F-theory on Quotient Threefolds and Their Discrete Superconformal Matter

Paul-Konstantin Oehlmann

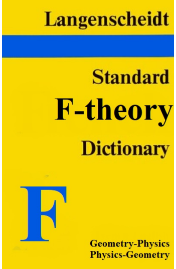
Virginia Polytechnic Institute and State University

Based on • arXiv:1801.XXXXX with: L. Anderson, J. Gray and A. Grassi

BIRS workshop on geometry and physics of F-theory, Banff
January 25th 2018

Deutsche Forschungsgemeinschaft

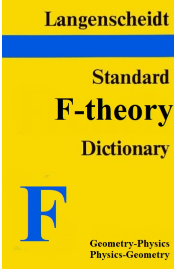
F-theory Dictionary



F-theory Dictionary

- **Geometry** of Torus fibered Calabi-Yau n-folds (Compact)

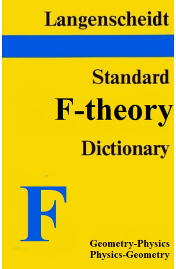
$\updownarrow$



F-theory Dictionary

- **Geometry** of Torus fibered Calabi-Yau n-folds (Compact)

↕



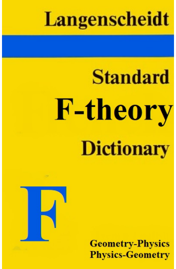
↕

- **Physics** of $12 - 2n$ Dimensional Supersymmetric Gauge Theories (+Gravity)

F-theory Dictionary

• **Geometry** of Torus fibered Calabi-Yau n-folds (Compact)

$\updownarrow$



Limits of the Dictionary:

• Physics obtained indirectly via String Dualities

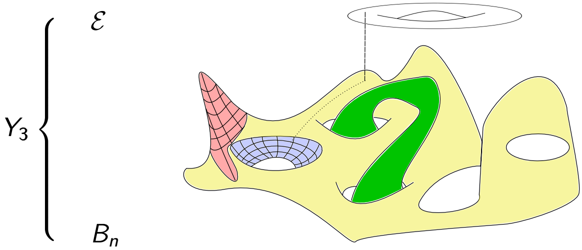
• No fundamental 12 D theory formulated

$\rightarrow$ Where are the limits of the Dictionary?

$\updownarrow$

• **Physics** of $12 - 2n$ Dimensional Supersymmetric Gauge Theories (+Gravity)

Introduction and Motivation

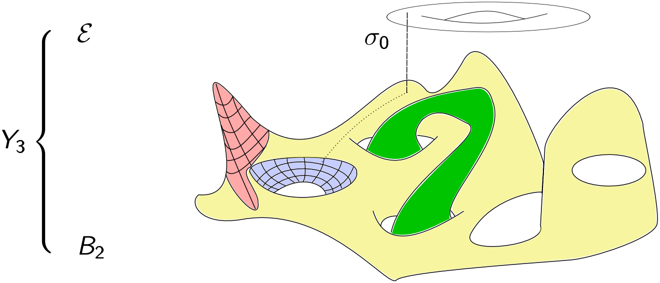


- Consider a torus $\mathcal{E}$ fibered CY-3fold Y_3 fold over a two dimensional Base B_2

$$\begin{array}{ccc} \mathcal{E} & \rightarrow & Y_3 \\ & & \downarrow \pi \\ & & B_2 \end{array}$$

- Treat τ of $\mathcal{E}$ as the **axio-dilaton of IIB** (forget the Vol($\mathcal{E}$))
- Power of F-theory: D7/O7 brane stacks
  - SL$(2,\mathbb{Z})$ monodromies of τ traced geometricaly
  - D7/O7 backreaction taken care of in B_n

Symmetries in F-theory

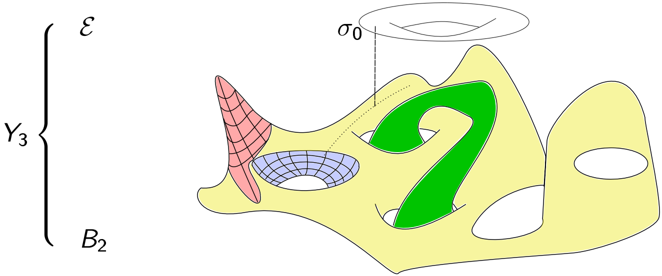


F-theory Setup

By **torus** $\mathcal{E}$ fibered Calabi-Yau 3-fold we actually mean:

- **Elliptic Fibration**: $\exists$ rational sections $S_r \cdot \mathcal{E} = 1 \rightarrow$ always a zero-section σ_0
- **Genus One Fibration**: $S_r \cdot \mathcal{E} = n_i \;\; n_i \neq 1 \; \forall i$ [Braun/Taylor, Morrison'14]
  $\rightarrow$ Jacobian map provides a **surjective** map to an elliptic fibration

Symmetries in F-theory

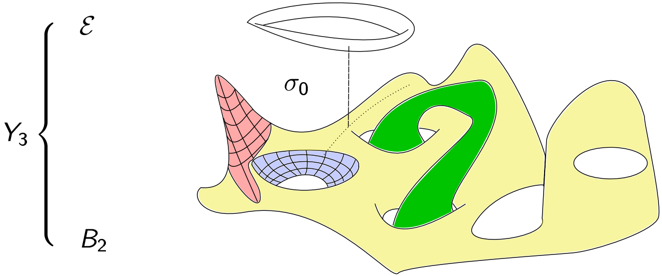


F-theory Setup

By **torus** $\mathcal{E}$ fibered Calabi-Yau 3-fold we actually mean:

- **Elliptic Fibration**: $\exists$ rational sections $S_r \cdot \mathcal{E} = 1 \rightarrow$ always a zero-section σ_0
- **Genus One Fibration**: $S_r \cdot \mathcal{E} = n_i \; n_i \neq 1 \; \forall i$ [Braun/Taylor, Morrison'14]
  $\rightarrow$ Jacobian map provides a **surjective** map to an elliptic fibration

Symmetries in F-theory

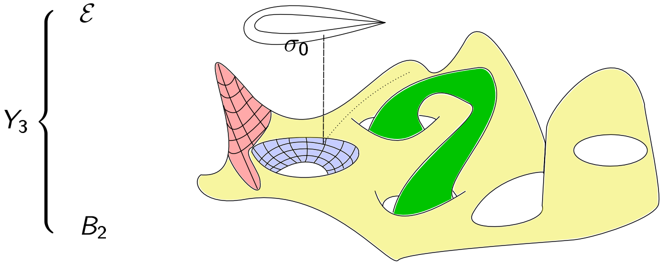


F-theory Setup

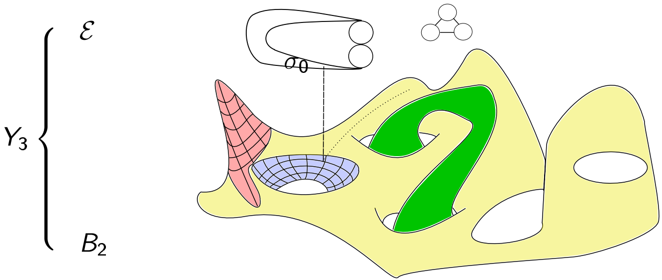
By **torus** $\mathcal{E}$ fibered Calabi-Yau 3-fold we actually mean:

