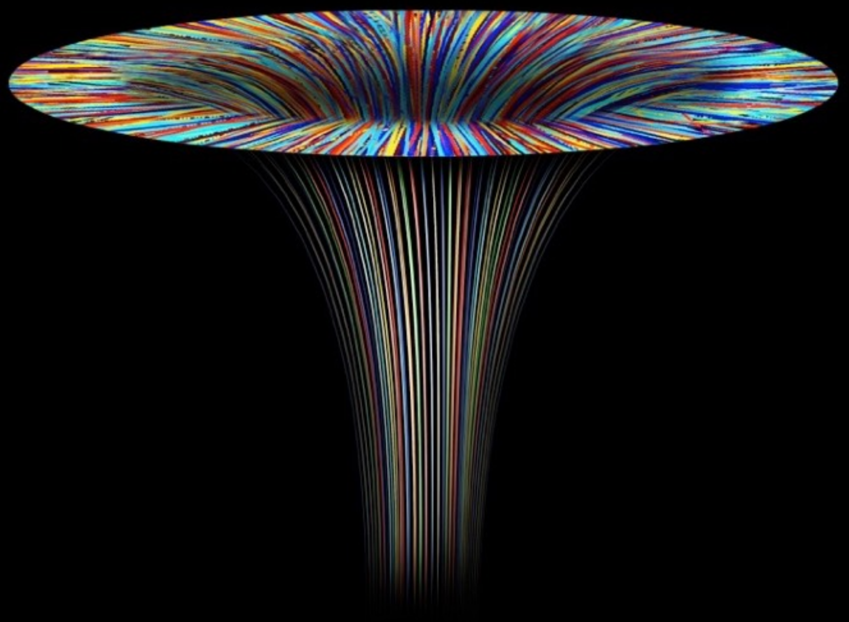
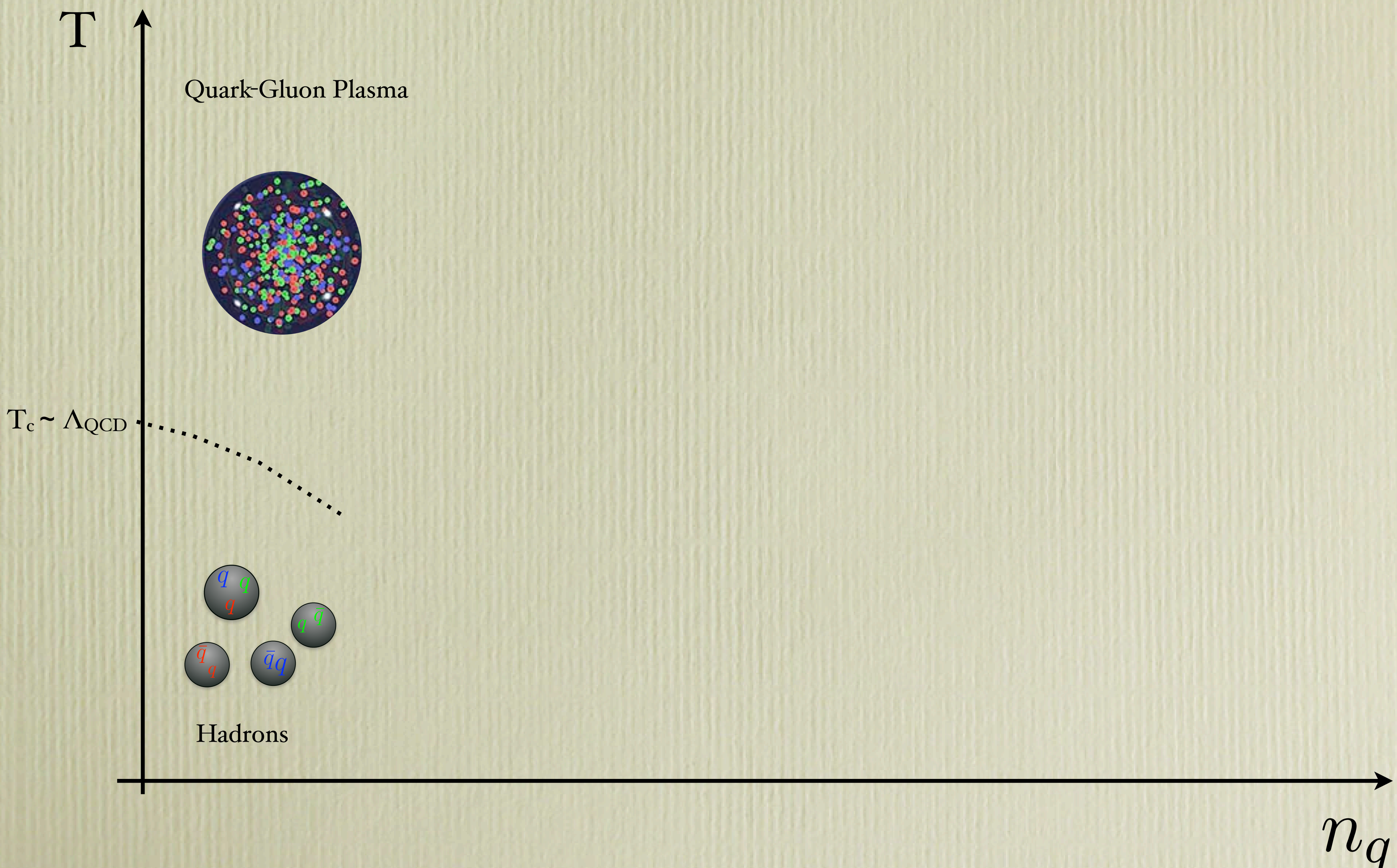
Hydrodynamics Near a Critical Point

