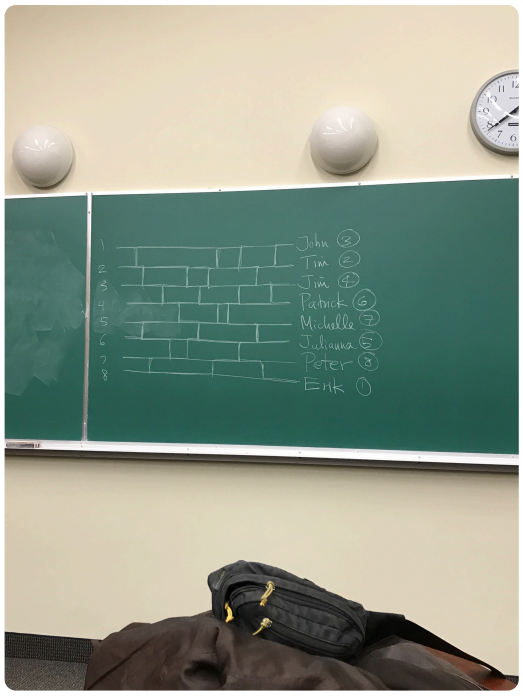
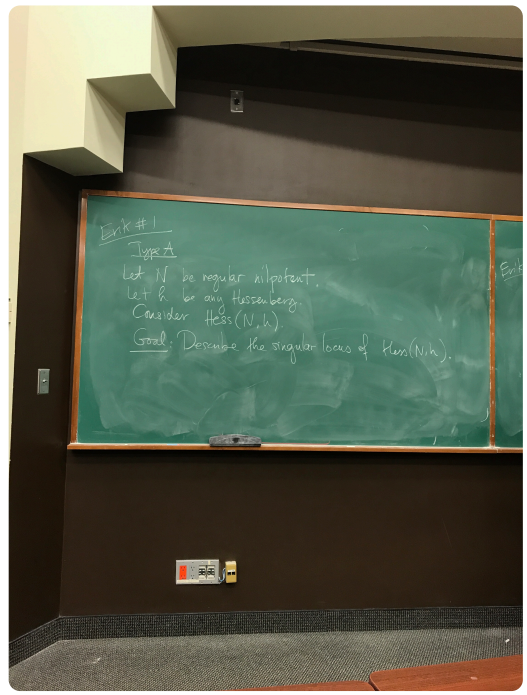
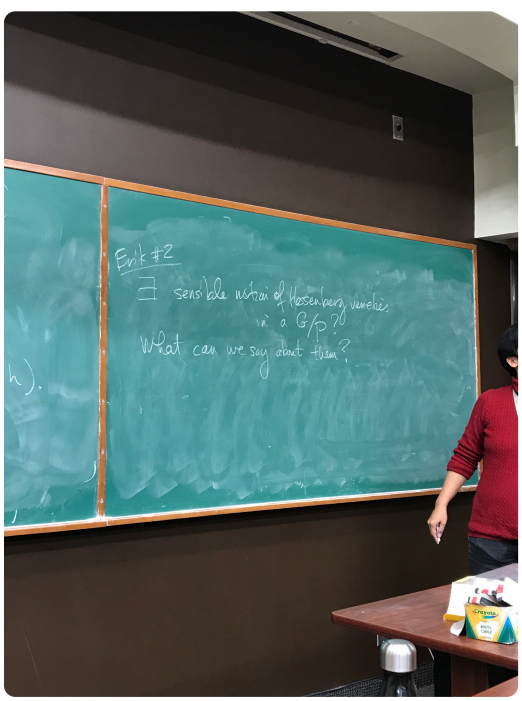