- **Elliptic Fibration**: $\exists$ rational sections $S_r \cdot \mathcal{E} = 1$ $\rightarrow$ always a zero-section σ_0
- **Genus One Fibration**: $S_r \cdot \mathcal{E} = n_i \;\; n_i \neq 1 \;\forall i$ [Braun/Taylor, Morrison'14]
  $\rightarrow$ Jacobian map provides a **surjective** map to an elliptic fibration

Symmetries in F-theory

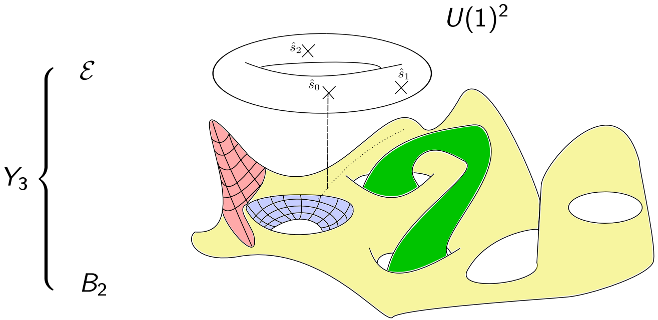


Symmetries in F-theory



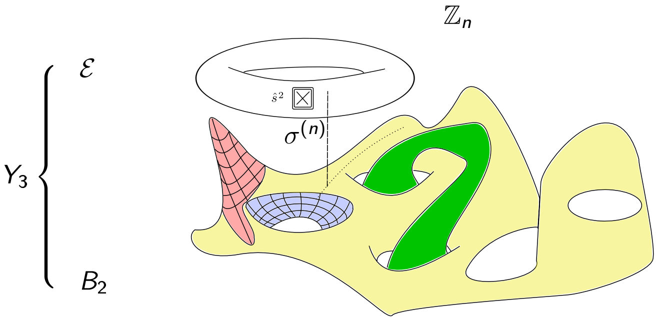
1. **Cartan Generators** D_i of **non-Abelian** ADE Group at codim 1 [Kodaira]

Symmetries in F-theory



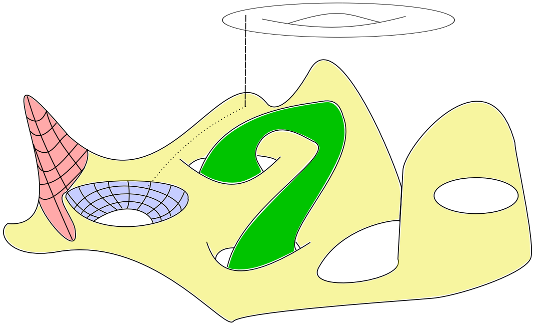
1. **Cartan Generators** D_i of **non-Abelian** ADE Group at codim 1 [Kodaira]
2. **Abelian Symmetries** from **free** part of the **Mordell-Weil group**
   [Mayrhover,Palti,Weigand; Morrison,Park'12...]

Symmetries in F-theory



1. **Cartan Generators** D_i of **non-Abelian** ADE Group at codim 1 [Kodaira]
2. **Abelian Symmetries** from **free** part of the **Mordell-Weil group** [Mayrhover,Palti,Weigand; Morrison,Park'12...]
3. **Discrete** $\mathbb{Z}_n$ **remnant** from a massive higgsed U(1) $\hat{A}_i$ [Braun/Taylor Morrison'14.....]

6D F-theory models



For Y_3 smooth, the $\mathcal{N} = (1, 0)$, 6D SUGRA theory is fully geometrized:

- **Tensors $\mathbf{T}_{(1,0)}$:** Supported in the Base by $h^{1,1}(B) - 1$
- **Hypers $\mathbf{H} = \mathbf{H}_{\text{uncharged}} + \mathbf{H}_{\text{charged}}$:**
  - $H_{\text{uncharged}} = h^{2,1}(Y_3) + 1$
  - $H_{\text{charged}} =$ **Codimension two** (points) in B_2 where $\mathcal{E}$ becomes further **reducible**
- **Anomalies:** strong constraints on matter and representations!

Motivation and Punshline

- Kodaira Singularities, codimension two non-flat fibers, Mordell-Weil group, Tate-Shafarevich group, terminal singularities
  all have a phyiscal counterpart

Does every subtle geometric property of F-theory fibrations X admit a physical counterpart?

Motivation and Punshline

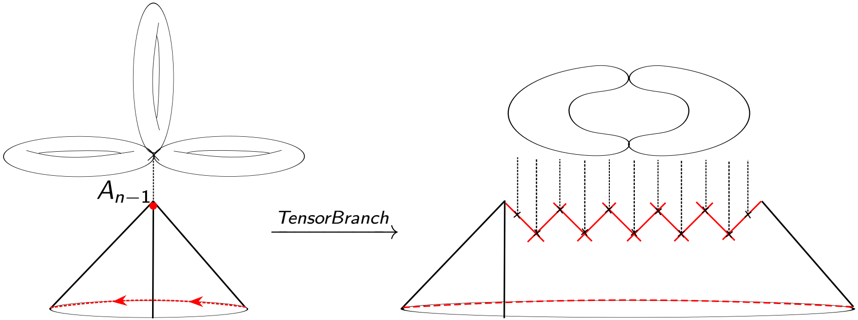
- Kodaira Singularities, codimension two non-flat fibers, Mordell-Weil group, Tate-Shafarevich group, terminal singularities
  all have a phyiscal counterpart

Does every subtle geometric property of F-theory fibrations X admit a physical counterpart?

What is F-theory Physics of a non-simply connected threefold?

Geometry	Physics
• **Fixed points** in the Base	• (2,0) Superconformal Matter
• with **multiple fibers**	• **Coupled** to $\mathbb{Z}_n$ Gauge Symmetry
• Sitting over a **Lens space**	• Visible at their Tensor Branch

Motivation and Punchline



Discrete Charged (2,0) Matter

- The Base contains (2,0) A_{n-1} **superconformal matter**
- At the **tensor branch**, there appear n I_2 fibers at codim 2
- These give n purely **discrete charged hypermultiplets**

They form a new type of 6D discrete charged (2,0) superconformal matter

Outline

1. Motivation and Punchline ✓
2. Geometric Setup
3. Example: Bi-Cubic-Quotient
   1. Covering Geometry and Quotient
   2. Spectrum, Anomalies and M5 branes
   3. Lens Spaces and Hyperconifold transitions
   4. Tensor branch theory
4. Summary and more

The starting point

Start with a Calabi-Yau threefold Y_3 realized as a complete intersection $P_i = 0$ in some ambient space Z that is torus-fibered and admits

- discrete
- free
- cyclic

Automorphism Γ_n (possibly inherited from the ambient space Z) of **order n**

The starting point

Start with a Calabi-Yau threefold Y_3 realized as a complete intersection $P_i = 0$ in some ambient space Z that is torus-fibered and admits

- discrete
- free
- cyclic

Automorphism Γ_n (possibly inherited from the ambient space Z) of **order n**

Take the **quotient** threefold $\hat{Y}_3 = Y_3/\Gamma_n$ such that

- $\widehat{Y}_3 = Y_3/\Gamma_n$ is still Calabi-Yau
- is smooth
- non-simply connected $\pi_1(\hat{Y}_3) = \mathbb{Z}_n$
- Torsion: $\mathrm{Tor}(H^2(\widehat{Y}_3, \mathbb{Z}) \sim B'(\widehat{Y}_3, \mathbb{Z}) = \mathbb{Z}_n$
- **Want it still to be** torus fibered

The starting point

Start with a Calabi-Yau threefold Y_3 realized as a complete intersection $P_i = 0$ in some ambient space Z that is torus-fibered and admits

- discrete
- free
- cyclic

Automorphism Γ_n (possibly inherited from the ambient space Z) of **order n**

Take the **quotient** threefold $\hat{Y}_3 = Y_3/\Gamma_n$ such that

- $\widehat{Y}_3 = Y_3/\Gamma_n$ is still Calabi-Yau
- is smooth
- non-simply connected $\pi_1(\hat{Y}_3) = \mathbb{Z}_n$
- Torsion: $\mathrm{Tor}(H^2(\widehat{Y}_3, \mathbb{Z}) \sim B'(\widehat{Y}_3, \mathbb{Z}) = \mathbb{Z}_n$
- **Want it still to be** torus fibered

- What are the **constraints** on the Γ_n quotient?