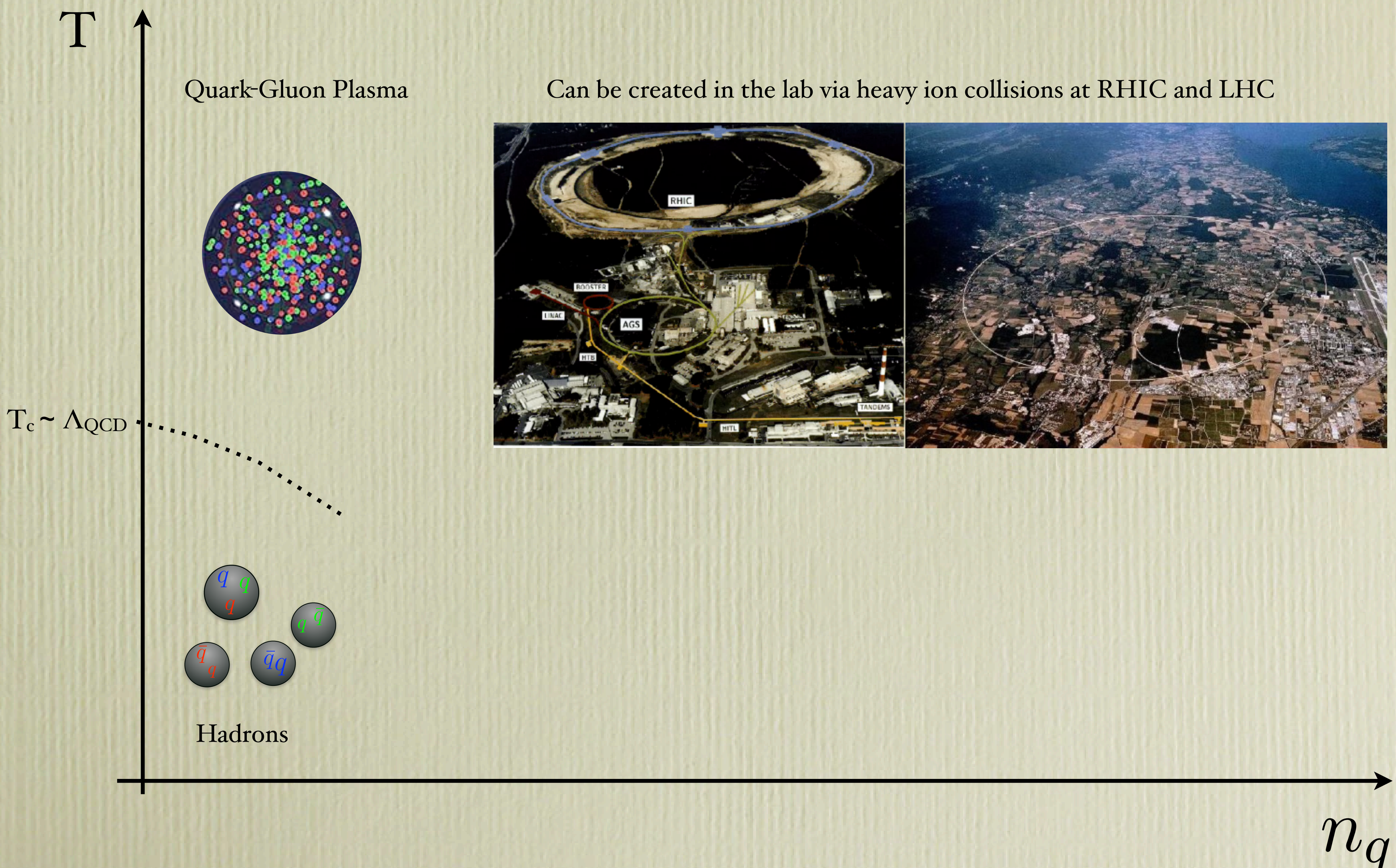


David Mateos
ICREA & University of Barcelona

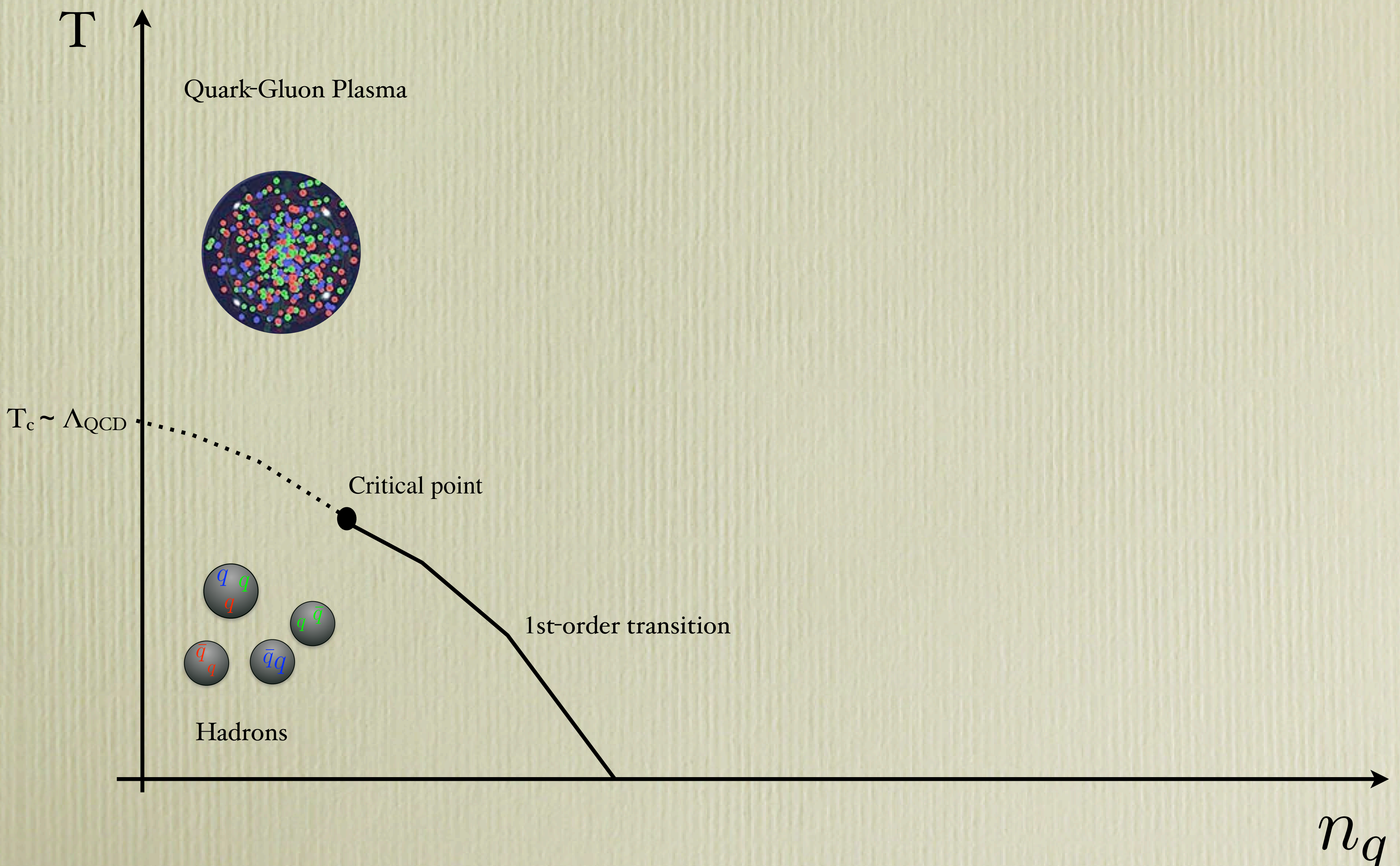
QCD phase diagram



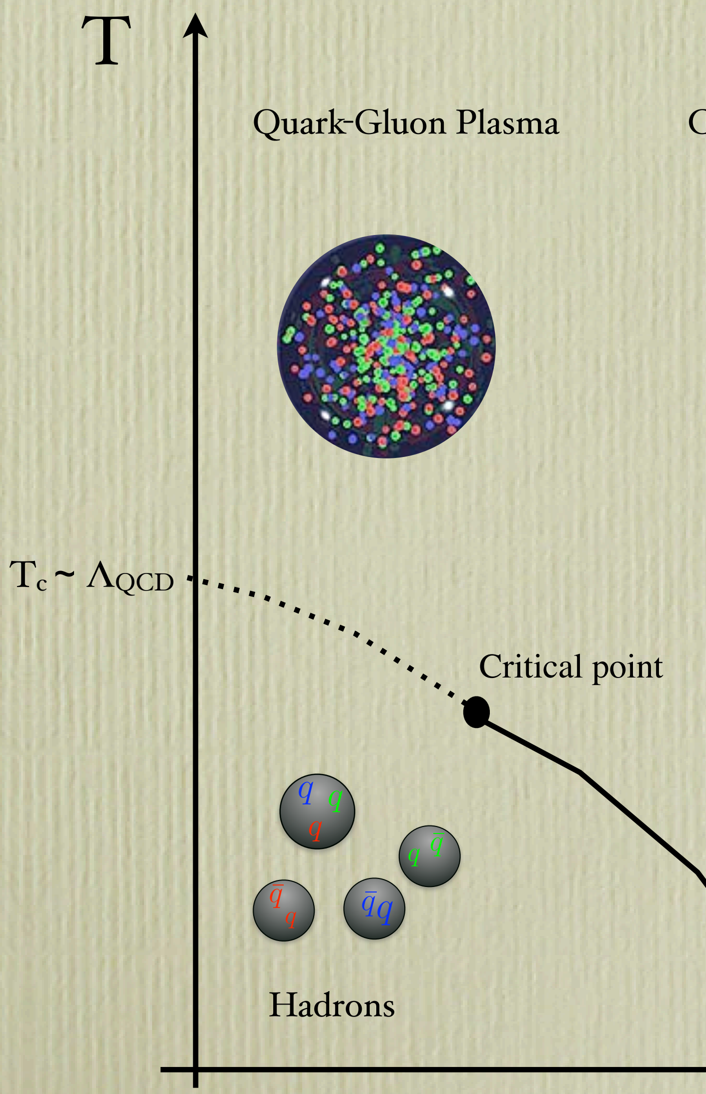
QCD phase diagram



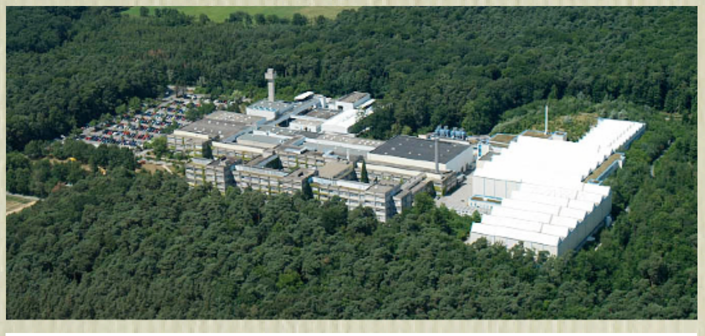
QCD phase diagram

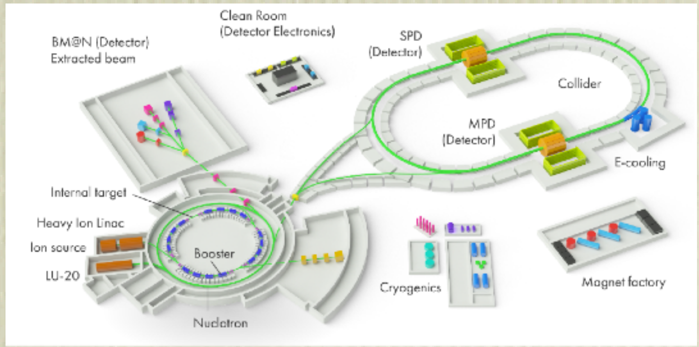


QCD phase diagram

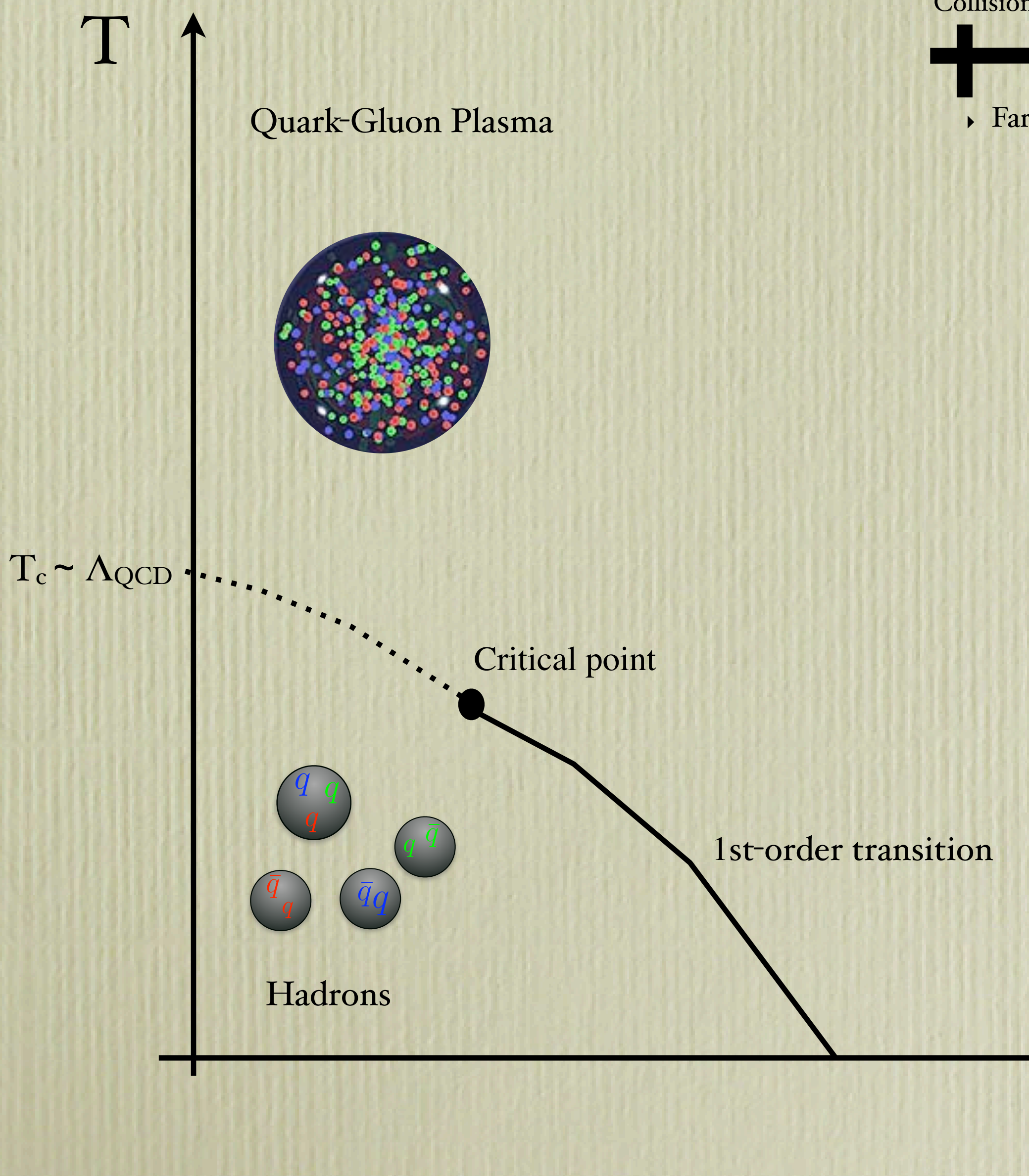


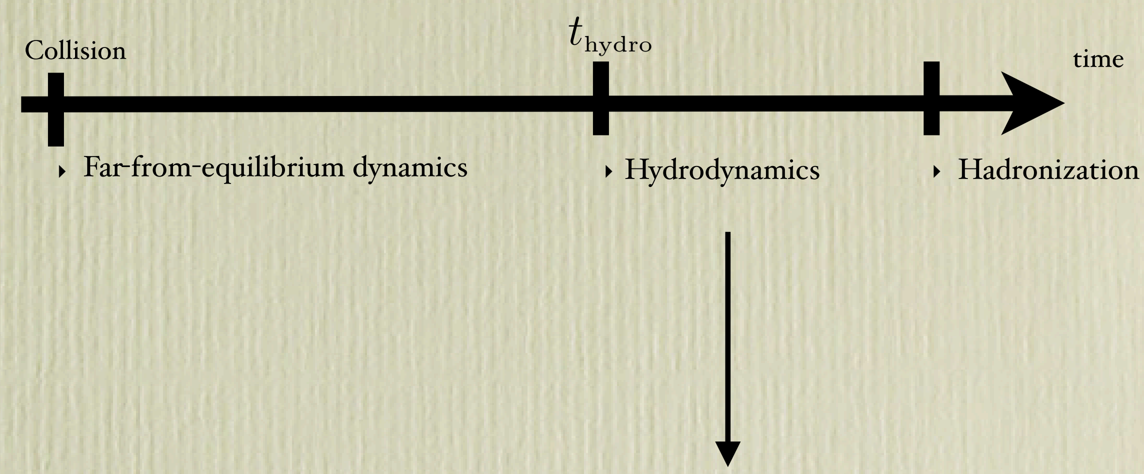
Critical point searched for at RHIC (now) and at FAIR and NICA (future).





QCD phase diagram





- HIC create new form of *matter.*
- Allows for *connection* between theory and experiment.

- Today: What can holography say near the CP?

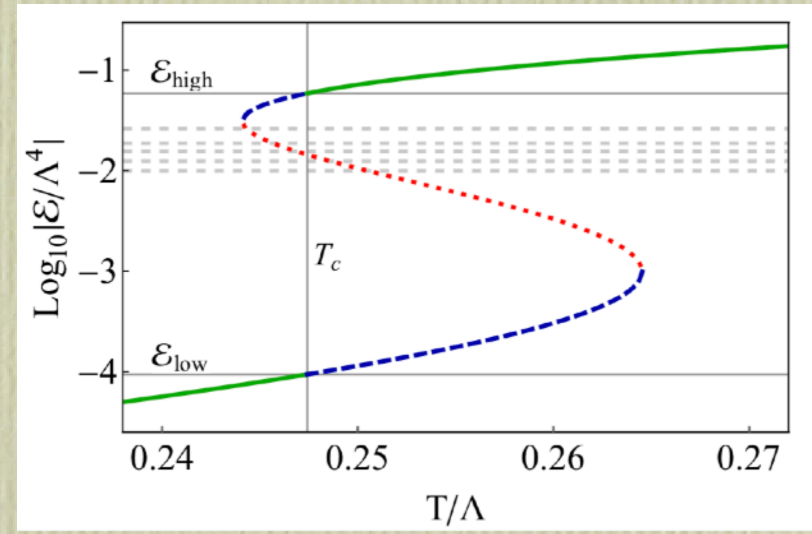
Plan

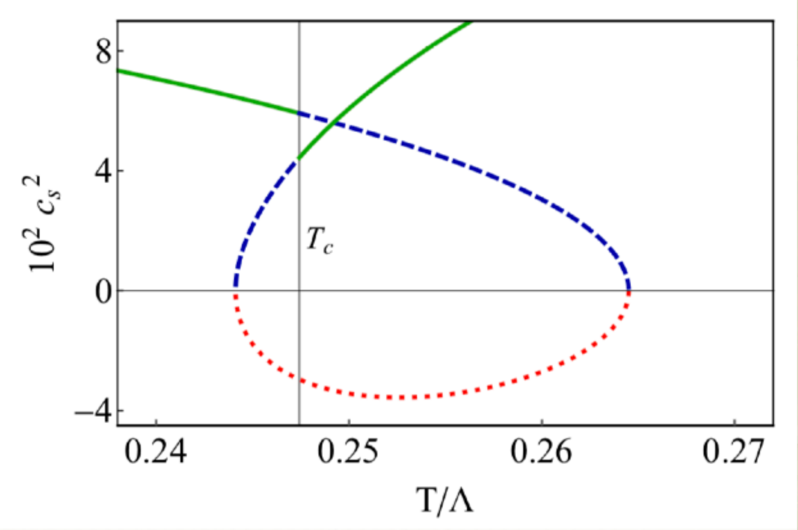
- Real-time dynamics of phase separation.
- Holographic collisions with phase transitions.
- What about critical fluctuations?

