Adiabatic hydrodynamization in the rapidly-expanding quark-gluon plasma

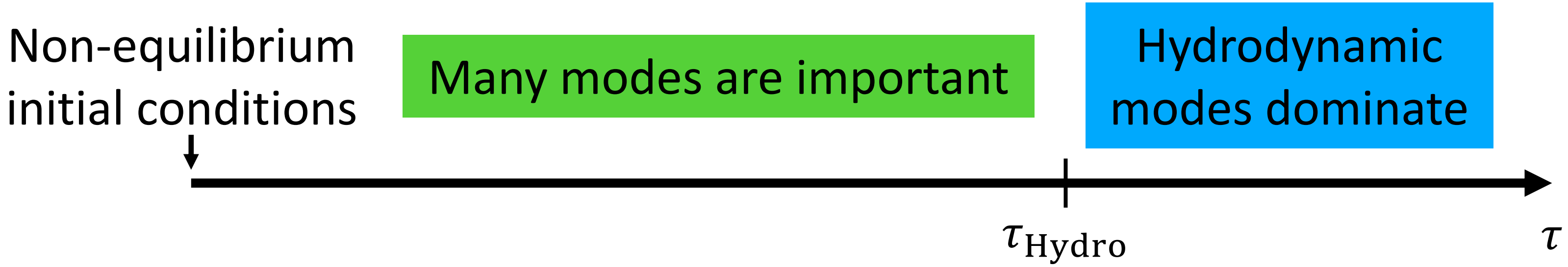
Jasmine Brewer

Based on:

JB, Li Yan, and Yi Yin [arXiv:1910.00021]

and ongoing work

Expected applicability of hydrodynamics

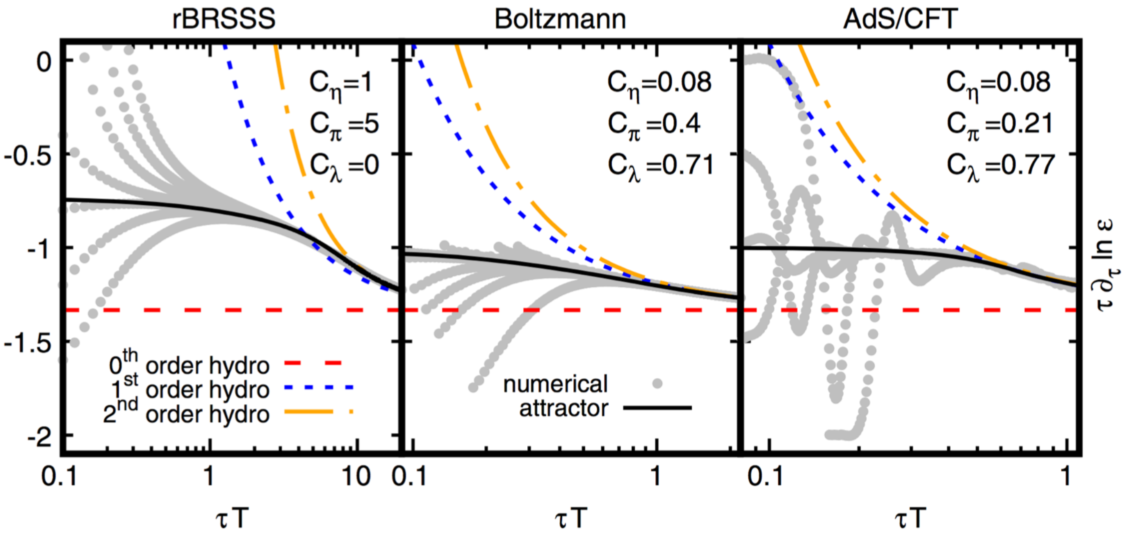


"Gradients are large so hydro doesn't work"

"Gradients are small so hydro works"

Observation of attractor behavior

Heller and Spalinski [1503.07514], many follow-ups



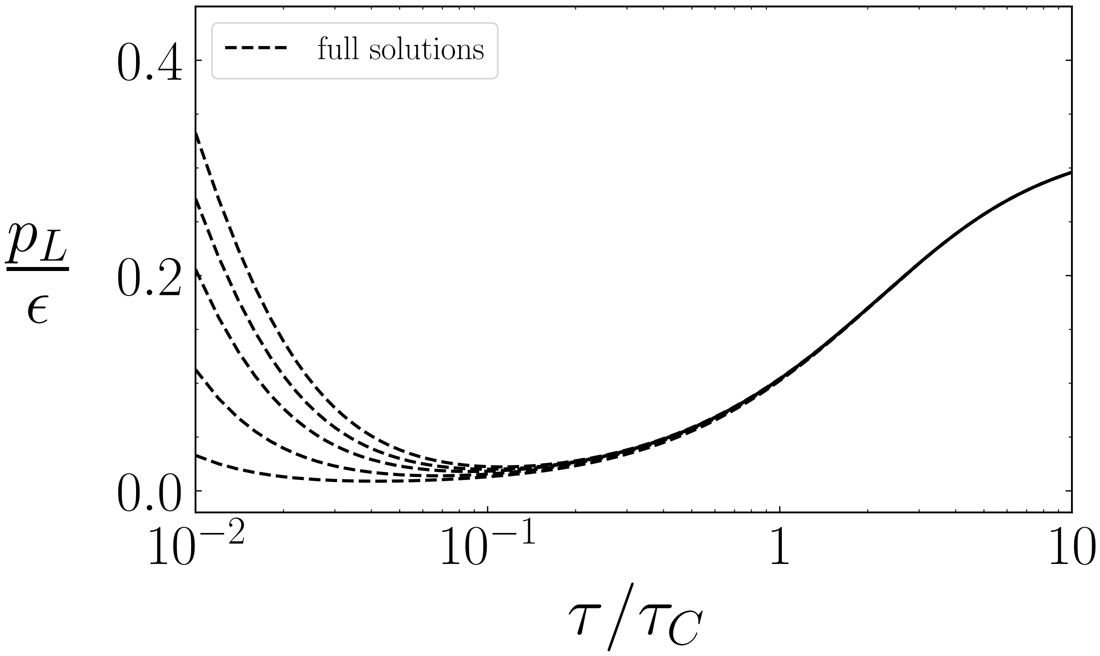
Suggestive of simplified bulk description before τ_{Hydro}

