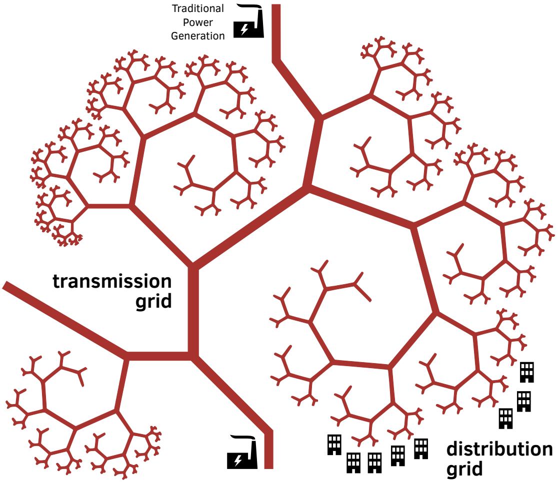
Networked Feedback Control for a Smart Power Distribution Grid

Saverio Bolognani

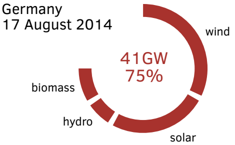
Future power distribution grids

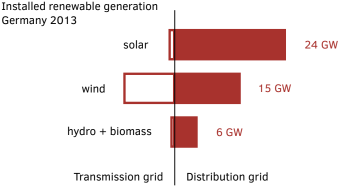


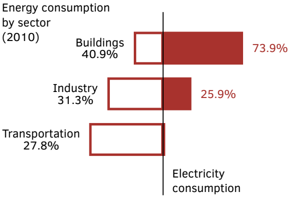
- It delivers power from the transmission grid to the consumers.
- **Very little sensing, monitoring, actuation.**
- The “easy” part of the grid: conventionally **fit-and-forget** design.

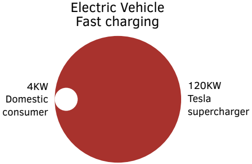
New challenges

- **Distributed microgenerators** (conventional and renewable sources)
- **Electric mobility** (large flexible demand, spatio-temporal patterns).





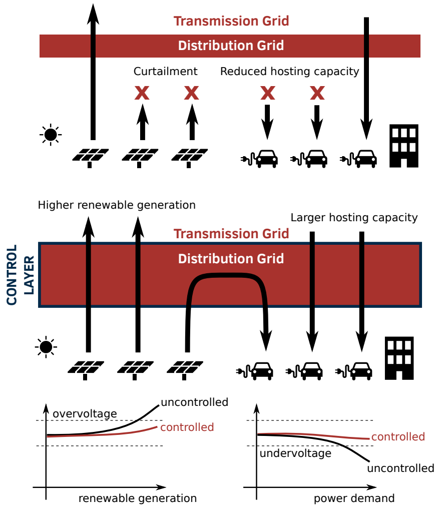


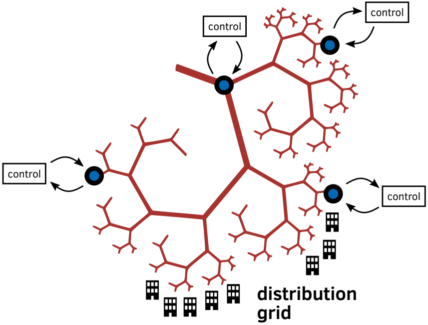


Physical grid limits → **grid congestion**

Fit-and-forget → unsustainable **grid reinforcement**

Virtual grid reinforcement





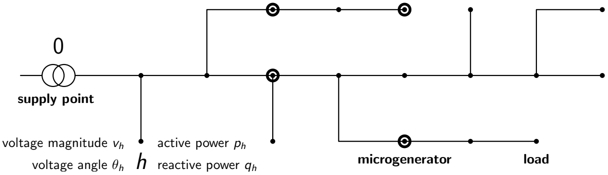
- **Virtual** grid reinforcement
  - same infrastructure
  - more sensors
  - controlled grid = stronger grid
  - distributed ancillary services
  - accomodate active power flows "transparently"

OVERVIEW

1. A feedback control perspective on power system operation
2. A tractable power grid model for feedback control design
3. Control design example: voltage regulation
   - Distributed “model-free” control
   - Centralized chance-constrained decision