Dynamics of phase separation

1st-order phase transition: Spinodal instability

Attems, Bea, Casalderrey, D.M., Triana & Zilhao '17
Janik, Jankowski, Soltanpanahi '17
Attems, Bea, Casalderrey, D.M. & Zilhao '19
Bellantuono, Janik, Jankowski, Soltanpanahi '19





- Thermodynamic instability implies dynamical instability:

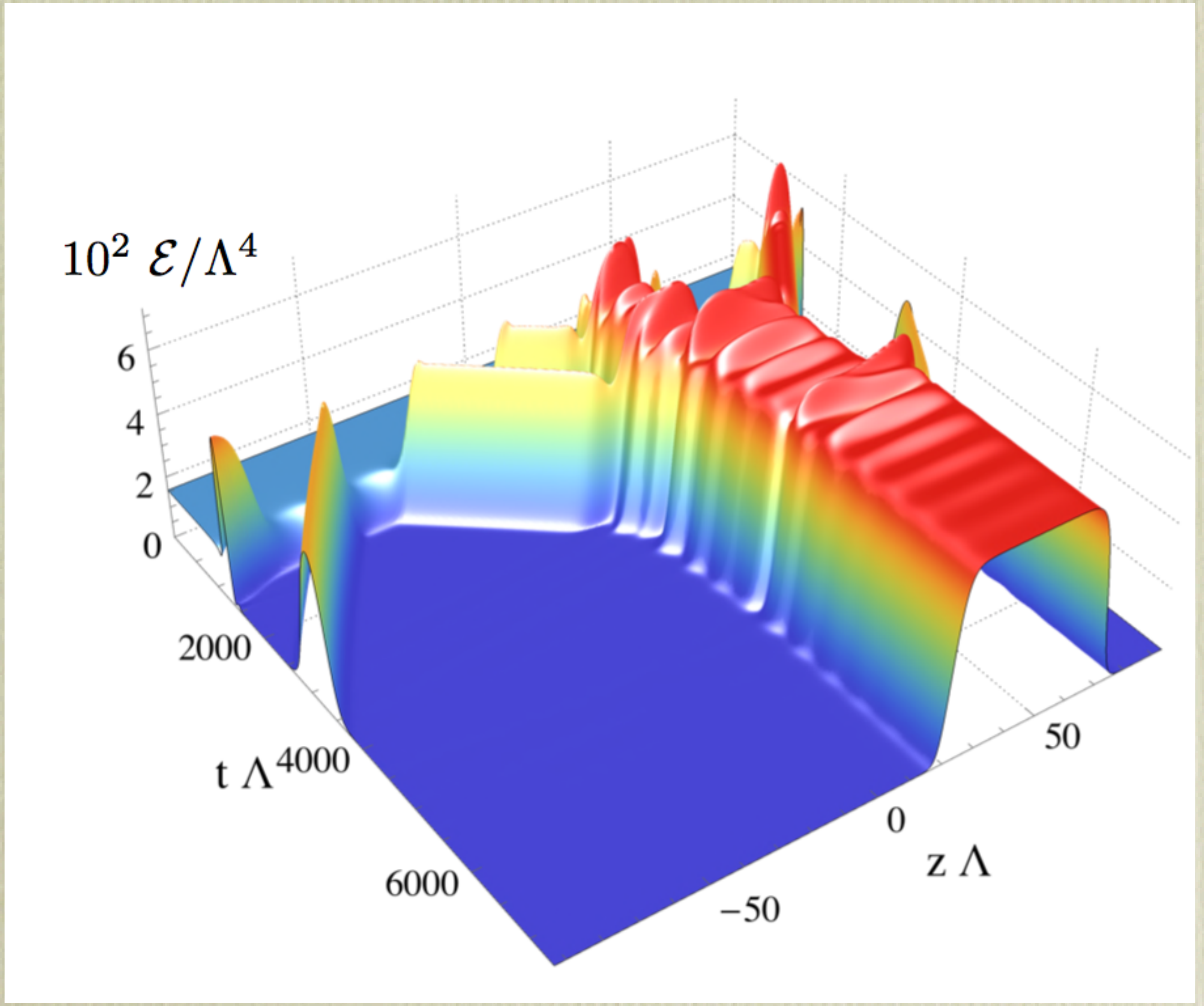
$$c_V < 0 \quad \rightarrow \quad c_s^2 = \frac{s}{c_V} < 0 \quad \rightarrow \quad c_s \text{ is imaginary}$$

$$\omega = c_s k \quad \rightarrow \quad e^{-i\omega t} = e^{+|c_s| k t}$$

1st-order phase transition: Phase separation

Attems, Bea, Casalderrey, D.M. & Zilhao '19

Perturbed homogeneous state evolves to phase-separated configuration:



1st-order phase transition: Phase separation

Attems, Bea, Casalderrey, D.M. & Zilhao '19

- Describing evolution in detail could fill an entire talk.

- Instead of that I will show you that entire evolution is well described by 2nd-order hydrodynamics.

Evolution described by 2nd-order hydrodynamics

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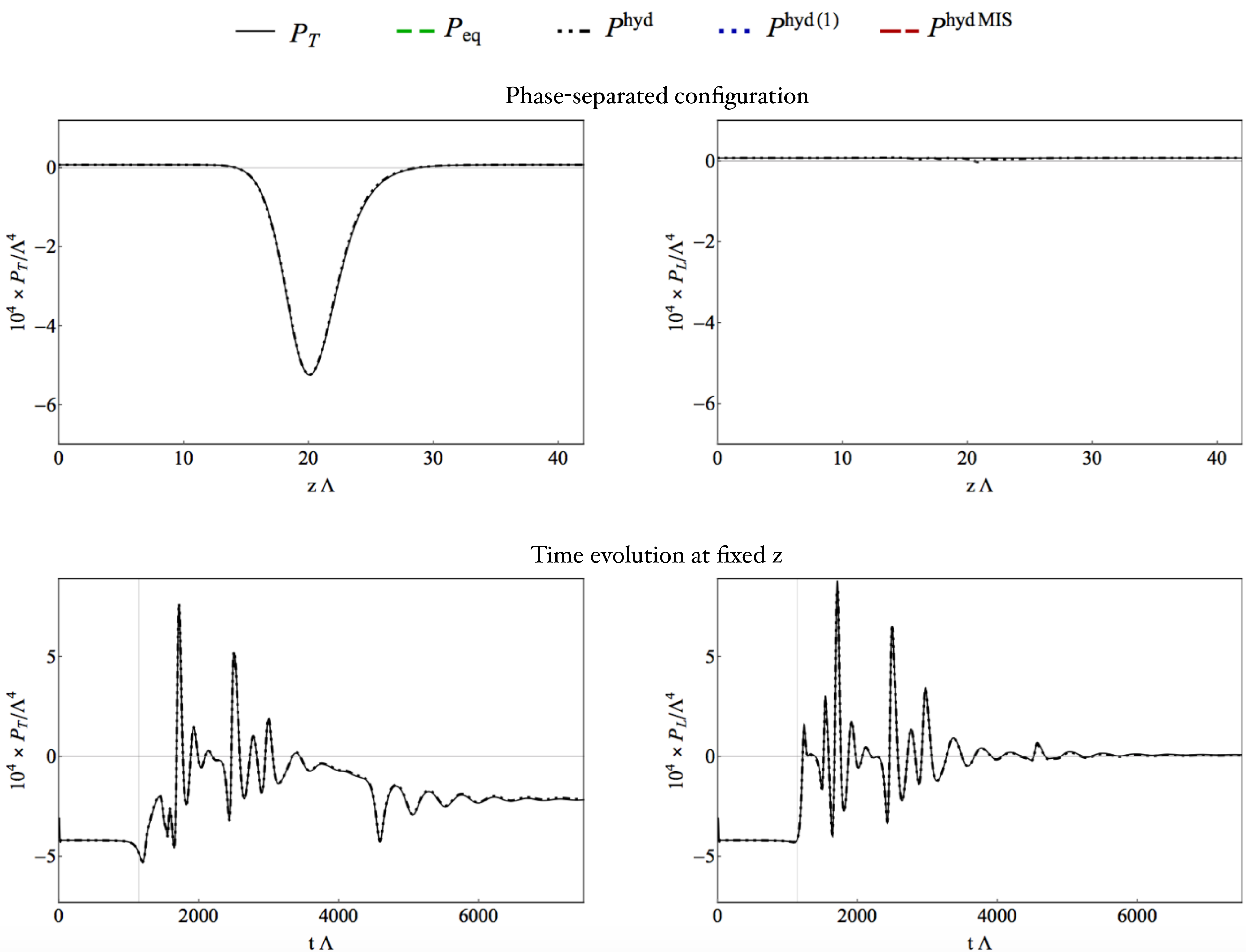
bulk & shear viscosities

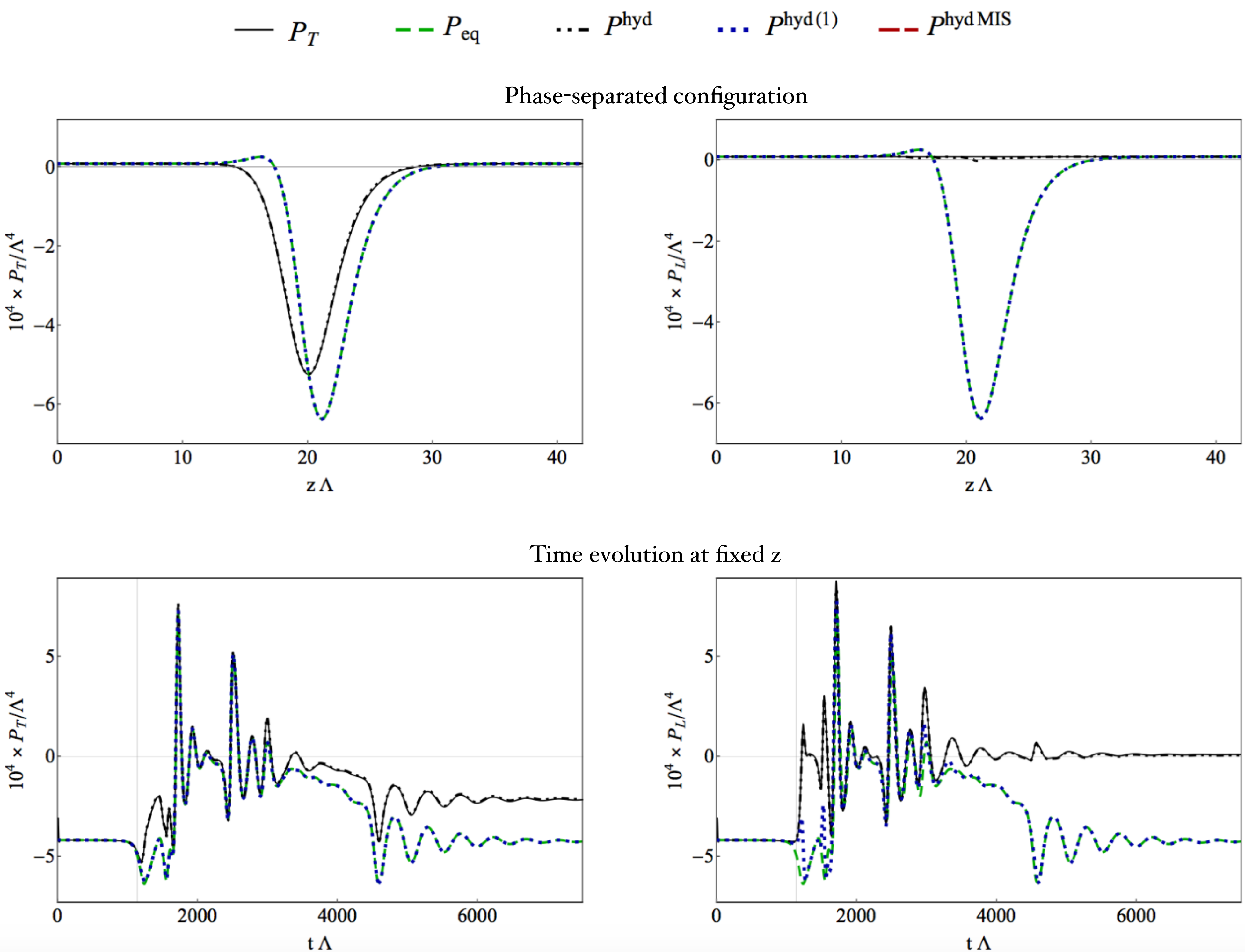
↓

$$T_{\mu\nu} = T_{\mu\nu}^{\text{ideal}} + \partial_{\text{spatial}} + \partial^2_{\text{spatial}}$$