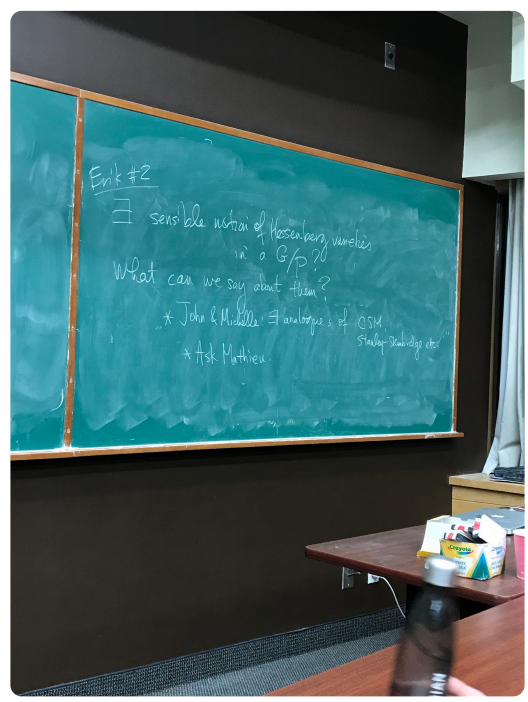
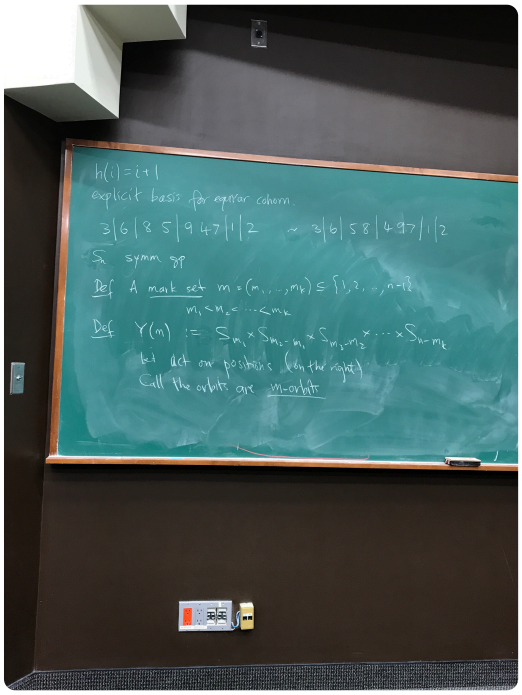
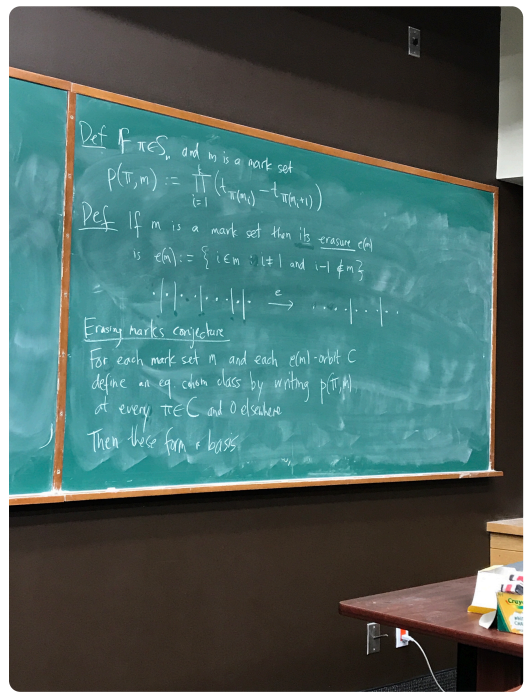