A FEEDBACK CONTROL PERSPECTIVE ON POWER SYSTEM OPERATION

Power distribution grid model



Grid model

Nonlinear complex valued power flow equations

$$\mathrm{diag}(u)\overline{Yu} = s$$

where

- $u_h = v_h e^{j\theta_h}$ complex bus voltages
- $s_h = p_h + jq_h$ complex bus powers

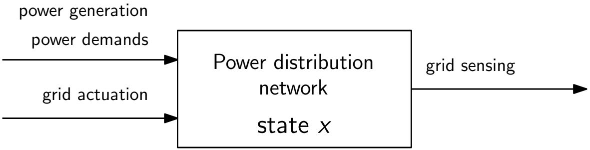
Actuation

- Tap changer – v_0
- Reactive power compensators – q_h
- Active power management – p_h

Sensing

- Voltage meters – v_h (sometimes θ_h)
- Line currents, transformer loading, . . .
- **Underdetermined**: few sensors

A control perspective on distributed grid operation



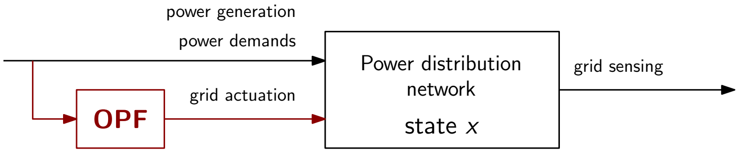
Ancillary services: voltage regulation / reactive power compensation / economic re-dispatch / loss minimization / line congestion control / energy balancing / ...

Control objective

Drive the system to a target state $x^* = \begin{bmatrix} v^* & \theta^* & p^* & q^* \end{bmatrix}$ subject to

- **soft constraints** $x^* = \textbf{argmin}_x \, J(x)$
- **hard constraints** $x \in \mathcal{X}$
- **chance constraints** $\mathbb{P}[x \notin \mathcal{X}] < \epsilon$

Feedforward control



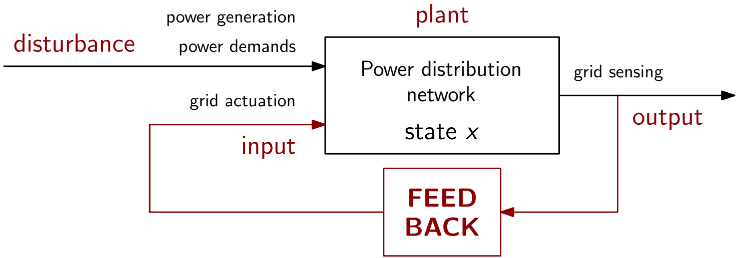
Conventional approach

- Core tool: **Optimal Power Flow**
- Fast OPF solvers in radial networks
- Many variants, including distributed implementations

However:

- Requires **full state measurement** - **full communication**
- Heavily **model based**

Feedback control



Control theory answer

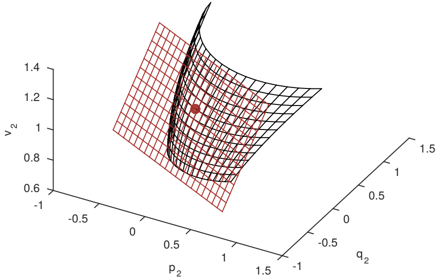
- **Disturbance rejection** $\equiv$ grid state regulated despite demand/generation
- **Model-free** design
- **Robustness** against uncertainty
- **Output** feedback

A TRACTABLE POWER GRID MODEL FOR FEEDBACK CONTROL DESIGN

Power flow manifold

Set of all states that satisfy the **grid equations** $\mathrm{diag}(u)\overline{Yu} = s$

$\rightarrow$ **power flow manifold** $\mathcal{M} := \{x \mid F(x) = 0\}$



Best linear approximant

Tangent plane at a nominal power flow solution $x^* \in \mathcal{M}$

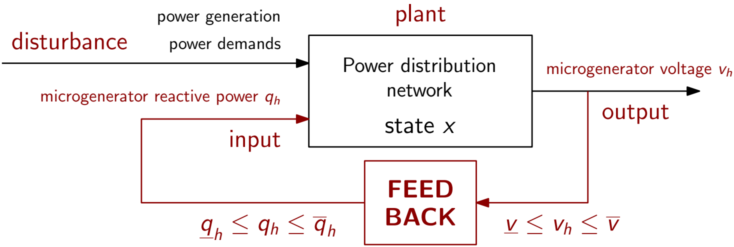
$$A_{x^*}(x - x^*) = 0 \qquad A_{x^*} := \left.\frac{\partial F(x)}{\partial x}\right|_{x=x^*}$$

