REPORT ON WORKSHOP 17W5118 "p-ADIC COHOMOLOGY AND ARITHMETIC APPLICATIONS"

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1. Overview

The subject of p-adic cohomology, a Weil cohomology theory for algebraic varieties defined over fields of positive characteristic p, has a long and distinguished history. It began with Dwork's proof in [Dwo60] of one of the most important results in arithmetic geometry, the rationality of zeta functions of algebraic varieties defined over finite fields, originally conjectured by Weil. This proof in fact predated that given by Artin and Grothendiek, and, very surprisingly at the the time, used p-adic analytic methods. Soon the cohomology theory that was hiding behind these methods was identified by Monsky and Washnitzer in [MW68]. At the same time, Grothendieck and Berthelot introduced crystalline cohomology [Ber74, Gro68], motivated by the desire to capture p-torsion phenomena in the cohomology of varieties in characteristic p. These two approaches were later unified, at least rationally, via Berthelot's theory of rigid cohomology [Ber97]. After a period of intensive work, due to the efforts of Kedlaya, Caro, and others, the theory reached a new level of sophistication with the proof of Grothendieck's six functors formalism, replicating those properties of ℓ-adic cohomology which occupied Grothendieck and his school for many years, and considered as a central achievement in arithmetic geometry.

While a major driving factor in the subject is often expected analogies with other cohomology theories, such as ℓ-adic étale cohomology or algebraic de Rham cohomology, in fact p-adic cohomology generally contains much more information, and displays a wealth of extra subtlety that both complicates and enriches the subject. Indeed, as Ogus has long argued, crystalline cohomology is often much more closely analogous to Hodge theory for complex varieties, for example in how one can use p-adic invariants to control certain deformation problems in mixed characteristic or equicharacteristic p.

This added subtlety makes p-adic cohomology a more challenging and intricate subject of study than the ℓ-adic theory, and is partially the reason why progress in the field has often been slower and more painstaking, usually lagging behind results obtained for other Weil cohomology theories. When the p-adic world does catch up, however, the results can be far more significant that their ℓ-adic counterparts. A good example of this is provided by the (full) Weil conjectures: although the ℓ-adic proof preceded the purely p-adic proof by over 30 years, it is only by using

p-adic cohomology that we now have algorithms that can actually compute the zeta functions of varieties [Ked04, Lau06].

Recent years have seen somewhat of a maturation of the subject; many basic foundational results are now settled, and interest has started to turn towards applications of these results to long standing arithmetic questions and conjectures, some of which were the original motivation for the development of p-adic cohomology. The most important of these are, in no particular order:

- various manifestations of p-adic Hodge theory;
- the p-adic Langlands correspondence both for number fields and function fields;
- applications of the theory of arithmetic $\mathcal{D}$-modules to p-adic representation theory;
- the p-adic properties of special values of L-functions, including refined conjectures of the Birch–Swinnerton-Dyer and Beilinson–Bloch–Kato type;
- Iwasawa theory in positive characteristic.

Another important new research direction is to explore some more refined aspects of the theory such as the question of integrality and p-torsion phenomena. New avenues are starting to be opened up in extending the scope of p-adic cohomology beyond the classical situation of perfect ground fields. There has also been a surge of interest in the more subtle cousin of p-adic cohomology theory, namely p-adic homotopy theory, which attempts to apply p-adic methods to study non-abelian phenomena on varieties in characteristic p.

These two directions of research are deeply interwoven. For example a major motivation for the study of integrality and p-torsion in p-adic cohomology is to develop tools for attacking refined Birch–Swinnerton-Dyer-, Beilinson–Bloch–Kato- and Iwasawa-type conjectures. The desire to develop p-adic homotopy theory is partially driven by problems encountered in the p-adic Langlands conjecture, and to have the usual homotopical tools available for the study of p-adic algebraic cycles, such as oriented cobordism theory. Part of the motivation for exploring overconvergent cohomology over non-perfect ground fields is to have an analogue of p-adic Hodge theory over Laurent series fields, especially for families of coefficients, and to develop tools for p-adic cycle and period maps, similar to those appearing in the regulators of Gros [Gro90] and the integration theory of Besser [Bes00].