Fig: Romatschke [1704.08699]

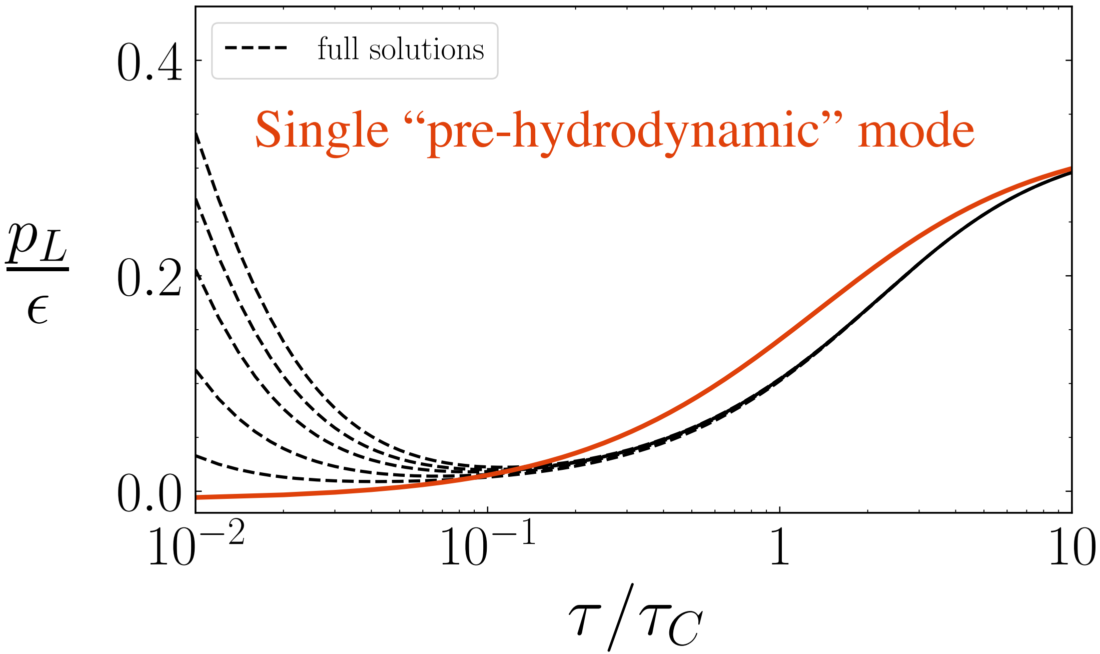
Observation of attractor behavior

What is the physical origin of this simplification?

What does attractor behavior imply about the system?



