A new look at Hydrodynamic Attractors

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November 27, 2019

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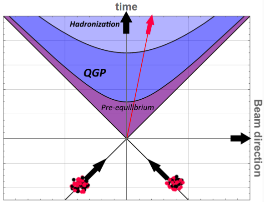
In this talk: Bjorken flow, BRSSS, conformal



Bjorken flow

- $ds^2 = -d\tau^2 + \tau^2 dx^2 + dy^2 + dz^2$

BRSSS, conformal hydrodynamics [Baier, Romatschke, Son, Starinets, Stephanov]

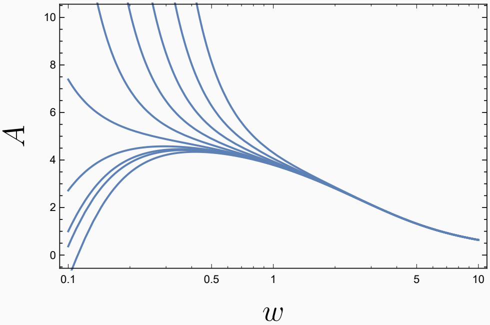
- Second order equation for temperature $T(\tau)$
- First order equation for pressure anisotropy $A(w)$, where $w \equiv \frac{\tau}{\tau_{\text{rel}}} = \tau T$

$$\frac{1}{12} wA(w)A'(w) + wA'(w) + \frac{C\lambda 1 wA(w)^2}{8C\eta^2 C\tau\Pi} + \frac{3wA(w)}{2C\tau\Pi} + \frac{A(w)^2}{3} - \frac{12C\eta}{C\tau\Pi} = 0$$

$$A'(w) = F[A(w), w]$$

The BRSSS attractor [Heller,Spalinski - 1503.07514]

- Solutions

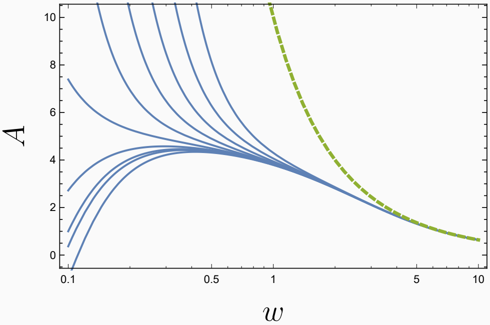


Solutions to the BRSSS equations for the pressure anisotropy

$$A'(w) = F[A(w), w]$$

The BRSSS attractor [Heller,Spalinski - 1503.07514]

- Solutions
- Gradient expansion



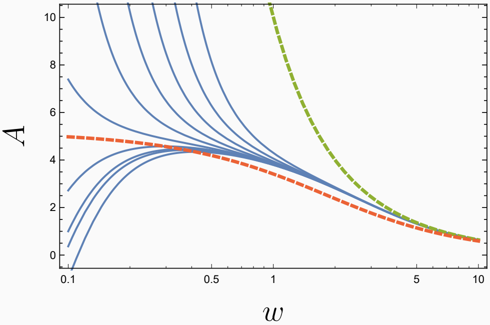
Gradient Expansion (= late time expansion)

$$A(w) = \sum_k \frac{A_k}{w^k}$$

- Can be solved to very high orders (but diverges)
- Describes solutions asymptotically as $w \to \infty$

The BRSSS attractor [Heller,Spalinski - 1503.07514]

- Solutions
- Gradient expansion
- Slow-roll



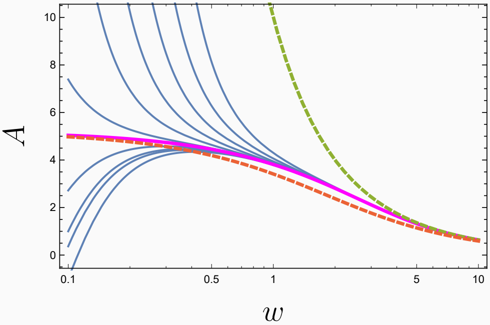
Slow-roll

$$A'(w) = F[A_{\text{slow-roll}}(w), w] = 0$$

- Can be improved in a series expansion, here we stick to zeroth order

The BRSSS attractor [Heller,Spalinski - 1503.07514]