1 John 3
2 Tim 2
3 Jim 4
4 Patrick 6
5 Michelle 7
6 Juliana 5
7 Peter 8
8 Erik 0

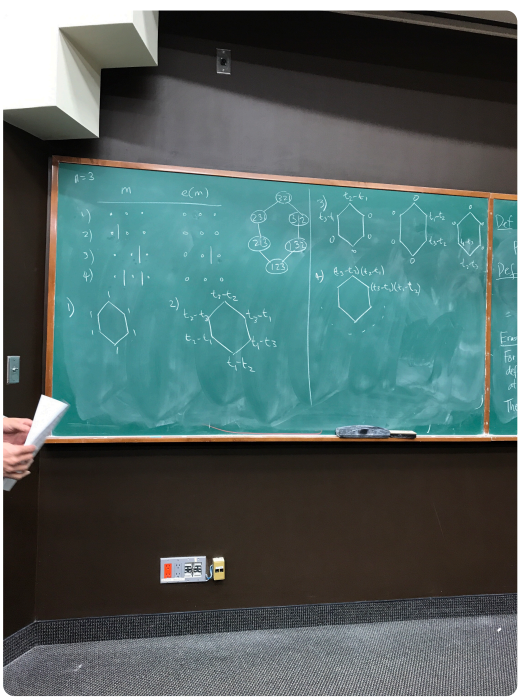

Erik #1
Type A
Let N be regular nilpotent.
Let h be any Hessenberg.
Consider Hess(N, h).
Goal: Describe the singular locus of Hess(N, h).

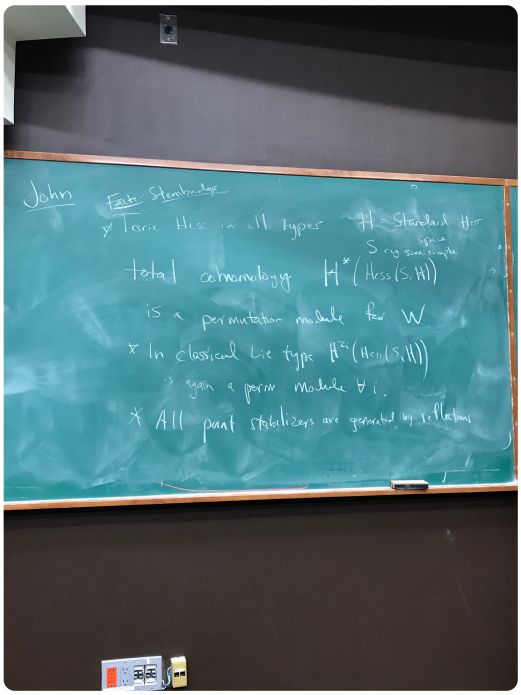

Erik #2
∃ sensible notion of Hessenberg varieties in a G/P ?
What can we say about them?

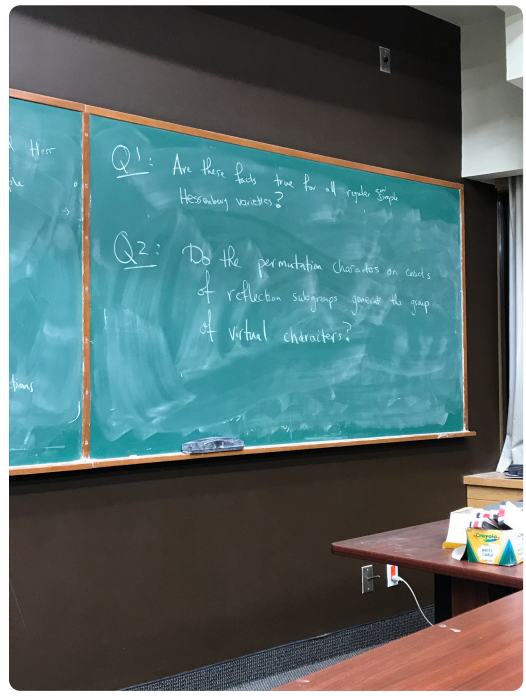

Erik #2
∃ sensible notion of Hessenberg varieties in a G/P ?
What can we say about them?
* John & Michelle: ∃ analogue's of CSM
Standard Knowledge etc.
* Ask Mathieu.