"Purely spatial formulation"



P_T
P_{eq}
P^{hyd}
$P^{hyd\,(1)}$
$P^{hyd\,MIS}$
Phase-separated configuration
$10^4 \times P_T/\Lambda^4$
$10^4 \times P_L/\Lambda^4$
z Λ
Time evolution at fixed z
t Λ



P_T
P_{eq}
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z Λ
Time evolution at fixed z
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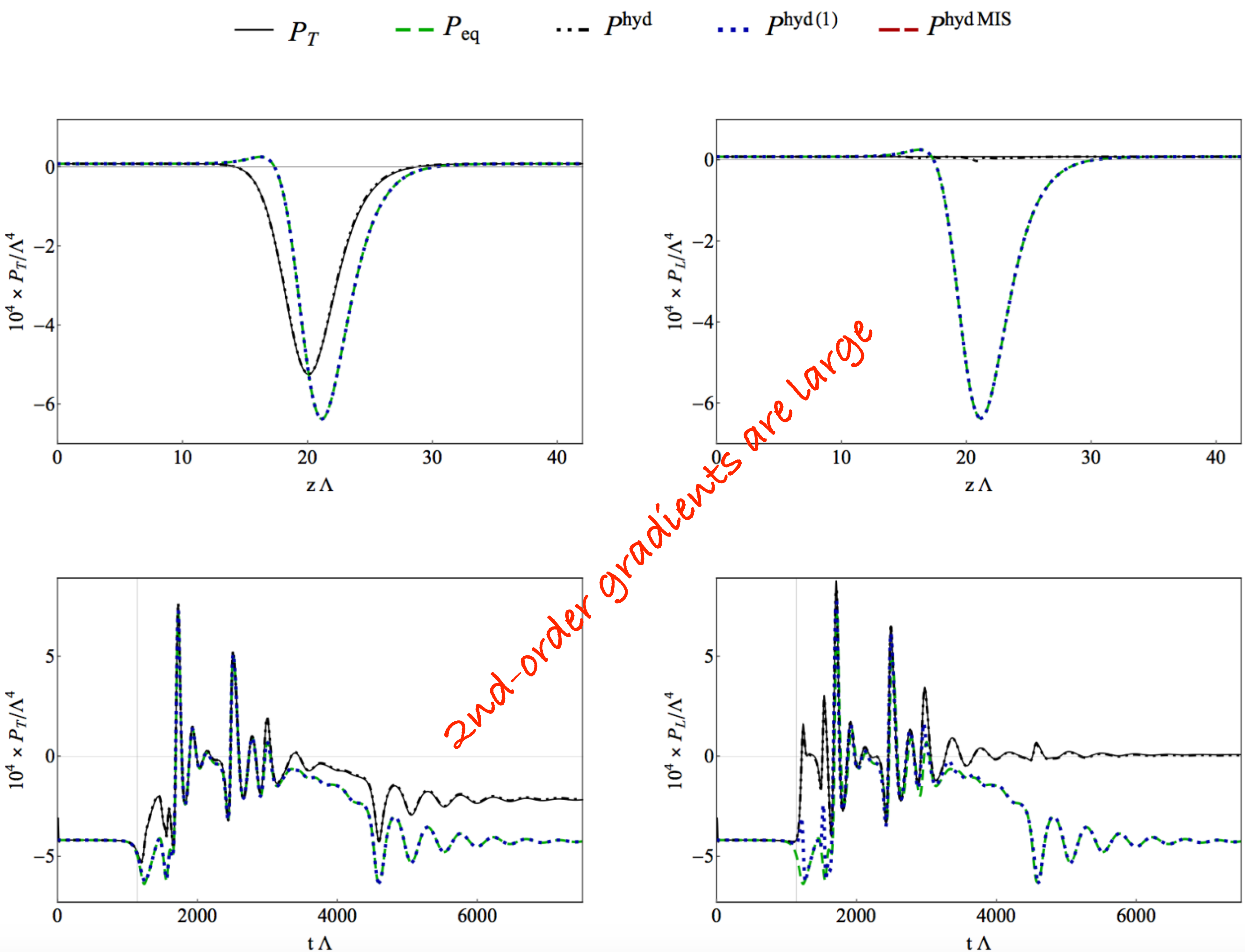

P_T
P_{eq}
P^{hyd}
$P^{hyd(1)}$
$P^{hyd\,MIS}$
$10^4 \times P_T/\Lambda^4$
$10^4 \times P_L/\Lambda^4$
z Λ
t Λ
2nd-order gradients are large

Evolution described by 2nd-order hydrodynamics

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Attems, Bea, Casalderrey, D.M. & Zilhao '19

- We are not doing time evolution, just checking constitutive relations.

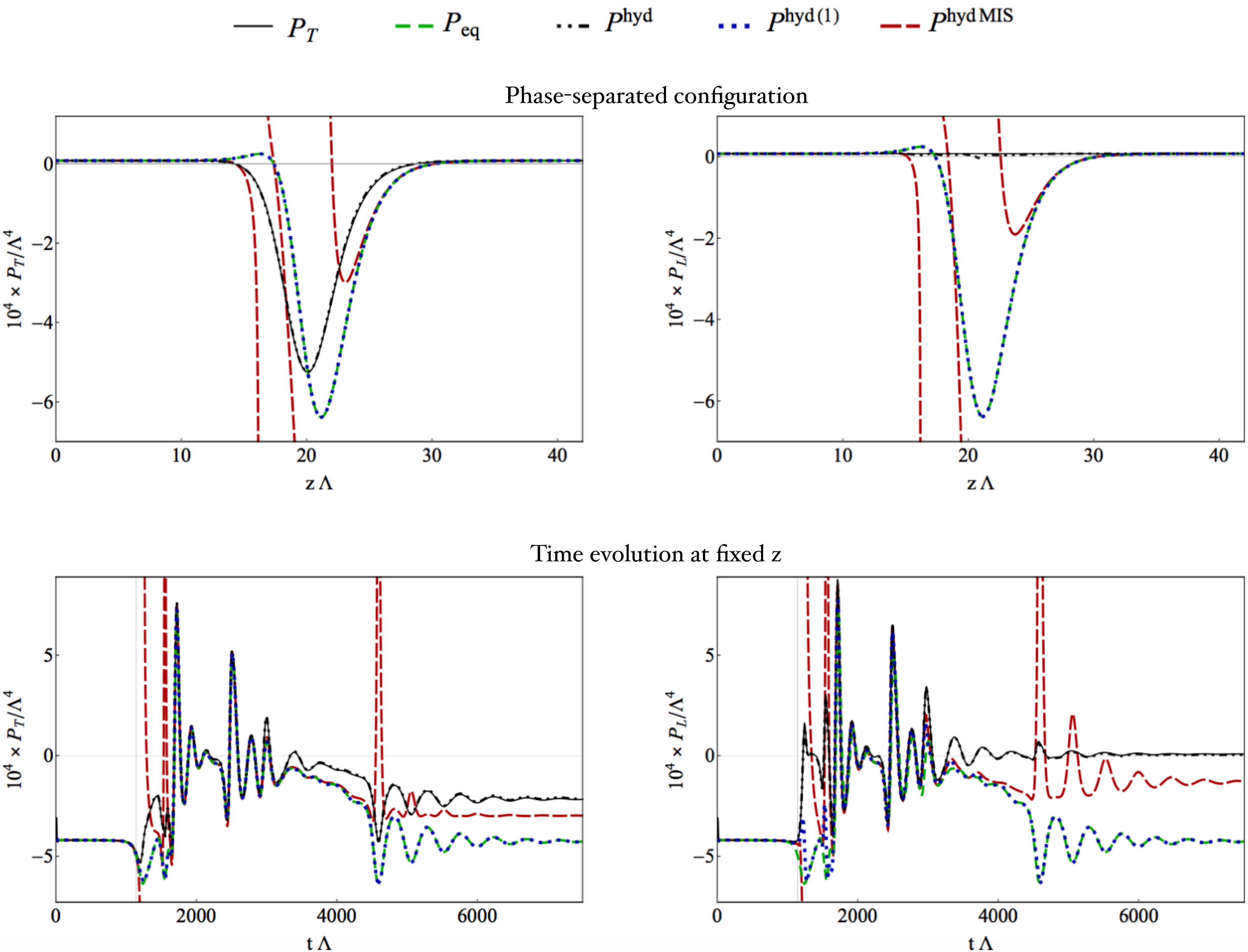
- Problem for time evolution: Hydrodynamics is acausal.

$$T_{\mu\nu} = T_{\mu\nu}^{\text{ideal}} + \partial_{\text{spatial}} + \partial^2_{\text{spatial}}$$

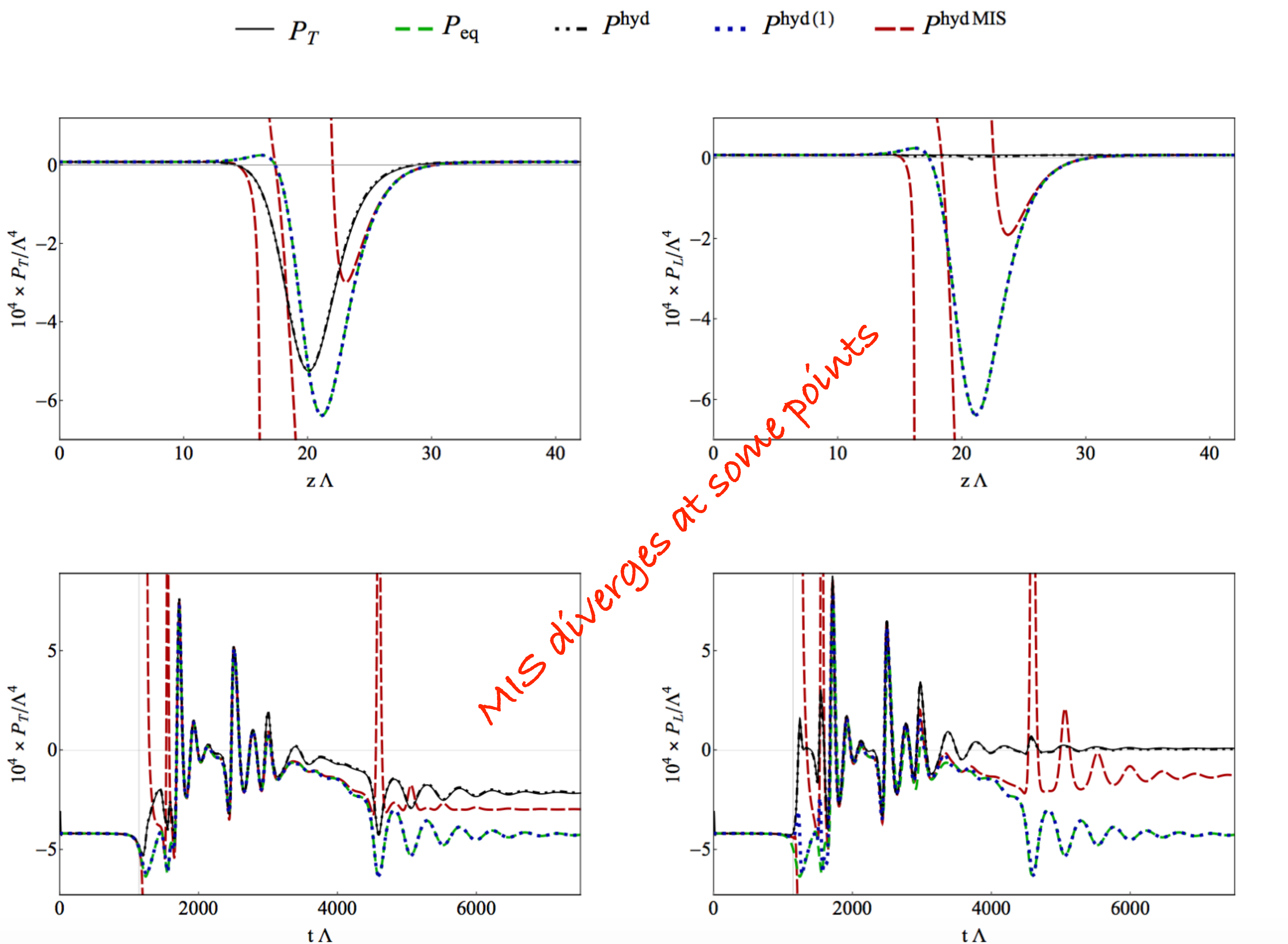
- One fix (Muller-Israel-Stewart): Use lower oder equations to get:

$$T_{\mu\nu}^{\text{MIS}} = T_{\mu\nu}^{\text{ideal}} + \partial_{\text{spatial}} + \partial_{\text{spatial}}\partial_{\text{time}}$$

- Produces equivalent descriptions if gradients are small, but not in our case.