- **Implicit** – No input/outputs (not a disadvantage)
- **Sparse** – The matrix A_{x^*} has the sparsity pattern of the grid graph
- **Structure preserving** – Elements of A_{x^*} depend on local parameters

$\rightarrow$ Bolognani & Dörfler (2015)
"Fast power system analysis via implicit linearization of the power flow manifold"

CONTROL DESIGN EXAMPLE: VOLTAGE REGULATION

Case 1: hard constraints



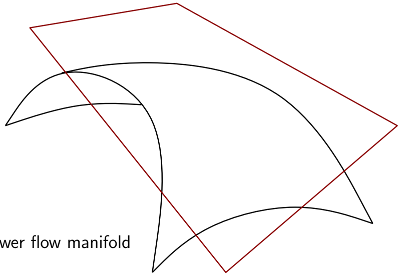
- **Inputs:** reactive power q_h of microgenerators
- **Outputs:** voltage measurement v_h at the microgenerators
- **Control objective:**
  - **Soft constraints**

    minimize $v^T L v$ (voltage drops on the lines)
  - **Hard constraints**

    $\underline{V} \leq v_h \leq \overline{V}$ at all sensors

    $\underline{q}_h \leq q_h \leq \overline{q}_h$ at all actuators

Case 1: hard constraints



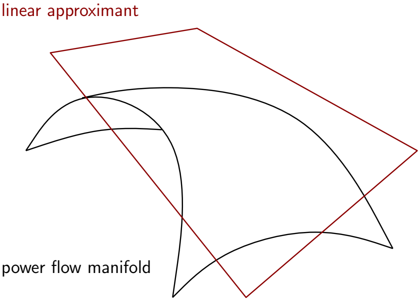
1. Modeling assumption

Modeling assumption

- on the parameters: constant R/X ratio ρ.
- on the structure: Kron reduction to controllable nodes

$$A_{x^*}(x - x^*) = 0 \qquad \rightarrow \qquad \left[\begin{array}{cc|cc} \rho L & -L & -I & 0 \\ -L & -\rho L & 0 & -I \end{array}\right] \begin{bmatrix} v \\ \theta \\ p \\ q \end{bmatrix} = \mathbb{0}$$

Case 1: hard constraints



1. Modeling assumption
2. Equilibrium

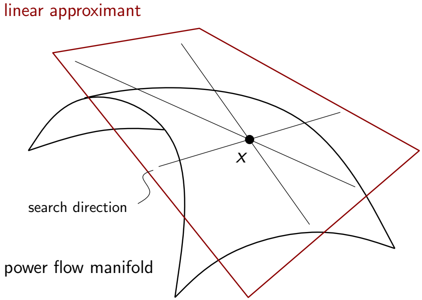
Equilibrium: Saddle point of the Lagrangian

$$\mathcal{L}(x, \lambda, \eta) = v^T L v + \lambda^T (v - \overline{v}) + \eta^T (\underline{q} - q) + \ldots$$

Stable for the discrete-time trajectories in which we alternate

- **exact minimization** in the primal variable x
- **projected gradient ascent** in the dual variables λ, η

Case 1: hard constraints

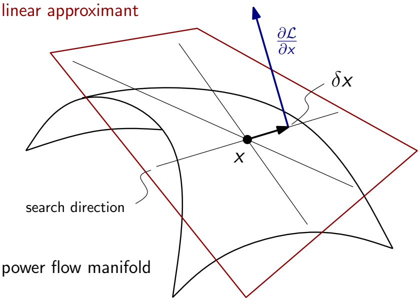


1. Modeling assumption
2. Equilibrium
3. Trajectory

Search directions: By projecting each possible direction δq on the linear manifold $\ker A_{x^*}$, we obtain feasible search directions in the state space.