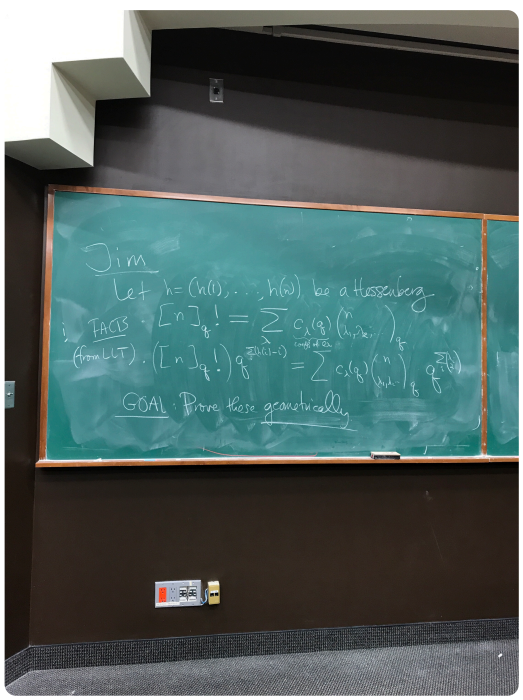

h(i) = i+1
explicit basis for equivar cohom.
3|6|8 5|9 4 7|1|2 ~ 3|6|5 8|4 9 7|1|2
S_n symm gp
Def A mark set m = (m_1, ..., m_k) ⊆ {1, 2, ..., n-1}
m_1 < m_2 < ... < m_k
Def Y(m) := S_{m_1} × S_{m_2 - m_1} × S_{m_3 - m_2} × ... × S_{n - m_k}
let act on positions (left to right)
Call the orbits are m-orbits

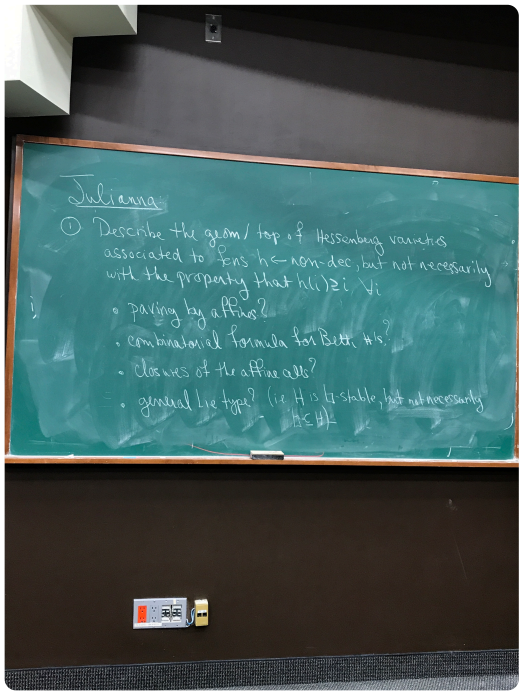

Def If π ∈ S_n and m is a mark set
p(π, m) := ∏_{i=1} (t_{π(m_i)} - t_{π(m_i+1)})
Def If m is a mark set then its erasure e(m)
is e(m) := { i ∈ m : i+1 and i-1 ∉ m }
e
Erasing marks conjecture
For each mark set m and each e(m)-orbit C
define an eq cohom class by writing p(π, m)
at every π ∈ C and 0 elsewhere
Then these form a basis