P_T
P_{eq}
P^{hyd}
$P^{hyd(1)}$
$P^{hyd\,MIS}$
Phase-separated configuration
$10^4 \times P_T/\Lambda^4$
$10^4 \times P_L/\Lambda^4$
z Λ
z Λ
Time evolution at fixed z
$10^4 \times P_T/\Lambda^4$
$10^4 \times P_L/\Lambda^4$
t Λ
t Λ



P_T
P_{eq}
P^{hyd}
$P^{hyd(1)}$
$P^{hyd\,MIS}$
$10^4 \times P_T/\Lambda^4$
$10^4 \times P_L/\Lambda^4$
z Λ
t Λ
MIS diverges at some points

Purely spatial coefficients are smooth and finite

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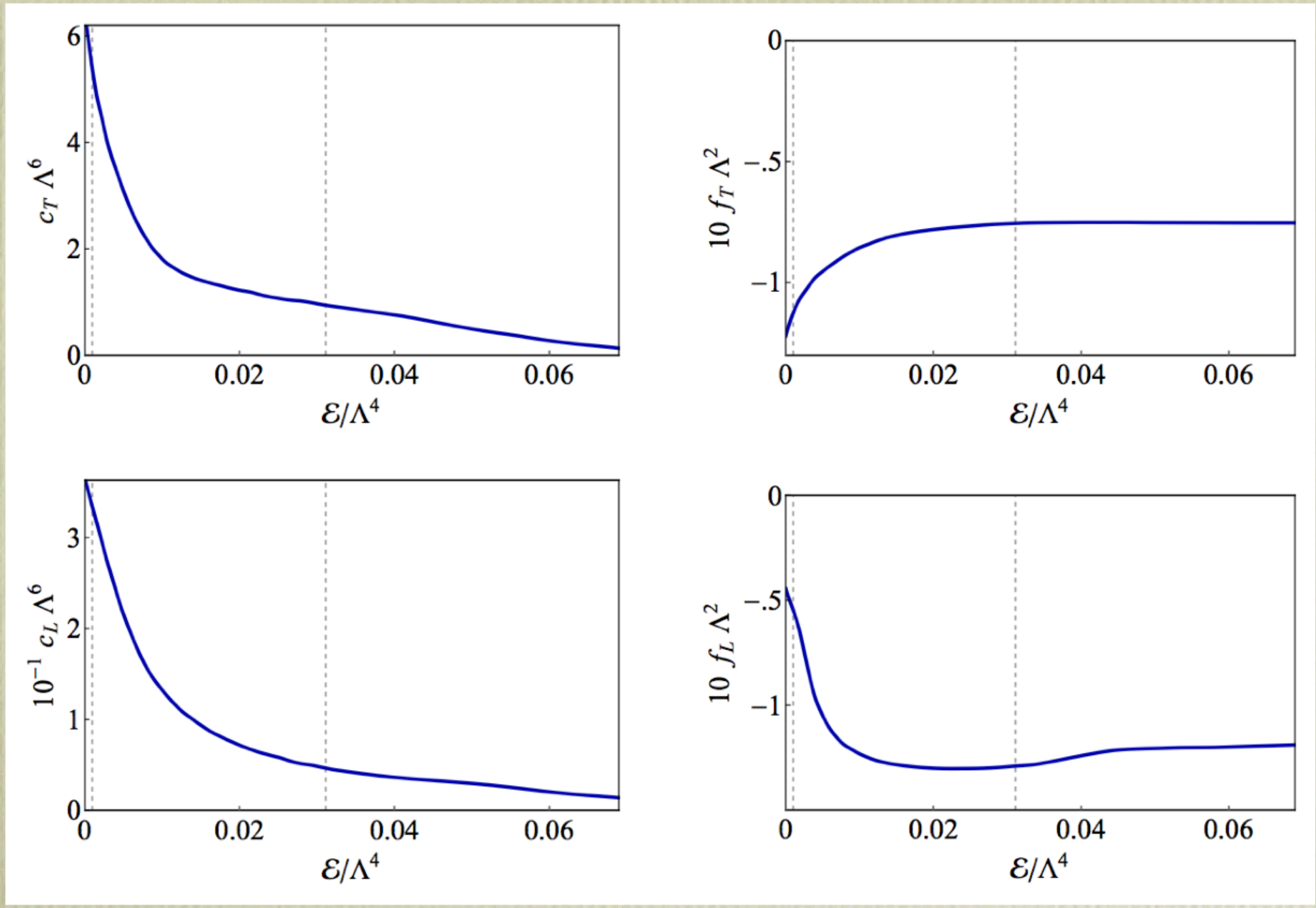

MIS coefficients diverge at points where c_s=0

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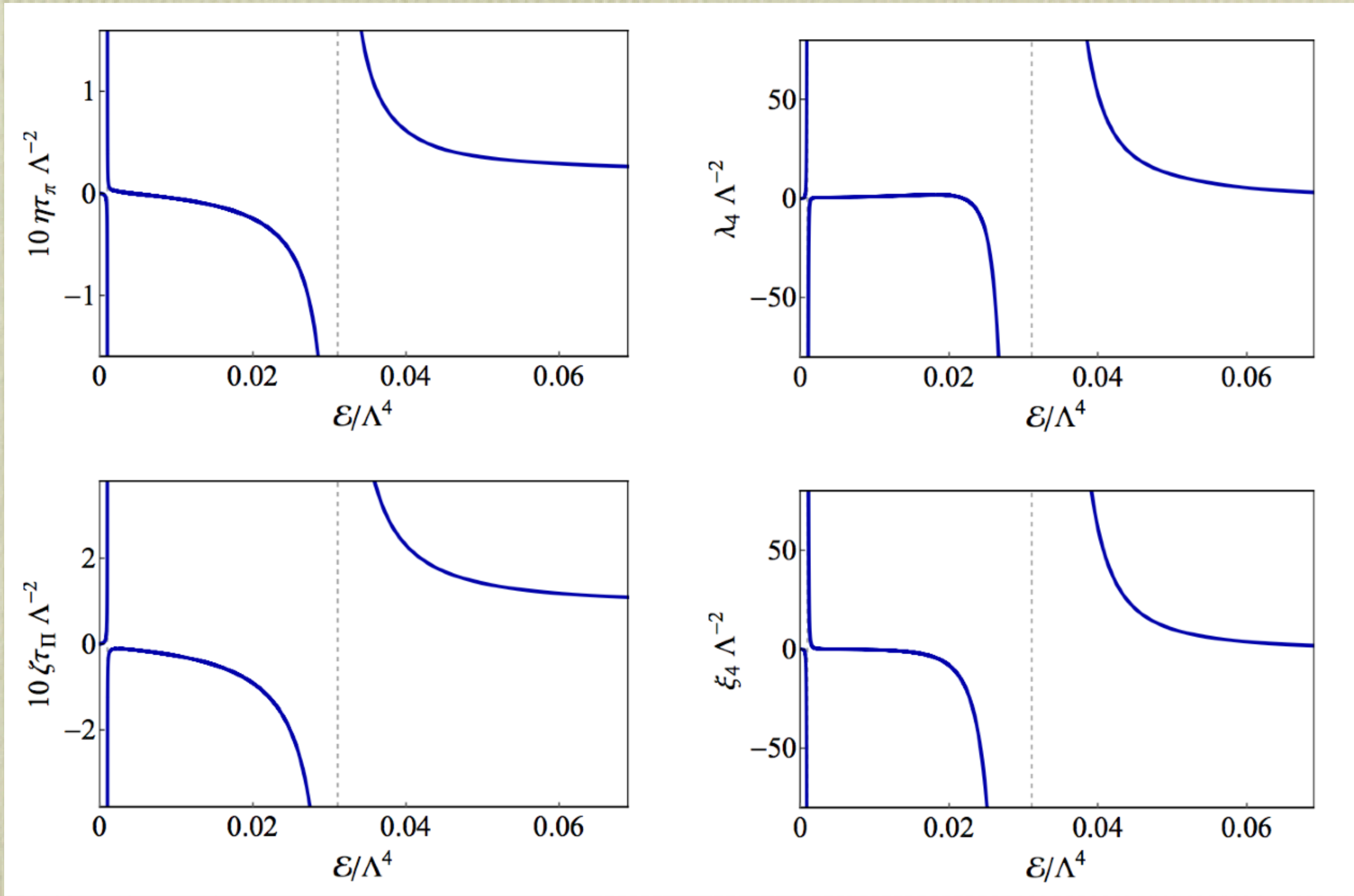
Attems, Bea, Casalderrey, D.M. & Zilhao '19



Change of basis involves powers of $1/c_s$

Collisions across a phase transition

Collisions across a 1st-order phase transition

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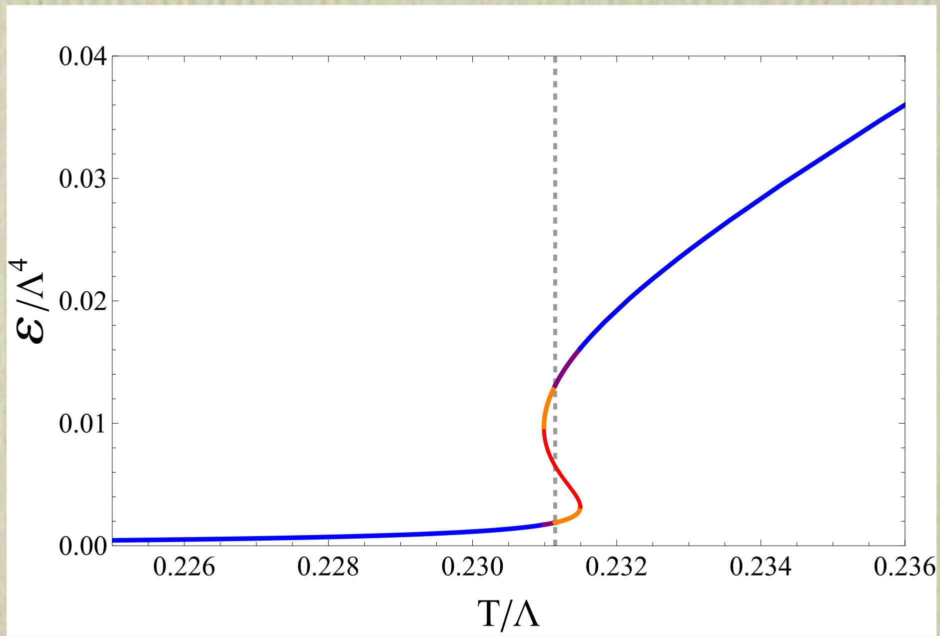