Quotient Calabi-Yau Geometries

Want $\widehat{Y}_3 = Y_3/\Gamma_n$ to be a **smooth** Calabi-Yau that is also **torus-fibered** in order to be relevant for F-theory. [Donagi, Ovrut, Pantev, Waldram'99]

Quotient Calabi-Yau Geometries

Want $\widehat{Y}_3 = Y_3/\Gamma_n$ to be a **smooth** Calabi-Yau that is also **torus-fibered** in order to be relevant for F-theory. [Donagi, Ovrut, Pantev, Waldram'99]

1. $\widehat{Y}$ stays Calabi-Yau: Γ_n leaves canonical divisor of Z invariant

Quotient Calabi-Yau Geometries

Want $\widehat{Y}_3 = Y_3/\Gamma_n$ to be a **smooth** Calabi-Yau that is also **torus-fibered** in order to be relevant for F-theory. [Donagi, Ovrut, Pantev, Waldram'99]

1. $\widehat{Y}$ stays Calabi-Yau: Γ_n leaves canonical divisor of Z invariant
2. $\hat{Y}_3 = Y_3/\Gamma_n$ is smooth (freely acting)

Quotient Calabi-Yau Geometries

Want $\widehat{Y}_3 = Y_3/\Gamma_n$ to be a **smooth** Calabi-Yau that is also **torus-fibered** in order to be relevant for F-theory. [Donagi, Ovrut, Pantev, Waldram'99]

1. $\widehat{Y}$ stays Calabi-Yau: Γ_n leaves canonical divisor of Z invariant
2. $\hat{Y}_3 = Y_3/\Gamma_n$ is smooth (freely acting)
3. Γ_n respects fibration/projecton π:

$$\begin{array}{ccc} T/\Gamma_{n,f} & \rightarrow & \hat{Y} = Y/\Gamma_n \\ & & \downarrow \pi \\ & & \hat{B} = B/\Gamma_{n,b} \end{array}$$

Action of Γ_n decomposable into a fiber and base part $\Gamma_n = \Gamma_{F,n} \oplus \Gamma_{b,n}$

Fiber and Base must not be mixed!

Quotient Calabi-Yau Geometries

Want $\widehat{Y}_3 = Y_3/\Gamma_n$ to be a **smooth** Calabi-Yau that is also **torus-fibered** in order to be relevant for F-theory. [Donagi, Ovrut, Pantev, Waldram'99]

1. $\widehat{Y}$ stays Calabi-Yau: Γ_n leaves canonical divisor of Z invariant
2. $\hat{Y}_3 = Y_3/\Gamma_n$ is smooth (freely acting)
3. Γ_n respects fibration/projecton π:

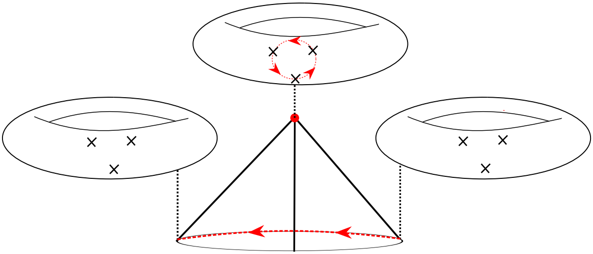
$$\begin{array}{ccc} T/\Gamma_{n,f} & \rightarrow & \hat{Y} = Y/\Gamma_n \\ & & \downarrow \pi \\ & & \hat{B} = B/\Gamma_{n,b} \end{array}$$

   Action of Γ_n decomposable into a fiber and base part $\Gamma_n = \Gamma_{F,n} \oplus \Gamma_{b,n}$

   Fiber and Base must not be mixed!

4. Quotient base $\hat{B}$ allowed to have **orbifold fixed points**
5. Singularity in the base must be compensated by a fiber translation to keep $\hat{Y}$ smooth

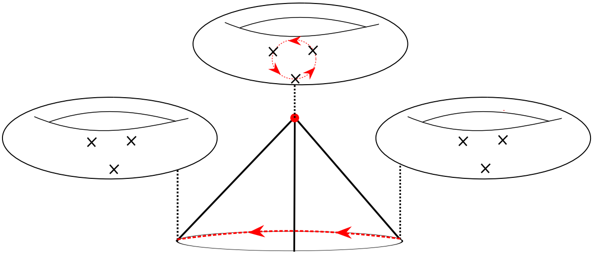
Quotient Fiber Action



Must **compensate** the $\Gamma_{n,b}$ **orbifold** fixed point in the base

- To **avoid a fixed fiber**, $\Gamma_{f,n}$ must act as a **fiber rotation**

Quotient Fiber Action



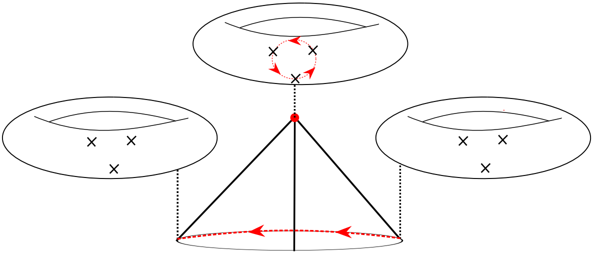
Must **compensate** the $\Gamma_{n,b}$ **orbifold** fixed point in the base

- To **avoid a fixed fiber**, $\Gamma_{f,n}$ must act as a **fiber rotation**

1. $T = \mathcal{C}$ **is a genus-one fibration:** Over the fixed point, there must be $n-$sections that **translate** into each other

$$s_n^i \xrightarrow{\Gamma_{n,f}} s_n^j \text{ with } j = i + 1 \bmod n \tag{1}$$

Quotient Fiber Action



Must **compensate** the $\Gamma_{n,b}$ **orbifold** fixed point in the base

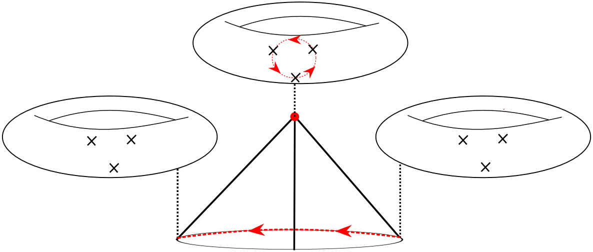
- To **avoid a fixed fiber**, $\Gamma_{f,n}$ must act as a **fiber rotation**