There are two more areas of mathematics which are closely connected to the main topic of the conference: p-adic or non-archimedean analytic geometry, via the study of p-adic differential equations, and function field arithmetic, especially its local and cohomological aspects (see for example [BP09, Tae12]). Non-archimedean analytic geometry went through something of a revolution in the last few years, with the methodical introduction of a new concept of points, due to Berkovich and Huber. This strongly extended the scope of its foundational results, but at the same time streamlined the proof of many classical theorems. In the setting of p-adic differential equations these methods also found applications, and via the work of Baldassarri, Poineau–Pulita and Kedlaya [Bal10, PP13, Ked16] now we have beautiful sufficient conditions for the finiteness of cohomology for p-adic local systems on curves.

All these various strands of research were represented at the workshop, which drew together a wide range of researchers all with a broad interest in p-adic cohomology. There were a large number of excellent talks given by a diverse selection of speakers, all generating lively further discussions among participants. A lot of the work discussed at the workshop concerned exciting progress towards major open problems in the area, as well as interesting new perspectives that will provide fertile ground for the continuing development of the subject.

2. Main Themes

As we have already explained, all these exciting new trends emerging in the field are of course deeply interwoven, and the aim of the workshop was to encourage new progress in these areas by promoting both predictable and unpredictable synergies between them. Here we describe in more details the specific topics that were covered during the workshop, and some of the interactions that were generated.

2.1. **Foundations and theory over non-perfect fields.** Traditionally, p-adic cohomology theories have been expressed for varieties over perfect ground fields of characteristic p. While much of the theory still works over non-perfect fields, arithmetic considerations (in particular the general phenomenon of semistable reduction, as well as analogies with the ℓ-adic theory) lead one to expect certain refinements of existing p-adic cohomologies (such as rigid cohomology) when working over such non-perfect fields. There have been several recent attempts to work out what such a more general theory might look like - for example in [LP16] the case of Laurent series fields over perfect fields was considered, and in [LS17] B. Le Stum has given a completely general definition of "overconvergent cohomology" for locally Noetherian schemes.

During the workshop, a major advance in these foundational matters was announced in the talk given by **Richard Crew** on **Rings of arithmetic differential operators on tubes**. He described an extension of Berthelot's theory of arithmetic differential operators to a class of morphisms of adic formal schemes that are not necessarily of finite type, or even adic, by introducing a new finiteness condition, that of being 'universally Noetherian'. After an overview of the necessary commutative algebraic foundations, he went onto to explain how one can use this theory to give generalisations of the theory of convergent and overconvergent isocrystals.

There are also other approaches to generalising the theory of arithmetic $\mathcal{D}^\dagger$-modules, as studied for example by Caro–Vauclair [CV15]. These are based on a slightly different finiteness condition - being 'p-smooth' rather than 'universally Noetherian'. These two conditions are somewhat orthogonal, and provide good frameworks in which to study Grothendieck's 6 operations in different situations. One particularly interesting question would be to see whether or not there is a natural way to unify these two approaches.

2.2. **The Langlands program and links with representation theory.** Berthelot's theory of arithmetic $\mathcal{D}^\dagger$-modules was created to try to give a good analogue of the theory of algebraic $\mathcal{D}$-modules in mixed characteristic or characteristic p situations, and thanks to recent work of Caro, many of the foundations are now essentially in place.

In the classical case, over the complex numbers, or more generally fields of characteristic zero, one especially important area of application is to the study of representation theory, the most famous example of this being the Beilinson–Bernstein correspondence. It is similarly hoped that the theory of arithmetic $\mathcal{D}^\dagger$-modules will prove to be a powerful tool in the study of representation theory, in particular that of p-adic Lie groups, and thus in the p-adic Langlands program. A similar such application of $\mathcal{D}$-module theory on rigid analytic spaces over p-adic fields has been already found by Ardakov–Wadsley [AW13], who used their theory to answer some representation theoretical problems which arose in the local Langlands program.

There is also closely related work of Huyghe, Patel, Schmidt and Strauch on localisation theorems in the setting of arithmetic $\mathcal{D}^\dagger$-modules [HPSS15], in which they show that there is an equivalence of categories between the category of locally analytic admissible representations of some split reductive group over a finite extension of $\mathbb{Q}_p$, and the category of coadmissible arithmetic $\mathcal{D}^\dagger$-modules over the rigid analytic space attached to the flag variety of the group. The talk by **Matthias Strauch** entitled **Arithmetic structures in sheaves of differential operators on formal schemes and $\mathcal{D}^\dagger$-affinity** reported on this exciting piece of work. First he defined certain integral structures, depending on a congruence level, for the sheaves of differential operators on a formal scheme which is a blow-up of a formal scheme which itself is formally smooth over a complete discrete valuation ring of mixed characteristic. When one takes the projective limit over all blow-ups, one obtains the sheaf of differential operators on the associated rigid space, introduced independently by K. Ardakov and S. Wadsley. In the second part he explained what it means for a formal model of a flag variety to be $\mathcal{D}^\dagger$-affine (this concept is analogous to that of Beilinson–Bernstein and Brylinski–Kashiwara in the algebraic context), and he then explained an example illustrating how to use these results together with methods from rigid cohomology to analyze locally analytic representations of p-adic groups.