Collisions across a 1st-order phase transition

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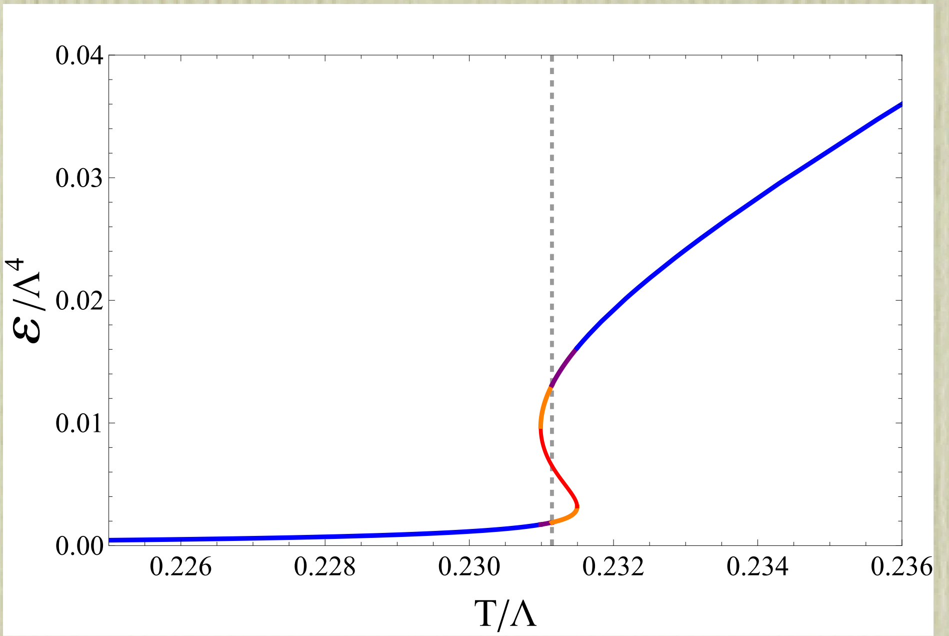

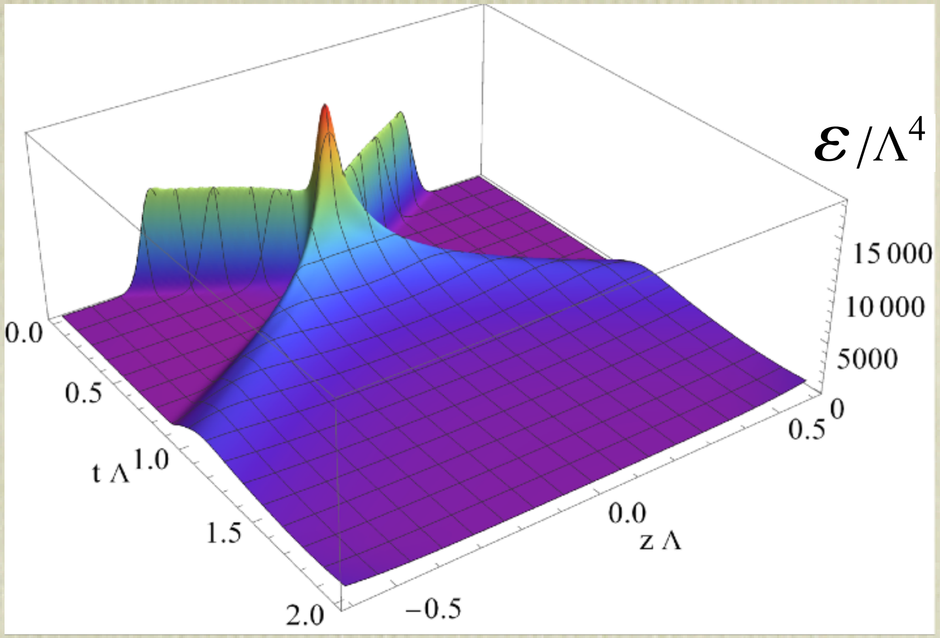

Extremely high energy:

Recover CFT result

Collisions across a 1st-order phase transition

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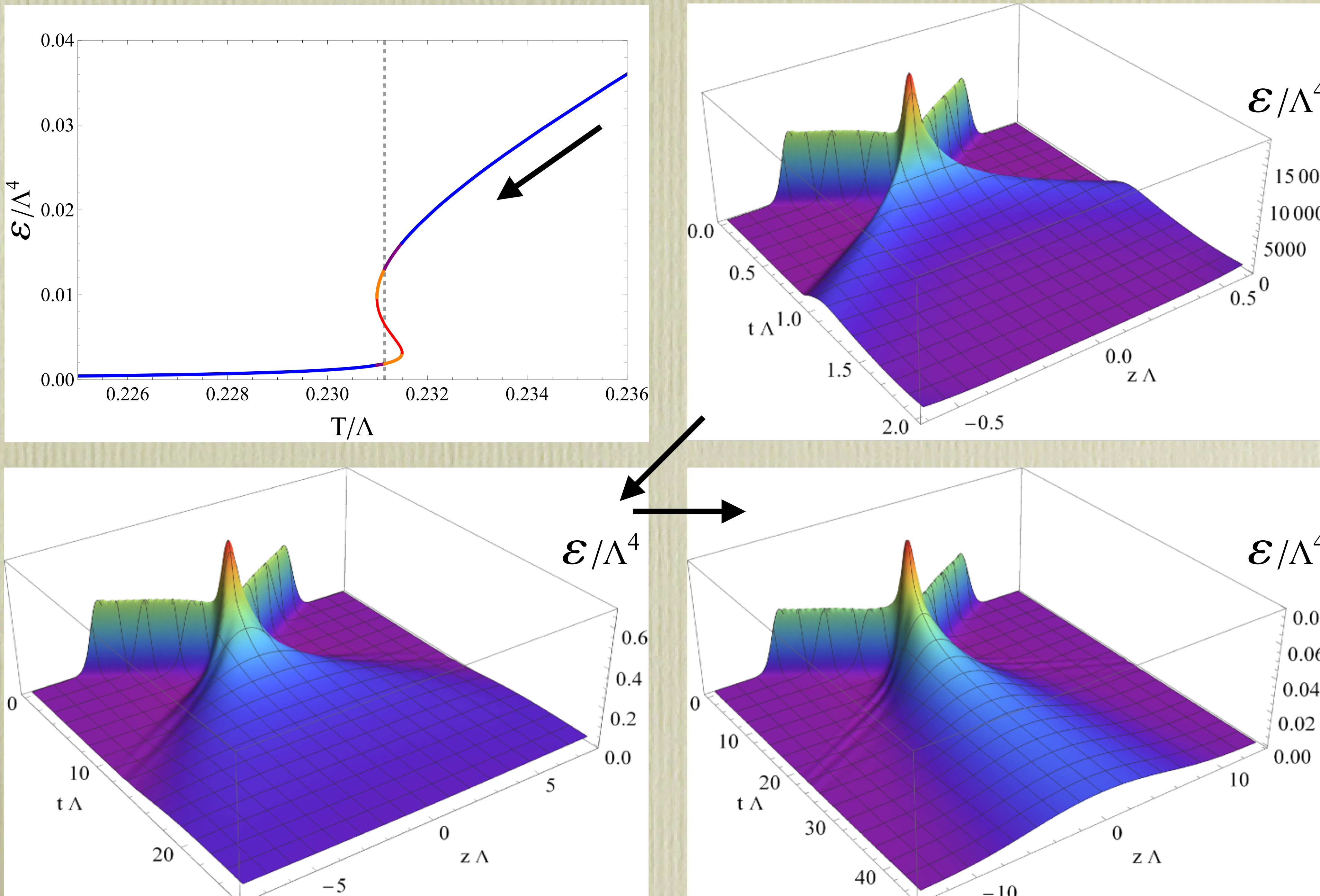