$$\delta x = \begin{bmatrix} -\frac{1}{1+\rho^2} L^{\dagger} \delta q \\ -\frac{\rho}{1+\rho^2} L^{\dagger} \delta q \\ \mathbb{0} \\ \delta q \end{bmatrix}$$

Case 1: hard constraints

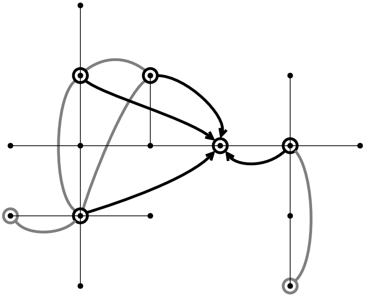


1. Modeling assumption
2. Equilibrium
3. Trajectory
4. Feedback law

Primal minimization step: we determine the step δx such that

$$\frac{\partial \mathcal{L}}{\partial x} = \begin{bmatrix} 2Lv + \lambda \\ \mathbb{0} \\ \mathbb{0} \\ -\eta \end{bmatrix} \quad \text{and} \quad \delta x = \begin{bmatrix} -\frac{1}{1+\rho^2} L^\dagger \delta q \\ -\frac{\rho}{1+\rho^2} L^\dagger \delta q \\ \mathbb{0} \\ \delta q \end{bmatrix} \quad \text{satisfy} \quad \frac{\partial \mathcal{L}}{\partial x}(x + \delta x, \lambda, \eta)^T \delta x = 0$$

Case 1: hard constraints



1. Modeling assumption
2. Equilibrium
3. Trajectory
4. Feedback law

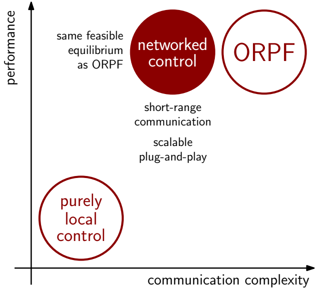
Output feedback control law

$$q \leftarrow q + (1+\rho^2)\,(Lv + \lambda) + (1+\rho^2)^2 L\eta \qquad \text{primal minimization}$$

$$\left.\begin{aligned} \lambda_h &\leftarrow [\lambda_h + \alpha(v_h - \overline{v})]_{\geq 0} \\ \eta_h &\leftarrow \left[\eta_h + \beta(\underline{q}_h - q_h)\right]_{\geq 0} \end{aligned}\right\} \text{dual ascent (integral action)}$$

Lv, $L\eta$ **Diffusion terms** that requires **nearest-neighbor communication**.

Case 1: hard constraints



Output feedback control law

- convergence to OPF solution
- no demand or generation measurement
- limited model knowledge
- no power flow solver
- interleaved sensing and actuation

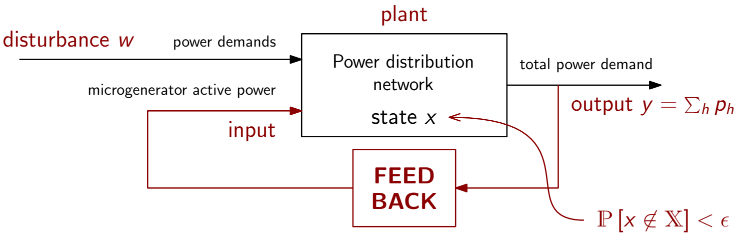
- Proof of **mean square convergence** (with randomized async updates)

→ S. Bolognani, R. Carli, G. Cavraro, & S. Zampieri (2015)
"Distributed reactive power feedback control for voltage regulation and loss minimization"

- **Communication is necessary:**
  No local strategy can guarantee convergence to a feasible voltage profile.

→ G. Cavraro, S. Bolognani, R. Carli, & S. Zampieri (2016)
"The value of communication in the voltage regulation problem"

Case 2: chance constraints



- **Inputs:** active power p_h of microgenerators
- **Outputs:** total grid demand $y = \sum_h p_h$
- **Control objective:**
  - **Soft constraints**

    $$\text{maximize} \sum_{\text{generators } h} p_h \qquad \text{(minimize curtailment)}$$

  - **Chance constraint**

    $\underline{V} \leq v_h \leq \overline{V}$ for **all** buses, with high probability