Similarly, the 6 operations formalism has been used by Abe [Abe13] to prove a p-adic Langlands correspondence in the function field setting, and thus verify Deligne's "petits camarades cristallins" conjecture on the existence of p-adic companions to compatible systems of ℓ-adic Galois representations (at least over curves). This has been the starting point of a lot of new research which we will talk about in more detail in the next section.

2.3. **Higgs bundles, Simpson correspondence, Simpson's conjecture and p-adic companions.** Since the foundational work of Grothendieck, Deligne, and Tate, it has been clear that ℓ-adic local systems (or more generally, complexes of ℓ-adic constructible sheaves) should be the ℓ-adic incarnation of a family of motives in characteristic p. The situation for p-adic coefficient objects was only more recently clarified, through foundational work of P. Berthelot and R. Crew and, more recently, breakthroughs due to D. Caro and especially T. Abe.

As we already mentioned above, Abe used his robust p-adic cohomology theory to resolve part of Deligne's Companions Conjecture from Weil II for curves: he constructs, for every sufficiently irreducible lisse ℓ-adic sheaf on a smooth curve $X/\mathbb{F}_q$, a compatible overconvergent F-isocrystal. However the p-adic Companions Conjecture for higher dimensional bases remains wildly open. Morally, this is because p-adic coefficent objects contain dramatically more information than their ℓ-adic cousins; they look and act much more like complex variations of Hodge structures. Nevertheless, in the talk by **Kiran Kedlaya** entitled **Update on the companion problem**, the current status of the problem was surveyed. He first described the problem in detail, and the previous work of Drinfeld, Deligne, and Abe–Esnault, then he talked about his new result, which establishes a weak form of the conjecture.

In Simpson's foundational work on non-abelian Hodge theory, he conjectures that rigid local systems on smooth, complex, quasi-projective varieties are of geometric origin. This problem was discussed in the talk given by **Hélène Esnault** entitled **Rigid systems and integrality**, where she reported on her remarkable joint work with M. Gröchenig. She described how they proved that the monodromy of a cohomologically rigid integrable connection (E, ∇) on a smooth complex projective variety X is integral, a statement sometimes known as Simpson's integrality conjecture. In order to achieve this, they first proved that the mod p reduction of a rigid integrable connection (E, ∇) has the structure of an isocrystal with Frobenius structure. She also explained how they proved that rigid integrable connections with vanishing p-curvatures are unitary, and how they used the latter result to prove new cases of Grothendieck's p-curvature conjecture. The result is both interesting for its philosophical implications and the link it creates between p-adic cohomology and Simpson's original conjecture.

A fundamental tool used by Esnault–Gröchenig is a certain p-adic analogue of Simpson's correspondence, relating local systems with Higgs bundles. The talk by **Bernard Le Stum** on **A quantum Simpson correspondence** was a report on a result of this type, but for 'quantum' local systems, i.e. non-commutative deformations through a formal parameter q. He first explained how the Simpson correspondence establishes an equivalence between a category of modules equipped with a connexion and a category of Higgs bundles. Then he talked about his joint work with Michel Gros and Adolfo Quirós, where they describe such an equivalence in the case of modules equipped with a q-connexion when q is a p-th root of unity. This is modelled on the characteristic p case and relies on the notion of quantum divided powers that he also discussed.

In contrast with what happens over the complex number field, for normal varieties over a finite field, recent work of Deligne, Drinfeld, Abe–Esnault, and Kedlaya establishes that for fixed ℓ, there are only finitely many ℓ-adic coefficient objects with finite determinant, bounded degree, and 'bounded ramification'. This suggests that they should exhibit some form of rigidity, and that some form of Simpson's conjecture should be true for them. Indeed, in the talk by **Subrahmanya Krishnamoorthy** entitled **Rank two F-isocrystals and abelian varieties** the speaker reported on his joint work-in-progress with Ambrus Pál on a Simpson-like conjecture for F-isocrystals. This conjecture states that for smooth varieties X over finite fields, any overconvergent F-isocrystal which 'looks' as though it should appear in the cohomology of a family of abelian varieties over X does in fact come from such a family. The methods are similar to the work above: it starts with the curve case, which can be proved via a refined form of the Langlands correspondence, then uses tools from geometry to extend the abelian variety from a sub-curve to the whole variety, using Serre–Tate type deformation theory and algebraization results.