1. $T = \mathcal{C}$ **is a genus-one fibration:** Over the fixed point, there must be $n-$sections that **translate** into each other

$$s_n^i \xrightarrow{\Gamma_{n,f}} s_n^j \text{ with } j = i + 1 \mod n \tag{1}$$

2. $T = \mathcal{E}$ **is an elliptic fibration:** Γ_n must be a homomorpshism into MW_{tor}

Quotient Fiber Action



Must **compensate** the $\Gamma_{n,b}$ **orbifold** fixed point in the base

- To **avoid a fixed fiber**, $\Gamma_{f,n}$ must act as a **fiber rotation**

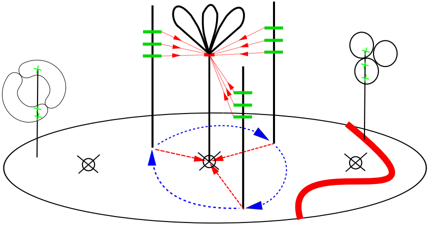
1. $T = \mathcal{C}$ **is a genus-one fibration:** Over the fixed point, there must be $n-$sections that **translate** into each other

$$s_n^i \xrightarrow{\Gamma_{n,f}} s_n^j \text{ with } j = i + 1 \text{ mod } n \tag{1}$$

2. $T = \mathcal{E}$ **is an elliptic fibration:** Γ_n must be a homomorpshism into MW_{tor}

Quotient Results in a **genus-one fibration with multiple fibers**

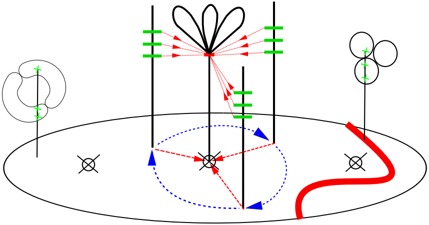
Quotient Geometry



Multiple Fiber

- Over a point in the base $s \in \widehat{B}$ the fiber is
$$\mathcal{C}_s = \pi^{-1}(s)$$
- The fiber $\mathcal{C}_s$ is singular **everywhere** $\rightarrow$ multiple fiber [Gross'93]

Quotient Geometry



Multiple Fiber

- Over a point in the base $s \in \widehat{B}$ the fiber is
$$\mathcal{C}_s = \pi^{-1}(s)$$
- The fiber $\mathcal{C}_s$ is singular **everywhere** $\rightarrow$ multiple fiber [Gross'93]

Fibration away from the fixed points

- **Genus-one** fibration **away from the fixed point**
- Allow for reducible fibers at codim 1 and 2
- Note: All ADE divisors miss the fixed points $\rightarrow$ Cartier in $H_2(\widehat{B}, \mathbb{Z})$.

Example: The bi-cubic

Take ambient space $Z = (\mathbb{P}^2 \times \mathbb{P}^2)$ with 4D polytope spanned by

x_0	x_1	x_2	y_0	y_1	y_2
1	0	-1	0	0	0
0	1	-1	0	0	0
0	0	0	1	0	-1
0	0	0	0	1	-1

- **Genus-one fibered** threefold with hypersurface
  $P = s_1 x_0^3 + s_2 x_0^2 x_1 + s_3 x_0 x_1^2 + s_4 x_1^3 + s_5 x_0^2 x_2 + s_6 x_0 x_1 x_2 + s_7 x_1^2 x_2 + s_8 x_0 x_2^2 + s_9 x_1 x_2^2 + s_{10} x_2^3$
- **Sections of the base** $s_i \in K_b^{-1} = 3H_b$
- they are generic cubic polynomials (too) $s_i = \sum_{i+j+k=3} a_{i,j,k} y_0^i y_1^j y_2^k$
- **Hodge numbers:** $(h^{(1,1)}, h^{(2,1)})_\chi = (2, 68)_{-163}$

Example: Bi-Cubic

- every toric divisor on Y **intersects fiber trice**: $\mathcal{C} \cdot D_{x_i} = 3$

Example: Bi-Cubic

- every toric divisor on Y **intersects fiber trice**: $\mathcal{C} \cdot D_{x_i} = 3$
- **Hodge numbers:** $(h^{(1,1)}, h^{(2,1)})_\chi = (2, 83)_{-162}$
- $\boxed{\mathbb{Z}_3}$ Gauge Symmetry

Example: Bi-Cubic

- every toric divisor on Y **intersects fiber trice**: $\mathcal{C} \cdot D_{x_i} = 3$
- **Hodge numbers:** $(h^{(1,1)}, h^{(2,1)})_\chi = (2, 83)_{-162}$
- $\boxed{\mathbb{Z}_3}$ Gauge Symmetry
- several charged matter singlets [Klevers, Mayorga,Piragua, P-K.O., Reuter]

Example: Bi-Cubic

- every toric divisor on Y **intersects fiber trice**: $\mathcal{C} \cdot D_{x_i} = 3$
- **Hodge numbers:** $(h^{(1,1)}, h^{(2,1)})_\chi = (2, 83)_{-162}$
- $\boxed{\mathbb{Z}_3}$ Gauge Symmetry
- several charged matter singlets [Klevers, Mayorga, Piragua, P-K.O., Reuter]

Full Spectrum

Tensors:	0		$H_{\text{uncharged}}$:	$h^{2,1}(Y) + 1$
Vectors:	0		H_{charged} :	$21(K_b^{-1})^2$

- Using $K_b^{-1} \cdot K_b^{-1} = 9$

Example: Bi-Cubic

- every toric divisor on Y **intersects fiber trice**: $\mathcal{C} \cdot D_{x_i} = 3$
- **Hodge numbers:** $(h^{(1,1)}, h^{(2,1)})_\chi = (2, 83)_{-162}$
- $\boxed{\mathbb{Z}_3}$ Gauge Symmetry
- several charged matter singlets [Klevers, Mayorga,Piragua, P-K.O., Reuter]

Full Spectrum

Tensors:	0	$H_{\text{uncharged}}$:	$h^{2,1}(Y) + 1 = 84$
Vectors:	0	H_{charged} :	$21(K_b^{-1})^2 = 189$

- Using $K_b^{-1} \cdot K_b^{-1} = 9$

Example: Bi-Cubic

- every toric divisor on Y **intersects fiber trice**: $\mathcal{C} \cdot D_{x_i} = 3$
- **Hodge numbers:** $(h^{(1,1)}, h^{(2,1)})_\chi = (2, 83)_{-162}$
- $\boxed{\mathbb{Z}_3}$ Gauge Symmetry
- several charged matter singlets [Klevers, Mayorga,Piragua, P-K.O., Reuter]

Full Spectrum

Tensors:	0		$H_{\text{uncharged}}$:	$h^{2,1}(Y) + 1 = 84$
Vectors:	0		H_{charged} :	$21(K_b^{-1})^2 = 189$

- Using $K_b^{-1} \cdot K_b^{-1} = 9$
- Check Gravitational Anomalies:

$$H - V + 29T = 273 \checkmark \qquad 9 - T = (\mathcal{K}_b^{-1})^2 \checkmark$$

Quotient Geometry

Toric **quotient** of ambient space Z: **refined polytope lattice**

x_0	x_1	x_2	y_0	y_1	y_2
1	0	-1	0	0	0
0	1	-1	0	0	0
0	0	0	1	0	-1
0	0	0	0	1	-1

Quotient Geometry

Toric **quotient** of ambient space Z: **refined polytope lattice**

x_0	x_1	x_2	y_0	y_1	y_2
1	0	-1	0	1	-1
0	1	-1	0	-1	1
0	0	0	1	1	-2
0	0	0	0	3	-3

.

