Ocean (Climate) Models
or
State of the Art in Ocean (Climate) Modeling

Workshop on Physics-Dynamics Coupling in ESMs

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CIMES

NOAA GFDL

Questions added under "Oceans" in document

- What is the total energy budget in the component? What are the fluxes in/out of the system? What are the energy source/sinks due to numerical errors? Where are the spurious source/sinks of energy put?
- Coupling (frequency, …): Which quantities are communicated between components? Are quantities missing? Consequences of having components on different grids? Are they using different time-steps? Dynamics and physics on different grids? Wind stress mapping? Error propagation between components. What is latent heat flux (physical understanding)?
- Ocean physics parameterizations: time-integration and conservation: physics-dynamics coupling, dycore time-stepping, component coupling
- Water cycle: mass exchange between components and associated heat exchange, what processes are we missing (for example, enthalpy of falling rain)

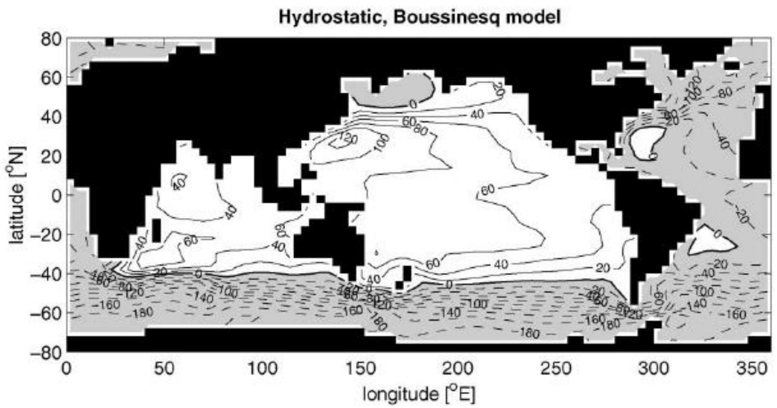
Outline

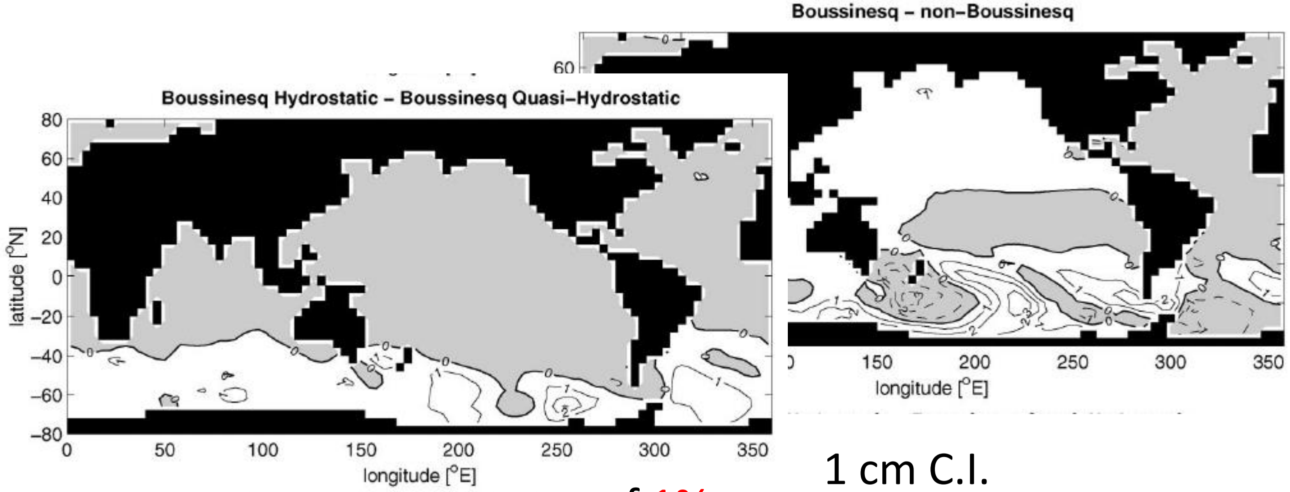
1. Some equations (because you asked)
2. Survey of methods used in ocean dynamical cores
3. Splitting and sub-cycling
4. Energy budget of ocean (to set up Remi)
5. Spurious mixing and the energy budget that matters
   – using energy to diagnose magnitude of problem
   – consequences for heat uptake and climate
6. Coupling the ocean and sea-ice

Approximations

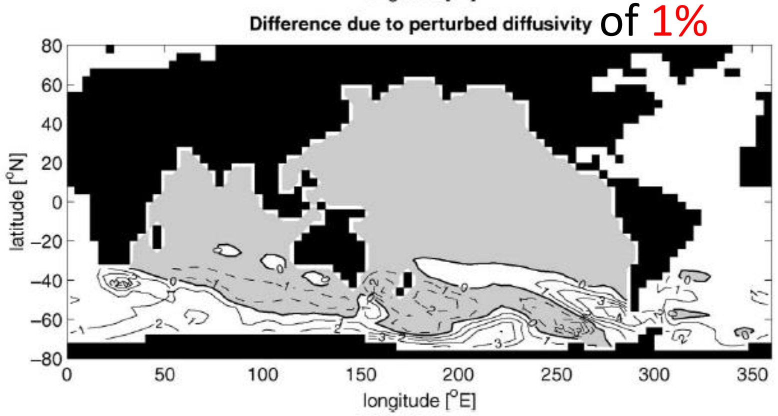
- Shallow ocean
  - Ocean is thin relative to radius of planet
- Hydrostatic balance
  - Non-hydrostatic motions normally associated with overturning (aspect ratio 1)
  - Systematic effects (non-overturning) are still small
- Boussinesq approximation
  - Avoids sound in the external mode
  - Avoids sound waves in non-hydrostatic models

20 cm C.I.





1 cm C.I.



Losch et al., 2004

Equations of oceanic motion

- Horizontal momentum

$$\rho_0\left(\frac{\partial \mathbf{u}}{\partial t} + (f+\zeta)\,\hat{\mathbf{z}}\wedge\mathbf{u} + w\frac{\partial \mathbf{u}}{\partial z} + \nabla_z K\right) = -\nabla_z p - \rho\nabla_z\Phi + \mathcal{F}$$

- Hydrostatic balance

$$\left(K = \frac{1}{2}\boldsymbol{u}.\boldsymbol{u} \text{ and } \zeta = \nabla\times\boldsymbol{u}\right)$$

$$\rho\frac{\partial \Phi}{\partial z} + \frac{\partial p}{\partial z} = 0$$

- Non-divergence

$$\nabla_z\cdot\mathbf{u} + \frac{\partial w}{\partial z} = 0$$

- Conservation of heat

$$\frac{\partial \theta}{\partial t} + \nabla_z\cdot(\theta\,\overset{*}{\mathbf{u}}) + \frac{\partial(\theta\,w)}{\partial z} = \boldsymbol{N}^{\gamma}_{\theta} - \frac{\partial J^{(z)}_{\theta}}{\partial z}$$

- Conservations of salts

$$\frac{\partial S}{\partial t} + \nabla_z\cdot(S\,\overset{*}{\mathbf{u}}) + \frac{\partial(S\,w)}{\partial z} = \boldsymbol{N}^{\gamma}_{S} - \frac{\partial J^{(z)}_{S}}{\partial z}$$

- Equation of state

$$\rho = \rho(S, \theta, -g\,\rho_0\,z)$$

- Free-surface

$$\frac{\partial \eta}{\partial t} + \nabla\cdot\left(\int_{-H}^{\eta}\mathbf{u}\,dz\right) = P - E$$