Finally let us mention the talk by **Paul Ziegler** entitled **Mirror symmetry for moduli spaces of Higgs bundles via p-adic integration**, which was a report on a very surprising application of p-adic methods to the study of Higgs bundles. The speaker talked about a recent proof, joint with M. Gröchenig and D. Wyss, of a conjecture of Hausel and Thaddeus which predicts the equality of suitably defined Hodge numbers of moduli spaces of Higgs bundles with SL_n- and PGL_n-structure. The proof, inspired by an argument of Batyrev, proceeds by comparing the number of points of these moduli spaces over finite fields via p-adic integration. This then provides information about Hodge numbers through the Weil conjectures and p-adic Hodge theory. Particularly exciting is the prospect of using these methods to shed new light on aspects of the Langlands program.

2.4. **The de Rham–Witt complex, Iwasawa theory and aspects of integral p-adic cohomology.** One of the original motivations of Grothendieck and Berthelot for inventing crystalline cohomology, as a p-adic companion to the family of ℓ-adic cohomologies produced by the étale theory, was to explain p-torsion phenomena. While integral crystalline cohomology achieves this for smooth and proper varieties, the extension to a 'good' cohomology theory for arbitrary varieties, which reached its zenith in the proof of the 6 operations formalism by Caro, has been achieved only for rational coefficients, i.e. after tensoring with $\mathbb{Q}$. This therefore still leaves open the question of what an integral p-adic theory should look like for open or singular varieties, which has been the subject of much recent work in the field, in particular the study of the overconvergent de Rham–Witt complex by Davis, Langer and Zink [DLZ11]. This now seems to provide a good candidate for smooth (but possibly open) varieties, although *a priori* it is not clear how it compares with other candidates such as integral Monsky–Washnitzer cohomology, or even if the latter is well-defined.

Such a comparison theorem between these two approaches was the subject of the talk given by **Veronika Ertl**, entitled **Integral Monsky-Washnitzer and overconvergent de Rham-Witt cohomology**. In it she reported on joint work with Johannes Sprang in which they first of all show that Monsky–Washnitzer cohomology is well defined on the integral level, and secondly prove a comparison isomorphism between this and the overconvergent de Rham–Witt cohomology. This extends previous work of Davis and Zurieck-Brown by removing all restrictions on the cohomological degree. Interesting further questions in this direction would be to calculate the p-torsion of these groups in new and interesting cases, and to give a geometric interpretation of this torsion.

The study of integral properties of p-adic cohomology is very closely related to that of the p-adic properties of L-functions in characteristic p. Most of the work recently has been done on 1-dimensional families of abelian varieties, for example [KT03], [Pál10] and [TV15] which look at the refined Birch–Swinnerton-Dyer conjecture, the integrality of p-adic L-functions and the equivariant Tamagawa number conjecture, respectively. What is common in these works is the crucial use of integral p-adic cohomology theories predating the construction in [DLZ11], typically log crystalline cohomology. Therefore they are forced either to reduce the general case to the semi-stable one, or worse, restrict to the situation when the abelian scheme is semi-stable and the considered Galois covers of the base are tame. This demonstrates the limitations of these methods, but with sufficient progress on the finiteness properties of the the overconvergent de Rham–Witt complex we expect that this area would start to develop very rapidly.

Another talk with connections to integral aspects of p-adic cohomology and Iwasawa theory was by **Ambrus Pál** on the subject of **Formal deformations of crystals and arithmetic applications**. The speaker first described two problems in the arithmetic of elliptic curves over function fields, the analogue of a classical conjecture of Stevens in the function field setting, originally asked by Mazur, and his own conjecture on the integrality properties of Hecke eigenforms attached to elliptic curves. He then spoke about his work in progress on these conjectures, using tools from p-adic cohomology. The first ingredient is a pro-representability theorem of what could be called arithmetic deformations of crystals and Dieudonné crystals. The second ingredient is the application of the Taylor–Wiles method in this setting, which shows that the first conjecture implies the second.