- $\mathbb{Z}_3$ **Lattice refinement** incorporates identification $\Gamma_3 = e^{(2\pi i/3)}$ **[Batyrev, Kreutzer'05]**
- Additional coordinate relation:
  $(x_0, x_1, x_2 | y_0, y_1, y_2) \sim (x_0, \Gamma_3 x_1, \Gamma_3^2 x_2 | y_0, \Gamma_3 y_1, \Gamma_3^2 y_2)$

Quotient Geometry

Toric **quotient** of ambient space Z: **refined polytope lattice**

x_0	x_1	x_2	y_0	y_1	y_2
1	0	-1	0	1	-1
0	1	-1	0	-1	1
0	0	0	1	1	-2
0	0	0	0	3	-3

.

- $\mathbb{Z}_3$ **Lattice refinement** incorporates identification $\Gamma_3 = e^{(2\pi i/3)}$ [Batyrev, Kreutzer'05]
- Additional coordinate relation: $(x_0, x_1, x_2 | y_0, y_1, y_2) \sim (x_0, \Gamma_3 x_1, \Gamma_3^2 x_2 | y_0, \Gamma_3 y_1, \Gamma_3^2 y_2)$
- Back to the hypersurface P $P = s_1 x_0^3 + s_2 x_0^2 x_1 + s_3 x_0 x_1^2 + s_4 x_1^3 + s_5 x_0^2 x_2 + s_6 x_0 x_1 x_2 + s_7 x_1^2 x_2 + s_8 x_0 x_2^2 + s_9 x_1 x_2^2 + s_{10} x_2^3$
- Not every monomial in P is Γ_3 invariant: $s_1 \ni a_1 y_0^3$ ✓ $+ a_2 y_0^2 y_1$ ✗ $+ \ldots$

Quotient Geometry

Toric **quotient** of ambient space Z: **refined polytope lattice**

x_0	x_1	x_2	y_0	y_1	y_2
1	0	-1	0	1	-1
0	1	-1	0	-1	1
0	0	0	1	1	-2
0	0	0	0	3	-3

.

- $\mathbb{Z}_3$ **Lattice refinement** incorporates identification $\Gamma_3 = e^{(2\pi i/3)}$ [Batyrev, Kreutzer'05]
- Additional coordinate relation:
  $(x_0, x_1, x_2 | y_0, y_1, y_2) \sim (x_0, \Gamma_3 x_1, \Gamma_3^2 x_2 | y_0, \Gamma_3 y_1, \Gamma_3^2 y_2)$
- Consider the Γ_3 invariant Calabi-Yau hypersurface P

$$P = s_1^{(0)} x_0^3 + s_2^{(2)} x_0^2 x_1 + s_3^{(1)} x_0 x_1^2 + s_4^{(0)} x_1^3 + s_5^{(1)} x_0^2 x_2 + s_6^{(0)} x_0 x_1 x_2 + s_7^{(2)} x_1^2 x_2 + s_8^{(2)} x_0 x_2^2 + s_9^{(1)} x_1 x_2^2 + s_{10}^{(0)} x_2^3$$

- The s_i transform Γ_3 **covariantly** $s_i^{(j)} \to \Gamma_3^j s_i^{(j)}$

Properties of Quotient Geometry

Generic structure of the **fiber** stays the same (still generic cubic)

$$P = s_1^{(0)} x_0^3 + s_2^{(2)} x_0^2 x_1 + s_3^{(1)} x_0 x_1^2 + s_4^{(0)} x_1^3 + s_5^{(1)} e_1 x_0^2 x_2 + s_6^{(0)} x_0 x_1 x_2 + s_7^{(2)} x_1^2 x_2 + s_8^{(2)} x_0 x_2^2 + s_9^{(1)} x_1 x_2^2 + s_{10}^{(0)} x_2^3$$

Properties of Quotient Geometry

Generic structure of the **fiber** stays the same (still generic cubic)

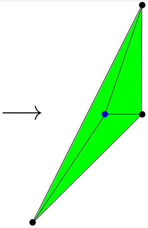
$$P = s_1^{(0)} x_0^3 + s_2^{(2)} x_0^2 x_1 + s_3^{(1)} x_0 x_1^2 + s_4^{(0)} x_1^3 + s_5^{(1)} e_1 x_0^2 x_2 + s_6^{(0)} x_0 x_1 x_2 + s_7^{(2)} x_1^2 x_2 + s_8^{(2)} x_0 x_2^2 + s_9^{(1)} x_1 x_2^2 + s_{10}^{(0)} x_2^3$$

Note:

- $(\mathbb{P}^2 \times \mathbb{P}^2)/\mathbb{Z}_3$ ambient space contains 9 codimension 4 orbifold singularities: $(x_0, x_1, x_2 | y_0, y_1, y_2) \sim (\underline{0, 0, 1} | \underline{0, 0, 1})$

Properties of Quotient Geometry

x_0	x_1	x_2	y_0	y_1	y_2
1	0	-1	0	1	-1
0	1	-1	0	-1	1
0	0	0	1	1	-2
0	0	0	0	3	-3



Generic structure of the **fiber** stays the same (still generic cubic)

$$P = s_1^{(0)} x_0^3 + s_2^{(2)} x_0^2 x_1 + s_3^{(1)} x_0 x_1^2 + s_4^{(0)} x_1^3 + s_5^{(1)} e_1 x_0^2 x_2 + s_6^{(0)} x_0 x_1 x_2 + s_7^{(2)} x_1^2 x_2 + s_8^{(2)} x_0 x_2^2 + s_9^{(1)} x_1 x_2^2 + s_{10}^{(0)} x_2^3$$

Note:

- $(\mathbb{P}^2 \times \mathbb{P}^2)/\mathbb{Z}_3$ ambient space contains 9 codimension 4 orbifold singularities: $(x_0, x_1, x_2 | y_0, y_1, y_2) \sim (\underline{0, 0, 1} | \underline{0, 0, 1})$
- Those project onto 3, A_2 singularities in the base: $\mathbb{P}^2/\mathbb{Z}_3$
- All **fixed points miss** the hypersurface $\rightarrow \widehat{Y}$ is smooth
- Justifies Euler number computation $\chi(\widehat{Y}) = \chi(Y)/3$

F-theory Physics of the Quotient

How does the F-theory **spectrum change**?

- **Before the quotient** we had
- $(h^{1,1}, h^{2,1})_\chi(\hat{Y}_3) = (2, 83)_{-163}$: $\mathbb{Z}_3$ gauge symmetry + 0 Tensor

F-theory Physics of the Quotient

How does the F-theory **spectrum change**?

- **Before the quotient** we had
- $(h^{1,1}, h^{2,1})_{\chi}(\hat{Y}_3) = (2, 83)_{-163}$: $\mathbb{Z}_3$ gauge symmetry + 0 Tensor
- with the intersection $(K_b^{-1})^2 = 9$

$$H_{\text{charged}}: \quad \boxed{\mathbf{1}_{(1)} : \quad 21(K_b^{-1})^2 \quad = 189}$$

$$H_{\text{uncharged}} \quad : \ 84$$

F-theory Physics of the Quotient

How does the F-theory **spectrum change**?

