6D SCFTs and Group Theory

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IAS

Based On

- 1502.05405/hep-th
  - with Jonathan Heckman, David Morrison, and Cumrun Vafa
- 1506.06753/hep-th
  - with Jonathan Heckman
- 1601.04078/hep-th
  - with Jonathan Heckman, Alessandro Tomasiello
- 1612.06399/hep-th
  - with Noppadol Mekareeya, Alessandro Tomasiello
- work in progress
  - with Darrin Frey

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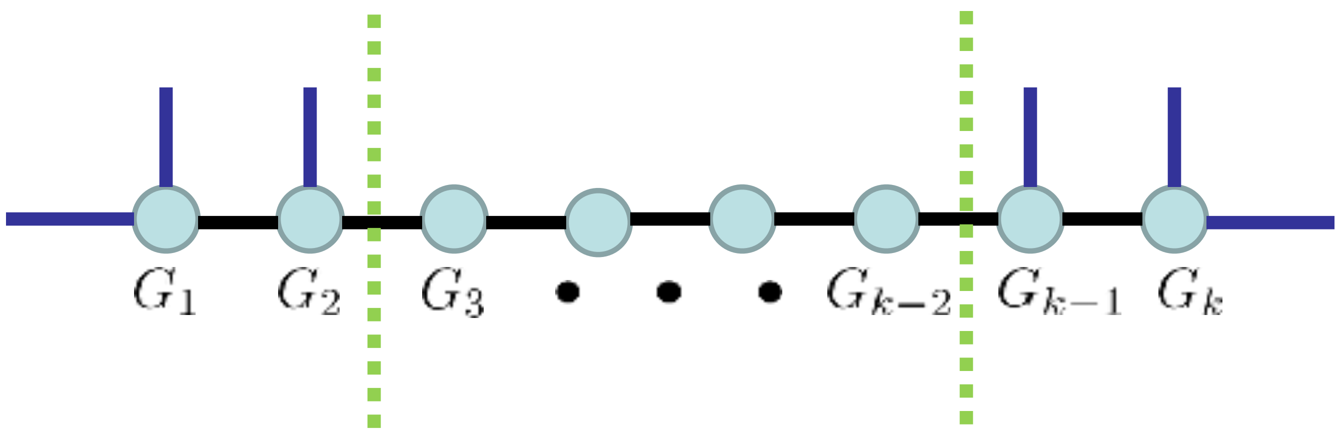
Outline

I. Classification of 6D SCFTs
   i. Tensor Branches/Strings
   ii. Gauge Algebras/Particles
II. 6D SCFTs and Homomorphisms
   i. $\mathfrak{su}(2) \to \mathfrak{g}_{\mathrm{ADE}}$
   ii. $\Gamma_{\mathrm{ADE}} \to E_8$
III. Implications for 6D SCFTs
   i. The a-theorem in 6D
   ii. Classification of RG Flows

Classification of 6D SCFTs

Classification of 6D SCFTs

- 6D SCFTs can be classified via F-theory
- Nearly all F-theory conditions can be phrased in field theory terms
- 6D SCFTs = Generalized Quivers

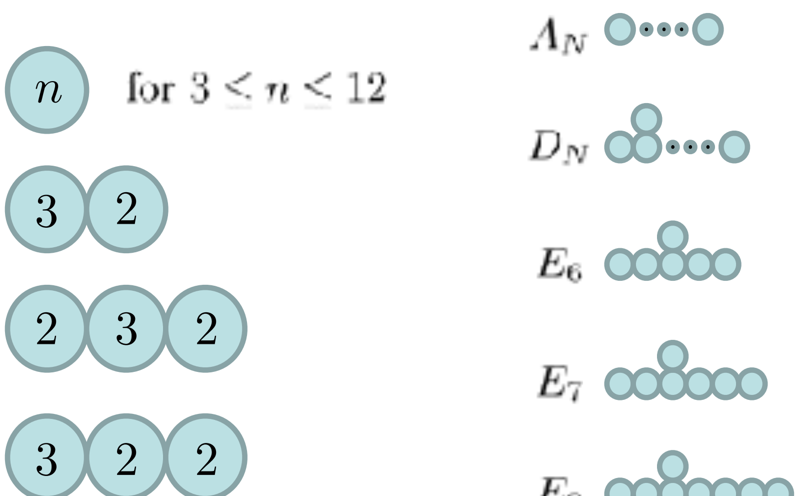


Classification of 6D SCFTs

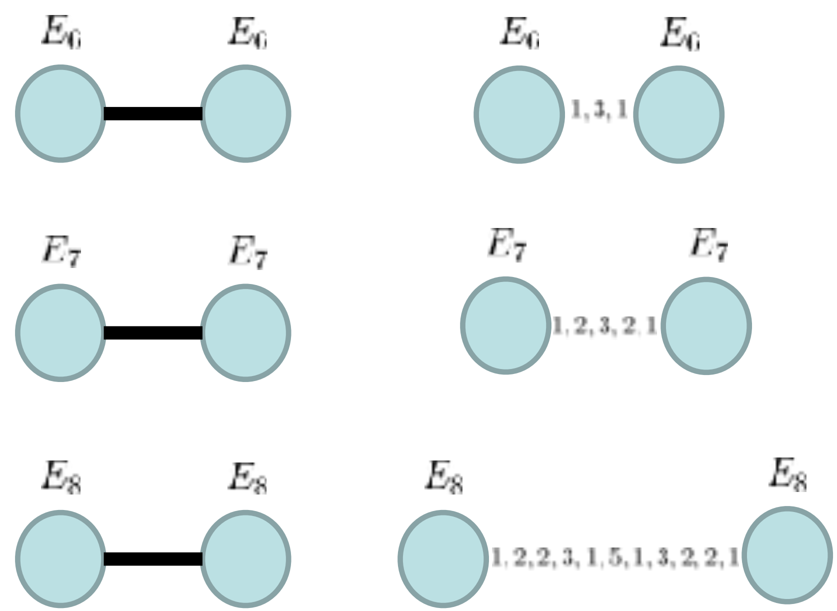
- Looks like chemistry

"Atoms"

c.f. Morrison, Taylor '12



"Radicals"



6D Theories and F-theory

Vafa '96, Vafa Morrison, I/II '96

All known 6D theories have F-theory avatar*

IIB: $\mathbb{R}^{5,1} \times B_2$ with pos. dep. coupling $\tau(z_B)$

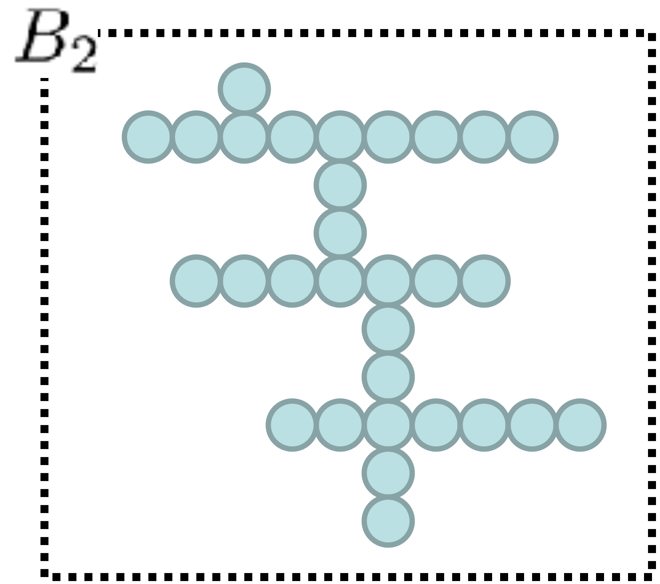
F-theory on $\mathbb{R}^{5,1} \times CY_3$

$$\begin{array}{ccc} T^2 & \rightarrow & CY_3 \\ & & \downarrow \\ & & B_2 \end{array}$$