Bjorken-expanding kinetic theory in the relaxation time approximation



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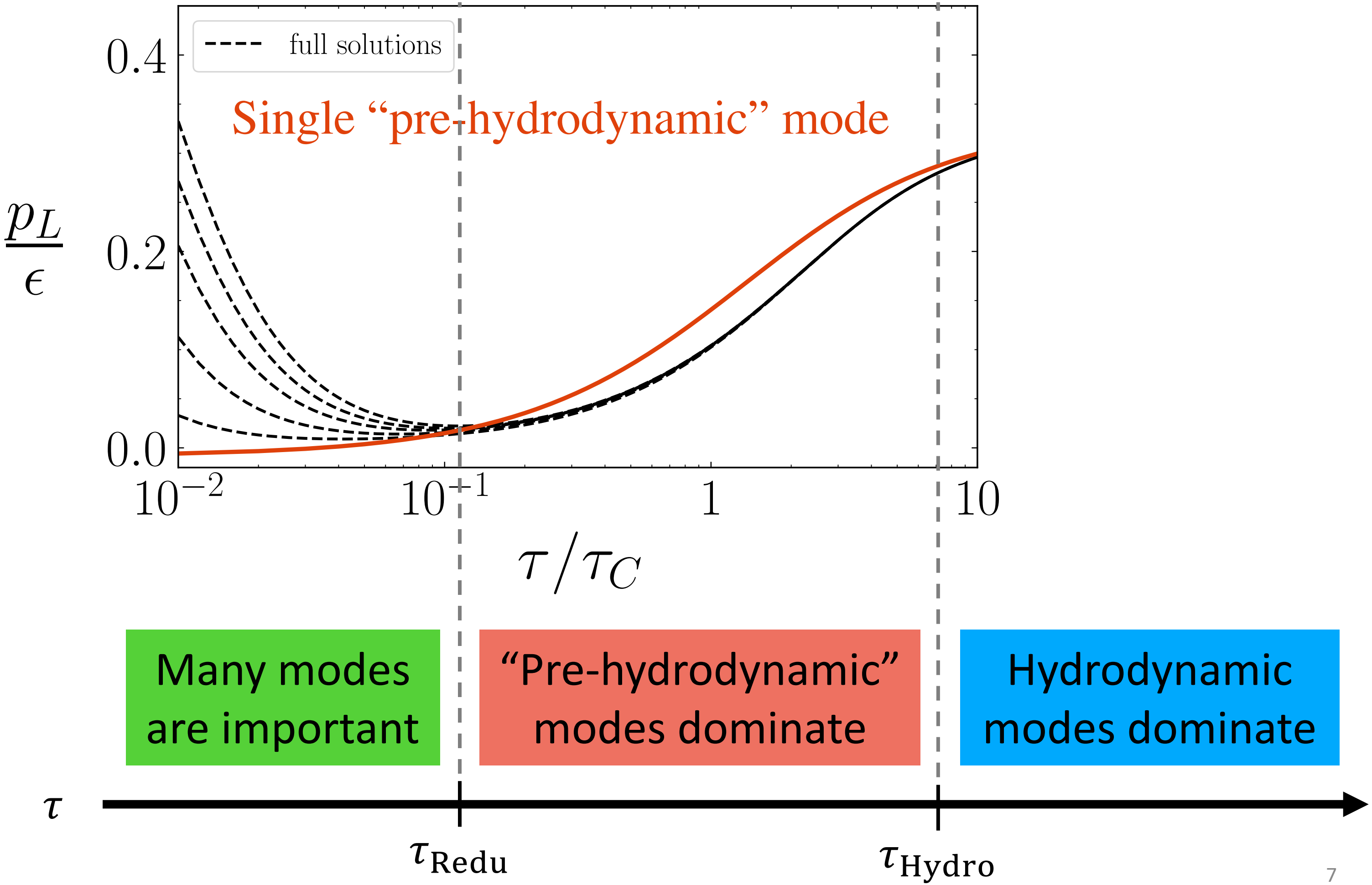
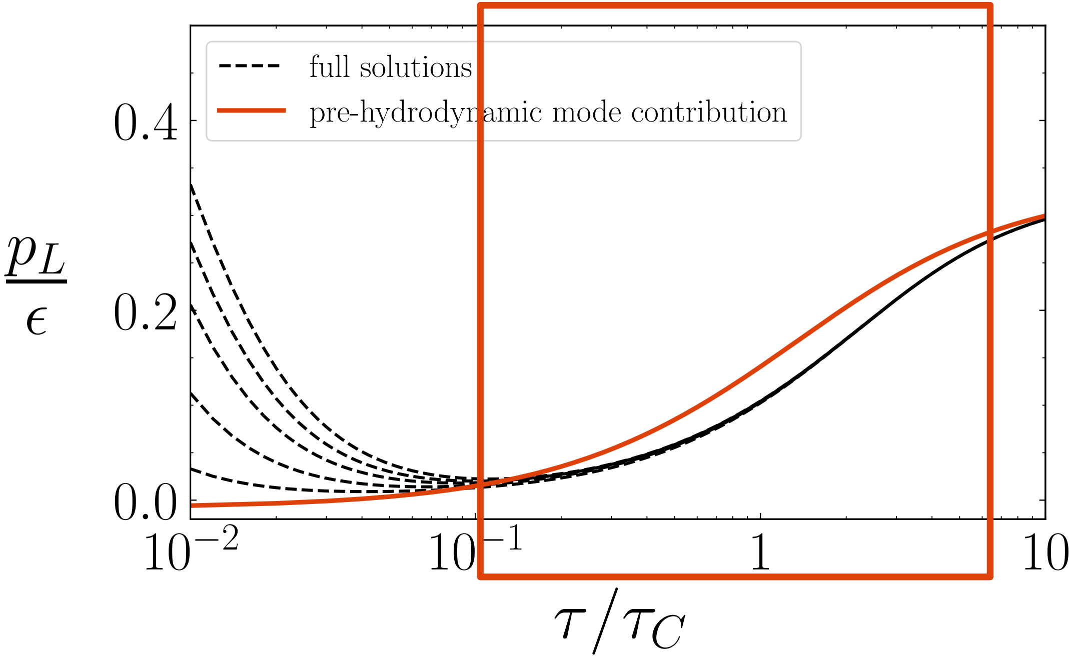


full solutions
Single “pre-hydrodynamic” mode
$\frac{p_L}{\epsilon}$
0.4
0.2
0.0
10^{-2}
10^{-1}
1
10
τ/τ_C
Many modes are important
“Pre-hydrodynamic” modes dominate
Hydrodynamic modes dominate
τ
τ_{Redu}
τ_{Hydro}

Adiabatic theorem:

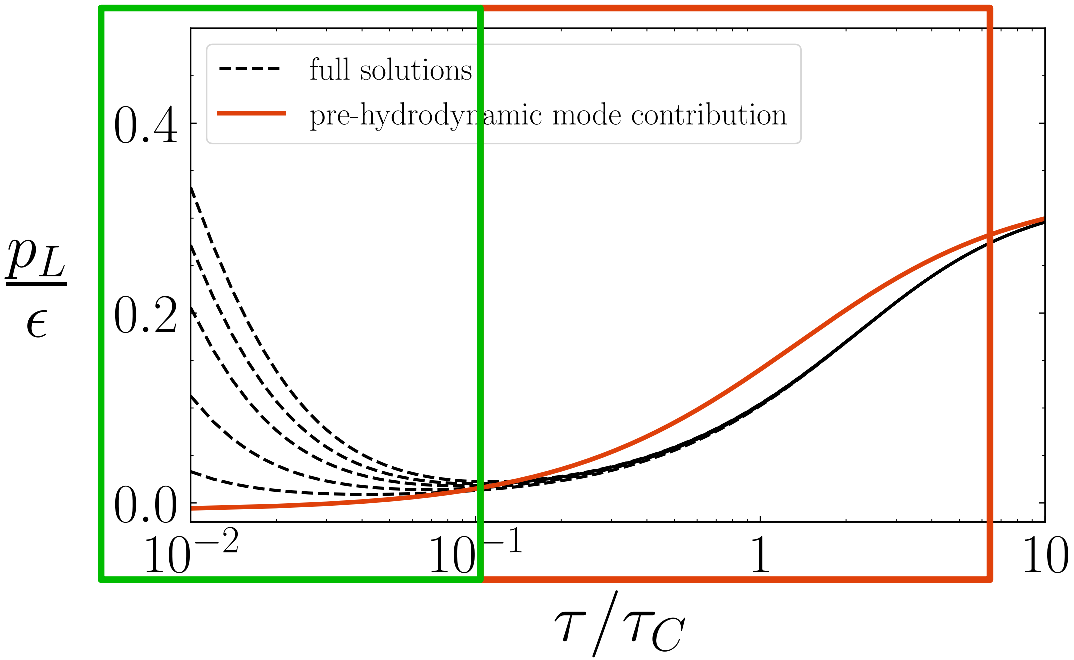
"A system prepared in its (instantaneous) ground state will remain in its (instantaneous) ground state under adiabatic evolution of the Hamiltonian"

Adiabatic interpretation of far-from-equilibrium behavior



Pre-hydrodynamic mode is instantaneous ground state of an effective Hamiltonian

Adiabatic interpretation of far-from-equilibrium behavior



Dominance of ground state at early times driven by rapid longitudinal expansion

Hamiltonian formulation from kinetic theory

Bjorken-expanding kinetic theory

$$\frac{\partial}{\partial \tau} f(p_z, p_\perp; \tau) = -\frac{p_z}{\tau}\frac{\partial}{\partial p_z} f(p_z, p_\perp; \tau) - \hat{C}[f]$$

longitudinal expansion $\longleftrightarrow$ collisions

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Moments of kinetic equation give evolution of more macroscopic quantities

$$\int_{\mathbf{p}} |\mathbf{p}| f(p_z, p_\perp; \tau) = \epsilon(\tau)$$

Energy density

$$\int_{|\mathbf{p}|} |\mathbf{p}| f(p_z, p_\perp; \tau) = F_\epsilon(\cos\theta; \tau)$$

Angular distribution contributing to energy density

Hamiltonian formulation from kinetic theory

$$\int_{|\mathbf{p}|} |\mathbf{p}| \left(\underbrace{\frac{\partial}{\partial\tau} f(p_z, p_\perp; \tau)}_{\displaystyle \frac{\partial}{\partial\tau} F_\epsilon} = -\frac{p_z}{\tau} \underbrace{\frac{\partial}{\partial p_z} f(p_z, p_\perp; \tau)}_{\displaystyle -\frac{1}{\tau}(\ldots) F_\epsilon} - \hat{C}[f] \right)$$

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For some $\hat{C}[f]$ $\quad \tau \frac{\partial}{\partial \tau} F_\epsilon = (\ldots) F_\epsilon \longleftrightarrow \partial_y \psi = -\mathcal{H}(y) \psi \qquad y = \log\left(\frac{\tau}{\tau_I}\right)$

Schrodinger-type evolution of generalized Hamiltonian $\mathcal{H}$

Hamiltonian formulation from kinetic theory

Expand F in Legendre polynomials

$$F_\epsilon(\cos\theta;\tau) = \epsilon(\tau) + \sum_{n=1} \frac{4n+1}{2}\mathcal{L}_n(\tau)P_{2n}(\cos\theta)$$

For RTA

$$\tau\frac{\partial}{\partial\tau}F_\epsilon = (\dots)F_\epsilon \quad \longleftrightarrow \quad \partial_y\mathcal{L}_n = -\left[a_n\mathcal{L}_n + b_n\mathcal{L}_{n-1} + c_n\mathcal{L}_{n+1}\right] - \frac{\tau}{\tau_C}(1-\delta_{n0})\mathcal{L}_n$$

Blaizot and Yan [1712.03856]

Hamiltonian formulation from kinetic theory

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$$\boxed{\partial_y\psi = -\left(\mathcal{H}_F + \frac{\tau}{\tau_C}\mathcal{H}_1\right)\psi} \qquad \psi = (\epsilon, \mathcal{L}_1, \mathcal{L}_2, \ldots)$$

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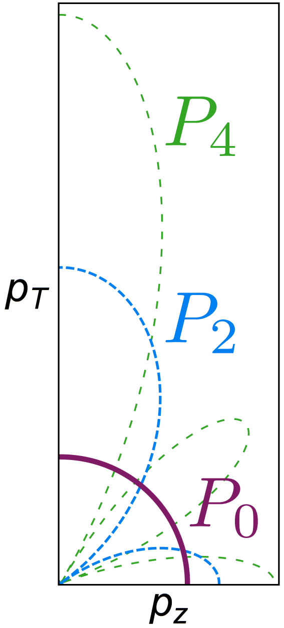

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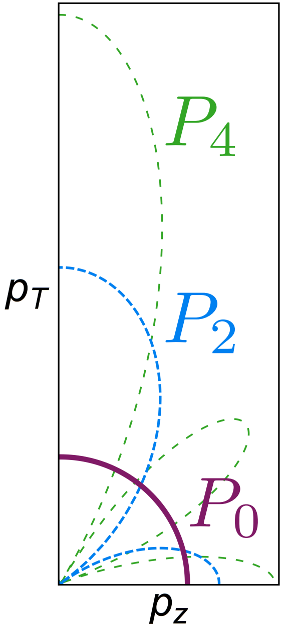