- **Before the quotient** we had
- $(h^{1,1}, h^{2,1})_\chi(\hat{Y}_3) = (2, 83)_{-163}$: $\mathbb{Z}_3$ gauge symmetry + 0 Tensor
- with the intersection $(K_b^{-1})^2 = 9$

$$H_{\text{charged}}: \quad \boxed{\mathbf{1}_{(1)} : \quad 21(K_b^{-1})^2 \quad = 189}$$

$$H_{\text{uncharged}} \quad : \ 84$$

- With **satisfied** anomalies

$$\text{Grav}^4 \qquad \underbrace{H}_{=273} - \underbrace{V}_{=0} - 29\underbrace{T}_{=0} - 273 = 0 \qquad 9 - \underbrace{T}_{=0} - \underbrace{(K_b^{-1})^2}_{=9} = 0 \checkmark$$

F-theory Physics of the Quotient

How does the F-theory **spectrum change**?

- **After taking the quotient** we have
- $(h^{1,1}, h^{2,1})_\chi(\hat{Y}_3) = (2, 29)_{-54}$: $\mathbb{Z}_3$ gauge symmetry + 0 Tensor

F-theory Physics of the Quotient

How does the F-theory **spectrum change**?

- **After taking the quotient** we have
- $(h^{1,1}, h^{2,1})_\chi(\hat{Y}_3) = (2, 29)_{-54}$: $\mathbb{Z}_3$ gauge symmetry + 0 Tensor
- Intersection change $(K_b^{-1})^2 = 9 \rightarrow (K_{\hat{b}}^{-1})^2 = 3$

$$H_{\text{charged}}: \quad \boxed{\mathbf{1}_{(1)} : \quad 21(K_b^{-1})^2 \quad = 63}$$

$$H_{\text{uncharged}} \quad = 30$$

F-theory Physics of the Quotient

How does the F-theory **spectrum change**?

- **After taking the quotient** we have
- $(h^{1,1}, h^{2,1})_\chi(\hat{Y}_3) = (2, 29)_{-54}$: $\mathbb{Z}_3$ gauge symmetry + 0 Tensor
- Intersection change $(K_b^{-1})^2 = 9 \to (K_{\hat{b}}^{-1})^2 = 3$

$$H_{\text{charged}}: \quad \boxed{\mathbf{1}_{(1)} : \quad 21(K_b^{-1})^2 \quad = 63}$$
$$H_{\text{uncharged}} \quad = 30$$

- with **unsatisfied** (gravitational) anomalies:

$$\underbrace{H}_{=93} - \underbrace{V}_{=0} - 29\underbrace{T}_{=0} - 273 = -(2\cdot 3)\cdot 30 \qquad 9 - \underbrace{T}_{=0} - \underbrace{(K_b^{-1})^2}_{=3} = 2\cdot 3$$

F-theory Physics of the Quotient

How does the F-theory **spectrum change**?

- **After taking the quotient** we have
- $(h^{1,1}, h^{2,1})_\chi(\hat{Y}_3) = (2, 29)_{-54}$: $\mathbb{Z}_3$ gauge symmetry + 0 Tensor
- Intersection change $(K_b^{-1})^2 = 9 \to (K_{\hat{b}}^{-1})^2 = 3$

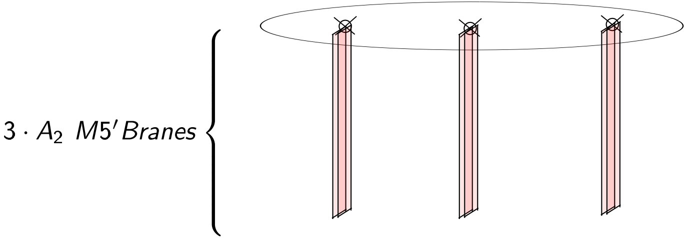
$$H_{\text{charged}}: \quad \boxed{\mathbf{1}_{(1)} : \quad 21(K_b^{-1})^2 \quad = 63}$$
$$H_{\text{uncharged}} \quad = 30$$

- with **unsatisfied** (gravitational) anomalies:

$$\underbrace{H}_{=93} - \underbrace{V}_{=0} - 29\underbrace{T}_{=0} - 273 = -(2 \cdot 3) \cdot 30 \qquad 9 - \underbrace{T}_{=0} - \underbrace{(K_b^{-1})^2}_{=3} = 2 \cdot 3$$

We need new **states** to cure the gravitational anomalies

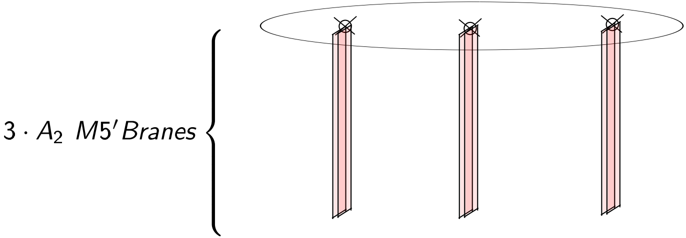
Cure for the gravitational Anomaly



- **M5 brane stacks** that probe the $\mathbb{C}^2/\mathbb{Z}_3$ singularities
- Each Γ_3 **orbifold fixed point** in $\widehat{B}$ contributes an A_2 free $\mathcal{N} = (2,0)$ Tensor multiplet $T_{(2,0)}$ [Harvey, Minasian, Moore'98]
- In a $\mathcal{N} = (1,0)$ language a free $\mathcal{N} = (2,0)$ **Tensor multiplet consists of:**

$$T_{(2,0)} \to H_{\mathbf{1_0}} \oplus T_{(1,0)}$$

Cure for the gravitational Anomaly



- **M5 brane stacks** that probe the $\mathbb{C}^2/\mathbb{Z}_3$ singularities
- Each Γ_3 **orbifold fixed point** in $\widehat{B}$ contributes an A_2 free $\mathcal{N} = (2,0)$ Tensor multiplet $T_{(2,0)}$ [Harvey, Minasian,Moore'98]
- In a $\mathcal{N} = (1,0)$ language a free $\mathcal{N} = (2,0)$ **Tensor multiplet consists of:**

$$T_{(2,0)} \rightarrow H_{1_0} \oplus T_{(1,0)}$$

$$\underbrace{H}_{=93} - \underbrace{V}_{=0} - 29\,\underbrace{T_{(1,0)}}_{=0} + 30\,\underbrace{T_{(2,0)}}_{3\cdot 2} = 273 \ , \quad 9 - \underbrace{T}_{=0} = \underbrace{(K_b^{-1})^2}_{=3} + \underbrace{T_{(2,0)}}_{3\cdot 2} \ \checkmark$$

Superconformal matter contribution

Summary of the Physics (up to now)

The quotient Γ_n action on the Base

- **Reduced matter spectrum** by 1/n consistent with all gauge anomalies
- **Introduces** $T_{(2,0)} = (K_b^{-1})^2(\frac{n-1}{n})$ free Tensor multiplets that **cures the gravitational anomaly** [del Zotto, Heckman,Morrison,Park'14]

Superconformal matter contribution

Summary of the Physics (up to now)

The quotient Γ_n action on the Base

- **Reduced matter spectrum** by 1/n consistent with all gauge anomalies
- **Introduces** $T_{(2,0)} = (K_b^{-1})^2(\frac{n-1}{n})$ free Tensor multiplets that **cures the gravitational anomaly** [del Zotto, Heckman,Morrison,Park'14]

Is that Everything?