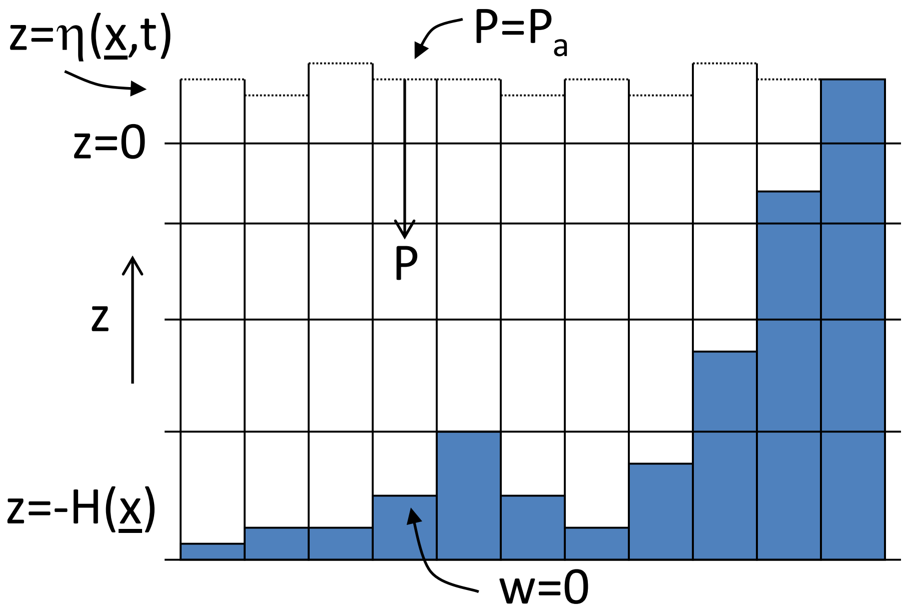
GM "physics"

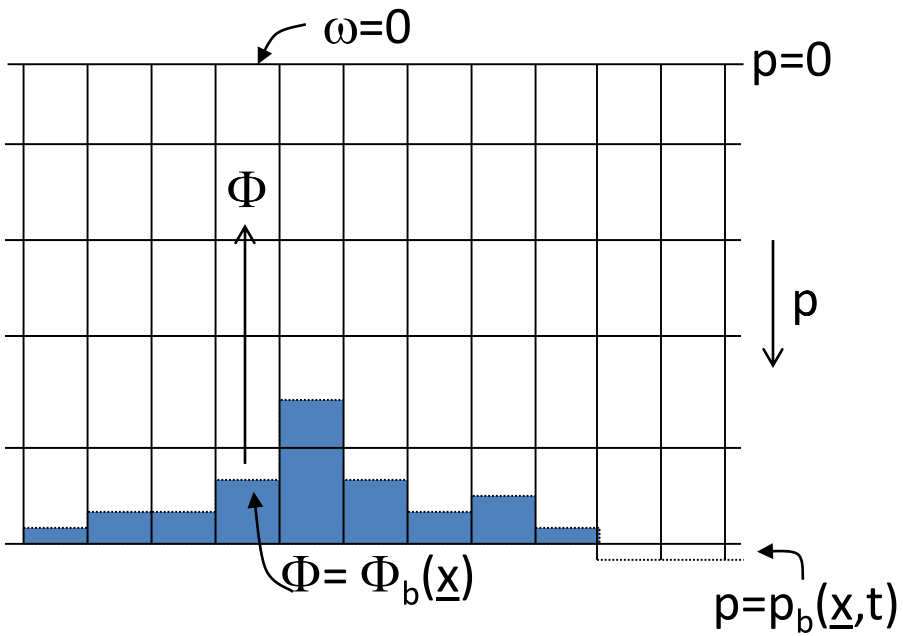
Neutral "physics"

Vertical "physics"

Equations of oceanic motion: the z-p Isomorphism

Boussinesq (z coordinates)	$z \leftrightarrow p$	Atmosphere (p coordinates)
$d_t\underline{v} + f \times \underline{v} + \underline{\nabla}_z P/\rho_o = \underline{F}$	$P \leftrightarrow \Phi$	$d_t\underline{v} + f \times \underline{v} + \underline{\nabla}_p \Phi = \underline{F}$
$g\rho + \partial_z P = 0$	$\rho \leftrightarrow \alpha$	$\alpha + \partial_p \Phi = 0$
$\underline{\nabla}_h \cdot \underline{v} + \partial_z w = 0$	$w \leftrightarrow \omega$	$\underline{\nabla}_p \cdot \underline{v} + \partial_p \omega = 0$
$d_t\theta = Q$	θ	$d_t\theta = Q$
$d_t s = S$	$s \leftrightarrow q$	$d_t s = S$
$\partial_t \eta + \underline{\nabla}\cdot(\eta + H)\underline{v} = P\text{-}E$	$\eta + H \leftrightarrow p_s$	$\partial_t p_b + \underline{\nabla}\cdot p_b \underline{v} = P\text{-}E$





Ocean Models (for Climate)

- MPAS-Ocean
- NEMO
- FESOM
- MOM6
- ICON-O
- MICOM
- POP
- MOM5
- HYCOM
- MITgcm
- ROMS

- All models **conserve heat, salt, and either mass or volume**
  - Models differ in conservation of momentum (angular/linear), PV, KE, enstrophy, …
  - Mimetic discretizations are typical
- All use hydrostatic approximation (in global mode)
- Most are still Boussinesq
  - Although many have non-Boussinesq option
- All are explicit in time for baroclinic equations
- All treat the external mode separately
- All stagger in space
  - but use same grid for dynamics/physics

Global ocean dynamic cores

	Method	Hor. grid	Vertical method	Coord.	External mode	Mom. eqns	Time integr.	Mom. transport	Tracer transport
MPAS-Ocean	FV, TRSK	Voronoi	ALE	z*	Split expl	VI	PC CN		FCT SG2011
NEMO	FV	C-grid	ALE	z~, s~	Split expl	FF or VI	MLF	FCT2 UP3	FCT2/4, UP3, Q
FESOM	FE/FV	Tri B-grid	ALE	z-σ	Semi-impl	FF or VI			PPM, PSM
MOM6	FV	C-grid	Lagr-remap	z-ρ	Split expl	VI	RK2	C2	PLM,PPM
ICON-O	FE?	Tri C-grid		z		VI	AB2		
MICOM	FD	C-grid	Layered	p-ρ	Split expl	VI	LF		
POP	FD	B-grid	Eulerian	z		FF			
MOM5	FD	B-grid	Eulerian	z*	Split expl	FF			U3, Q
HYCOM	FD	C-grid	Lagr-remap	p-ρ					PLM
MITgcm	FV (NH)	C-grid	Eulerian	z*	Semi-impl	FF or VI	AB3	C2 C4	U3, U7, …
ROMS	FD	C-grid	Eulerian	σ	Split expl				

Modes of motion

$$\rho_0\left(\frac{\partial \mathbf{u}}{\partial t} + (f+\zeta)\hat{\mathbf{z}}\wedge\mathbf{u} + w\frac{\partial \mathbf{u}}{\partial z} + \nabla_z K\right) = -\nabla_z p - \rho\nabla_z\Phi + \mathcal{F}$$