*up to subtleties involving frozen singularities, see Alessandro's talk

SCFT Limit

Start: A smooth base B_2

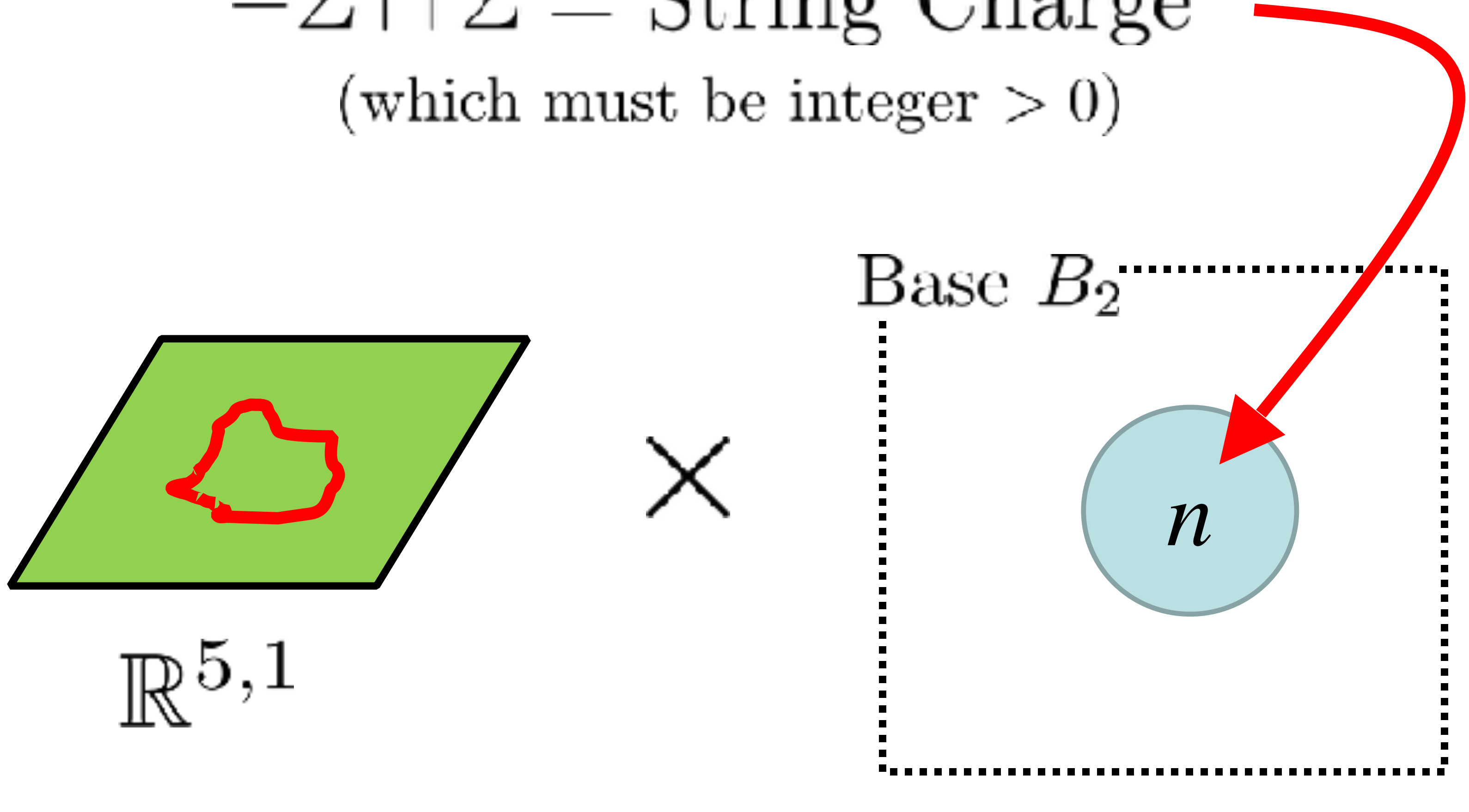


End: To get a CFT, sim. contract curves of B_2

Strings from D3 on a $\mathbb{P}^1$

$$-\Sigma \cap \Sigma = \text{String Charge}$$

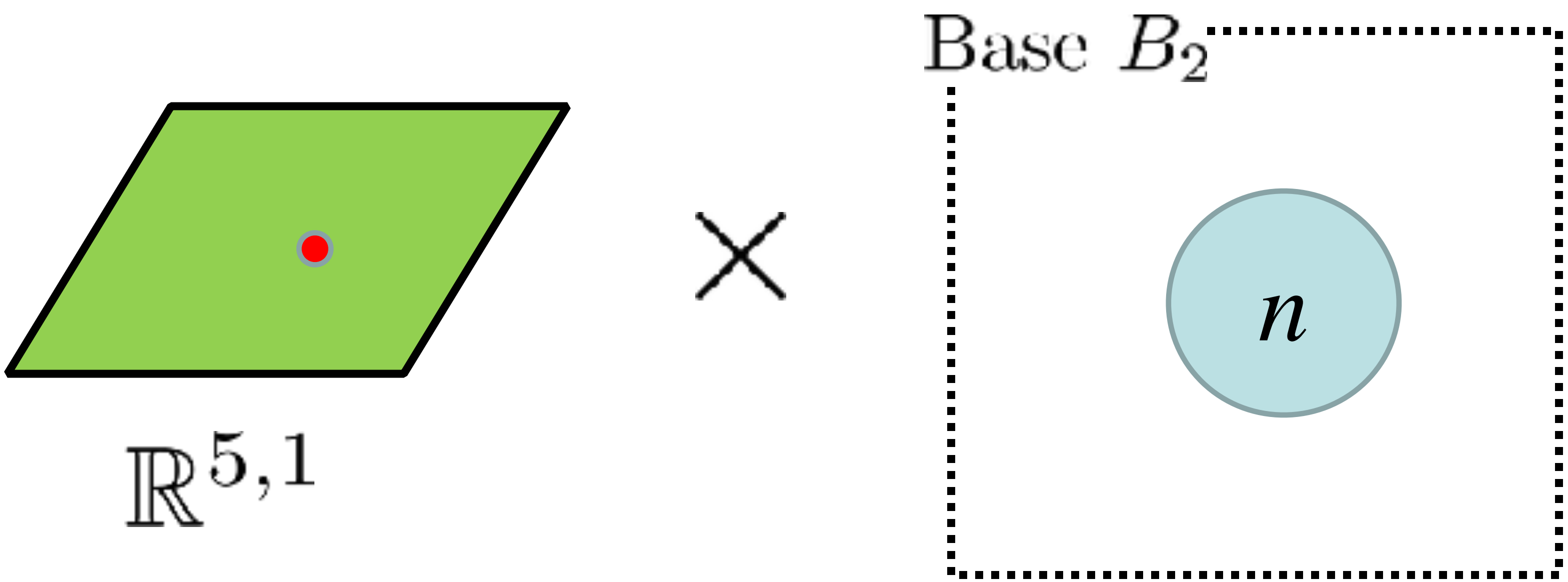
(which must be integer > 0)



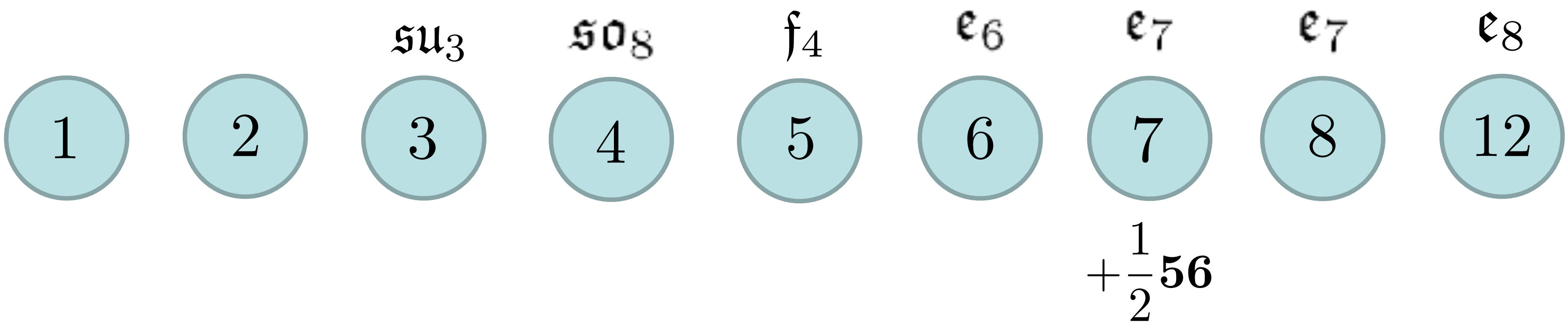
Particles from D7's on a $\mathbb{P}^1$

$3 \leq n \leq 12 \Rightarrow$ always have gauge fields

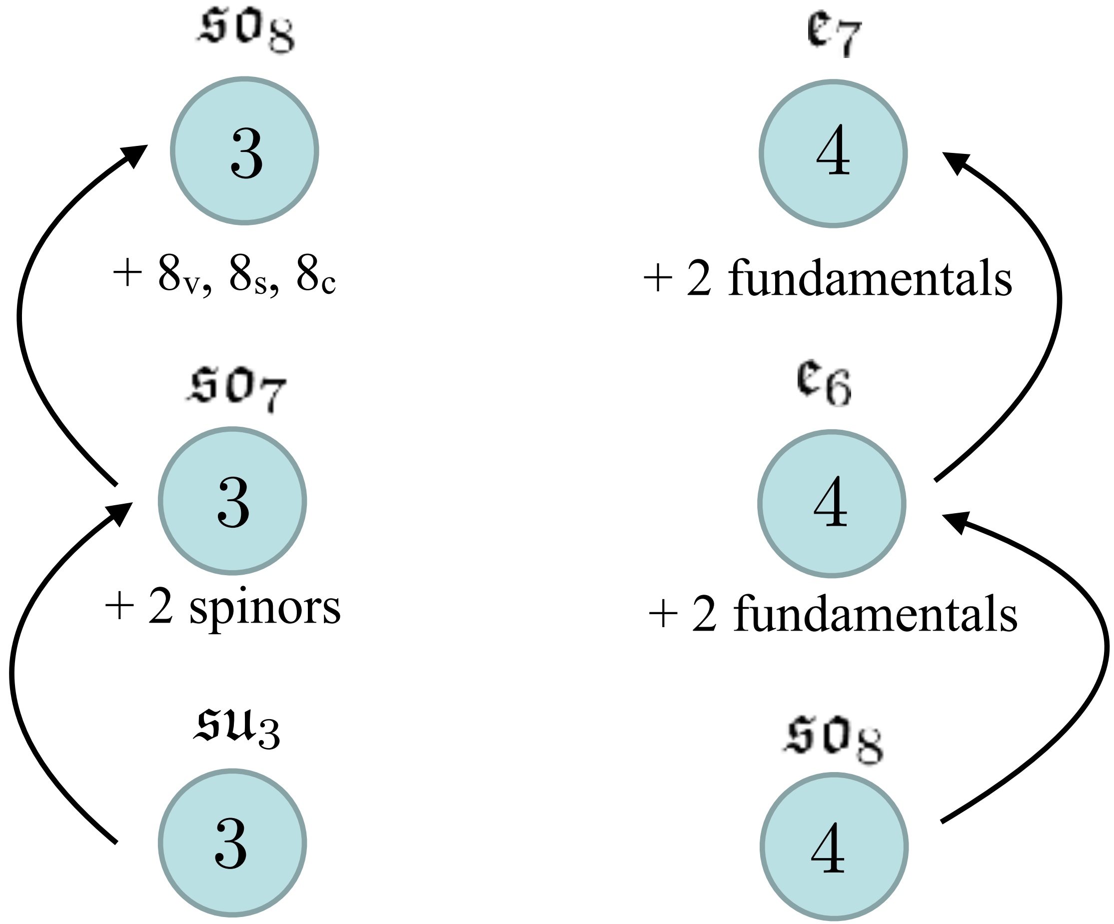
(elliptic fiber is singular: Morrison Taylor '12)