Collisions across a 1st-order phase transition

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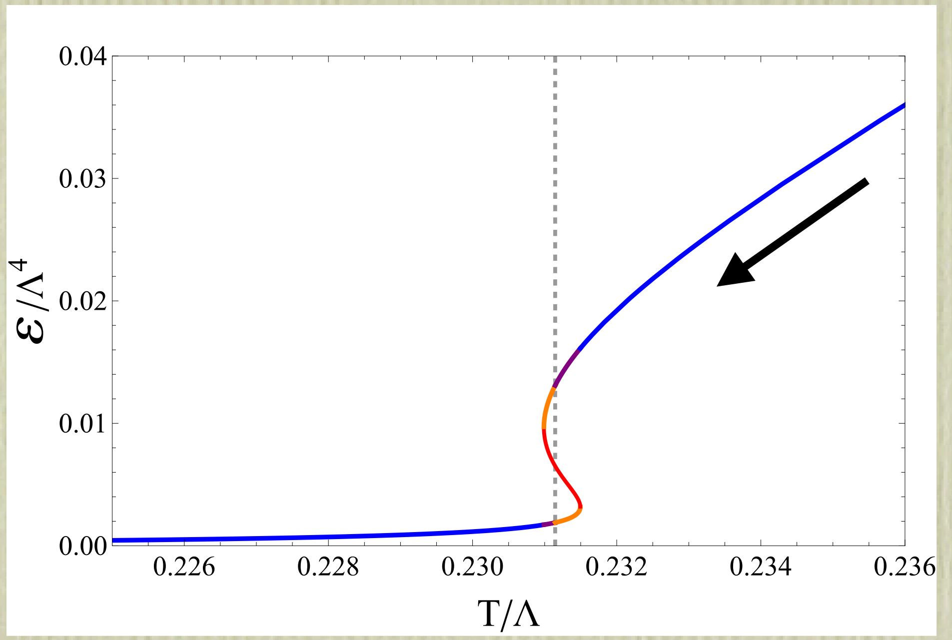
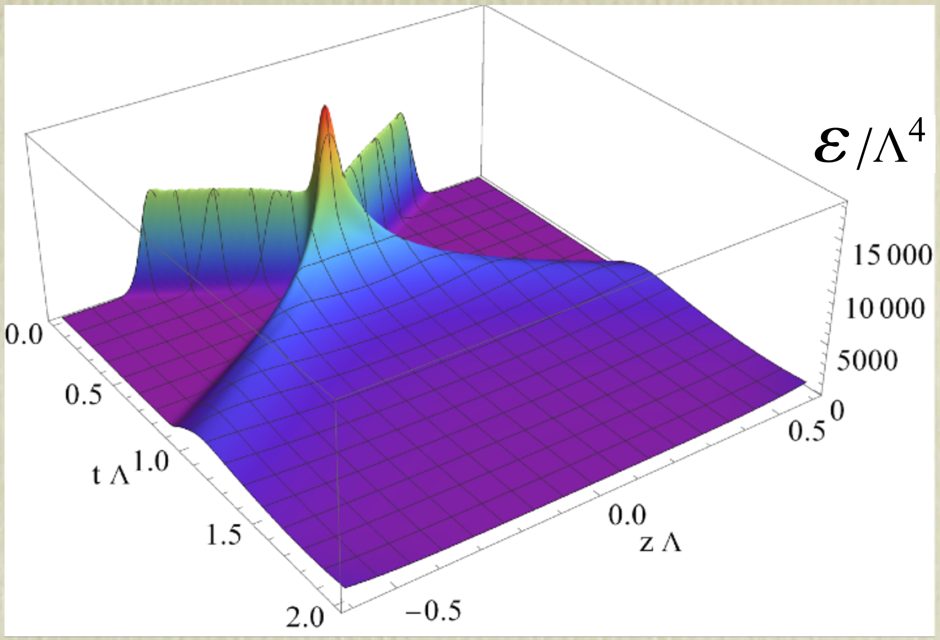
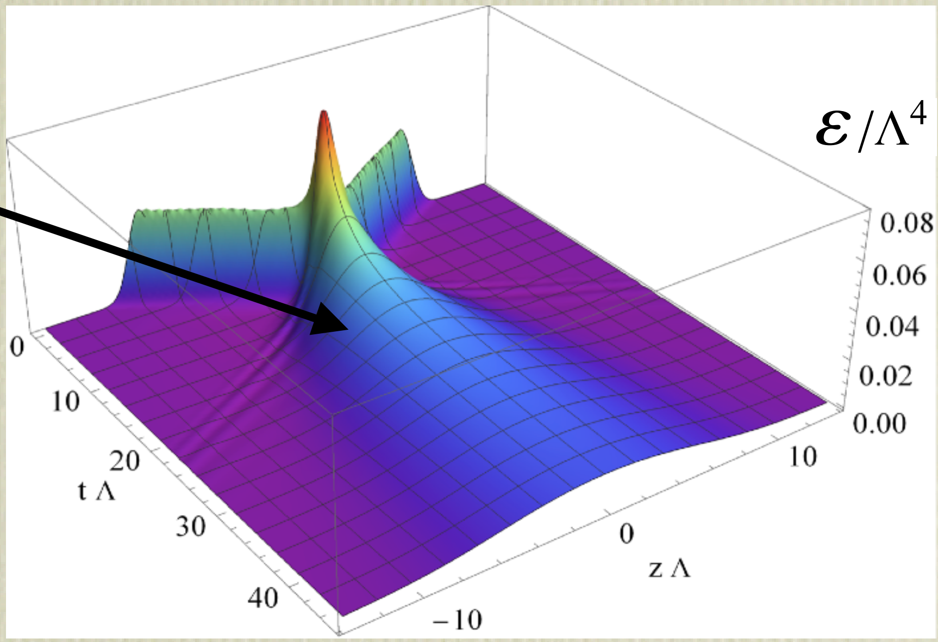
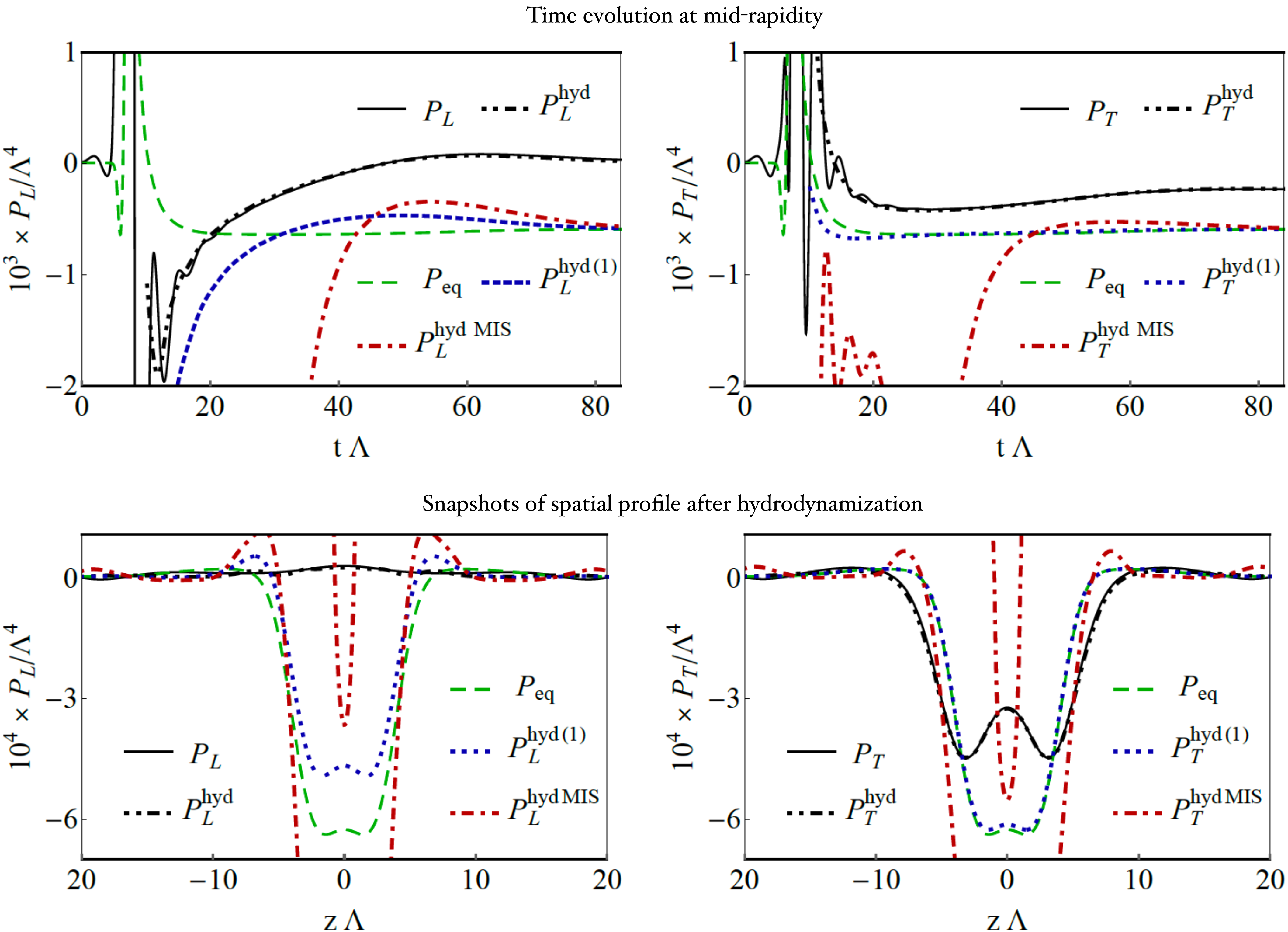




- Long-lived, quasi-static blob

- Well described by 2nd order purely spatial hydro but not MIS





Time evolution at mid-rapidity
$10^3 \times P_L/\Lambda^4$
P_L
$P_L^{\rm hyd}$
$P_{\rm eq}$
$P_L^{\rm hyd\,(1)}$
$P_L^{\rm hyd\ MIS}$
t Λ
$10^3 \times P_T/\Lambda^4$
P_T
$P_T^{\rm hyd}$
$P_{\rm eq}$
$P_T^{\rm hyd\,(1)}$
$P_T^{\rm hyd\ MIS}$
t Λ
Snapshots of spatial profile after hydrodynamization
$10^4 \times P_L/\Lambda^4$
$P_{\rm eq}$
P_L
$P_L^{\rm hyd\,(1)}$
$P_L^{\rm hyd}$
$P_L^{\rm hyd\,MIS}$
z Λ
$10^4 \times P_T/\Lambda^4$
$P_{\rm eq}$
P_T
$P_T^{\rm hyd\,(1)}$
$P_T^{\rm hyd}$
$P_T^{\rm hyd\,MIS}$
z Λ

From 1st-order to 2nd-order to crossover

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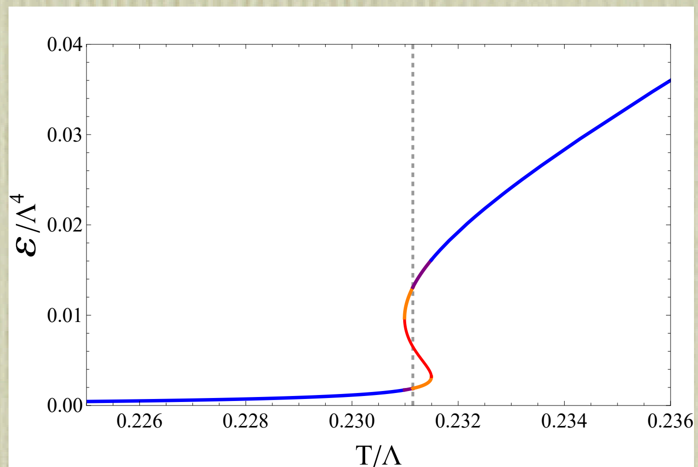
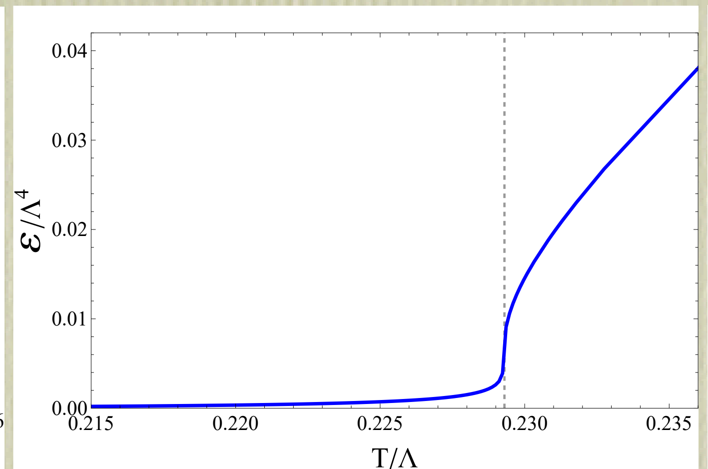
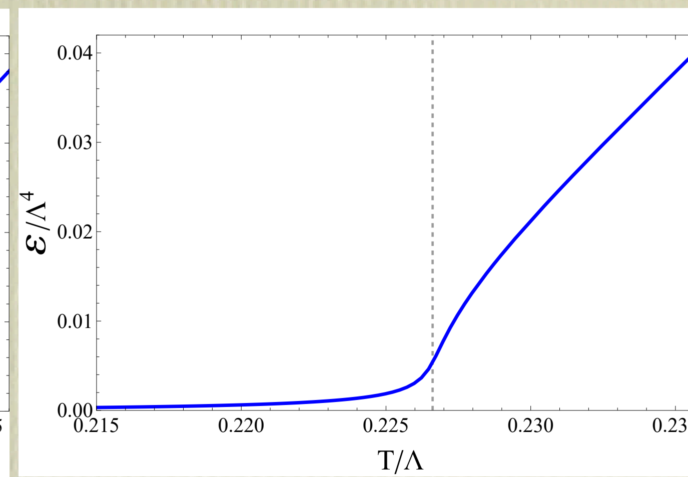
Continous parameter →







1st order

2nd order

Crossover

From 1st-order to 2nd-order to crossover

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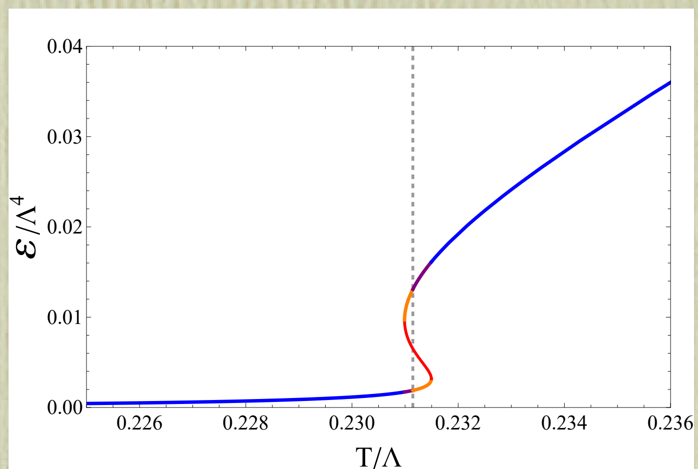
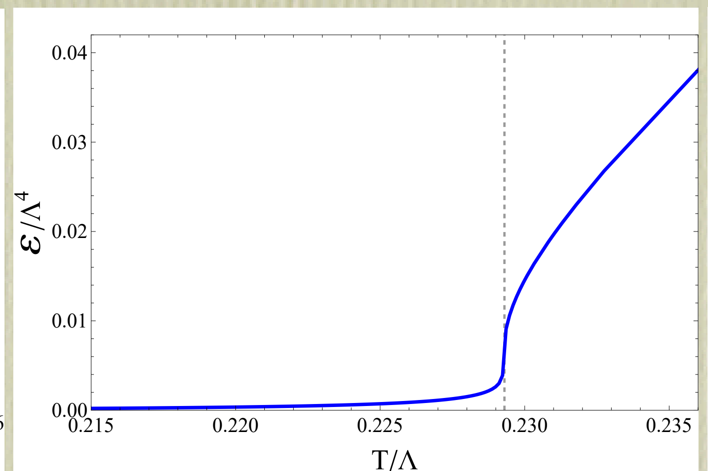
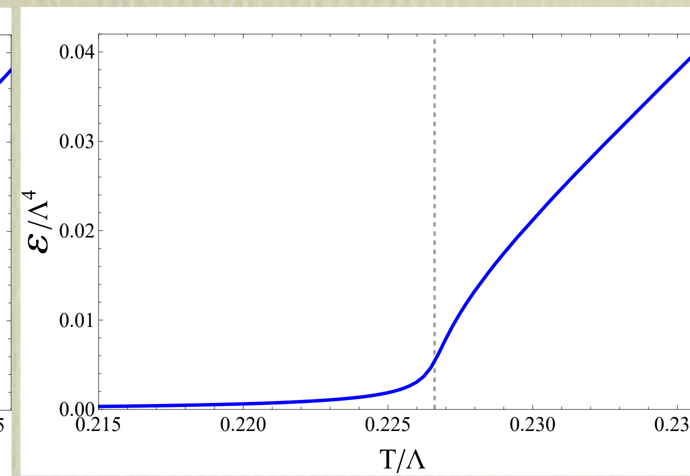
Continous parameter →