One exciting recent trend in integral p-adic cohomology, and in particular integral p-adic Hodge theory, has seen the application of methods and techniques from higher category theory and higher algebra to the study of crystalline cohomology and K-theory. The talk given by **Lars Hesselholt** on **Higher algebra and arithmetic** provided an entertaining introduction to some of these ideas, and how they can be used to circumvent problems in arithmetic geometry, in particular in characteristic p, caused by the introduction of denominators.

As explained in the talk, these often arise because of the fact that the natural numbers record only the result of counting, and not the process of counting. The higher algebra of Joyal, Lurie and others replaces the initial ring of algebra $\mathbb{Z}$ by a more fundamental object, the sphere spectrum $\mathbb{S}$, which also retains information about the counting *process*. The 'brave new algebra' long advocated by Waldhausen uses $\mathbb{S}$ as the basis for arithmetic, and doing so can often lead to the elimination of 'unwanted' denominators.

Notable manifestations of this vision include the Bökstedt–Hsiang–Madsen topological cyclic homology, which receives a denominator-free Chern character, and the related Bhatt–Morrow–Scholze integral p-adic Hodge theory, which makes it possible to exploit torsion cohomology classes in arithmetic geometry. In the talk, the speaker explained the construction of a certain 'higher' analogue of de Rham cohomology, and gave some calculations in p-adic and characteristic p settings. In the former case, one recovers quite naturally certain constructions that appear in integral p-adic Hodge theory. In the latter, one can use this theory to give a cohomological interpretation of the Hasse-Weil zeta function of smooth and proper varieties over finite fields by regularized determinants, as envisioned by Deninger.

The recent revolution in the higher category theory and higher algebra has the potential to become a powerful tool in all sorts of areas of mathematics. This talk gave some excellent concrete examples of this potential in the particular fields of algebraic K-theory and p-adic Hodge theory.

2.5. ***F*-isocrystals and *p*-adic representations, and homotopy theory.** Most current research on the homotopy-theoretical aspects of p-adic cohomology concentrates on two particular strands: the theory of the p-adic, or crystalline, fundamental group via various Tannakian categories of isocrystals, and applications of the latter to Diophantine problems. The talk by **Atsushi Shiho** entitled **On de Jong conjecture** concerned the first aspect, covering a fascinating conjecture due to de Jong predicting that any isocrystal on a geometrically simply connected smooth projective variety over a perfect field of characteristic $p > 0$ should be constant. The speaker reported on his joint work in progress with Hélène Esnault where they proved several results related to this conjecture, including some special cases and a reduction to the case of convergent isocrystals.

On the other hand the talk by **Ishai Dan-Cohen** entitled **Rational motivic path spaces** concerned the second aspect. A central ingredient in Kim's work on integral points of hyperbolic curves is the "unipotent Kummer map" which goes from integral points to certain torsors for the pro-unipotent completion of the fundamental group, and which, roughly speaking, sends an integral point to the torsor of homotopy classes of paths connecting it to a fixed base-point. The speaker reported on recent joint work of his with Tomer Schlank, which provides liftings of Kim's construction to more refined rational homotopy types, and thus gives rise to factorizations of Kim's original period map. He explained the construction of a certain space Ω of rational motivic loops, and use this to provide a factorization of the unipotent Kummer map which can be summarized schematically as

$$\text{points} \to \text{rational motivic points} \to \Omega\text{-torsors} \to \pi_1\text{-torsors}.$$

For affine curves, Ω-torsors are the same thing as π_1-torsors, essentially because these spaces are $K(\pi, 1)$'s, however, their approach has the potential to give more refined information for higher dimensional varieties.

A key link between the p-adic, and the usual étale fundamental groups is given by the Katz correspondence, or the study of unit root F-isocrystals, and the associated p-adic Galois representations. The talk by **Joe Kramer-Miller** on **Slope filtrations of F-isocrystals, log decay, and genus stability for towers of curves** reported on work which shed new light on this classical, but very important connection. The speaker introduced a notion of F-isocrystals with logarithmic decay, gave a conjectural description of how this should relate to the slope filtrations, and sketched a proof of this conjecture when the unit-root subcrystal has rank one. This then leads to a new proof of a recent theorem of Drinfeld–Kedlaya, as well as a generalized version of Wan's conjecture on genus stability for towers of curves coming from geometry.