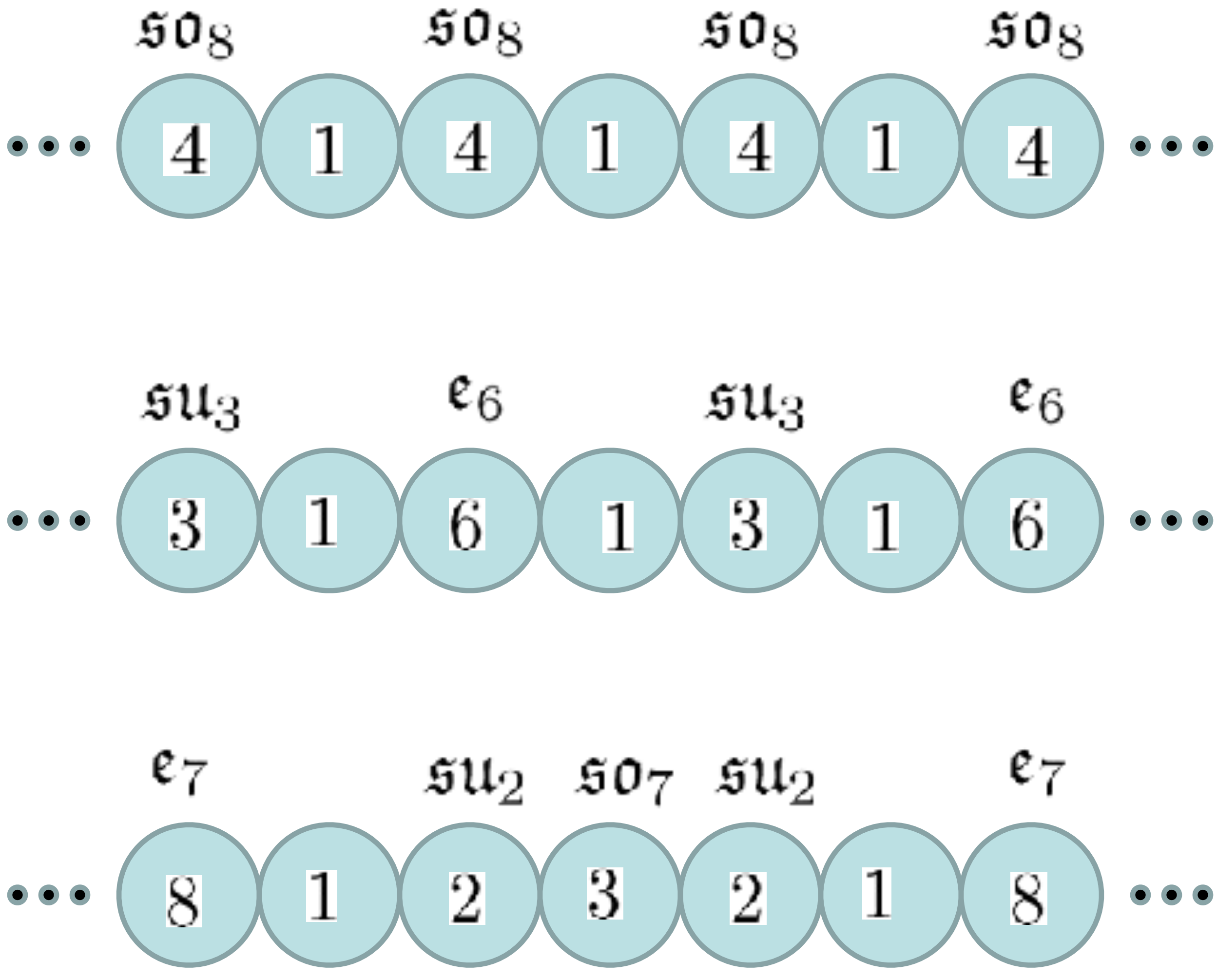
Minimal Gauge Algebras



Fiber Enhancements



Examples

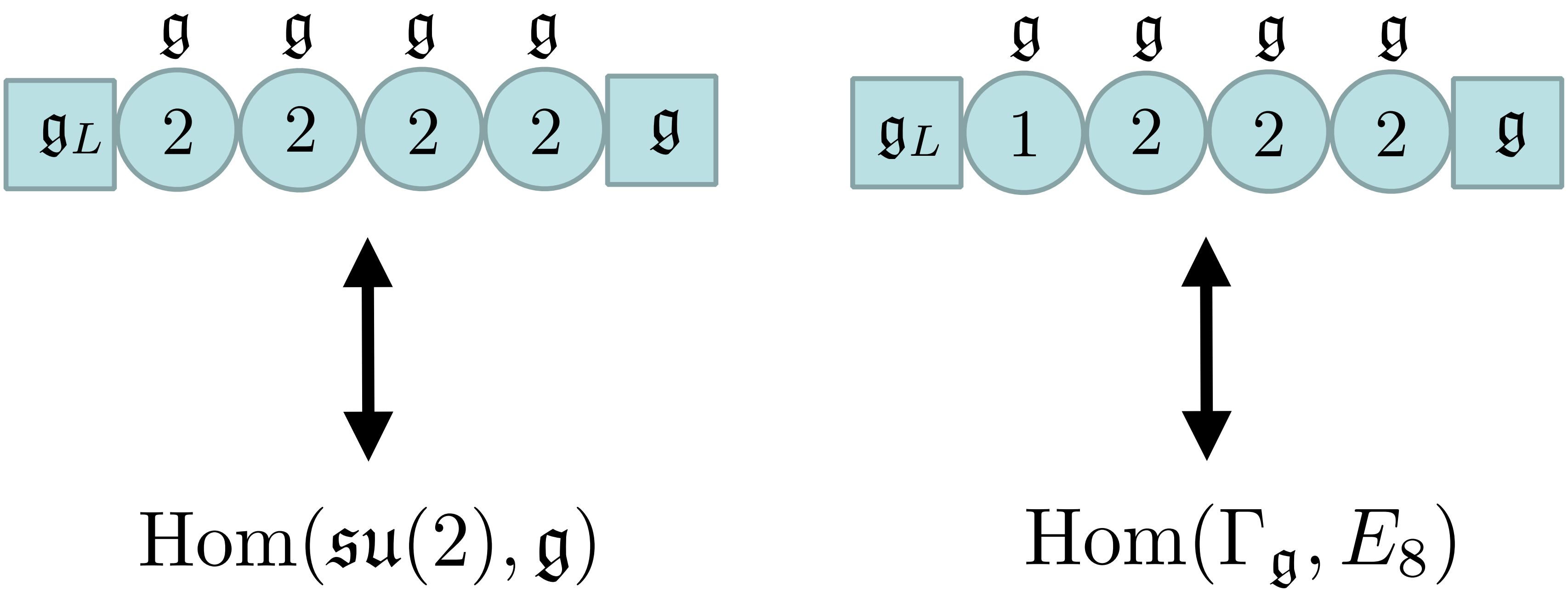


$\mathfrak{so}_8$ $\mathfrak{so}_8$ $\mathfrak{so}_8$ $\mathfrak{so}_8$
4 1 4 1 4 1 4
$\mathfrak{su}_3$ $\mathfrak{e}_6$ $\mathfrak{su}_3$ $\mathfrak{e}_6$
3 1 6 1 3 1 6
$\mathfrak{e}_7$ $\mathfrak{su}_2$ $\mathfrak{so}_7$ $\mathfrak{su}_2$ $\mathfrak{e}_7$
8 1 2 3 2 1 8