$g\rho_o\eta$

$$\rho\frac{\partial \Phi}{\partial z} + \frac{\partial p}{\partial z} = 0$$

$$\nabla_z\cdot\mathbf{u} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial \theta}{\partial t} + \nabla_z\cdot(\theta\,\mathbf{u}) + \frac{\partial(\theta\,w)}{\partial z} = \mathcal{N}_\theta^\gamma - \frac{\partial J_\theta^{(z)}}{\partial z}$$

$$\frac{\partial S}{\partial t} + \nabla_z\cdot(S\,\mathbf{u}) + \frac{\partial(S\,w)}{\partial z} = \mathcal{N}_S^\gamma - \frac{\partial J_S^{(z)}}{\partial z}$$

$$\rho = \rho(S,\theta,-g\,\rho_0\,z)$$

$$\frac{\partial\eta}{\partial t} + \nabla\cdot\left(\int_{-H}^{\eta}\mathbf{u}\,dz\right) = P - E$$

- $f\sim10^{-4}$ 1/s ; $g\sim10$ m/s ; $H\sim6000$ m ; $\frac{N}{f}\sim10$

  $u\sim2$ m/s

  - $\sqrt{gH}\sim$ 250 m/s $NH\sim$5 m/s

- $\Delta x\sim$100 km | $\Delta x\sim$10 km
  - $\Delta t_{gH}\sim$400 s | $\Delta t_{gH}\sim$40 s
  - $\Delta t_f\sim$1 hr | $\Delta t_f\sim$1 hr
  - $\Delta t_{NH}\sim$5 hr | $\Delta t_{NH}\sim$30 mn
  - $\Delta t_u\sim$12 hr | $\Delta t_u\sim$1 hr

Typical Eulerian algorithm

$$\delta_k p = -\rho(z, S^n, \theta^n)\delta_k \Phi$$

$$v_h^{m+1} = v_h^m + \frac{\Delta t}{\rho_o}(-\nabla_\mathrm{h} p + \cdots)$$

$$U^{l+1} = U^l + \tfrac{1}{L}\Delta t\left(-\nabla \eta^l + \cdots\right)$$

$$\eta^{l+1} = \eta^m - \tfrac{1}{L}\Delta t \nabla_\mathrm{r} \cdot (H U^{l+1})$$

$\times L$

$$\delta_k w^{m+1} = -\nabla_h \cdot v_h^{m+1}$$

$$C^{n+1} = C^n - \Delta t\left[\nabla \cdot h^m v_h^{m+1} C^n\right]$$

Internal gravity waves: $\frac{\Delta t c_{ig}}{\Delta x} < 1$

Barotropic gravity waves: $\frac{\Delta t\sqrt{gH}}{L\Delta x} < 1$

CFL: $\frac{\Delta t u_h}{\Delta x} < 1$

- Barotropic-baroclinic split usually designed for consistency with baroclinic equations
  - Energetics of split system are normally not a primary consideration?
- Alternative is to make the free-surface implicit in time (Dukowicz, 1994 ; MITgcm)

Shchepetkin & McWilliams, 2005

General coordinates $r = r(x, y, z, t)$

$$\rho_0 \left(\frac{\partial \mathbf{u}}{\partial t} + (f + \zeta)\,\hat{\mathbf{z}} \wedge \mathbf{u} + \dot{r}\,\frac{\partial \mathbf{u}}{\partial r} + \nabla_r K \right) = -\nabla_r\, p - \rho \nabla_r\, \Phi + \mathcal{F}$$

$$\rho\,\frac{\partial \Phi}{\partial r} + \frac{\partial p}{\partial r} = 0$$

$$\frac{\partial z_r}{\partial t} + \nabla_r \cdot (z_r\,\mathbf{u}) + \frac{\partial (z_r\,\dot{r})}{\partial r} = 0$$

$$z_r = \frac{\partial z}{\partial r}$$

$$\frac{\partial (\theta\, z_r)}{\partial t} + \nabla_r \cdot (\theta z_r\,\mathbf{u}) + \frac{\partial (\theta\, z_r\,\dot{r})}{\partial r} = z_r \boldsymbol{N}_\theta^\gamma - \frac{\partial J_\theta^{(z)}}{\partial r}$$

$$\frac{\partial (S\, z_r)}{\partial t} + \nabla_r \cdot (S\, z_r\,\mathbf{u}) + \frac{\partial (S\, z_r\,\dot{r})}{\partial r} = z_r \boldsymbol{N}_S^\gamma - \frac{\partial J_S^{(z)}}{\partial r}$$

$$\rho = \rho\,(S, \theta, -g\rho_0 z(r))$$

Starr, 1945; Kasahara, 1974; …

Lagrangian method in the vertical

$$g\rho_o\eta$$

$$\rho_0\left(\frac{\partial \mathbf{u}}{\partial t} + \frac{(f+\zeta)}{h}\hat{\mathbf{z}}\wedge h\,\mathbf{u} + \dot{r}\,\frac{\partial \mathbf{u}}{\partial r} + \nabla_r K\right) = -\nabla_r\, p - \rho\nabla_r\,\Phi + \mathcal{F}$$

$$\rho\,\delta_r\Phi + \delta_r p = 0$$

$$\frac{\partial h}{\partial t} + \nabla_r\cdot(h\,\mathbf{u}) + \delta_r(z_r\dot{r}) = 0$$

$$\frac{\partial(\theta\,h)}{\partial t} + \nabla_r\cdot(\theta h\,\mathbf{u}) + \delta_r(\theta\,z_r\dot{r}) = h\boldsymbol{N}_\theta^\gamma - \delta_r J_\theta^{(z)}$$

$$\frac{\partial(S\,h)}{\partial t} + \nabla_r\cdot(S\,h\,\mathbf{u}) + \delta_r(S\,z_r\dot{r}) = h\boldsymbol{N}_S^\gamma - \delta_r J_S^{(z)}$$

$$\rho = \rho\,(S,\theta,-g\rho_0 z(r))$$

$$\frac{\partial\eta}{\partial t} + \nabla\cdot\left(\int_{-H}^{\eta}\mathbf{u}\,dz\right) = P - E$$

Use $\dot{r} = 0$ intermittently followed by vertical remap

Sub-cycling in a Lagrangian-remap algorithm

$$\delta_k p = -\rho(z, S^n, \theta^n)\delta_k \Phi$$

$$v_h^{m+1} = v_h^m + \frac{1}{M}\frac{\Delta t}{\rho_o}(-\nabla_r p - \rho \nabla_r \Phi + \cdots)$$