Case 2: chance constraints

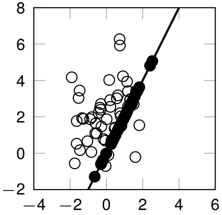
Scenario approach

Convert **stochastic** constraint into large set of **determistic** ones

$$\mathbb{P}\left[x \notin \mathbb{X}(w)\right] < \epsilon \qquad \rightarrow \qquad x \in \mathbb{X}(w^{(i)}), \quad i = 1, \ldots, N$$

Two sources of information on the unknown w

- Historical **samples** $w^{(i)}$ of the prior distribution
- **Online measurements** $y = Hw$ from the system

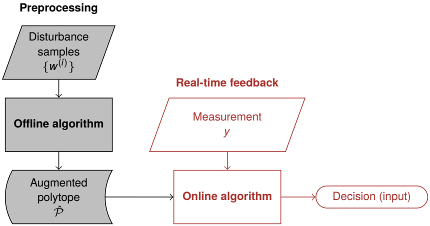


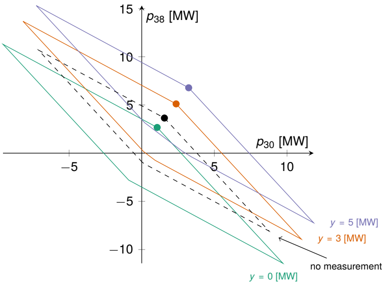
Scenario approach based on conditional distribution

- High computational demand
- Large memory footprint

$\rightarrow$ Not suited for real-time feedback control

Case 2: chance constraints





Two-phase algorithm

- Express posterior distribution as a projection: $\hat{w}_y = w + K(y - Hw)$
- Construct a feasible region parametrized in y **offline**
- Compute the conditional feasible polytope **online**

Computation time

Offline	Compute Σ and K	
	Construct augmented polytope $\hat{\mathcal{P}}$	
	Compute minimal representation of $\hat{\mathcal{P}}$	
	Total offline computation time	*55 min*
Online	Slice $\hat{\mathcal{P}}$ at $y = y^{\text{meas}}$ to obtain $\hat{\mathcal{P}}_y$	
	Solve LP defined on $\hat{\mathcal{P}}_y$	
	Total online computation time	*1.8 ms*

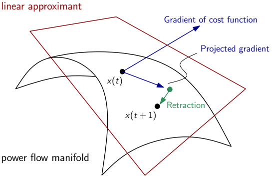
Memory footprint

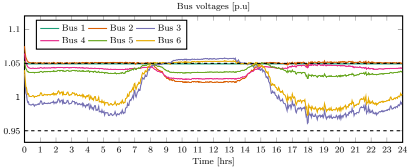
Offline	Augmented polytope $\hat{\mathcal{P}}$	48620 constraints
Online	Minimal representation of $\hat{\mathcal{P}}$	12 constraints

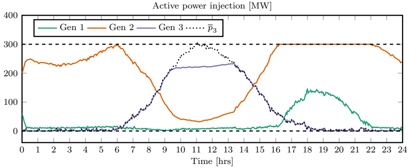
CONCLUSIONS

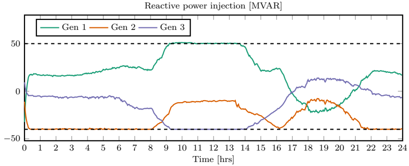
Conclusions

- **A tractable linear model**
  - structure preserving
  - computationally efficient
- **Ancillary services via feedback control**
  - model-free and robust
  - limited measurement
  - need for communication
- **Next step**
  - Feedback on the power flow manifold









→ A. Hauswirth, A. Zanardi, S. Bolognani, F. Dörfler, & G. Hug (2017)
"Online Optimization in Closed Loop on the Power Flow Manifold"

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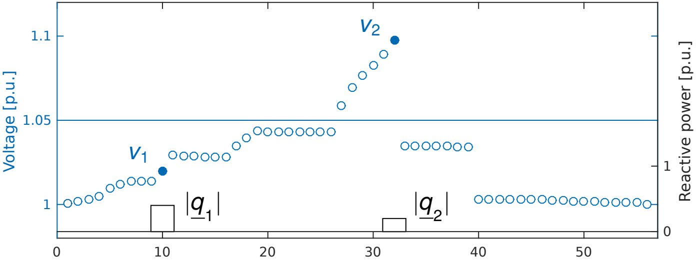
THE VALUE OF COMMUNICATION IN VOLTAGE REGULATION