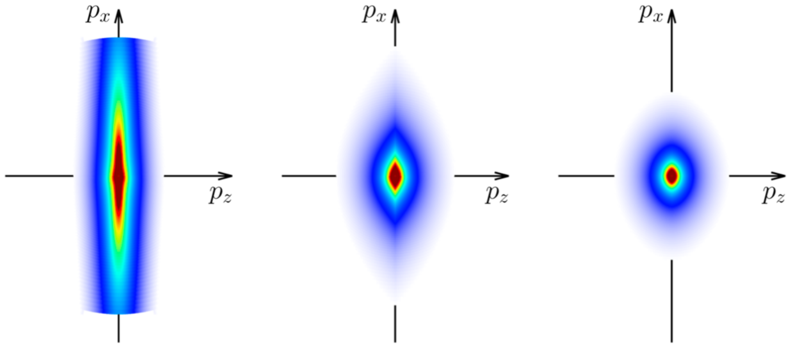

$$(\epsilon, \mathcal{L}_1, \mathcal{L}_2, \ldots) \longrightarrow (\epsilon, 0, 0, \ldots)$$

hydrodynamization

Fig adapted from KMPST [1805.00961]

Is the system prepared in the ground state?

At early times $\hat{C}[f] = 0 \longrightarrow \partial_y \psi = -\mathcal{H}_F \psi$

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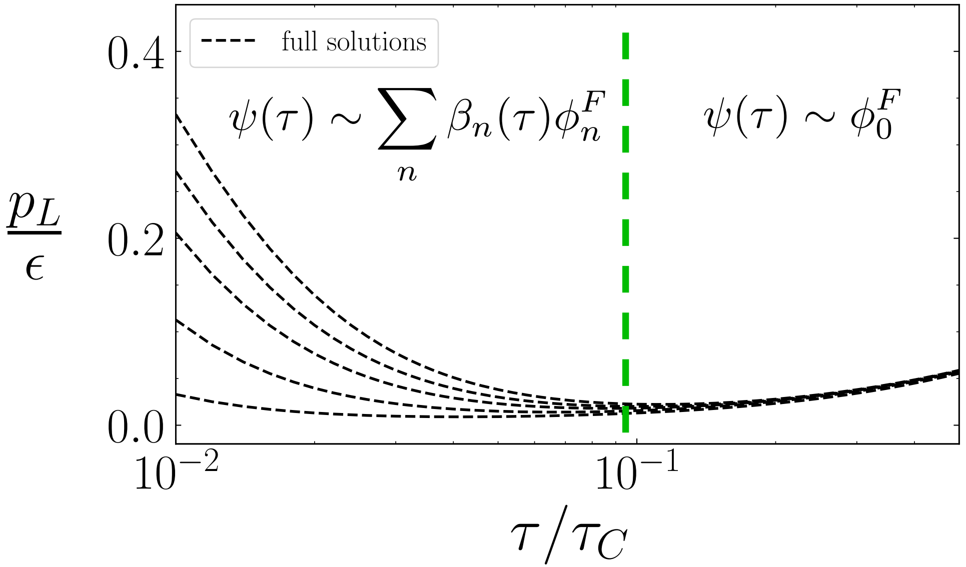

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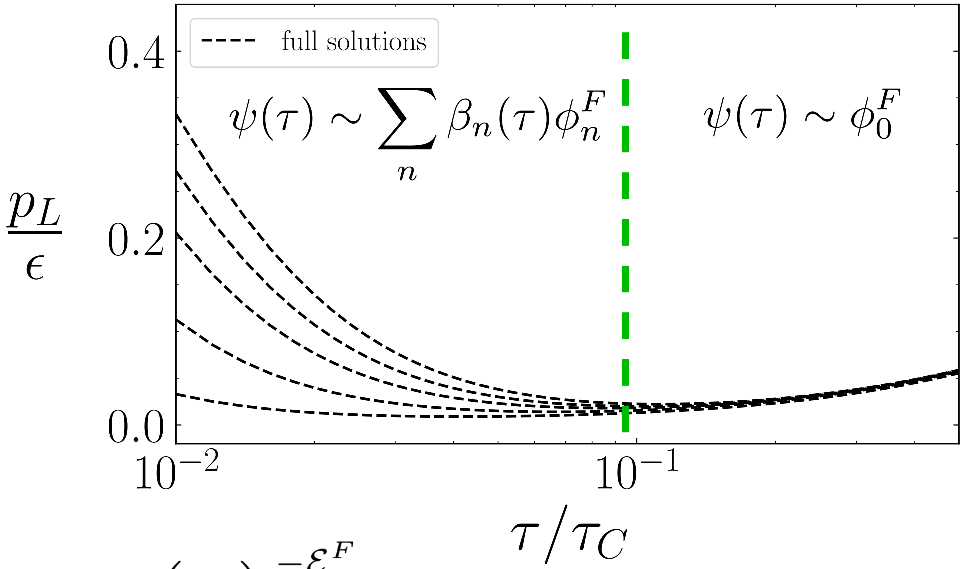

$$\beta_n(\tau) \sim \beta_n(\tau_I)\left(\frac{\tau}{\tau_I}\right)^{-\mathcal{E}_n^F}$$

gives time scale for decay of initial conditions

Similar result to Kurkela, van der Schee, Wiedemann, Wu [1907.08101]