2.6. **Other topics.** Finally we report on those activities of the workshop which cannot be easily classified into one of the categories mentioned above, but nevertheless demonstrate the richness of the subject and its many connections to other areas of algebraic geometry and number theory.

The talk by **Gebhard Böckle** entitled **Compatible systems of Galois representations of global function fields** reported on joint work of the speaker with W. Gajda and S. Petersen on the important open image conjecture. If K is a finitely generated infinite field, then it was shown recently by Cadoret, Hui and Tamagawa that for almost all ℓ, the cohomology group $H^i_{\text{ét}}(X_{K^s}, \mathbb{F}_\ell)$ is semisimple as a representation of the geometric fundamental group $G_K^{\text{geo}} = \text{Gal}(K^s/K\mathbb{F}_p^s)$. This has an important consequences for the image of G_K^{geo} under its action on the adèlic module $H^i_{\text{ét}}(X_{K^s}, \mathbb{A}_\mathbb{Q})$. The speaker described how to extend these results to the case where $\mathbb{F}_\ell$ is replaced by the mod-ℓ reduction of a compatible system of Galois representations, using automorphic methods.

The talk given by **Masha Vlasenko** on **Atkin and Swinnerton-Dyer congruences for toric hypersurfaces** was about a new result on a classical application of p-adic cohomology to arithmetic. It concerned certain kinds of explicit crystals called Dwork modules, which in the 1990's were used by V. Batyrev to describe the mixed Hodge structure on the middle cohomology of affine hypersurfaces in algebraic tori. In the talk the speaker reported on her work in progress done jointly with Frits Beukers, in which they they used these crystals to show several p-adic congruences for the coefficients of powers of a Laurent polynomial. This then leads to congruences involving the L-function of toric exponential sums, and yields new p-adic unit-root formulas.

One of the early indications of the power and utility of p-adic methods was the Serre–Tate theorem on the deformation theory of abelian varieties, showing that this is in fact controlled by the deformation theory of its p-divisible group, or equivalently its first crystalline cohomology. This basic idea of using p-adic invariants to control deformation problems has proved to be extremely fruitful, and the talk given by **Ananth Shankar** provided an excellent example of this. Entitled **Serre-Tate theory for Shimura varieties of Hodge type**, it was a report on work of the speaker and his coauthor Rong Zhou in which they studied the formal neighbourhood of a point in μ-ordinary locus of an integral model of a Hodge type Shimura variety. They proved the existence of a structure analogous to the Serre–Tate structure on the deformation space of an ordinary abelian variety, in particular providing an analogue of 'canonical lifts' in these situations.

Finally, the talk by **Edgar Costa** entitled **Computing zeta functions of nondegenerate toric hypersurfaces** was a very useful update on the ongoing revolution on the point counting problem via p-adic methods. The speaker reported on an ongoing joint project with Kiran Kedlaya and David Harvey on the computation of zeta functions of nondegenerate toric hypersurfaces over finite fields, using p-adic cohomology. By exploiting a new way of computing sparse approximations to overconvergent cohomology classes, they were able to drastically improve the feasibility of computing zeta functions of certain smooth varieties, in particular for much larger values of p than were previously accessible.

3. Outcomes

The subject of p-adic cohomology is often characterised by a plethora of different approaches to the subject, each of which has its own particular perspective and scope of application. The workshop successfully drew together people working on all aspects of the theory, and the various topics generated many discussions, kick-starting new research and collaborations across the whole breadth of the subject.

For example, the talk given by Veronika Ertl inspired much speculation on the expected properties and possible scope of any integral theory of p-adic cohomology. Richard Crew in particular suggested that in fact a fully robust theory cannot exist for finite coefficients, and sketched during discussions the reasons why.

Another topic that was constantly in the air during the workshop concerned problems surrounding the Companion Conjecture, the main remaining case now being how to produce p-adic companions on higher dimensional varieties. This is a topic that several of the participants at the workshop have been working on recently, and one in which we can hope to see some more progress in the near future.

Let us also mention several interesting discussions that took place concerning the p-adic homotopical properties of varieties in characteristic p, and to what extent these mirror or deviate from 'classical' topological behaviour. As well as basic foundational matters, such as deformation invariance, or the existence of moduli spaces for p-adic local systems, there were also interesting problems posed on the behaviour of particular kinds of isocrystals, often inspired by Hodge theory and Simpson-like conjectures. This reflects a general shift in focus in the subject, and is likely to become an important area of future research. Some of these debates have been followed up since the conference, and have formed the germ of potential new collaborations.

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