FUTURE TARGETS IN THE CLASSIFICATION PROGRAM FOR AMENABLE C^*-ALGEBRAS

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A list of open problems and goals recorded during the workshop 'Future Targets in the Classification Program for Amenable C^*-algebras', BIRS, Banff, 4. to 8. September 2017.

(1) The UCT-problem: Does every nuclear C^*-algebra satisfy the universal coefficient theorem (UCT)?
(2) How bad can crossed products be? When is $C(X) \rtimes G$ classifiable?
(3) Let D be a strongly self-absorbing (s.s.a.) C^*-algebra with $D \not\cong \mathcal{O}_2$. Is there a unital embedding $D \hookrightarrow \mathcal{O}_\infty \otimes \mathcal{Q}$, where $\mathcal{Q}$ denotes the universal UHF-algebra? (This can be considered as an infinite version of the quasidiagonality problem.)
(4) Develop applications of classification. Establish Giordano-Putnam-Skau (GPS) type theorems for spaces that are not Cantor spaces.
(5) More examples of actions of $\mathbb{Z}^d$ on Cantor sets.
(6) Sell classification. Simplify the proof and make it accessible to other areas of mathematics.
(7) Is conjugacy of shifts of finite type (SFT) decidable? Can this be rephrased in terms of C^*-algebras?
(8) Do simple C^*-algebras with real rank zero have strict comparison of positive elements? Are they $\mathcal{Z}$-stable?
(9) Range results for Cuntz semigroups.
(10) Which rigid C^*-tensor categories embed into the representation theory of $\mathcal{Z}$?
(11) Let D be s.s.a. C^*-algebra, let G be a torsion-free amenable group. Does there exists a unique, strongly outer action of G on D?
(12) Develop a theory of subfactors for inclusions of classifiable C^*-algebras.
(13) Let G be an étale, amenable groupoid with $G^{(0)}$ a Cantor set such that $C^*(G)$ is simple. Is $C^*(G)$ $\mathcal{Z}$-stable?
(14) Let A be a simple, classifiable C^*-algebra, and let G be a (finite) group of automorphisms of the Elliott invariant $\mathrm{Ell}(A)$. Does G lift to an action on A? Does the map $\mathrm{Aut}(A) \to \mathrm{Aut}(\mathrm{Ell}(A))$ split?
(15) Let A be a simple, classifiable C^*-algebra, let $a \in A$, and let $\pi\colon A \to B(H)$ be an irreducible representation. Does H have an $\pi(a)$-invariant subspace?
(16) Is every simple, classifiable C^*-algebra a groupoid C^*-algebra? Is every simple (exact) C^*-algebra a groupoid C^*-algebra?
(17) Develop analogies of Popa's rigidity theory (for actions of property (T) groups on s.s.a. algebras). Is there a theory of intertwining by bimodules for C^*-algebras?
(18) Classify non-simple TAF algebras.
(19) Is every real rank zero ASH algebra with slow dimension growth TAF?
(20) Develop a better understanding of completely positive approximations. In particular, when is an inductive limit of finite-dimensional C^*-algebras with fixed order units and c.p. maps that preserve order units, order-isomorphic to a C^*-algebra?
(21) What are the possible Cowling-Haagerup invariants for separable, simple, exact C^*-algebras?

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