6D SCFTs and Homomorphisms

6D SCFTs and Group Theory

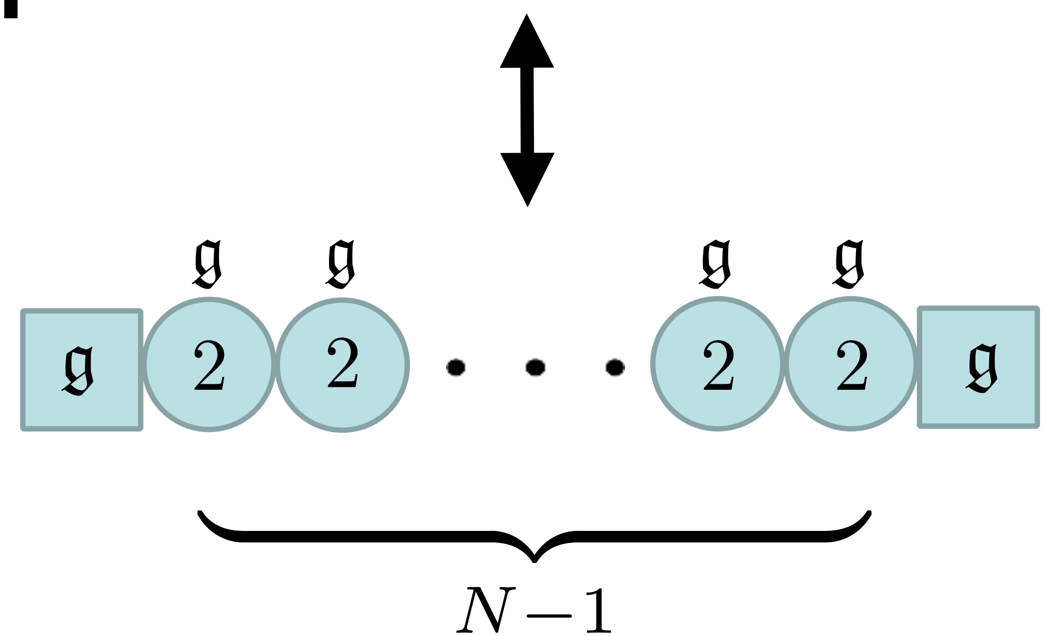
- Large classes of 6D SCFTs have connections to structures in group theory
- The correspondence has been verified explicitly



M5-Branes Probing $\mathbb{C}^2/\Gamma_{ADE}$

$\mathbb{R}^{5,1}$ (directions 0–5) $\quad$ $\mathbb{C}^2/\Gamma_{ADE}$ (directions 6–9)

	0	1	2	3	4	5	6	7	8	9	10
N M5s	×	×	×	×	×	×	•	•	•	•	•



Nilpotent Deformations

- Matrix of normal deformations Φ characterizes positions of 7-branes
- View intersection points of $\mathbb{CP}^1$ in base as marked points
- Can let adjoint field Φ have singular behavior at marked points $\Rightarrow$ Hitchin system coupled to defects:

$$\partial_A \Phi = \sum_p \mu_{\mathbb{C}}^{(p)} \delta_{(p)} \qquad F + [\Phi, \Phi^\dagger] = \sum_p \mu_{\mathbb{R}}^{(p)} \delta_{(p)}$$

Nilpotent Deformations

- Split $\mu_{\mathbb{C}} = \mu_s + \mu_n$, consider nilpotent part μ_n, get $\mathfrak{su}_2$ algebra:

$$J_+ = \mu_{\mathbb{C}} \qquad J_- = \mu_{\mathbb{C}}^\dagger \qquad J_3 = \mu_{\mathbb{R}}$$

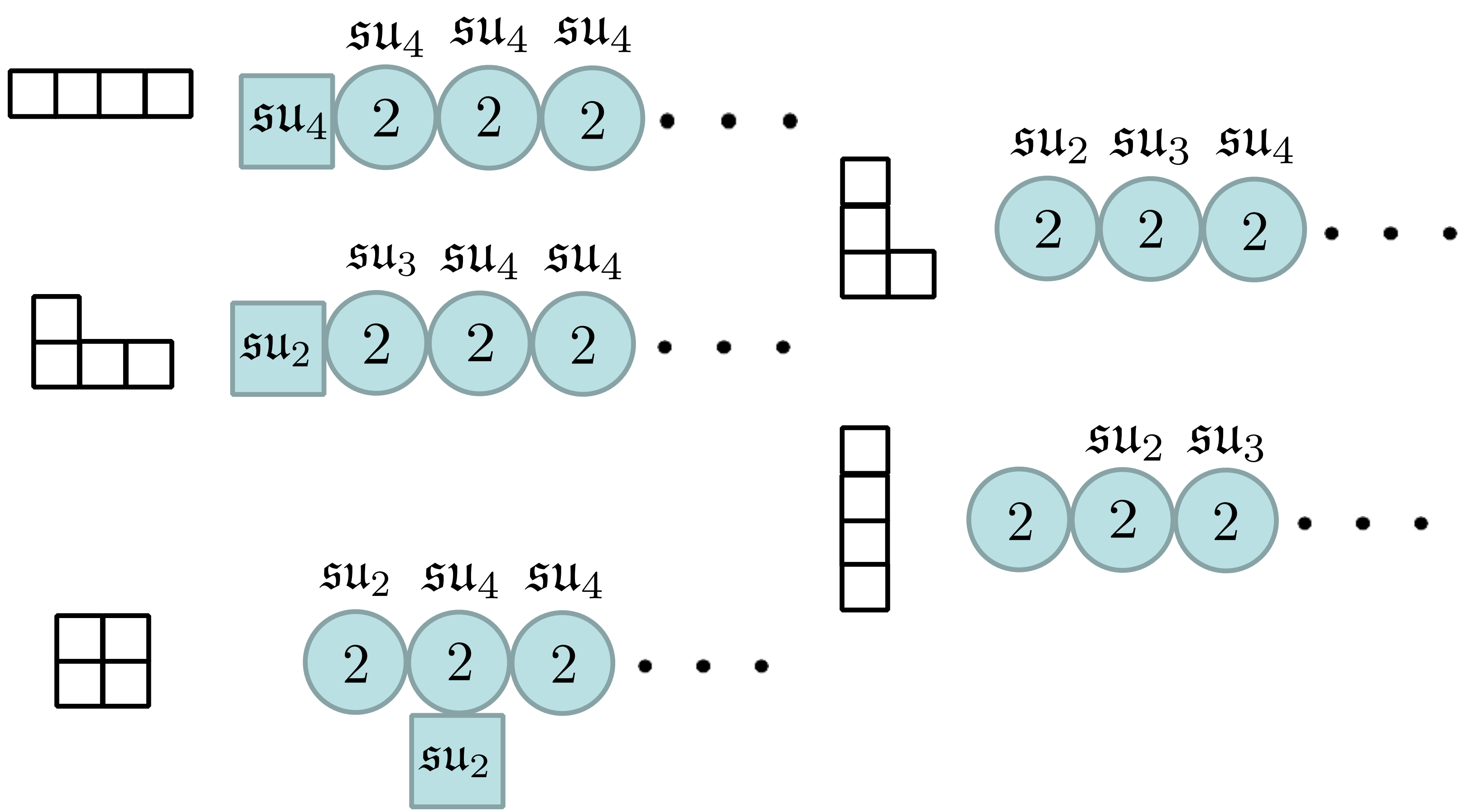
- Adjoint vevs $\Phi \sim \mu_{\mathbb{C}} \frac{dz}{z}$

$$\Rightarrow \text{ Classified by Hom}(\mathfrak{su}(2), \mathfrak{g})$$

$$\text{(equivalently, by nilpotent orbits } J_+)$$

6D SCFTs and $\mathrm{Hom}(\mathfrak{su}(2), A_{k-1})$

$\mathrm{Hom}(\mathfrak{su}(2), A_{k-1})$ labeled by partitions of k:

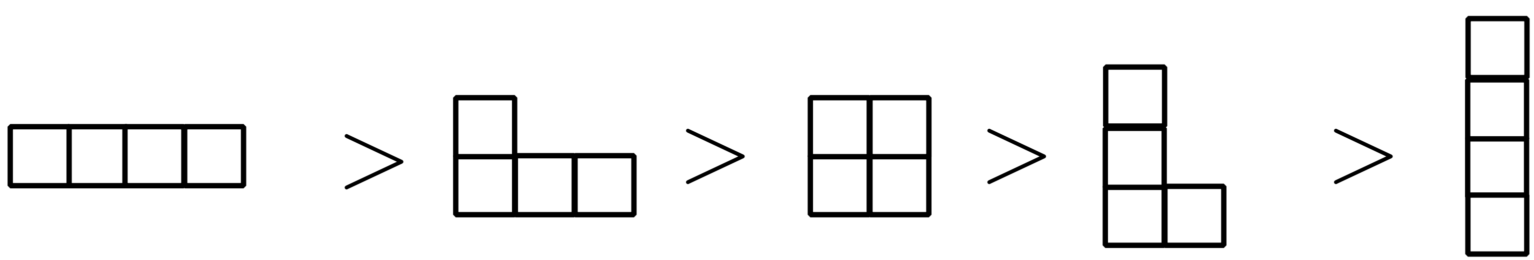


Partial Ordering of Nilpotent Orbits

$$\mathcal{O}_\mu \geq \mathcal{O}_\nu \Leftrightarrow \bar{\mathcal{O}}_\mu \supset \mathcal{O}_\nu$$

$$\Leftrightarrow \mu \geq \nu$$

$$\Leftrightarrow \sum_{i=1}^{m} \mu_i^T \geq \sum_{i=1}^{m} \nu_i^T \ \forall m$$