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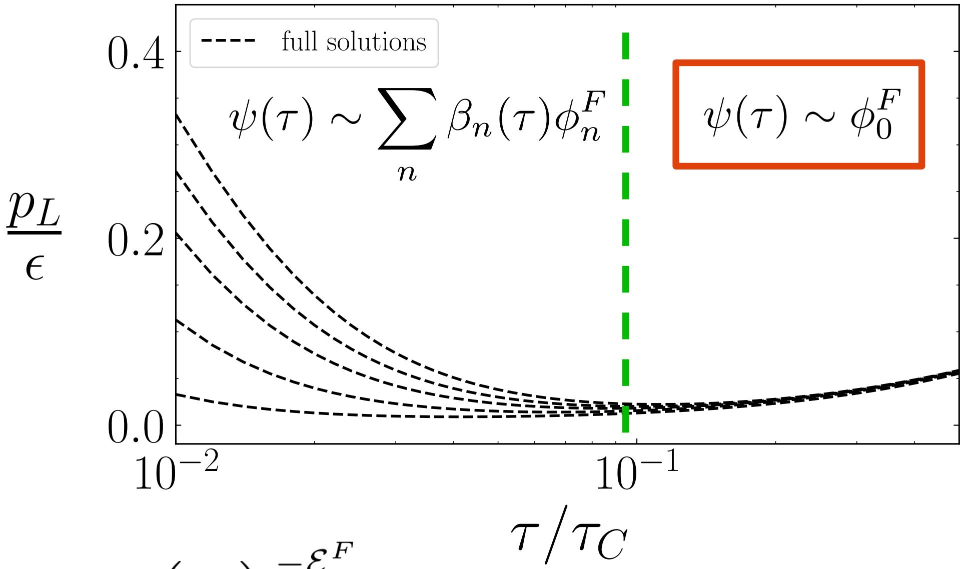

Since $\mathcal{H}_F$ is gapped, ψ decays toward the ground state

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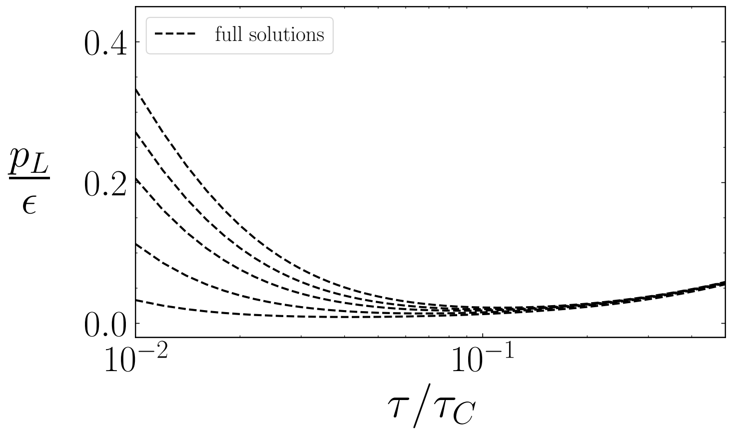
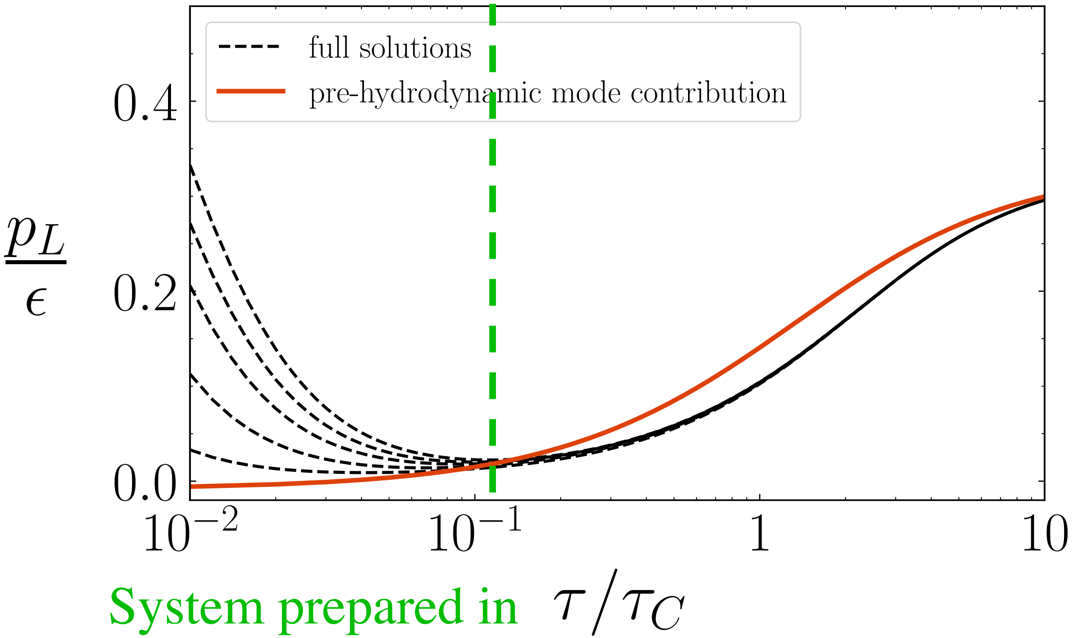
Interlude: physical interpretation of the attractor

No collisions →



Attractor behavior need not imply collectivity or equilibration



System prepared in ground state

Is the subsequent evolution adiabatic?