Simulations and comparison

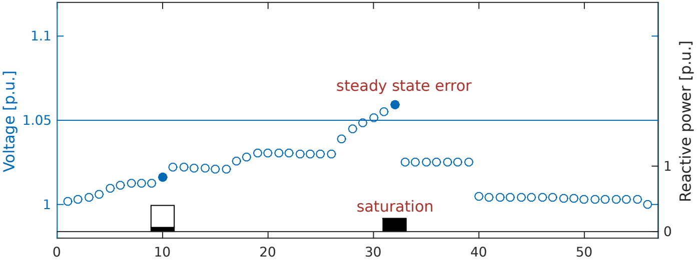


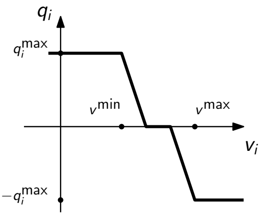
Modified IEEE 123 Distribution Test Feeder → `github`

Light load + 2 microgenerators → **overvoltage**

2 sets of constraints:
- **voltage limits** $v_h \leq \overline{v}$
- **power converter limits** $\underline{q}_h \leq q_h$

Simulations and comparison



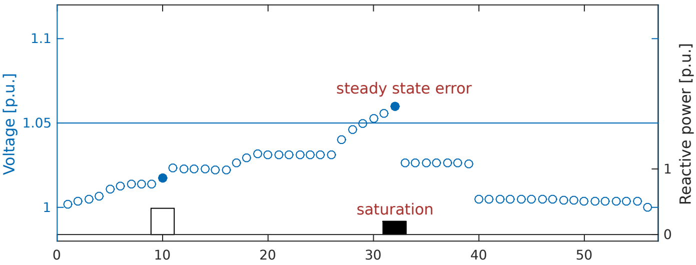


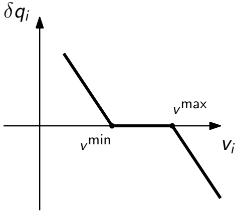
Fully decentralized, proportional controller.

$$q_h(t) = -f(v_h(t))$$

- Latest grid code draft
- Turitsyn (2011)
- Low (2012)
- Aliprantis (2013)
- Hiskens (2013)
- Kekatos (2015)

Simulations and comparison



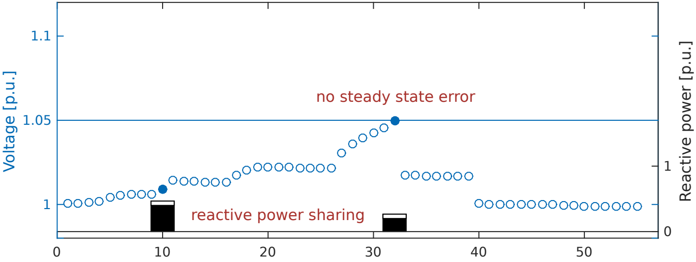


Fully decentralized, integral controller.

$$q_h(t+1) = q_h(t) - f(v_h(t))$$

- Li (2014)
- Farivar (2015)

Simulations and comparison



Networked feedback control **(neighbor-to-neighbor communication)**

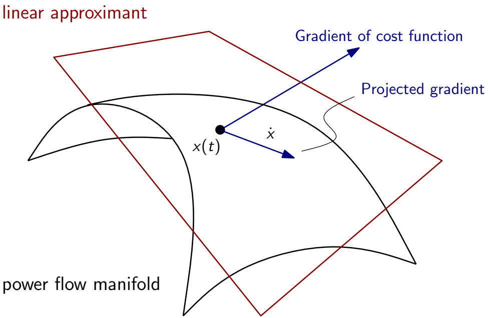
$$\lambda_h \leftarrow [\lambda_h + \alpha(v_h - \overline{v})]_{\geq 0}$$

$$\eta_h \leftarrow \left[\eta_h + \beta(\underline{q}_h - q_h)\right]_{\geq 0}$$

$$q \leftarrow q - \gamma \nabla J(q) - \lambda - \tilde{L}\eta$$

FEEDBACK OPTIMIZATION ON THE POWER FLOW MANIFOLD

Gradient descent on the power flow manifold



Target state

$$x^* = \arg\min_{x \in \mathcal{M}} J(x)$$

local minimizer on the
power flow manifold

Continuous time trajectory on the power flow manifold

1. $\nabla J(x)$: gradient of the cost function **(soft constraints)** in ambient space
2. $\Pi_x \nabla J(x)$: projection of the gradient on the linear approximant in x
3. Flow on the manifold: $\dot{x} = \gamma \Pi_x \nabla J(x)$