Renormalization Group Flows

High Energy | Short Distance

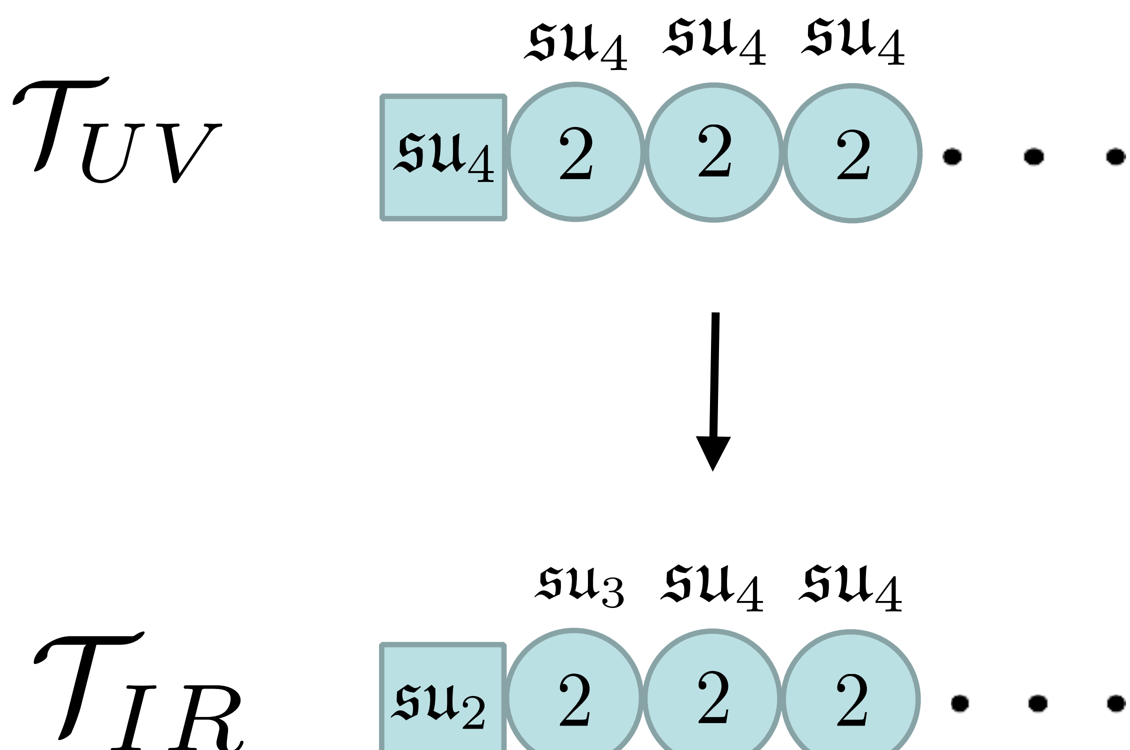
$$\mathcal{T}_{UV}$$

$$\downarrow$$

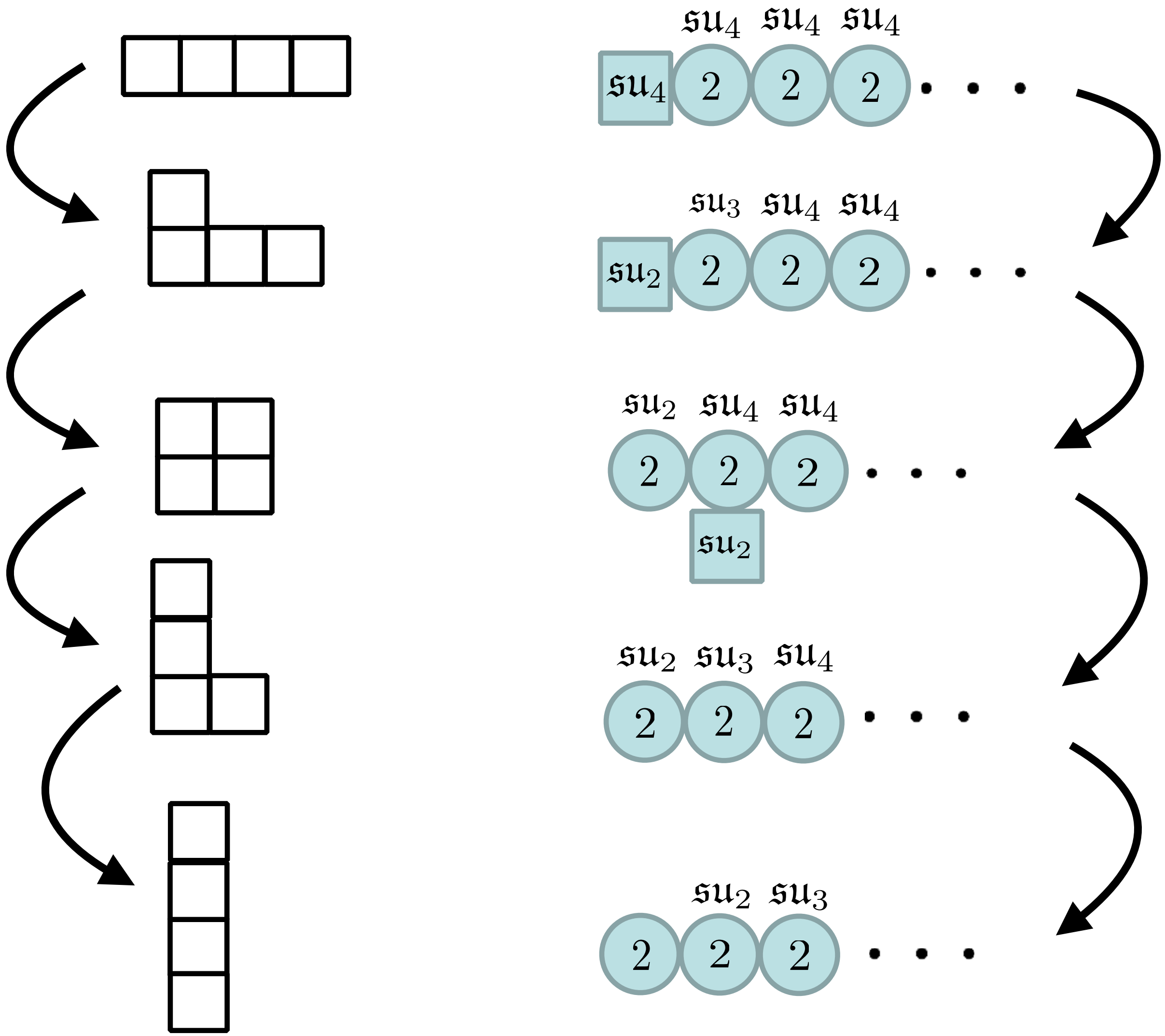
$$\mathcal{T}_{IR}$$

Low Energy | Long Distance

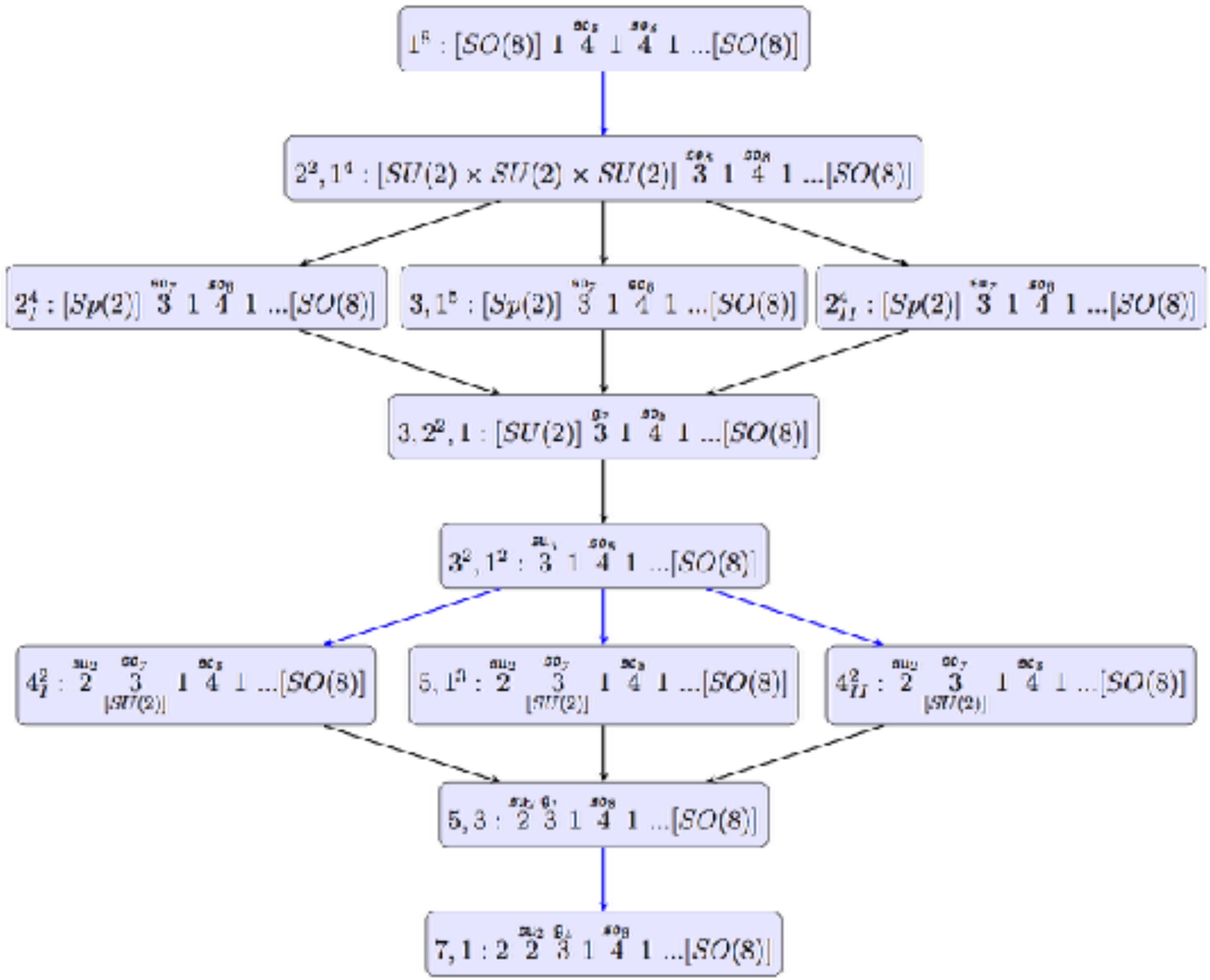
RG Flows in 6D SCFTs



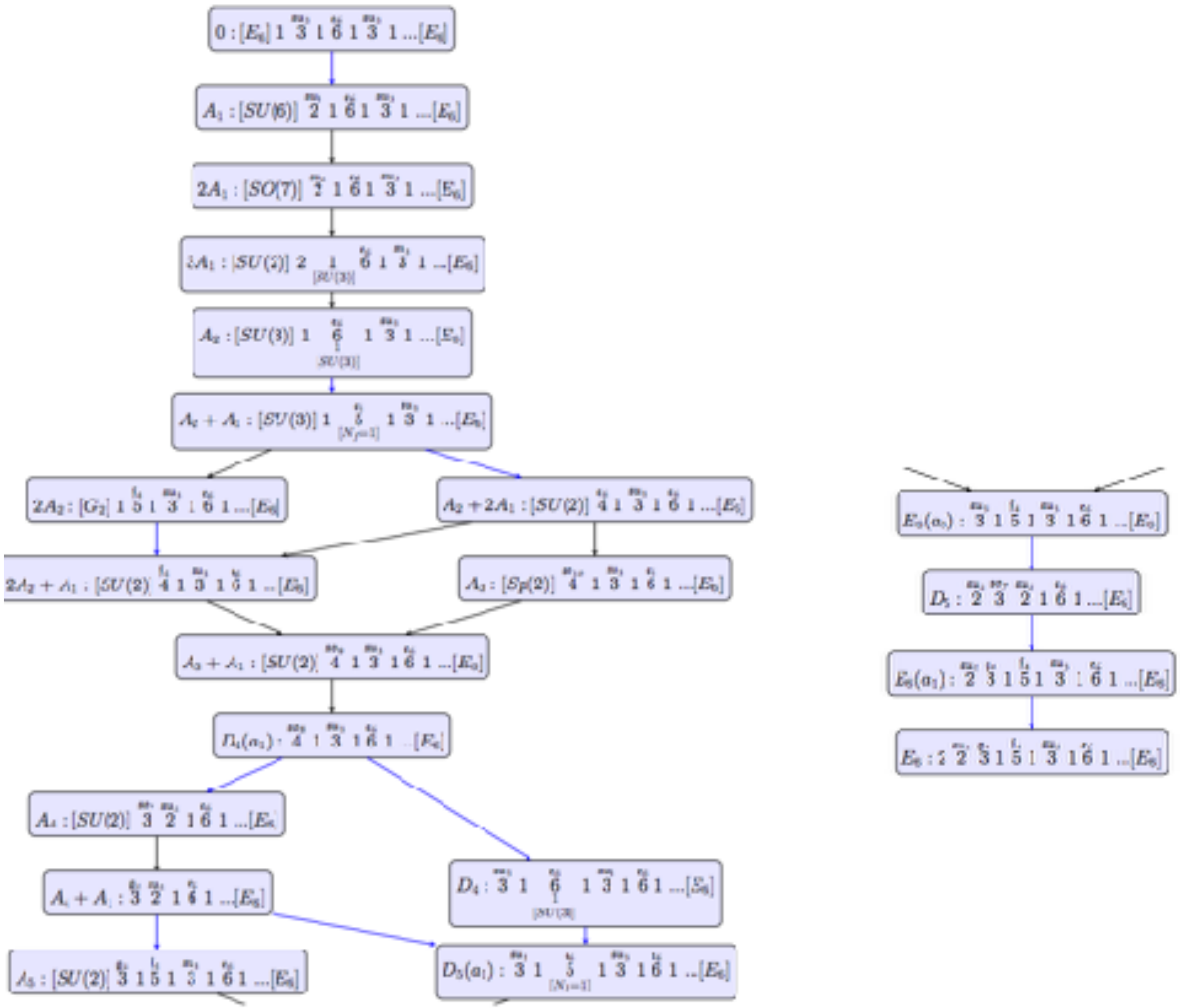
Nilpotent Hierarchy Matches RG Hierarchy!



6D SCFTs and $\mathrm{Hom}(\mathfrak{su}(2), D_k)$

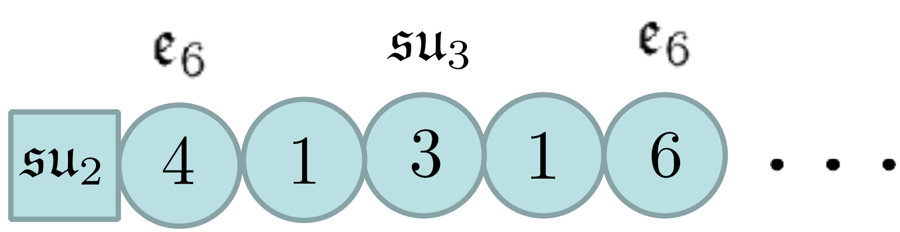


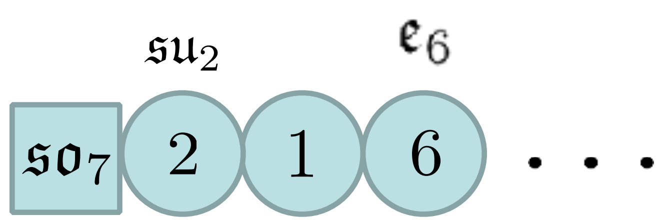
6D SCFTs and $\mathrm{Hom}(\mathfrak{su}(2), E_6)$



Nilpotent Orbits and Global Symmetries

- Consider nilpotent orbit $\mathcal{O}_\mu \in \mathfrak{g}$