$$h^{m+1} = h^m - \frac{1}{M}\Delta t \nabla_r \cdot (h^m v_h^{m+1})$$

Internal gravity waves $\frac{\Delta t c_{ig}}{\Delta x} < 1$

$$U^{l+1} = U^l + \frac{1}{L}\Delta t\left(-\nabla \eta^l + \cdots\right)$$

$$\eta^{l+1} = \eta^m - \frac{1}{L}\Delta t \nabla_r \cdot (H U^{l+1})$$

Barotropic gravity waves $\frac{\Delta t \sqrt{gH}}{L \Delta x} < 1$

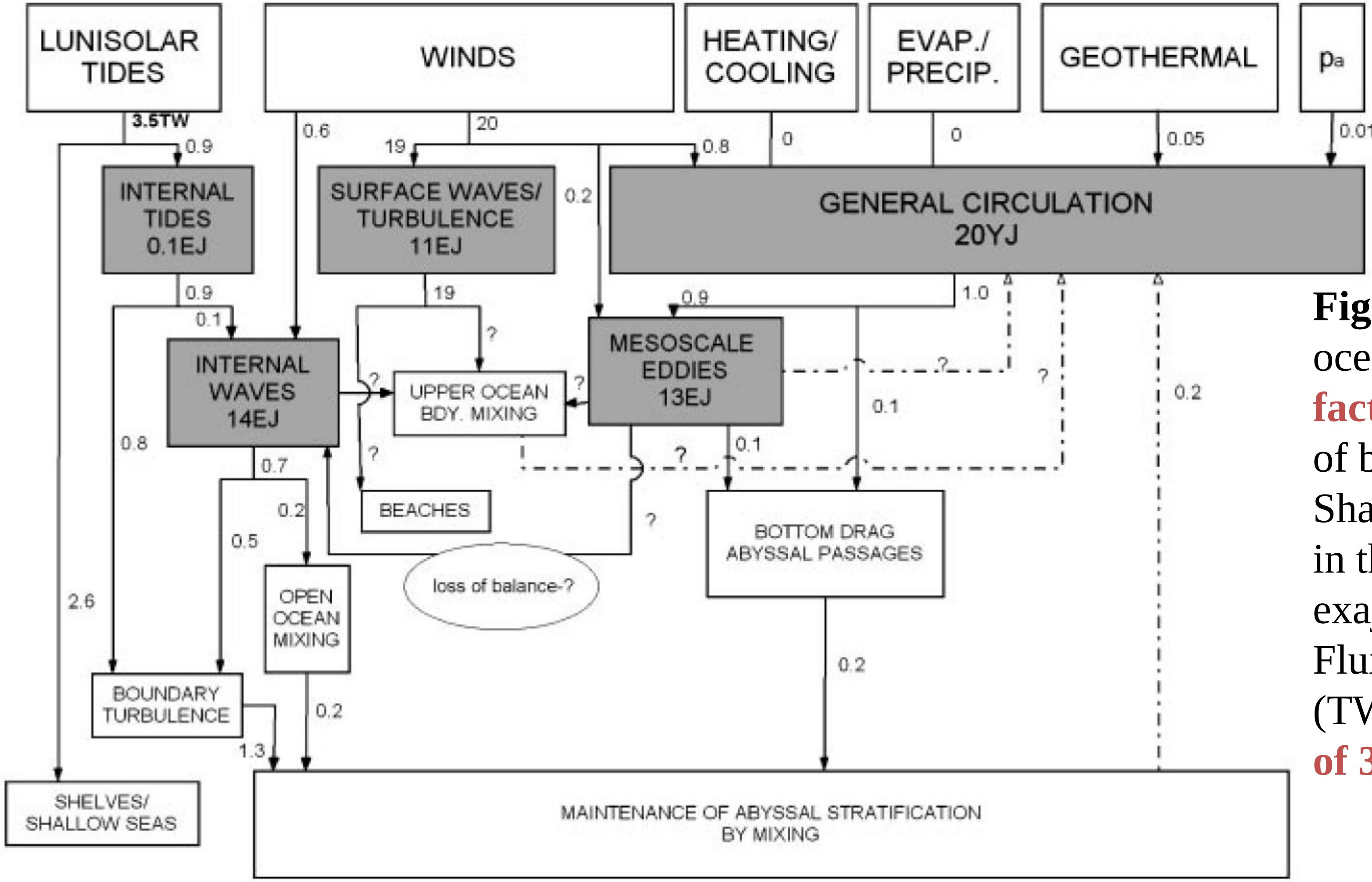
$$h^* C^* = h^n C^n - M\Delta t \left[\nabla \cdot \left(\sum_{m=1}^{M} h^m v_h^{m+1} C^n\right)\right]$$

Tracers $\frac{M \Delta t u_h}{\Delta x} < 1$

$$h^{n+1} \leftarrow \delta_k Z(h^*)\,;\, C^{n+1} = C^*(Z(h^*))\,;\, \ldots$$

Vertical remap

What is the total energy budget in the component?



10 EJ / 10 TW = 11.5 days
20 YJ / 1 TW = 630,000 years

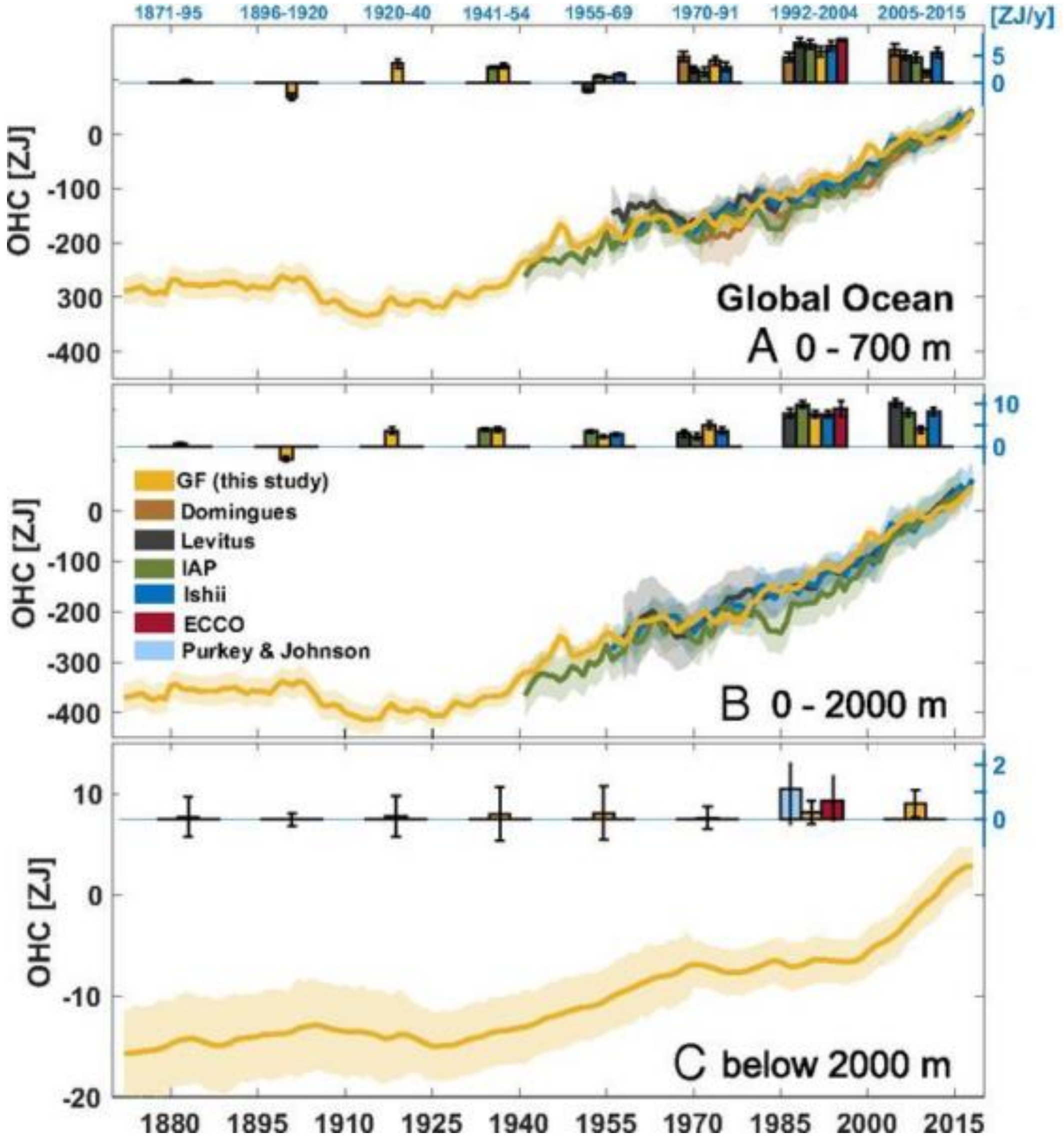
Figure 5 Strawman energy budget for the global ocean circulation, with **uncertainties of at least factors of 2 and possibly as large as 10.** Top row of boxes represent possible energy sources. Shaded boxes are the principal energy reservoirs in the ocean, with crude energy values given [in exajoules (EJ) 10^{18} J, and yottajoules (YJ) 10^{24} J]. Fluxes to and from the reservoirs are in terrawatts (TWs). **Tidal input (see Munk & Wunsch 1998) of 3.5 TW is the only accurate number here.** …