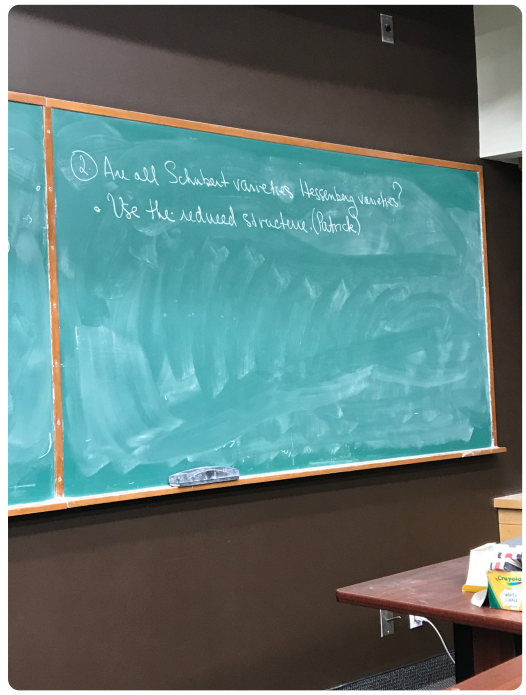

n = 3
m
e(m)
1)
2)
3)
4)
t_2 - t_1
t_3 - t_2
t_1 - t_3
t_1 - t_2

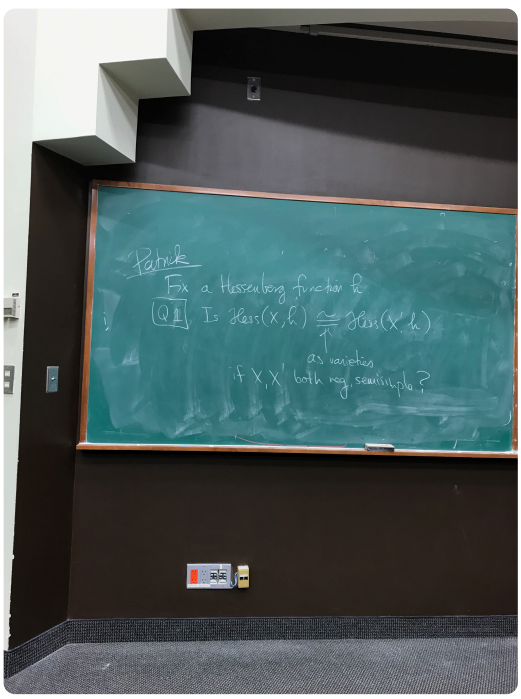

John
Facts: Steinbergs
* Toric Hess in all types
H = Standard Herm space
S reg semi-simple
total cohomology $H^*(\mathrm{Hess}(S,H))$
is a permutation module for W
* In classical Lie types $H^{2i}(\mathrm{Herm}(S,H))$
is again a perm module $\forall i$
* All point stabilizers are generated by reflections

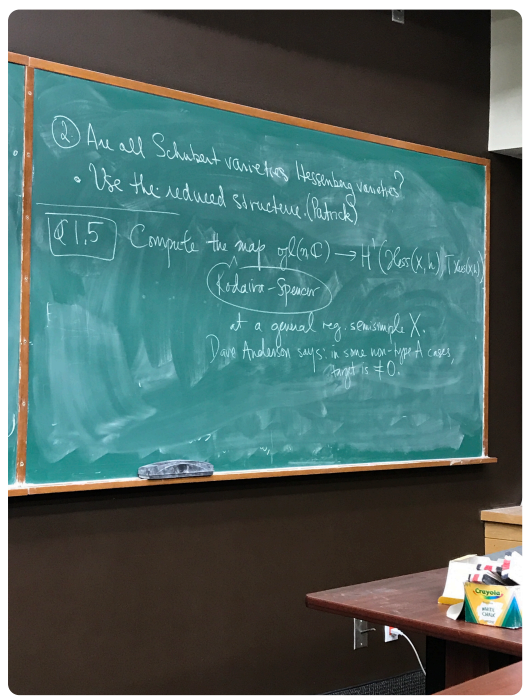

Q1: Are these facts true for all regular semi-simple Hessenberg varieties?
Q2: Do the permutation characters on cosets of reflection subgroups generate the group of virtual characters?

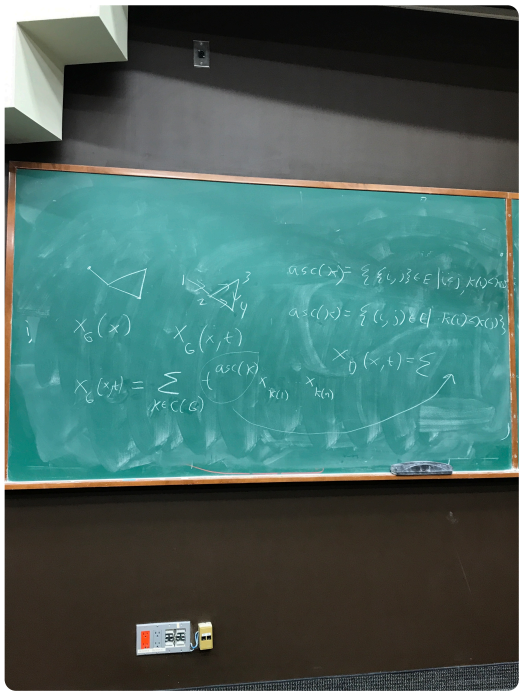

Jim
Let h = (h(1), ..., h(n)) be a Hessenberg
FACTS (from LLT): $[n]_q! = \sum_{\text{coeff of } \mathfrak{S}_\lambda} c_\lambda(q) \binom{n}{\lambda_1, \lambda_2, \ldots}_q$
$([n]_q!)\, q^{\sum h(i)-i} = \sum c_\lambda(q) \binom{n}{\lambda_1, \ldots}_q q^{\sum \lambda_i i}$
GOAL: Prove these geometrically