- Solutions
- Gradient expansion
- Slow-roll
- Attractor/regular solution



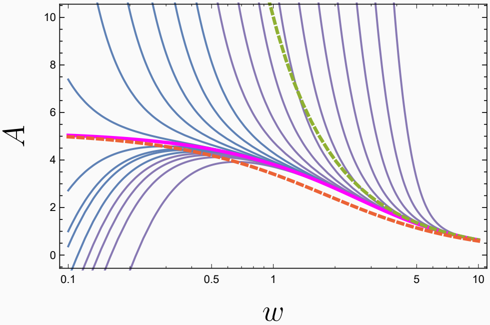
The attractor/regular solution

$$\lim_{w \to 0} A(w) \text{ is finite}$$

- Close to slow-roll
- Solutions decay to it, even before the gradient expansion

The BRSSS attractor [Heller,Spalinski - 1503.07514]

- Solutions
- Gradient expansion
- Slow-roll
- Attractor/regular solution

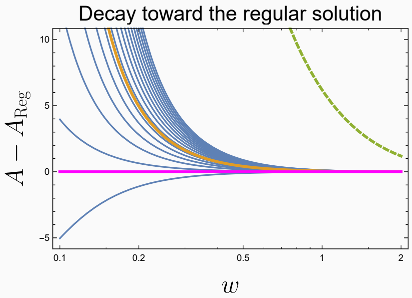


The attractor/regular solution

$$\lim_{w \to 0} A(w) \text{ is finite}$$

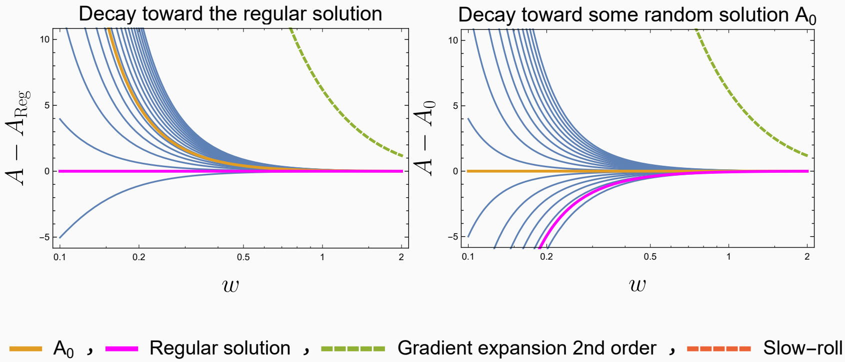
- Close to slow-roll
- Some solutions decay to it, even before the gradient expansion

Is the attractor more attractive than others?



A_0 , Regular solution , Gradient expansion 2nd order , Slow–roll

Is the attractor more attractive than others?



The regular solution attracts in the same way as every other solution

Which solution is the most attractive?

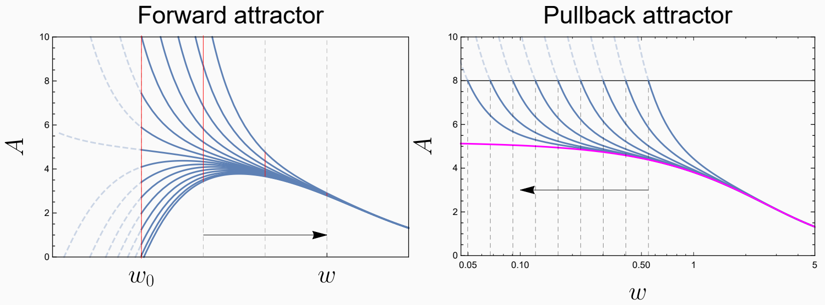
Attraction depends on choice of metric
Usually left implicit as flat metric in plot variables.