1st order

Non-zero latent heat

2nd order

Infinite correlation length

Crossover

Neither of the above

Equilibrium physics is qualitatively very different

From 1st-order to 2nd-order to crossover

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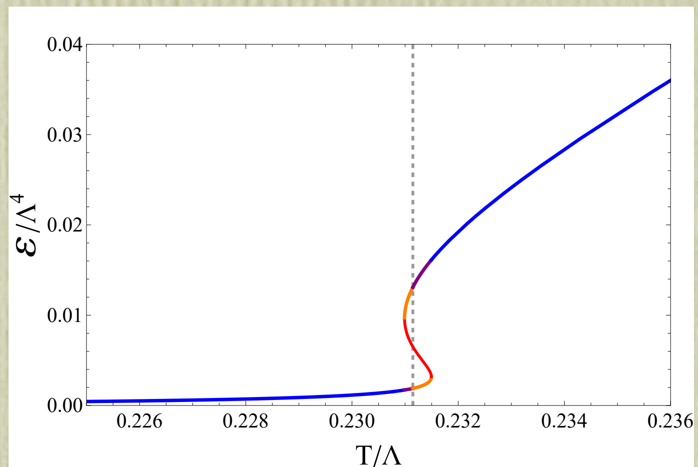
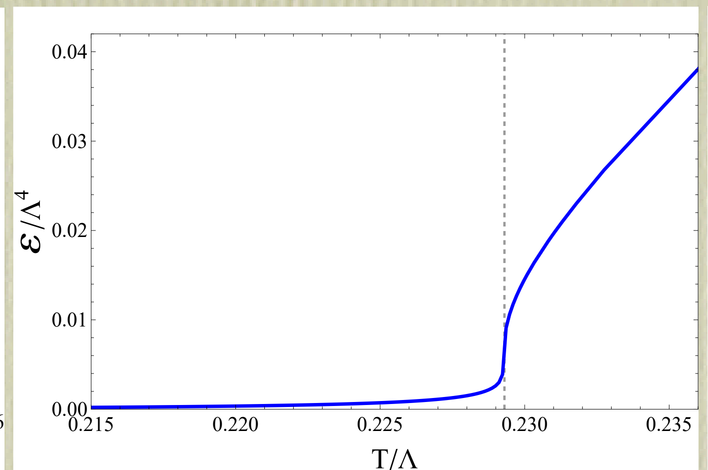
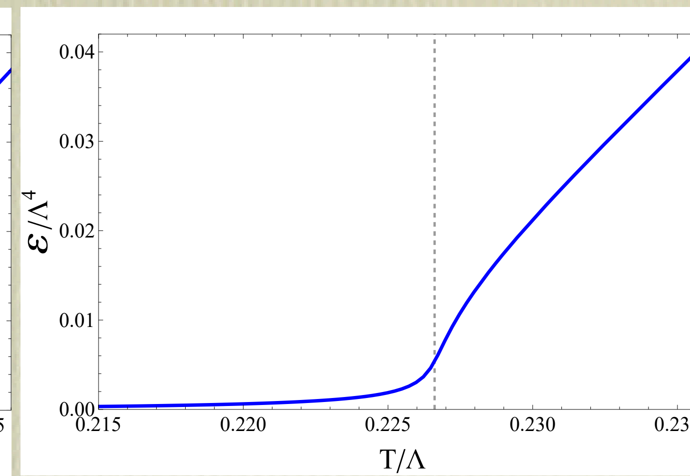
Continous parameter



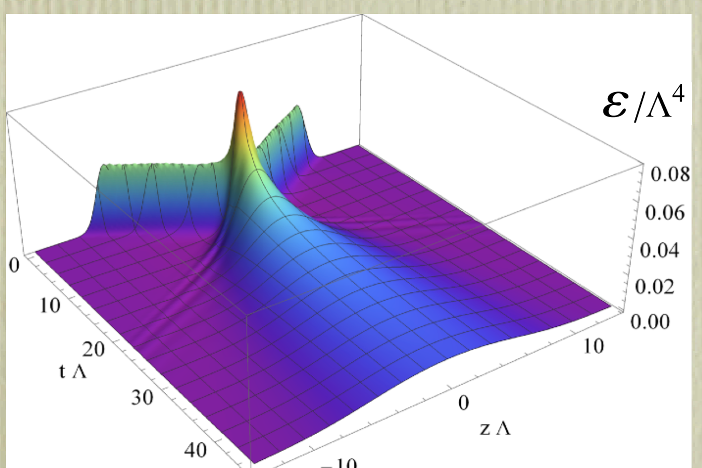


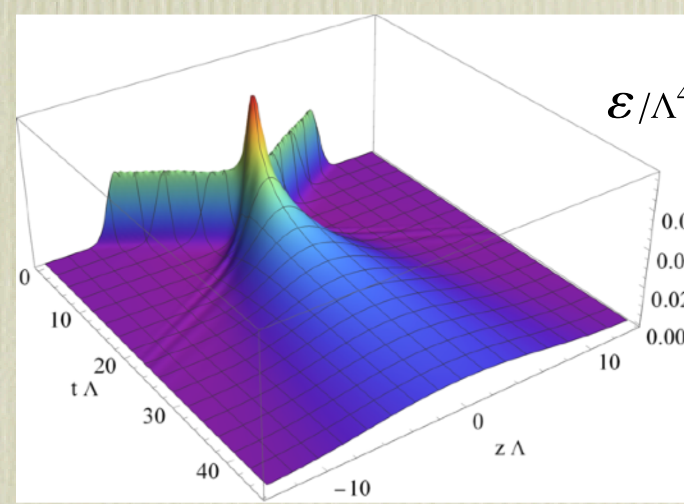


1st order

2nd order

Crossover



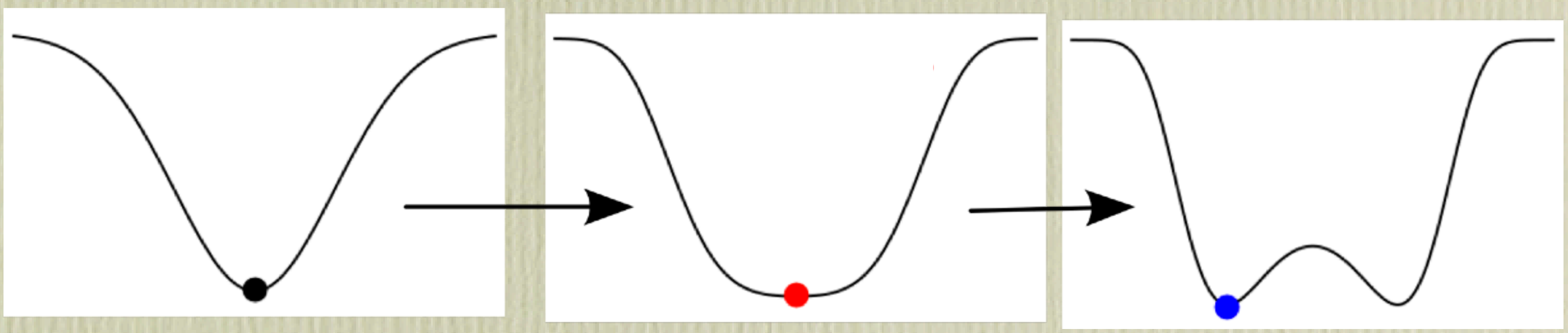


But off-equilibrium physics is qualitatively very similar

Critical fluctuations

Critical fluctuations

- Potential for order parameter flattens out at critical point:



T > Tc　　T = Tc　　T < Tc

- At the critical point:

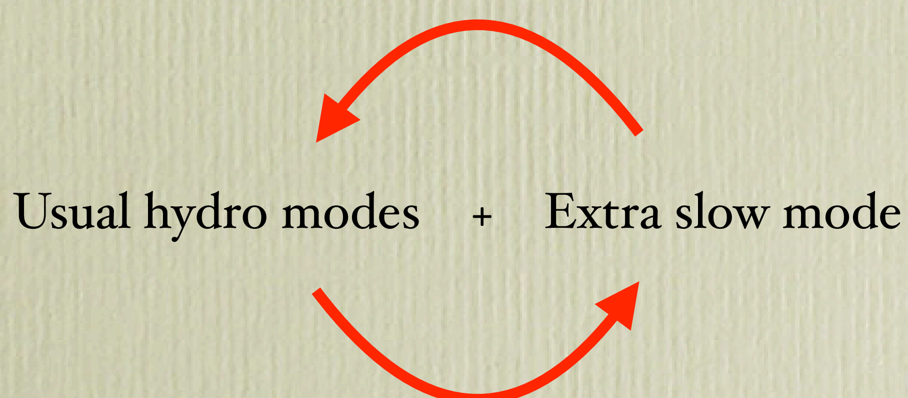
$$m \to 0\,, \qquad \xi = m^{-1} \to \infty \qquad \tau_\xi \to \infty$$

Critical fluctuations

- This leads to divergences also in transport coefficients (e.g. viscosities) because of mode-mode coupling.

- Near the critical point fluctuations of the order parameter are light and must be added to usual hydro: HYDRO+

Stephanov & Yin '17



Critical fluctuations

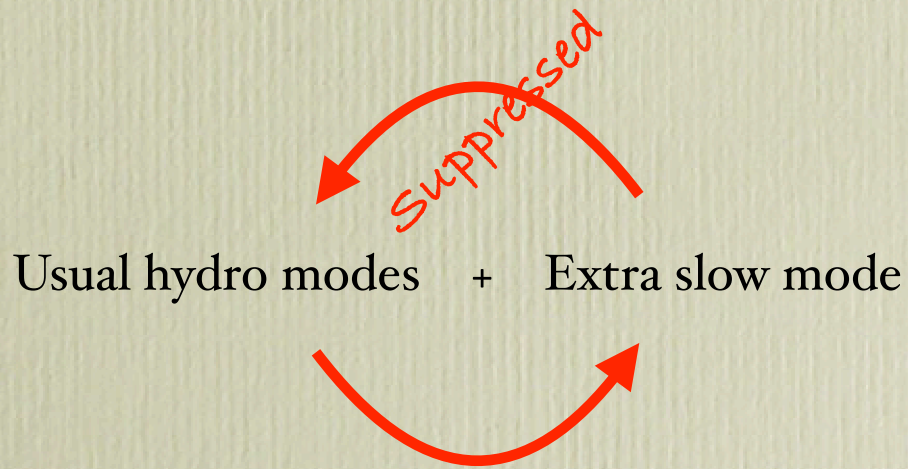
- However, at large-N:

- This leads to divergences also in transport coefficients (e.g. viscosities) because of mode-mode coupling.

Suppressed

- Near the critical point fluctuations of the order parameter are light and must be added to usual hydro: HYDRO+

Stephanov & Yin '17



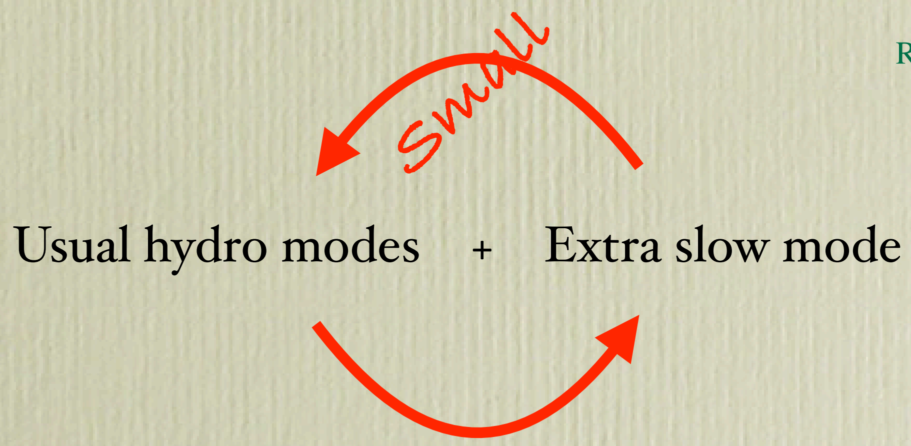
Critical fluctuations

- Moreover, even at finite-N:

- Near the critical point fluctuations of the order parameter are light and must be added to usual hydro: HYDRO+

Stephanov & Yin '17

Rajagopal, Ridgwaya, Weller & Yin '17



Thank you