Gradient descent on the power flow manifold

$$x = \begin{bmatrix} x_{\text{exo}} \\ x_{\text{endo}} \end{bmatrix}$$

Exogenous variables

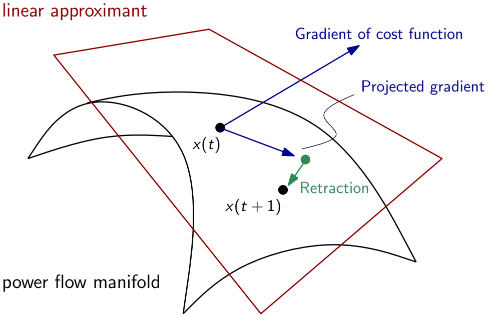
Inputs/disturbances

Reactive power injection q_i

Endogenous variables

Determined by the physics of the grid.

Voltage v_i



From **gradient descent flow** to **discrete-time feedback control**:

1. Compute $\Pi_x \nabla J(x)$
2. Actuate system based on $\delta x = \gamma \Pi \nabla J$ (exogeneous variables / inputs)
3. Retraction step $x(t+1) = R_{x(t)}(\delta x)$ $\Rightarrow x(t+1) \in \mathcal{M}$.

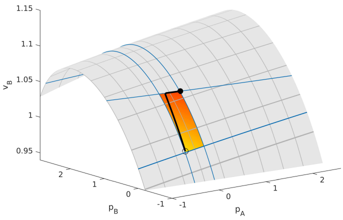
Hard constraints: need for a new theory

Feasible input region

- Not a smooth manifold
- Projected gradient descent
- Retraction preserves feasibility

$\rightarrow$ A. Hauswirth, S. Bolognani, G. Hug, & F. Dörfler (2016) **"Projected Gradient Descent on Riemannian Manifolds with Applications to Online Power System Optimization"**

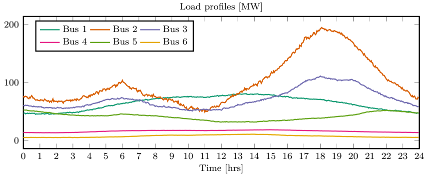
$$\min_{x \in \mathcal{M} \cap \mathcal{K}} J(x)$$

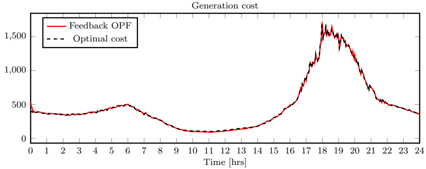


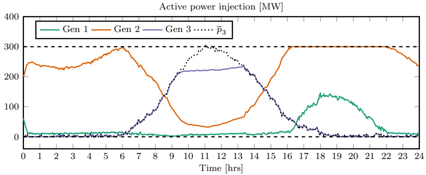
Output constraints

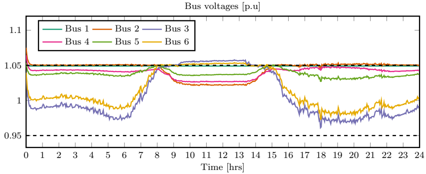
- No barrier function (backtracking not allowed)
- No time-varying penalty (persistent feedback control)
- **Dualization:** saddle / primal-dual trajectories on manifolds

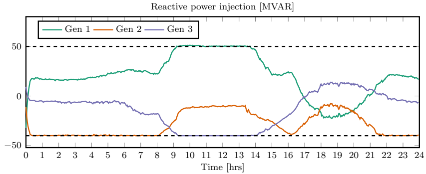
Feedback optimization on the power flow manifold











- Optimal tracking
- Feasible trajectory
- Joint economic dispatch / volt-VAR optimization

→ A. Hauswirth, A. Zanardi, S. Bolognani, F. Dörfler, & G. Hug (2017)
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