For each w, distance is $|A_1(w) - A_2(w)|$

Solutions do not depend on choice of metric
Attraction and repulsion are not intrinsic properties of solutions

Attractors for non-autonomous dynamical systems [Kloeden, Rasmussen - Nonautonomous dynamical systems]

In hydro [Behtash, Kamata, Martinez, Shi - 1911.06406 + earlier papers]



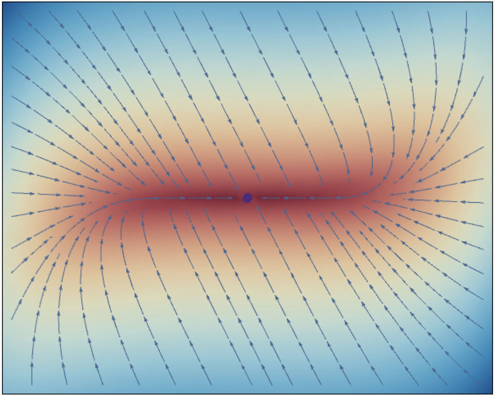
Every solution

Regular solution

Pullback attractor needs $w \to 0$ limit and boundedness

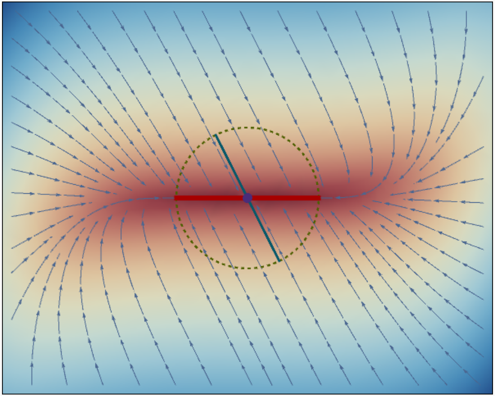
Center manifold captures asymptotic dynamics

- Fixed point



Center manifold captures asymptotic dynamics

- Fixed point
- Linear regime - dynamics determined by eigenvectors of a matrix
- Stable subspace - negative eigenvalue
- Center subspace - vanishing eigenvalue



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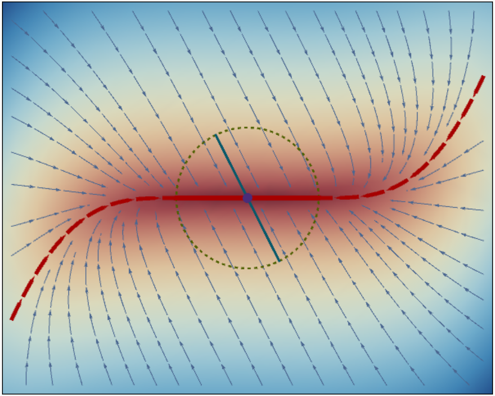

Center manifold - defined by matching onto the center subspace

Center manifold captures asymptotic dynamics

- Fixed point
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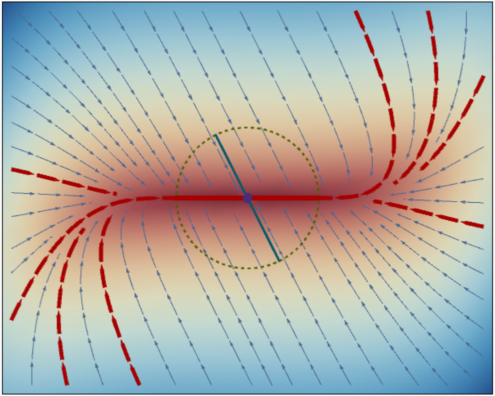


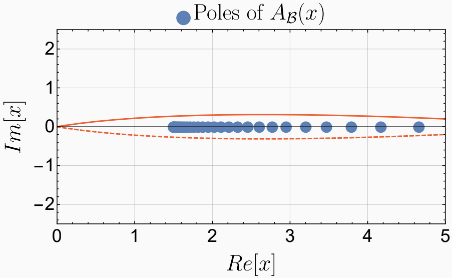
Center manifold - defined by matching onto the center subspace