Wunsch & Ferrari, 2004

This picture is all about mixing

Ocean Heat Uptake

Zanna et al, 2019



- By taking up heat, the ocean reduces the transient climate response (TCR)
- OHU is has been hard to reconstruct
  - Absence of data
  - Models have unknown **spurious heat uptake** (due to numerical diffusion)
  - Models do not properly represent processes that govern OHU (eddies)
  - The real world OHU is governed by a balance between mixing and eddies
- OHU of 4 ZJ/yr ~ 120 TW ~ 0.35 W/m2

Quantifying spurious mixing using energetics

- Potential energy

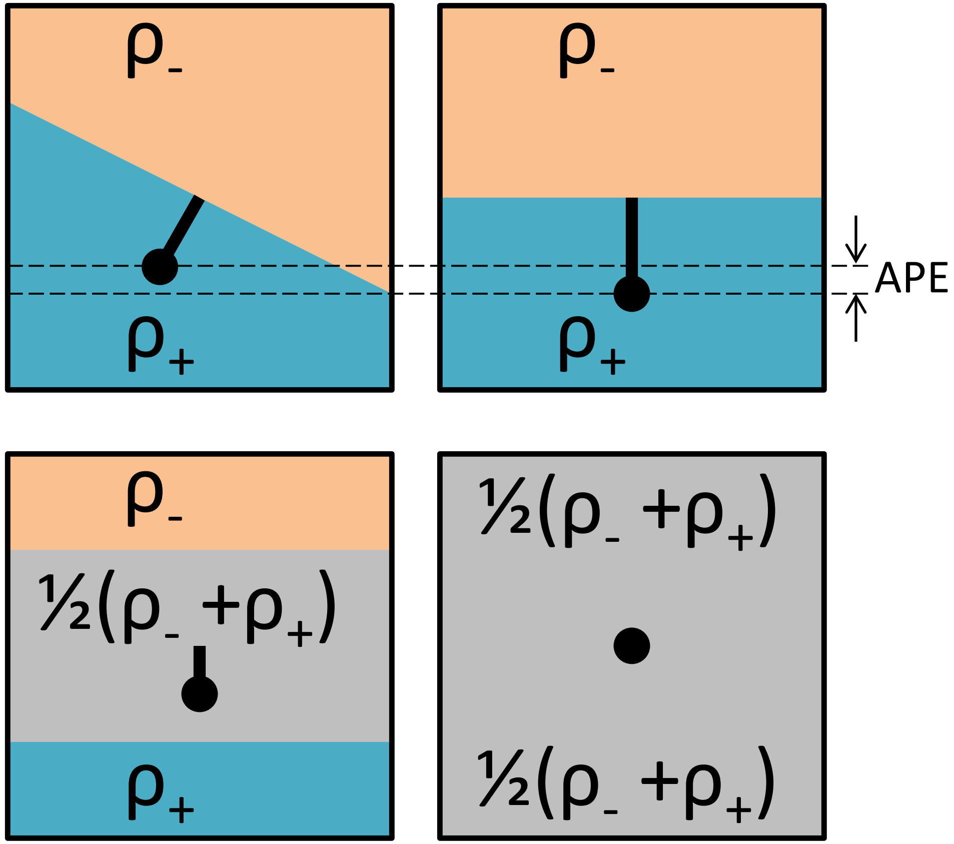
$$PE = g \iiint \rho z \, dV$$

- Available potential energy (APE)

$$APE = PE - RPE$$

$$RPE = g \iiint \rho^* z \, dV$$

- ρ^* is the adiabatically re-arranged state with minimal potential energy
- **RPE can only be changed by diapycnal mixing**
  - Mixing raises center of mass



Winters et al., JFM 1995
Ilicak et al., OM 2012

Global spin-down

- CM2G is the "right amount of mixing"
- MOM5 1°
  - $\kappa_V=0$ about 20% of CM2G
  - Very acceptable IMHO
- MOM5 ¼°
  - $\kappa_V=0$ as large as CM2G
- POP 1°
  - !!!
- MPAS-O
  - $\tilde{z}$–coordinate
  - Is this convergence, or good choice of dissipation?