- All massless degrees of freedom captured
- What about the multiple fiber over the fixed points?
- Is that a regular A_{n-1} (2,0) theory?

Superconformal matter contribution

Summary of the Physics (up to now)

The quotient Γ_n action on the Base

- **Reduced matter spectrum** by 1/n consistent with all gauge anomalies
- **Introduces** $T_{(2,0)} = (K_b^{-1})^2(\frac{n-1}{n})$ free Tensor multiplets that **cures the gravitational anomaly** [del Zotto, Heckman,Morrison,Park'14]

Is that Everything?

- All massless degrees of freedom captured
- What about the multiple fiber over the fixed points?
- Is that a regular A_{n-1} (2,0) theory?

Check the Tensor Branch

- When **blowing up** the A_{n-1} points: do we obtain anything **in addition** to the blow-up modes (additional singular fibers?) (Yes we do)

Back to the Bicubic

Ambient Space Polytope

x_0	x_1	x_2	y_0	y_1	y_2
1	0	-1	0	1	-1
0	1	-1	0	-1	1
0	0	0	1	1	-2
0	0	0	0	3	-3

Hyperconifold Resolution

Let **three ambient space** fixed points hit the threefold Y and resolve

Back to the Bicubic

x_0	x_1	x_2	y_0	y_1	y_2	$e_{1,1}$	$e_{1,2}$	$e_{2,1}$	$e_{2,2}$	$e_{3,1}$	$e_{3,2}$
1	0	-1	0	1	-1	1	0	0	0	1	1
0	1	-1	0	-1	1	0	1	1	1	0	0
0	0	0	1	1	-2	0	-1	-1	0	1	1
0	0	0	0	3	-3	1	-1	-2	-1	1	2

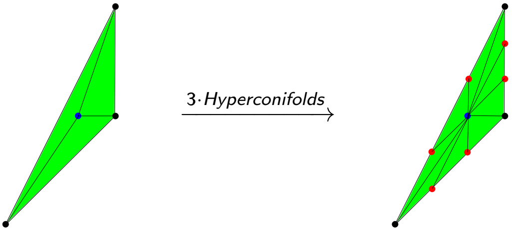
.

Hyperconifold Resolution

Let **three ambient space** fixed points hit the threefold Y and resolve

- **Resolution of base** $\mathbb{P}^2/\mathbb{Z}_3 \to dP_6$
- Obtain a smooth *simply connected* CY (by removal of 3 **Lens Spaces**)
  $(h^{1,1}, h^{2,1})_\chi = (2, 29)_{-54} \xrightarrow{3\cdot Hyperconifold} (h^{1,1}, h^{2,1})_\chi = (8, 26)_{-36}$

Back to the Bicubic

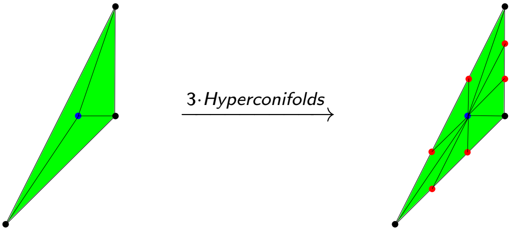


Hyperconifold Resolution

Let **three ambient space** fixed points hit the threefold Y and resolve

- **Resolution of base** $\mathbb{P}^2/\mathbb{Z}_3 \to dP_6$
- Obtain a smooth *simply connected* CY (by removal of 3 **Lens Spaces**)
  $(h^{1,1}, h^{2,1})_\chi = (2, 29)_{-54} \xrightarrow{3\cdot Hyperconifold} (h^{1,1}, h^{2,1})_\chi = (8, 26)_{-36}$
- The **discriminant** (of the Jacobian) factorizes resolution divisors
  $\Delta = e_{1,1} e_{1,2} e_{2,1} e_{2,2} e_{3,1} e_{3,1} \left(P_1 + \mathcal{O}((e_{1,1} e_{1,2} e_{2,1} e_{2,2} e_{3,1} e_{3,1})^2)\right)$

Back to the Bicubic

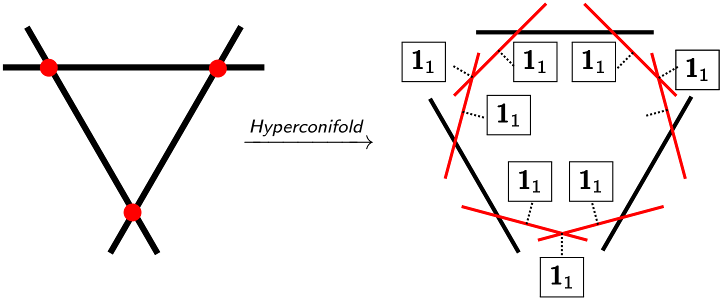


Hyperconifold Resolution

Let **three ambient space** fixed points hit the threefold Y and resolve

- **Resolution of base** $\mathbb{P}^2/\mathbb{Z}_3 \to dP_6$
- Obtain a smooth *simply connected* CY (by removal of 3 **Lens Spaces**)
  $(h^{1,1}, h^{2,1})_\chi = (2, 29)_{-54} \xrightarrow{3\cdot Hyperconifold} (h^{1,1}, h^{2,1})_\chi = (8, 26)_{-36}$
- The **discriminant** (of the Jacobian) factorizes resolution divisors
  $\Delta = e_{1,1} e_{1,2} e_{2,1} e_{2,2} e_{3,1} e_{3,1} \left(P_1 + \mathcal{O}((e_{1,1} e_{1,2} e_{2,1} e_{2,2} e_{3,1} e_{3,1})^2)\right)$
- I_2 fibers over $e_{i,1} = e_{i,2} = 0$ and $e_{i,j} = P_1 = 0$

$\mathcal{A}_{n-1}$ tensor branch matter



Hyperconifold Tensor Branch

- **Additional purely discrete charged** states appear, all **anomalies satisfied** ✓
- I_2 **Factorization** of the smooth genus-one curve explicitly confirmed ✓

Summary and ....

We have taken **freely acting quotients** Γ_n of genus-one fibered CY three-folds Y_3

Summary and ....

We have taken **freely acting quotients** Γ_n of genus-one fibered CY three-folds Y_3

1. $\hat{Y}_3 = Y_3/\Gamma_n$ is **non-simply connected**, $\pi_1(\hat{Y}_3) = n$

Summary and ....

We have taken **freely acting quotients** Γ_n of genus-one fibered CY three-folds Y_3

1. $\hat{Y}_3 = Y_3/\Gamma_n$ is **non-simply connected**, $\pi_1(\hat{Y}_3) = n$
2. $\hat{Y}_3$ has no section but only **multi-sections** $\rightarrow \mathbb{Z}_n$ symmetries.

Summary and ....

We have taken **freely acting quotients** Γ_n of genus-one fibered CY three-folds Y_3

1. $\hat{Y}_3 = Y_3/\Gamma_n$ is **non-simply connected**, $\pi_1(\hat{Y}_3) = n$
2. $\hat{Y}_3$ has no section but only **multi-sections** $\to \mathbb{Z}_n$ symmetries.
3. Fibration contains **isolated multiple fibers** over $\mathbb{Z}_n$ orbifold singularities in the base.