- Let $F(\mu)$ be subgroup of G commuting with nilpotent element
- Claim: $F(\mu)$ is the global symmetry of the 6D SCFT associated with $\mathcal{O}_\mu$
- E.g.





6D SCFTs and $\mathrm{Hom}(\Gamma_{\mathrm{ADE}}, E_8)$

- Consider M5-branes probing Horava-Witten wall and $\mathbb{C}^2/\Gamma_{ADE}$ singularity

$\mathbb{R}^{5,1}$ (directions 0–5) $\quad$ $\mathbb{C}^2/\Gamma_{ADE}$ (directions 6–9)

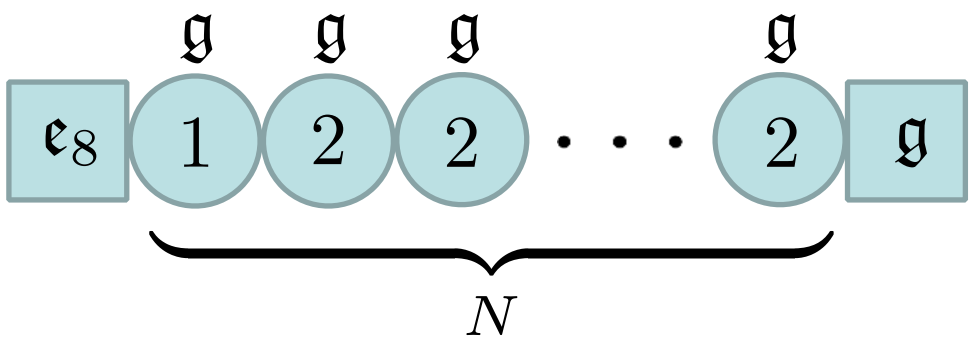
	0	1	2	3	4	5	6	7	8	9	10
N M5s	×	×	×	×	×	×	•	•	•	•	•
E_8 Wall	×	×	×	×	×	×	×	×	×	•	×

- Boundary data $\simeq$ flat E_8 connections on $S^3/\Gamma_{\mathrm{ADE}}$

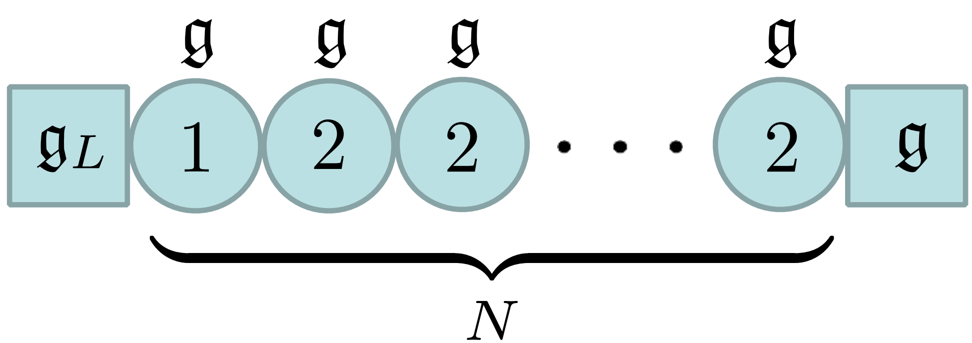
$$\simeq \mathrm{Hom}(\Gamma_{\mathrm{ADE}}, E_8)$$