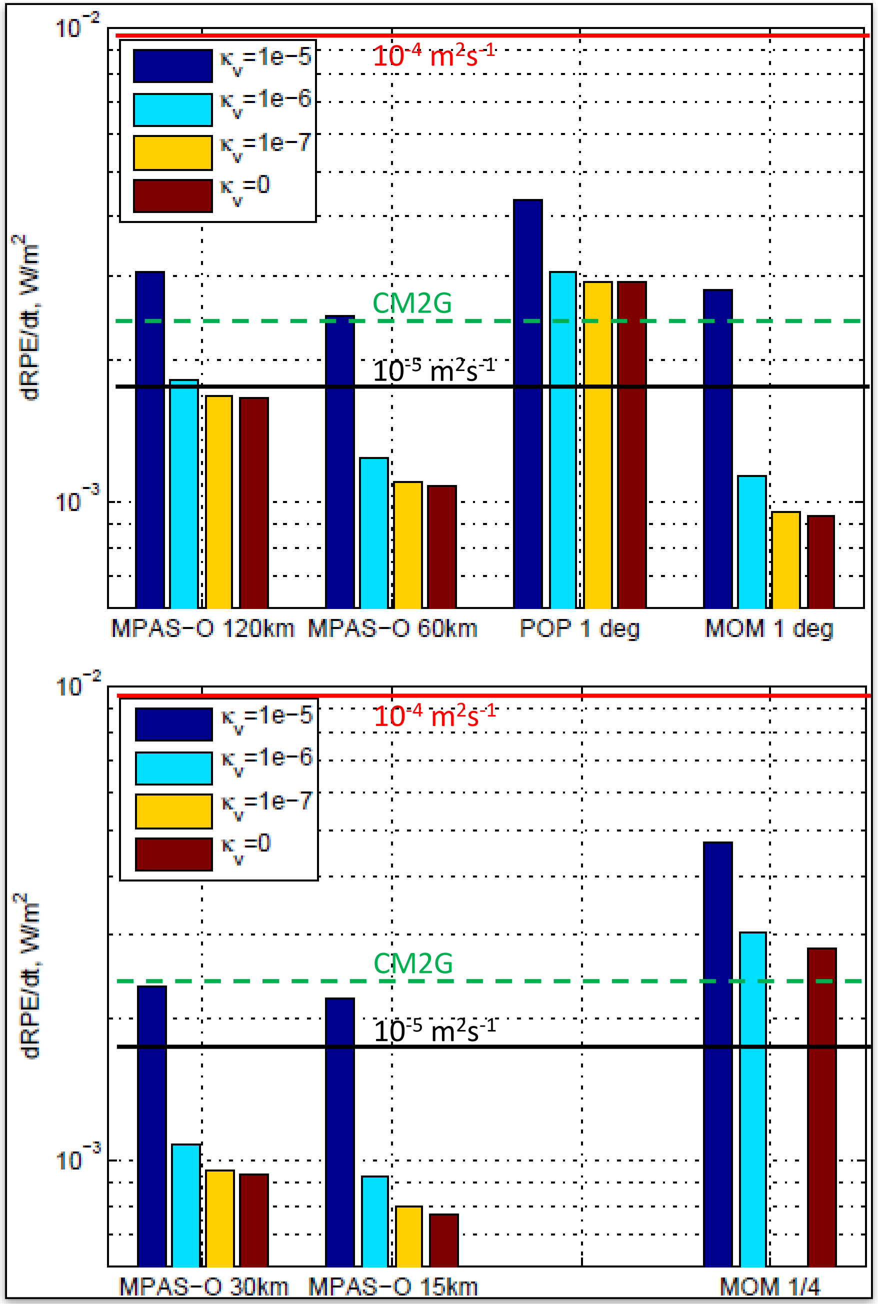


Fig 13, Petersen et al., 2014

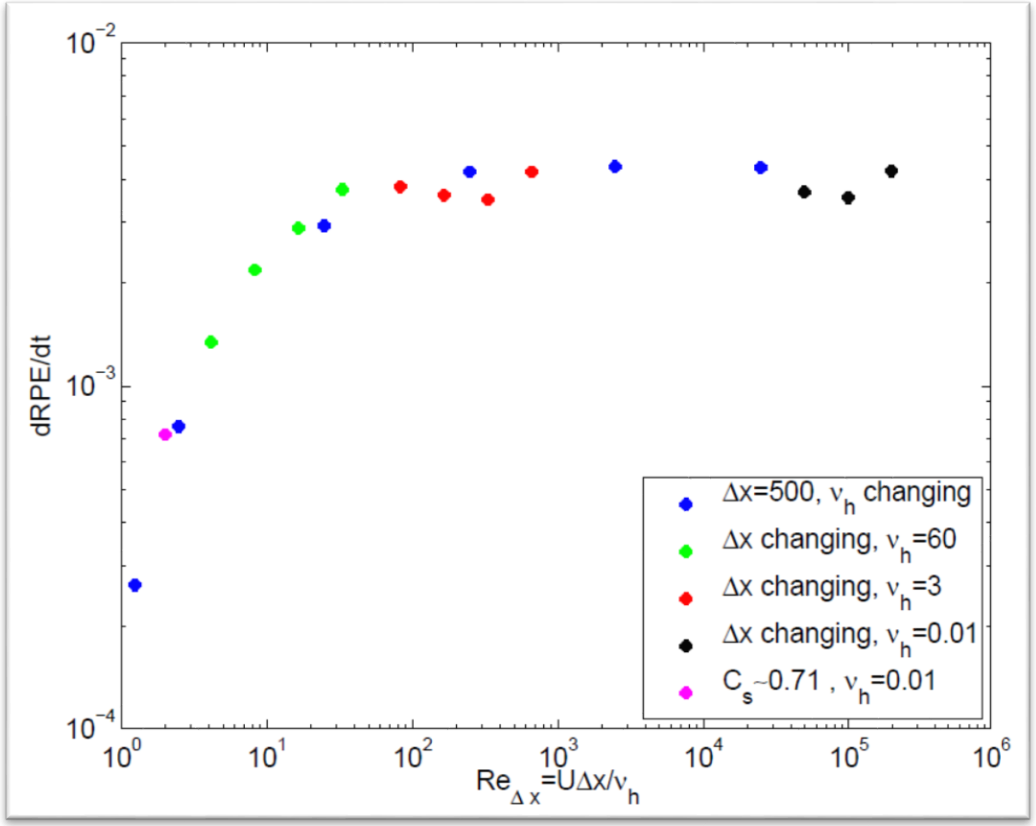
What controls spurious mixing

1. Accuracy of transport scheme most significant at low orders
   – Large difference between 1st and 2nd order
   – Small difference between 3rd and 7th order
2. Noise in flow field
   – Controlled by grid Reynolds number

$$Re_{\Delta} = \frac{U\Delta x}{\nu}$$

   – **Usual practice is to use largest Re_{Δ} that is stable!**

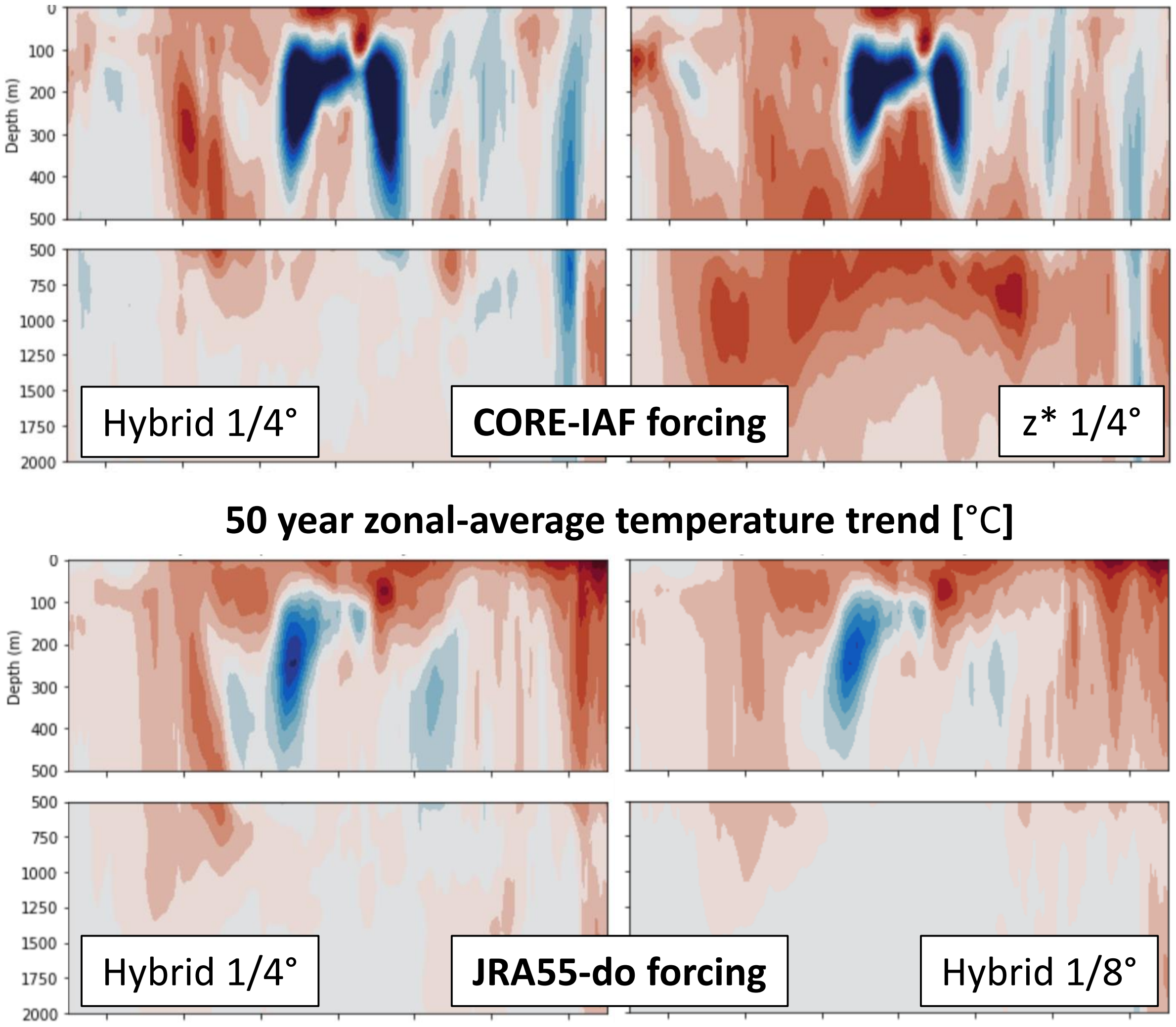
Note: this primarily concerns 3D transport in non-isopycnal coordinates