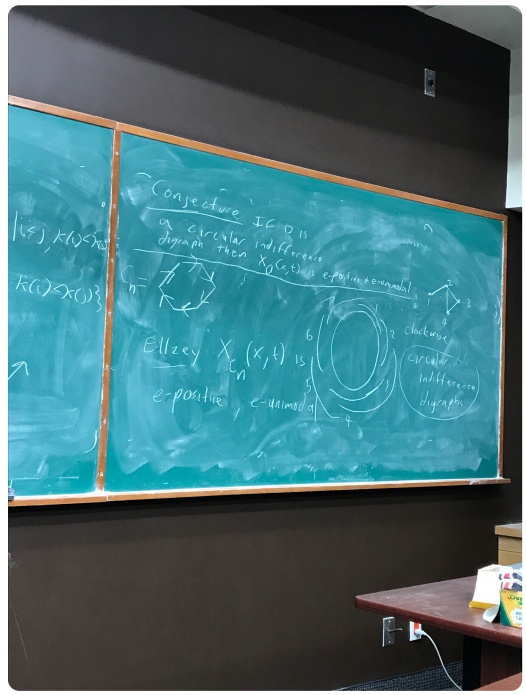

Julianna:
① Describe the geom / top of Hessenberg varieties associated to fns h ← non-dec, but not necessarily with the property that h(i) ≥ i ∀i
• paving by affines?
• combinatorial formula for Betti #'s?
• closures of the affine cells?
• general Lie type? (i.e. H is L-stable, but not necessarily L ⊆ H)

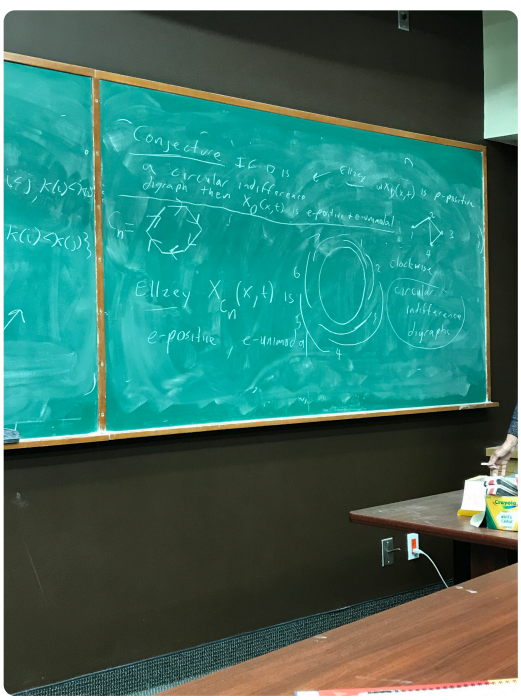

② Are all Schubert varieties Hessenberg varieties?
• Use the reduced structure (Patrick)