6D SCFTs and $\mathrm{Hom}(\Gamma_{\mathrm{ADE}}, E_8)$

- For trivial boundary data, get 6D SCFT:

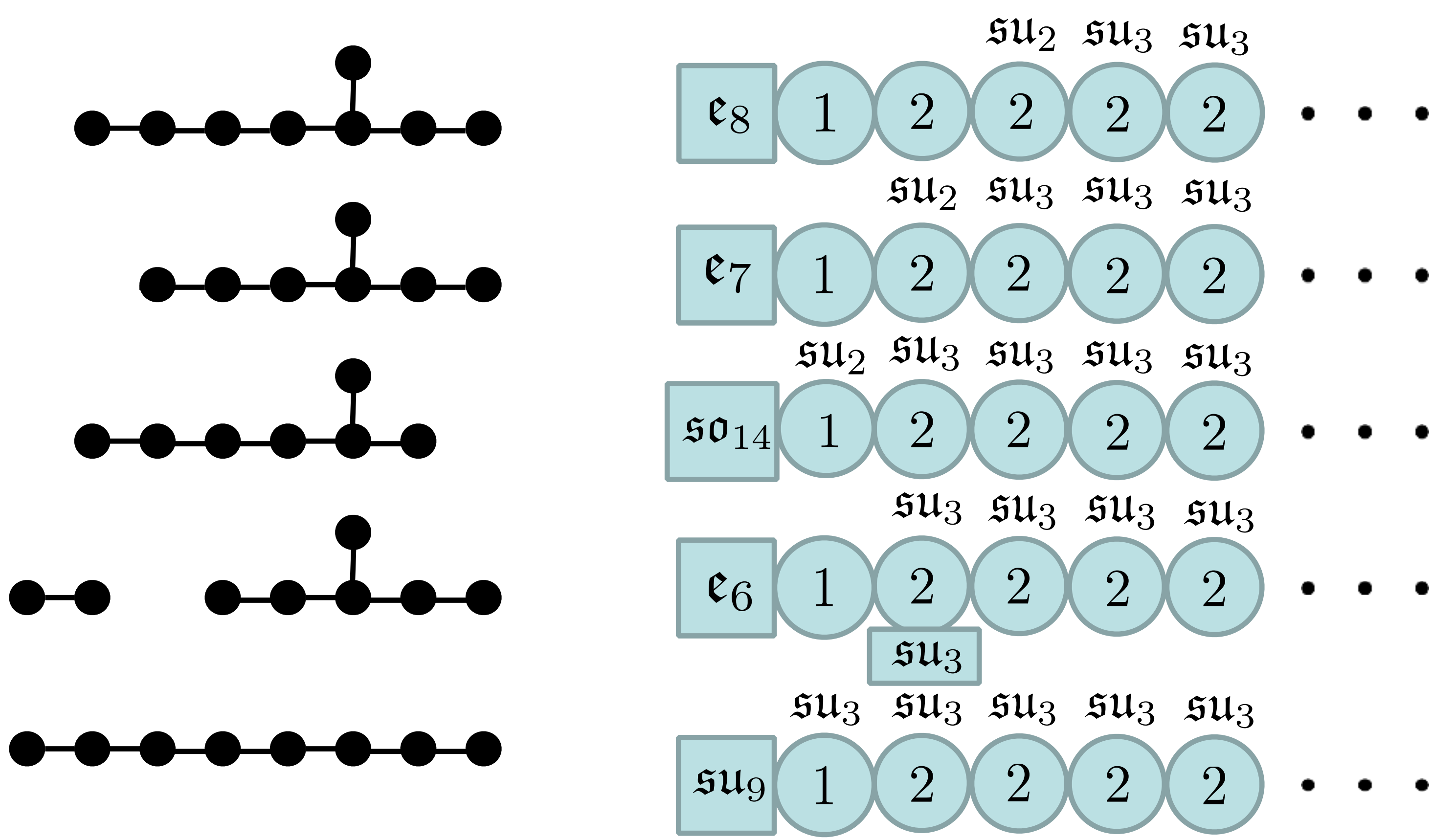


- For non-trivial boundary data, global symmetry is broken to a subgroup



6D SCFTs and $\mathrm{Hom}(\Gamma_{\mathrm{ADE}}, E_8)$

E.g. Γ_{A_2}, $\mathrm{Hom}(\mathbb{Z}_3, E_8)$:

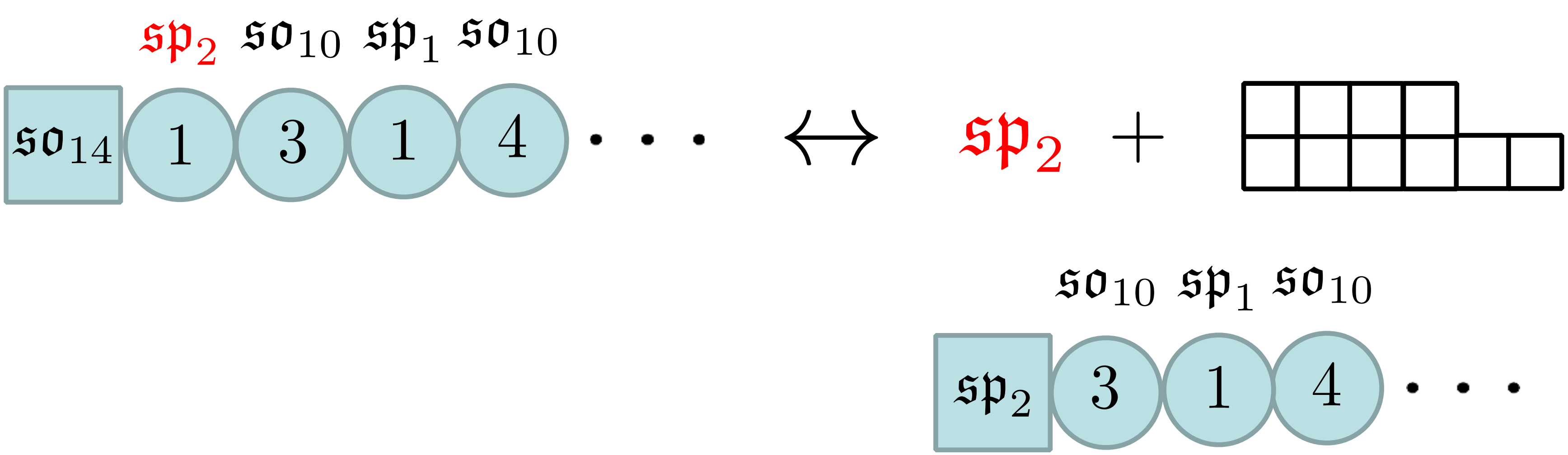


Classification of $\mathrm{Hom}(\Gamma_{\mathrm{A}DE}, E_8)$

- A_n case: done (Kac '83)
- E_8 case: done (Frey '98)
- D_n case: open!
- E_6 case: open!
- E_7 case: open!

Classification of $\mathrm{Hom}(\Gamma_{D_n}, E_8)$

- $\mathrm{Hom}(\Gamma_{D_n} \simeq \mathrm{Dic}_{n-2}, E_8)$ are uniquely labeled by a nilpotent orbit of D_n together with a simple Lie algebra!
- E.g. $\Gamma_{D_5} \to E_8$:



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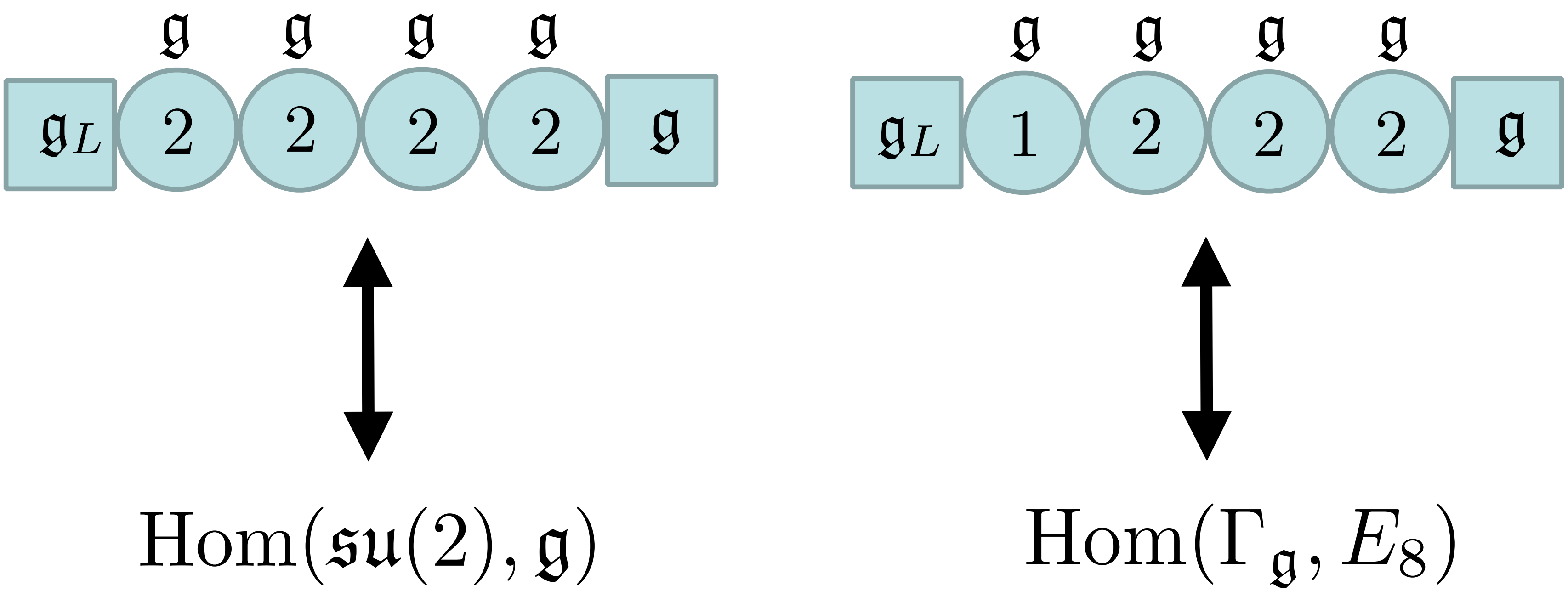
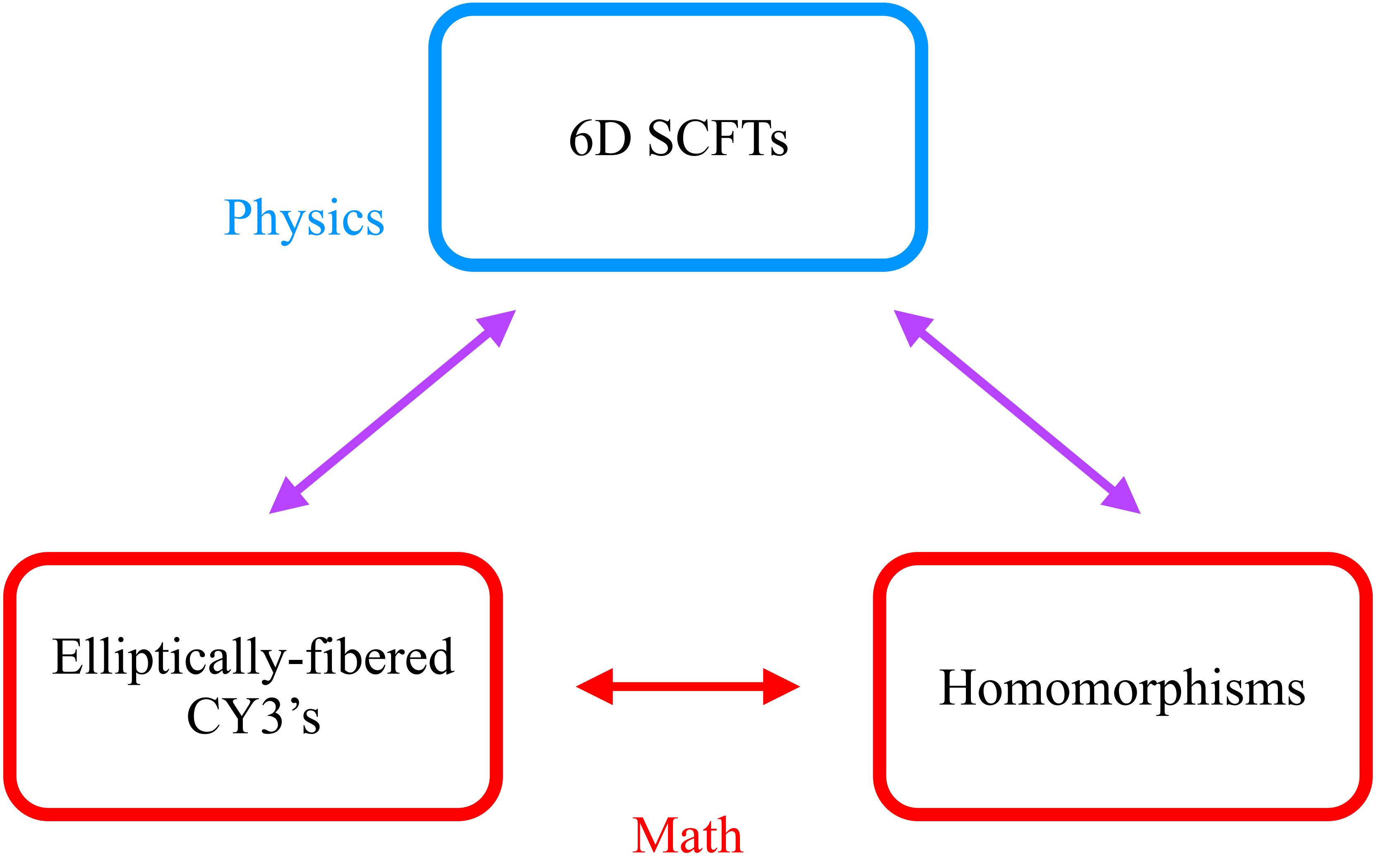


Geometry and Group Theory



Implications for 6D SCFTs

Implications for 6D SCFTs

- There is significant evidence for the a-theorem (and an infinite collection of other c-theorems) in 6D SCFTs
- Connections to group theory provide a proof in certain classes of RG flows
- We speculate that a full classification of RG flows among 6D SCFTs is possible through these connections to group theory

’t Hooft Anomalies in 6D SCFTs

- Anomaly polynomial calculable for any 6D SCFT

Ohmori, Shimizu, Tachikawa, Yonekura ’14

$$I = \alpha c_2(R)^2 + \beta c_2(R) p_1(T) + \gamma p_1(T)^2 + \delta p_2(T) + \ldots$$

- Trace anomaly related to 6D Euler density

$$\langle T^{\mu}_{\mu} \rangle = \left(\frac{1}{4\pi}\right)^3 a E_6 + \ldots$$

- Can be expressed in terms of anomaly polynomial:

$$a = \frac{8}{3}(\alpha - \beta + \gamma) + \delta$$

Cordova, Dumitrescu, Intriligator ’15

Two Deformation Types

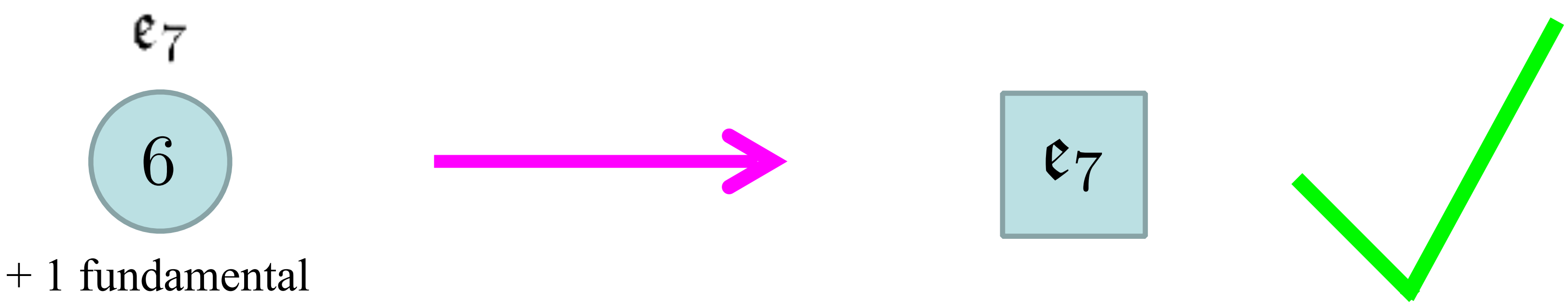
Expand a curve in base to large size / Tensor Branch

Complex Structure Deformation / Higgs Branch

Evidence for the a-theorem

- Tensor branch flows: a-theorem proven!

Cordova, Dumitrescu, Intriligator ’15

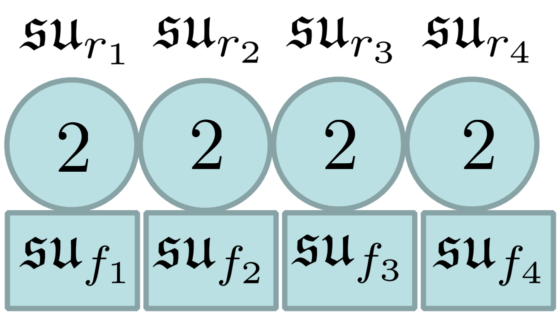


- Higgs branch flows: numerical sweep

Heckman, T.R. ’15

Nilpotent Orbit SCFTs

- Can relate anomalies to data of nilpotent orbit



$\Rightarrow$

$$\alpha = 12 \sum_{i,j} C^{-1}_{i,j} r_i r_j + 2(N-1) - \sum_i r_i^2$$

$$\beta = N - 1 - \frac{1}{2} \sum_i r_i^2$$

$$\gamma = \frac{1}{240} \left(\frac{7}{2} \sum_i r_i f_i + 30(N-1) \right)$$

$$\delta = -\frac{1}{120} \left(\sum_i r_i f_i + 60(N-1) \right)$$

Cremonesi, Tomasiello ’15

- $\Delta d_H \sim -\Delta \delta \sim -\Delta d_{\mathcal{O}}$
- Allows for proof of a-theorem for these flows

Summary and Future Research

- So far…
  - Classified 6D SCFTs in terms of CY3's
  - Found relationships between 6D SCFTs and two classes of homomorphisms
  - Found strong evidence for the a-theorem in 6D

Summary and Future Research

- In the future…
  - Can mathematics give deeper insight into the geometry-group theory correspondence?
  - Can we classify full set of 6D RG Flows in terms of group theory data?
  - Can we prove a-theorem in full generality?