Lock exchange test problem

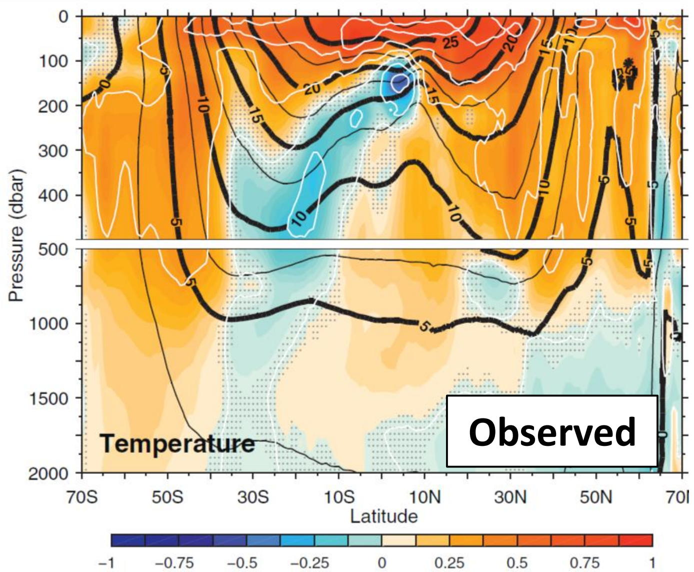
Ilicak et al., OM 2012

An era where we get the mixing right: observed trends?



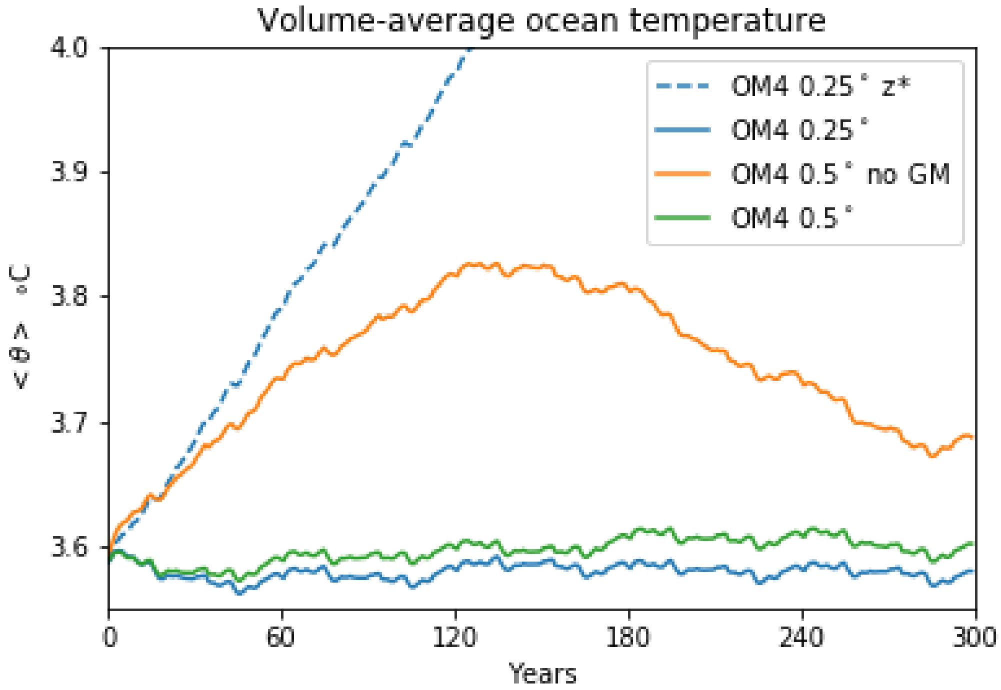
50 year zonal-average temperature trend [°C]

- Understanding/control of numerical mixing
  – High fidelity
  – We might now know when we get right answer for wrong reasons