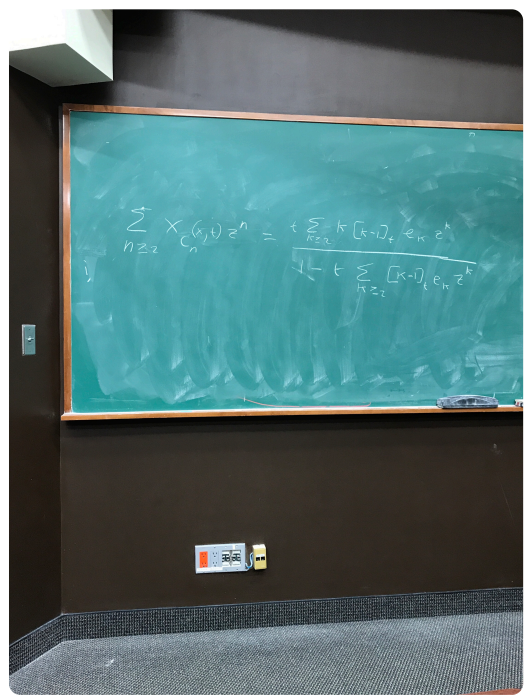

Patrick
Fix a Hessenberg function h
Q1 Is Hess(X, h) ≅ Hess(X', h)
as varieties
if X, X' both reg. semisimple?

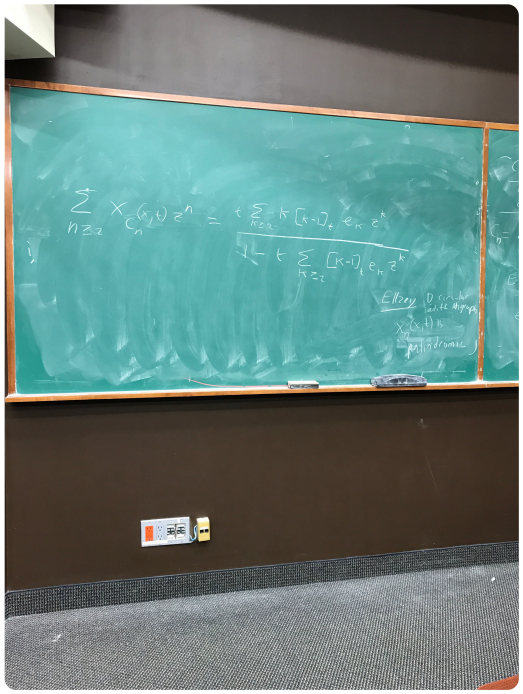

② Are all Schubert varieties Hessenberg varieties?
• Use the reduced structure (Patrick)
Q1.5 Compute the map gr(n,D) → H^•(Hess(X,h), Tweddle)
Kodaira-Spencer
at a general reg. semisimple X.
Dave Anderson says: in some non-type A cases, target is ≠ 0.

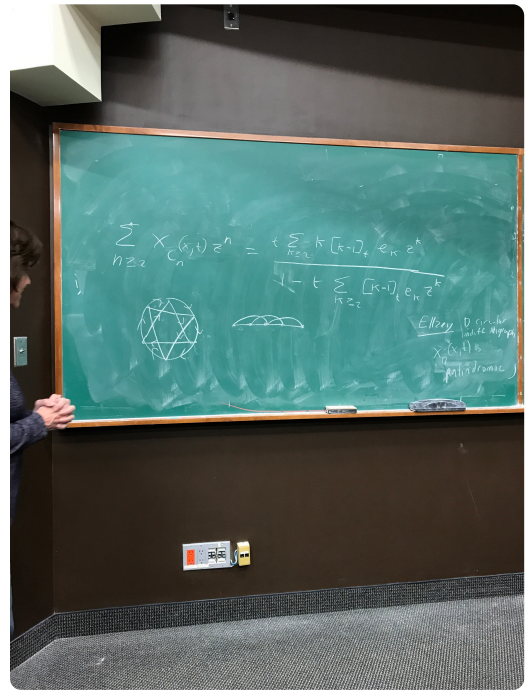

asc(x) = {(i,j) ∈ E | i<j, k(i)>k(j)}
asc(x) = {(i,j) ∈ E | i<j, k(i)>x(j)}
X_G(x)
X_G(x,t)
X_G(x,t) = Σ_{x∈C(G)} t^{asc(x)} x_{k(1)} x_{k(n)}
X_G(x,t) = Σ