Perturbative matching ⇒ non-perturbative ambiguities

Resummation of gradient expansion is not unique [Basar, Dunne - 1509.0504]

$$A(w) = \sum_k \frac{A_k}{w^{k+1}}$$
$$= \int_0^\infty dx\, e^{-xw} A_{\mathcal{B}}(x)$$

where $A_{\mathcal{B}}(x) = \sum_k \frac{A_k}{k!} x^{k+1}$



Resummation gives family of solutions

$$A_\sigma(w) - A'_\sigma(w) \sim (\sigma - \sigma')e^{-\frac{3}{2}w}$$

The amplitude σ of non-hydro modes is free

Is there anything that selects a preferred σ?
Not in the large w regime

Modifying the expansion rate

Complementary analysis to [Kurkela, Schee, Wiedemann, Wu - 1907.08101] for BRSSS

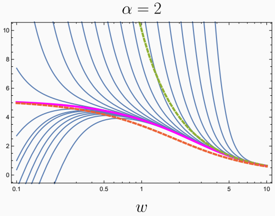
$$ds^2 = -d\tau^2 + g(\tau)dx^2 + dy^2 + dz^2$$

$$g(\tau) = \tau^\alpha$$

- $\alpha = 0$ is flat
- $\alpha = 2$ is Bjorken flow
- Some kind of transition at $\alpha = 6$

τ^α expansion: Slow and fast limits

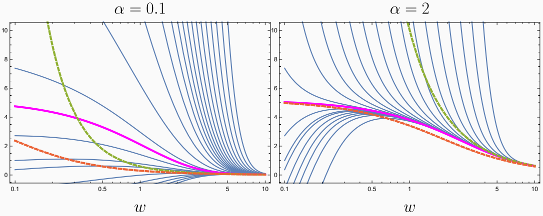
Solutions Gradient Expansion Slow-roll Regular



Bjorken

τ^α expansion: Slow and fast limits

Solutions Gradient Expansion Slow-roll Regular

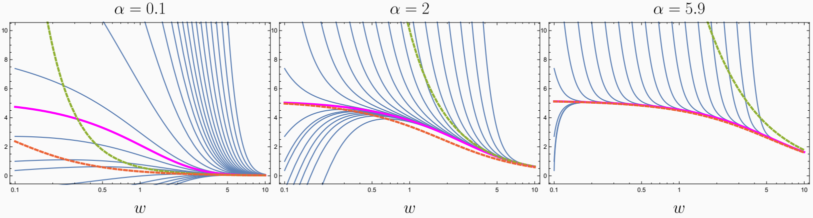


Slow expansion: convergence to gradient expansion

Bjorken

τ^{α} expansion: Slow and fast limits

Solutions Gradient Expansion Slow-roll Regular



Slow expansion: convergence to gradient expansion

Bjorken

Fast expansion: convergence to Regular and Slow-roll

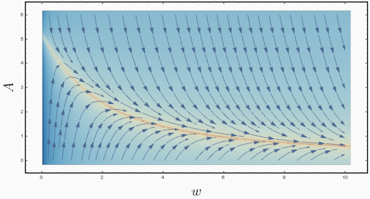
Slow-roll is not an approximation to the regular solution. The regular solution is an approximation to slow-roll!

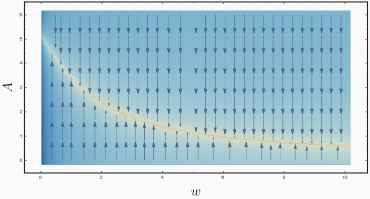
- Slow-roll is defined locally at each w
- Identifies a region in phase space, rather than a solution
- Easy to generalize to higher dimensional phase spaces

Slow-roll and adiabatic hydrodynamization [Brewer, Yan, Yin - 1910.00021], [Blaizot, Yan - 1904.08677]