Definition of adiabatic hydrodynamization:

System evolution determined by
the instantaneous ground state

$$\psi(y) \sim \phi_0(y)$$

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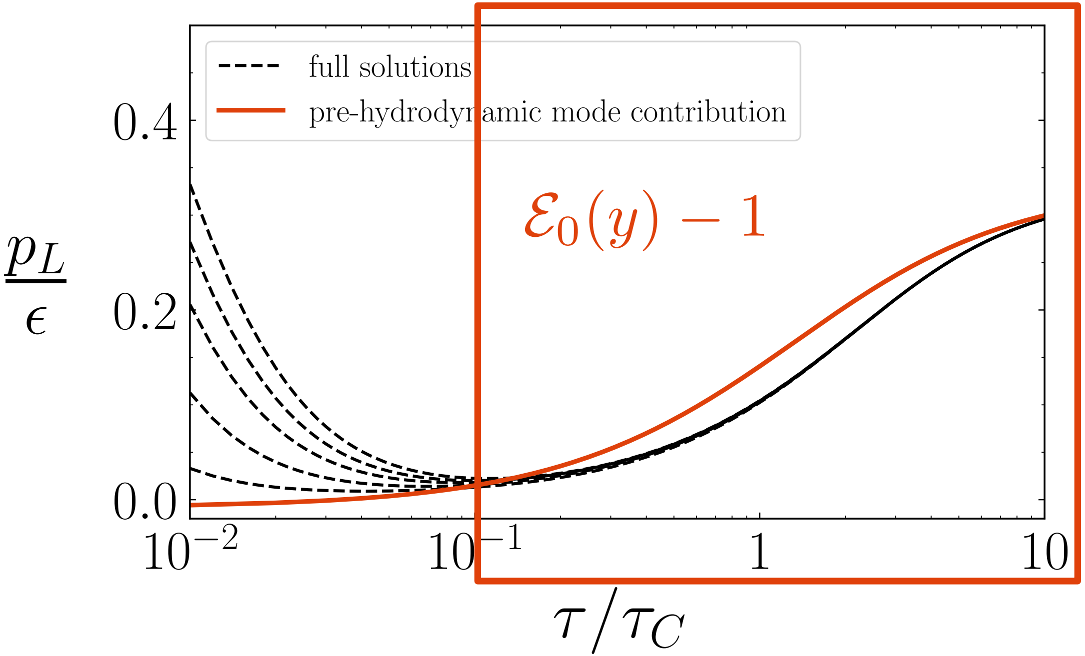
Predicts non-trivial relations between components of ψ

$$\partial_y \psi = -\mathcal{H}(y)\psi \longrightarrow \partial_y \phi_0(y) = -\mathcal{E}_0(y)\phi_0(y)$$

e.g. $$g(y) \equiv 1 + p_L/\epsilon = \mathcal{E}_0(y)$$

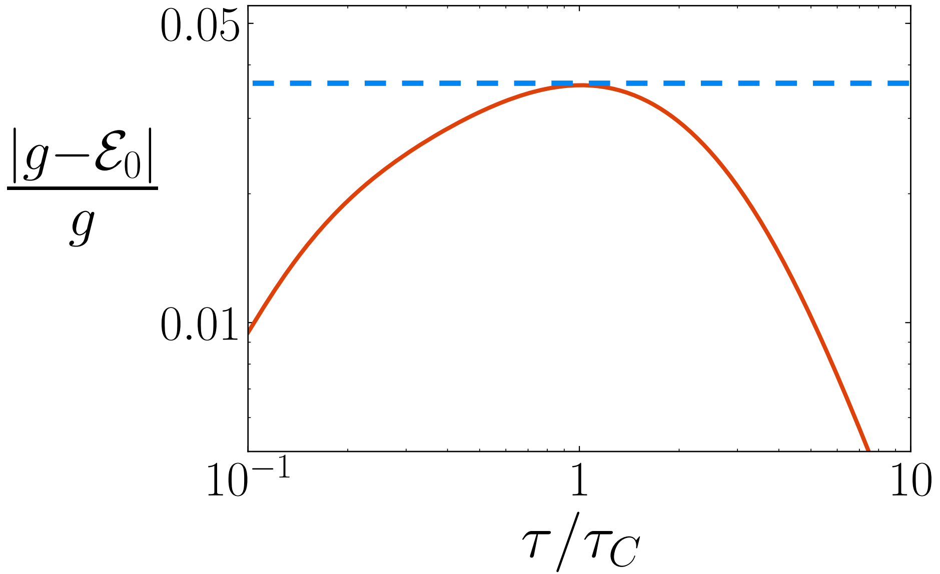
Test extent to which these relations are satisfied!

Hydrodynamization in RTA kinetic theory is adiabatic!



Far-from-equilibrium evolution dominated just by evolution of instantaneous ground state ("pre-hydrodynamic") mode!

Implies presence of a small "adiabatic" parameter that suppresses contributions from other modes



$>$ 95% of g described by instantaneous ground state mode

Why would the rapidly-expanding QGP be adiabatic?

Suppression of excited states:

$$\delta_A \sim \frac{\partial_\tau \log \lambda}{\Delta E_n} \langle 0(\tau)|H|n(\tau)\rangle$$

