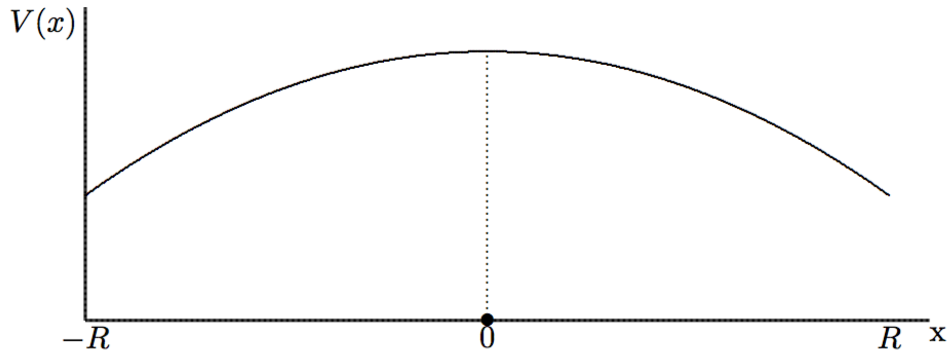
- G_τ: Gaussian heat kernel
- $\phi(K,\beta,\tau;d,R)$: explicit nonlinear function
- $\mathcal{A}$: admissible set

$$(K,\beta,\tau) \in \mathcal{A} \quad \longleftrightarrow \quad \mathcal{P}J \geq 0$$

Pseudo-optimum computed with help of Matlab
$\rightarrow$ rigorous lower estimate for inf by monotone discretization
$\rightarrow$ usual functions + erf

QUESTION 3: DOES TOUCHDOWN CONCENTRATE NEAR POINTS OF MAXIMUM OF V ?

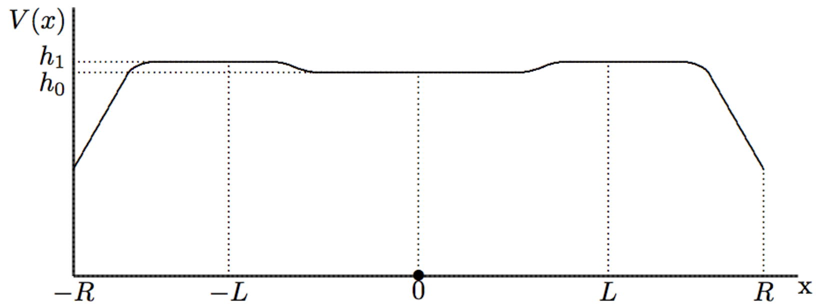
1- Positive results [Y. Guo, 2008] $\Omega = B_R \subset \mathbb{R}^n$



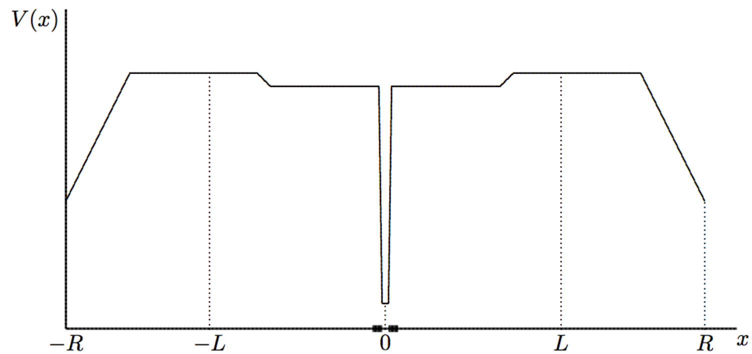
Touchdown at the origin only, for symmetric monotone profile.

$\rightarrow$ Idea: moving planes

2- Negative results [Esteve-S., 2017] $\Omega = B_R \subset \mathbb{R}^n$



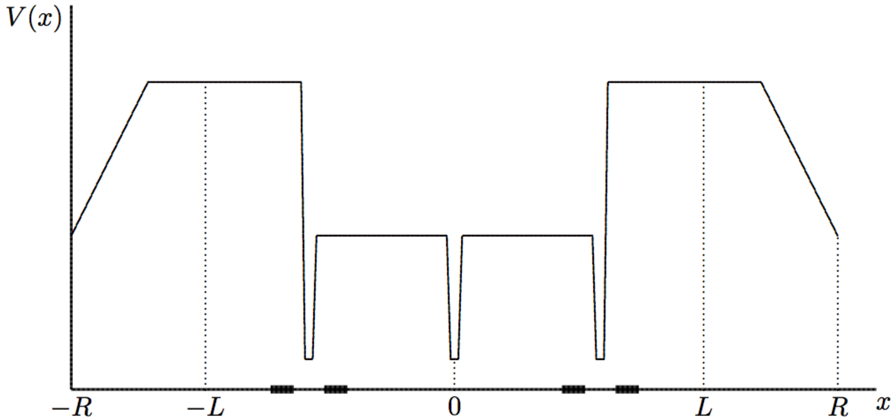
Touchdown at the origin only, for nonmonotone profile with $h_1 - h_0$ small.



Touchdown set does not contain 0, but concentrated near the origin and far from the maxima of V

→ Idea: continuity estimates w.r.t. V in L^q for the touchdown time and touchdown set

→ Rem: confirms numerical predictions of [Esposito-Ghoussoub-Y.Guo, 2010]



Touchdown set has at least 2 connected components and contains at least two spheres.

Open questions:

- finer structure of touchdown set ?
- Can it contain an interval ? (cf. numerical predictions of [Esposito-Ghoussoub-Y.Guo, 2010])