Adiabatic approximation
A evolves much faster than w

$$A'(w, w_\star) = F[A(w), w_\star]$$

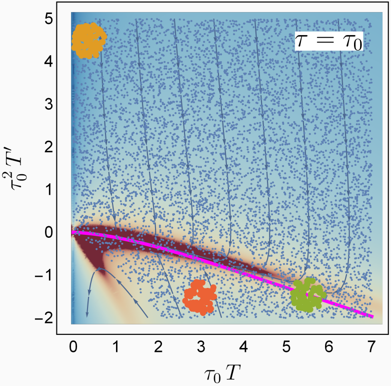




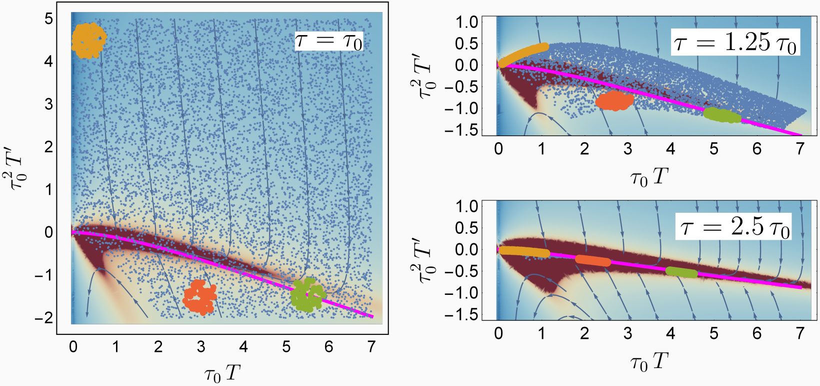
For each $w_\star$, the system evolves to a fixed point where

$$A'(\infty, w_\star) = 0 = F[A_{\text{slow-roll}}(w_\star), w_\star]$$
$$A(w, w_\star) = A_{\text{slow-roll}}(w_\star) + \sigma e^{-\lambda(w_\star) w}$$
$$A_{\text{adiabatic}}(w) = \underbrace{A_{\text{slow-roll}}(w)}_{A_{\text{pre-hydro}}} + \underbrace{\sigma e^{-\lambda(w) w}}_{A_{\text{pre-non-hydro}}}$$

BRSSS with T and τ: Phase space is two-dimensional



BRSSS with T and τ: Phase space is two-dimensional



- Two-dimensional clouds become one-dimensional
- Hard to visualize for higher dimensions, but can be quantified using e.g. PCA
- End up in the slow region

Summary

Attractor from late time regime. :(
Resummation of gradient expansion / Center manifold / Forward attractor

Attractor from $w = 0$. : |
Pullback attractor, but this requires singular limit $w \to 0$

Attractor from slow-roll/adiabatic hydrodynamization. :)
Works at any w

- Expansion is important for attractor beyond gradient expansion [Blaizot, Yan - 1904.08677], [Kurkela, Schee, Wiedemann, Wu, 1907.08101]
- Attraction is not an intrinsic property of a solution, need metric on phase space
- Phase space can show the attractor without relying on A(w)
- Phase space may have higher dimensional attractors

Attractors in dynamical systems: Autonomous case

Include w as a state variable to make the system autonomous

$$\frac{\partial A}{\partial s} = F[A(s), w(s)]$$
$$\frac{\partial w}{\partial s} = 1$$

Attractors of autonomous systems: fixed points, periodic cycles

Attractors in dynamical systems: Autonomous case

Include w as a state variable to make the system autonomous

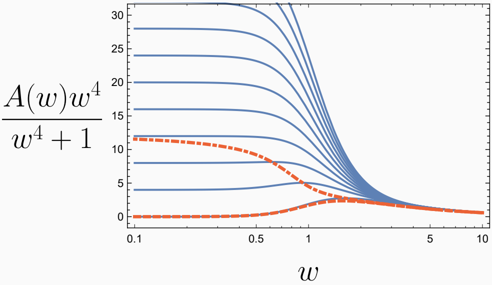
$$\begin{aligned}\frac{\partial A}{\partial s} =& F[A(s), w(s)] \\ \frac{\partial w}{\partial s} =& 1\end{aligned}$$

Attractors of autonomous systems: fixed points, periodic cycles

In this setting, the attractor is thermal equilibrium
Fixed point at $w = \infty\,, A = 0$

Dependence on parametrization

Non-linear changes of variables or mixing of time (w) and state (A) variables changes things



Slow-roll for A and for $\frac{A(w)w^4}{w^4+1}$