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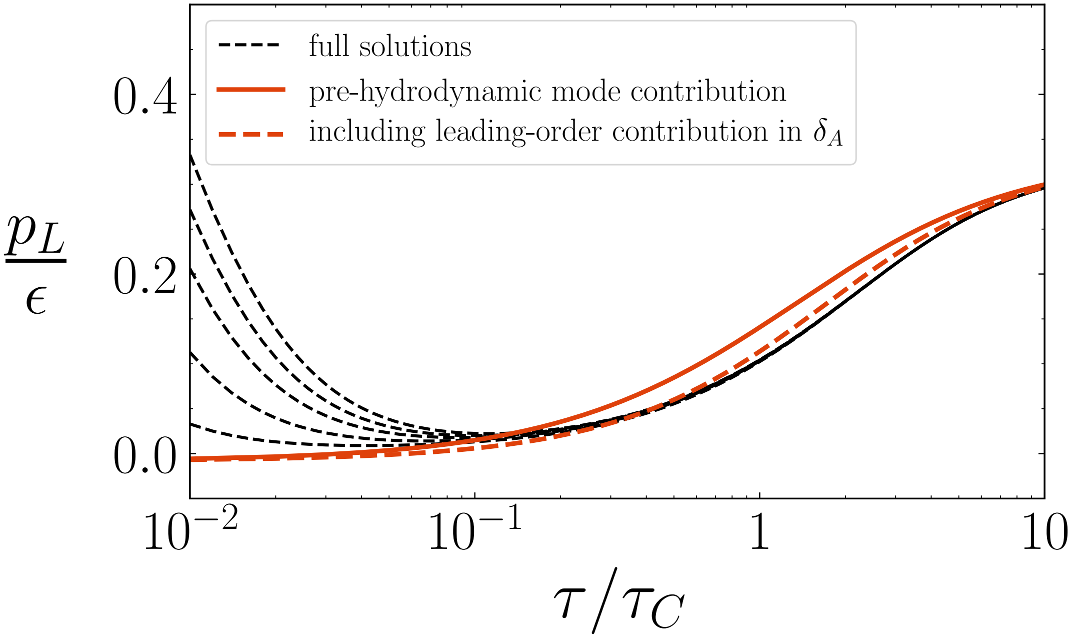
Hamiltonian evolves slowly compared to energy gap

Small overlap between ground and excited states

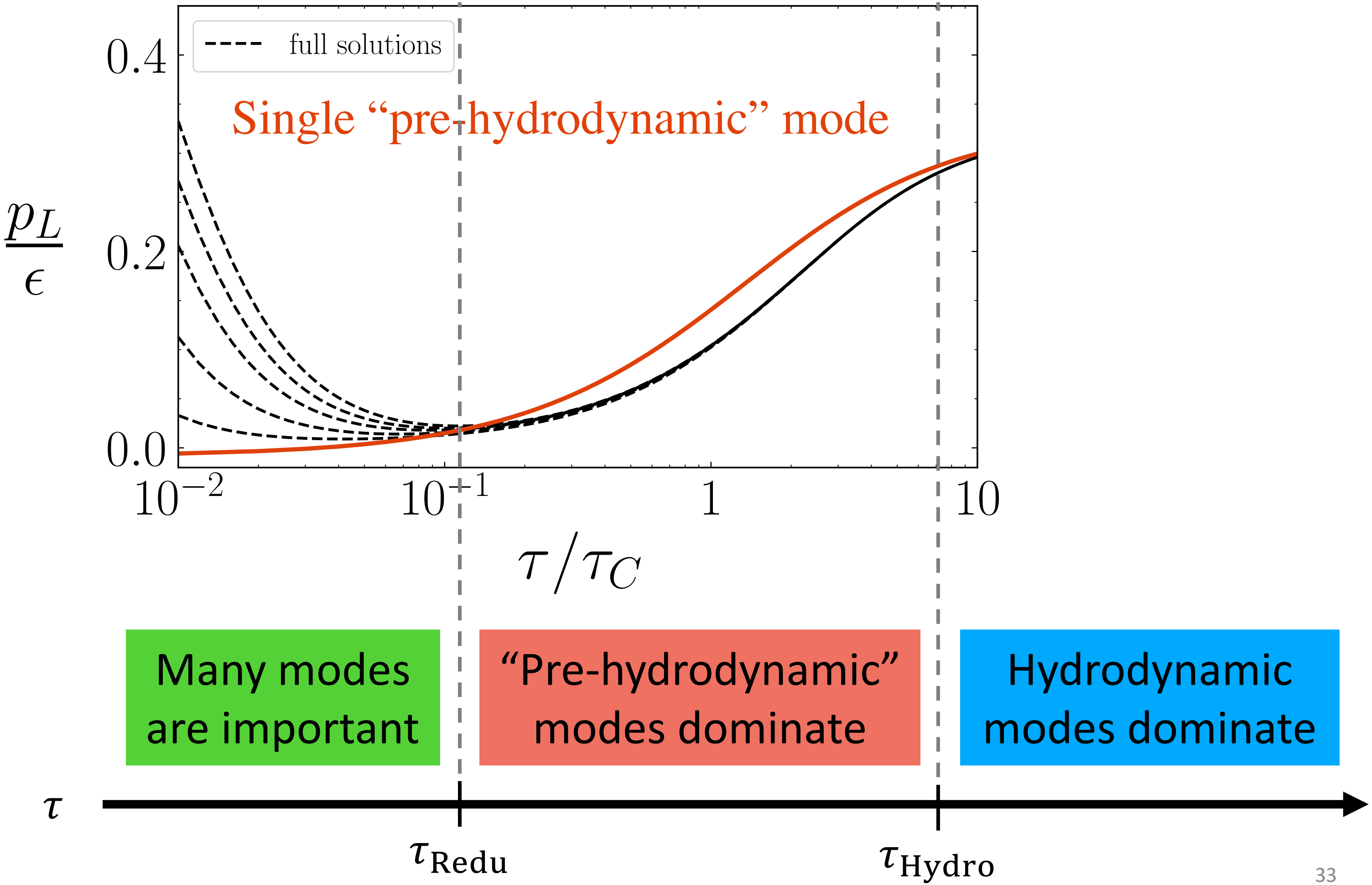
δ_A small near hydro limit

δ_A small at early times

A new perturbative expansion



Including contributions from the excited states to the evolution at $\mathcal{O}(\delta_A)$ shows explicitly that they are a small correction



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