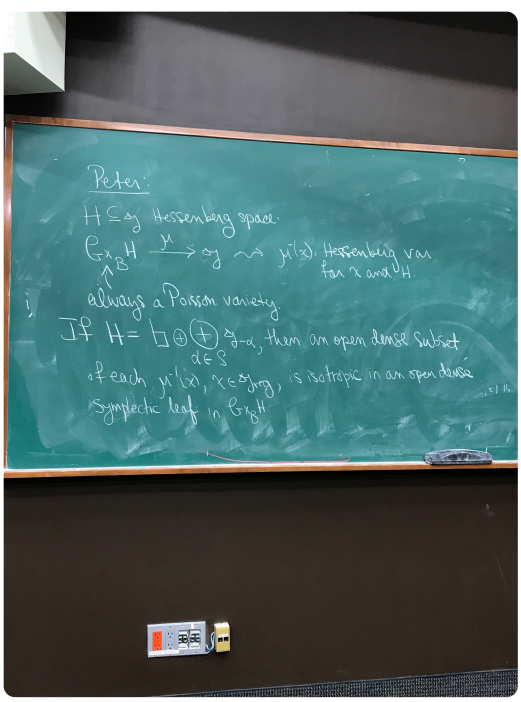

Conjecture If G is a circular indifference digraph then X_G(x,t) is e-positive e-unimod!
C_n
Ellzey X_{C_n}(x,t) is
e-positive, e-unimod
clockwise
circular indifference digraphs

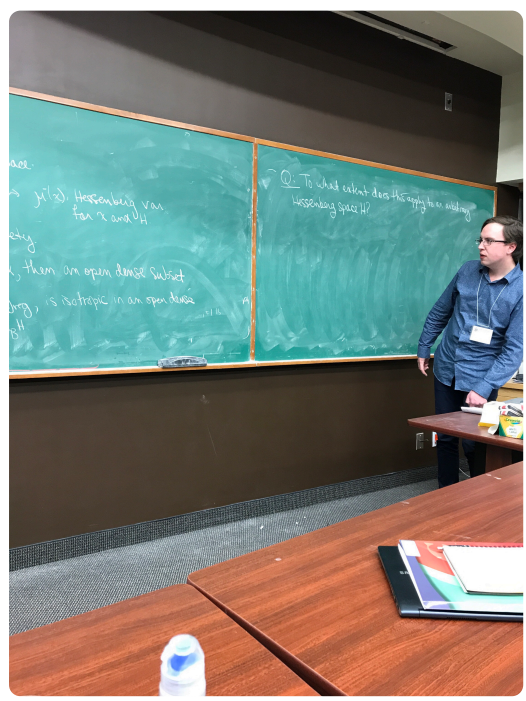

Conjecture If G is a circular indifference digraph then X_G(x,t) is e-positive e-unimod!
Ellzey unif(x,t) is e-positive
C_n
Ellzey X_{C_n}(x,t) is
e-positive, e-unimod
clockwise
circular indifference digraphs


Σ_{n≥2} X_{C_n}(x,t) z^n = t Σ_{k≥2} k [k-1]_t e_k z^k / (1 - t Σ_{k≥2} [k-1]_t e_k z^k)


$\sum_{n\geq 2} X_{C_n}(x,t) z^n = \frac{t \sum_{k\geq 2} k\,[k-1]_t\, e_k z^k}{1 - t \sum_{k\geq 2} [k-1]_t\, e_k z^k}$
Ellzey


$\sum_{n\geq 2} X_{C_n}(x,t) z^n = \frac{t \sum_{k\geq 2} k\,[k-1]_t\, e_k z^k}{1 - t \sum_{k\geq 2} [k-1]_t\, e_k z^k}$
Ellzey


Peter:
$H \subseteq \mathfrak{g}$ Hessenberg space.
$G \times_B H \xrightarrow{\mu} \mathfrak{g} \rightsquigarrow \mu^{-1}(x)$, Hessenberg var for x and H
always a Poisson variety.
If $H = \mathfrak{b} \oplus \bigoplus_{\alpha \in S} \mathfrak{g}_{-\alpha}$, then an open dense subset of each $\mu^{-1}(x)$, $x \in \mathfrak{g}_{reg}$, is isotropic in an open dense symplectic leaf in $G \times_B H$


$\mu^{-1}(x)$, Hessenberg var for x and H
Q: To what extent does this apply to an arbitrary Hessenberg space H?