Summary and ....

We have taken **freely acting quotients** Γ_n of genus-one fibered CY three-folds Y_3

1. $\hat{Y}_3 = Y_3/\Gamma_n$ is **non-simply connected**, $\pi_1(\hat{Y}_3) = n$
2. $\hat{Y}_3$ has no section but only **multi-sections** $\to \mathbb{Z}_n$ symmetries.
3. Fibration contains **isolated multiple fibers** over $\mathbb{Z}_n$ orbifold singularities in the base.

Summary and ....

We have taken **freely acting quotients** Γ_n of genus-one fibered CY three-folds Y_3

1. $\hat{Y}_3 = Y_3/\Gamma_n$ is **non-simply connected**, $\pi_1(\hat{Y}_3) = n$
2. $\hat{Y}_3$ has no section but only **multi-sections** $\rightarrow \mathbb{Z}_n$ symmetries.
3. Fibration contains **isolated multiple fibers** over $\mathbb{Z}_n$ orbifold singularities in the base.

The Physics of the Multiple Fiber

- Contributes the d.o.f. of a free A_{n-1} (2,0) **superconformal point**
- **Tensor branch** obtained by resolution of a **Lens space**
- **Over the Tensor branch**, n hypers appear charged under the discrete $\mathbb{Z}_n$ gauged symmetry
  $\rightarrow$ **new discrete charged superconformal matter**

...More

Further Highlights in the paper

- **Coupling** the (2,0) theory **to** the **un-Higgsed U(1)**
- General **anomaly cancellation proven**
- **More concrete examples** with additional gauge symmetries

...More

Further Highlights in the paper

- **Coupling** the (2,0) theory **to** the **un-Higgsed U(1)**
- General **anomaly cancellation proven**
- **More concrete examples** with additional gauge symmetries

Further Directions

- Understand the precise connection of **multiple fibers**, **Tate-Shafarevich VS. Weil-Châtelet**
- **New** class of **SCFTs** or **Little String Theories** after decompactification?
- Understand the **M-theory - F-theory** lift with multiple fibers
- Many more.....

...More

Further Highlights in the paper

- **Coupling** the (2,0) theory **to** the **un-Higgsed U(1)**
- General **anomaly cancellation proven**
- **More concrete examples** with additional gauge symmetries

Thank You Very Much!

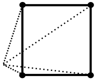
Further Directions

- Understand the precise connection of **multiple fibers**, **Tate-Shafarevich VS. Weil-Châtelet**
- **New** class of **SCFTs** or **Little String Theories** after decompactification?
- Understand the **M-theory** - **F-theory** lift with multiple fibers
- Many more.....

The Hyperconifold Transition

Tune in an ambient space fixed point onto $\widehat{Y}$

The Hyperconifold Transition



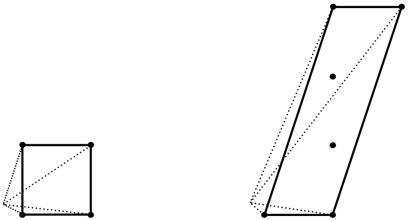
Tune in an ambient space fixed point onto $\widehat{Y}$

A Conifold Transition on the covering space

Tuning in a **cusp singularity** on the **covering** Calabi-Yau Y

- $p = y_1 y_4 - y_2 y_3 = 0$
- With Toric Fan
  $\Sigma_1 : \{v_1 = (1,0,0)\,, v_2 = (1,1,0)\,, v_3 = (1,0,1)\,, v_4 = (1,1,1)\,\}$
- $\mathbb{S}^3$ Deformation phase, $\mathbb{S}^2$ resolution phase

The Hyperconifold Transition



Tune in an ambient space fixed point onto $\widehat{Y}$

A Hyperconifold Transition

This is the **quotient** of a conifold transition [Davis'13]

- Quotient: $(y_1, y_2, y_3, y_4) \sim (\Gamma_n y_1, \Gamma_n^k y_2, \Gamma_n^{-k} y_3 \Gamma_n^{-1} y_4)$
- Refined lattice fan: $\Sigma_1' = \{(1,0,0),(1,1,0),(1,k,n),(1,k+1,n)\}$
- The two phases correspond to:
    - **Deformation Phase**: lens space $L(n,k)$ (twisted $\mathbb{S}^3$)
    - **Resolution Phase**:Chain of $n-1$ $\mathbb{P}^1$'s

Topological Properties of a Hyperconifold

Global Properties of a Hyperconifold transition $\widehat{Y} \to X$ [Davis'13]

- Change in Hodge Numbers
$$(\Delta h^{1,1}, \Delta(h^{2,1}))_{\Delta\chi} = (n-1, -1)_{2n}$$
- **Seifert-van Kampen theorem**: $\pi_1(X) = \pi_1(\widehat{Y})/\pi_1(L(m,k)) = \mathbb{Z}_{n/m}$.
- **Resolves an orbifold singularity in the Base**
$$h^{1,1}(B_{\text{res}}) = h^{1,1}(\widehat{B}) + n - 1$$

Change in Spectrum

Change in the gravitational anomaly
$H - V \quad +29\,T_{(1,0)} + 30\,T_{(2,0)} - 273 = 0$

Topological Properties of a Hyperconifold

Global Properties of a Hyperconifold transition $\widehat{Y} \to X$ [Davis'13]

- Change in Hodge Numbers
$$(\Delta h^{1,1}, \Delta(h^{2,1}))_{\Delta X} = (n-1, -1)_{2n}$$
- **Seifert-van Kampen theorem**: $\pi_1(X) = \pi_1(\widehat{Y})/\pi_1(L(m,k)) = \mathbb{Z}_{n/m}$.
- **Resolves an orbifold singularity in the Base**
$$h^{1,1}(B_{\text{res}}) = h^{1,1}(\widehat{B}) + n - 1$$

Change in Spectrum

Change in the gravitational anomaly

$$\begin{array}{ll} H - V & +29T_{(1,0)} + 30T_{(2,0)} - 273 = 0 \\ & \downarrow \\ H - 1 & -V + 29(T_{(1,0)} + n - 1) + 30(T_{(2,0)} - n + 1) - 273 = -n \end{array}$$

Topological Properties of a Hyperconifold

Global Properties of a Hyperconifold transition $\widehat{Y} \to X$ [Davis'13]

- Change in Hodge Numbers

$$(\Delta h^{1,1}, \Delta(h^{2,1}))_{\Delta_X} = (n-1, -1)_{2n}$$

- **Seifert-van Kampen theorem**: $\pi_1(X) = \pi_1(\widehat{Y})/\pi_1(L(m,k)) = \mathbb{Z}_{n/m}$.
- **Resolves an orbifold singularity in the Base**

$$h^{1,1}(B_{\text{res}}) = h^{1,1}(\widehat{B}) + n - 1$$

Change in Spectrum

Change in the gravitational anomaly

$$H - V \quad +29T_{(1,0)} + 30T_{(2,0)} - 273 = 0$$

$$\downarrow$$

$$H - 1 \quad -V + 29(T_{(1,0)} + n - 1) + 30(T_{(2,0)} - n + 1) - 273 = -n$$

- The hyperconifold implies a necessary missmatch of n **charged hypers**
- **All gauge divisors** are **Cartier**: No change in the matter
- genus-one symmetry $\leftrightarrow \mathbb{Z}_n$ gauged matter is the only candidate!