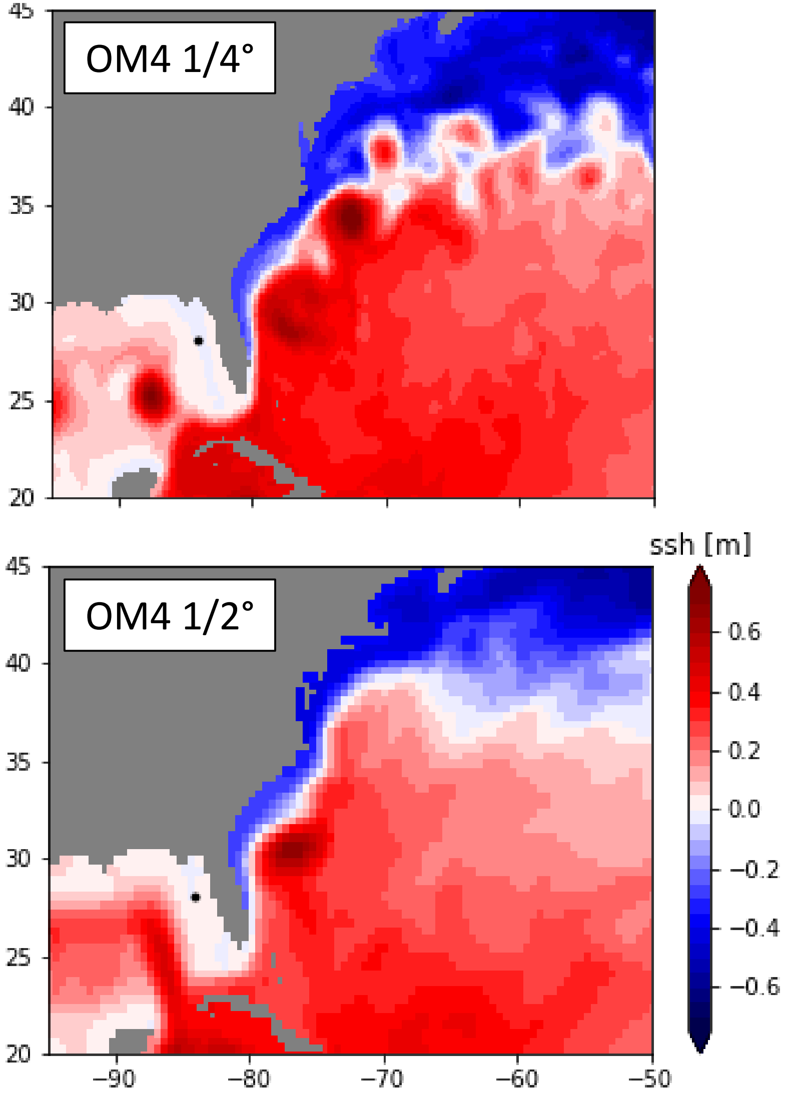


OM4.0: Role of eddies

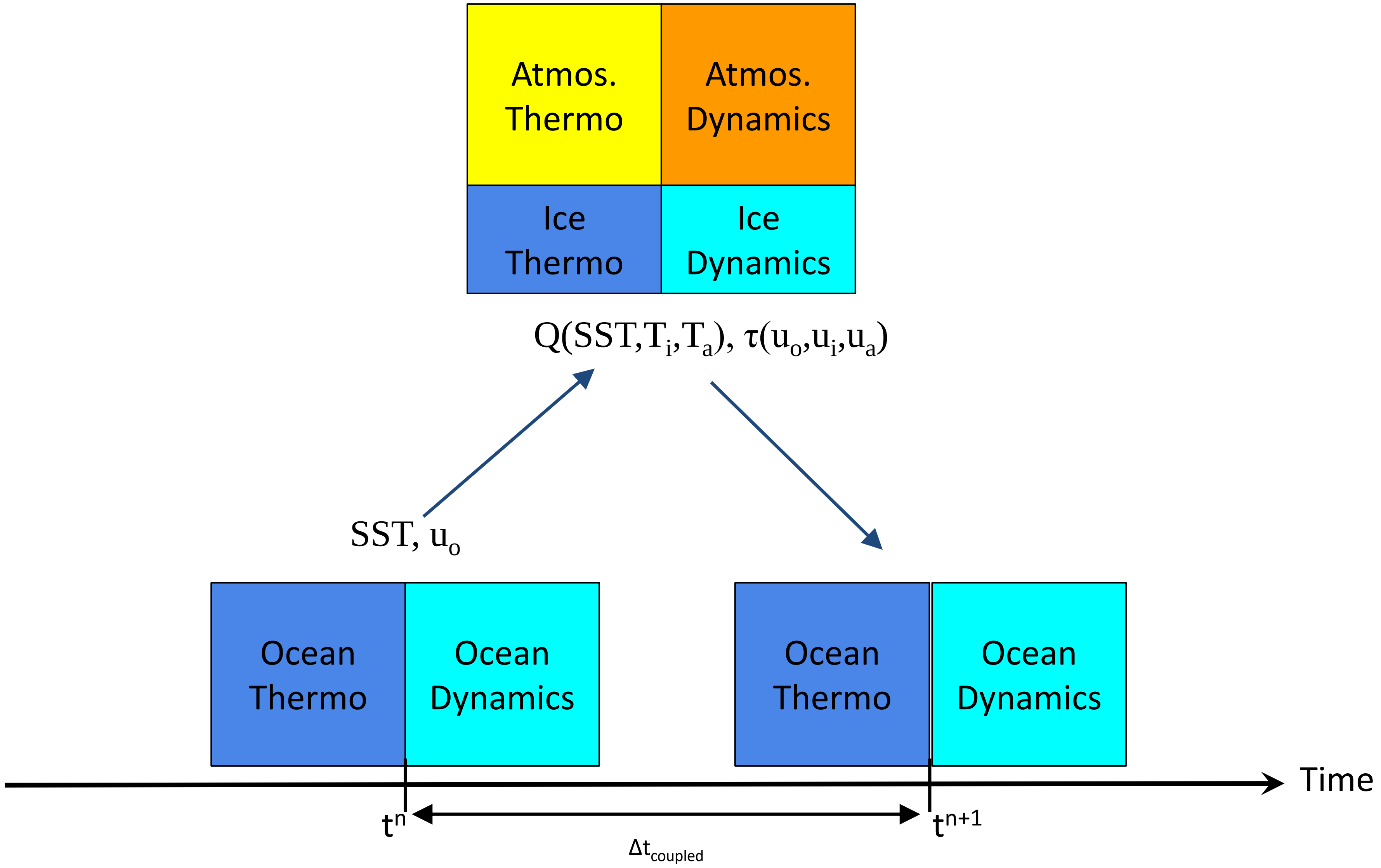
- Transition of laminar to eddying motion at mid-latitudes happens between ½°-¼° resolutions
- Mesoscale eddies in coarse resolution models must be parameterized



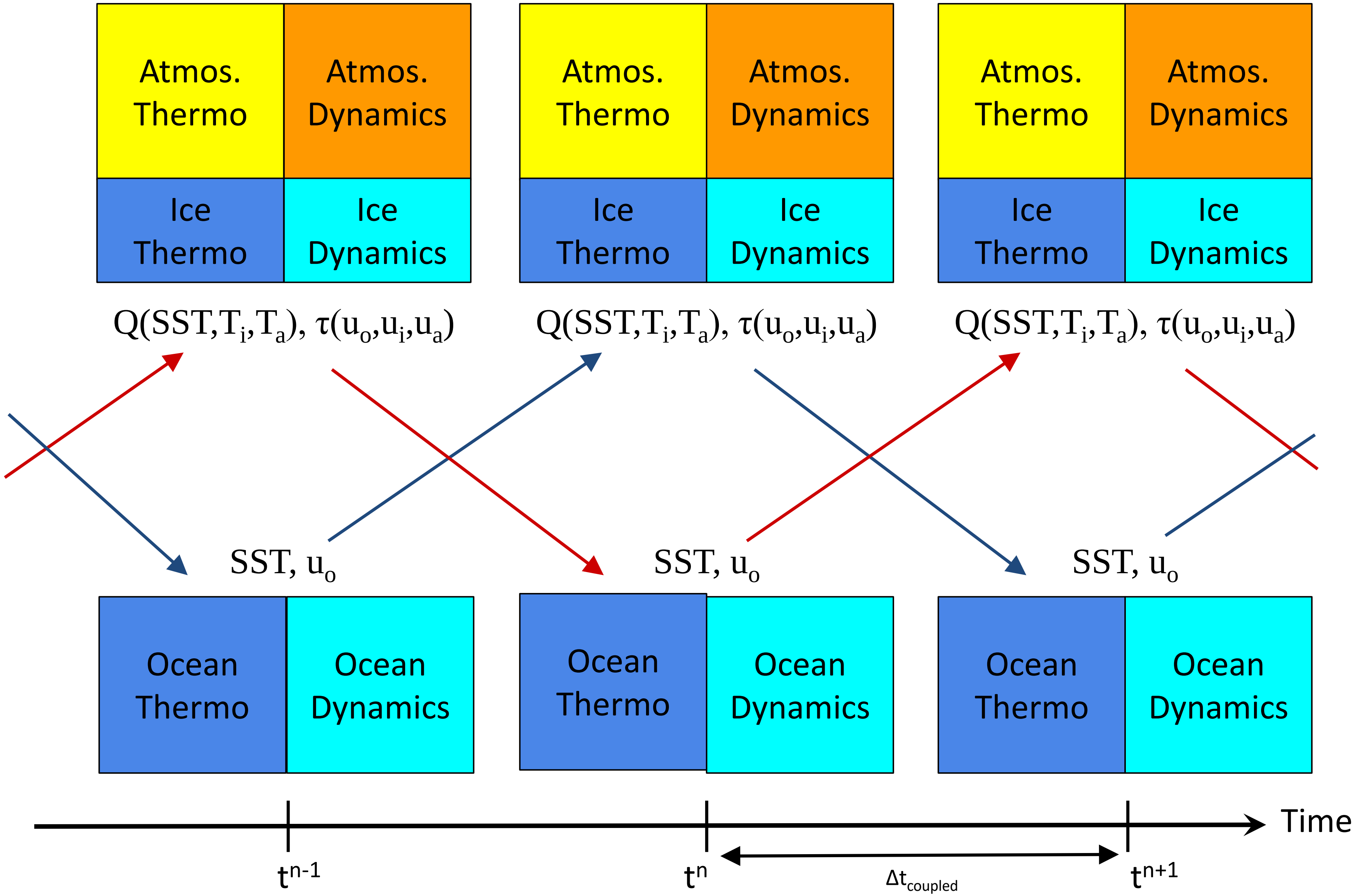
Snapshot of sea-surface height



Sequential Coupling



Concurrent Coupling



A coupled gravity-wave toy model

From Hallberg, 2014

2-layer (sea-ice & ocean) linear nonrotating flat-bottom channel flow with no viscosity.

$$\frac{\partial u_1}{\partial t} = -g\frac{\partial \eta_{1/2}}{\partial x}$$

$$= -g\frac{\partial}{\partial x}(h_1 + h_2)$$

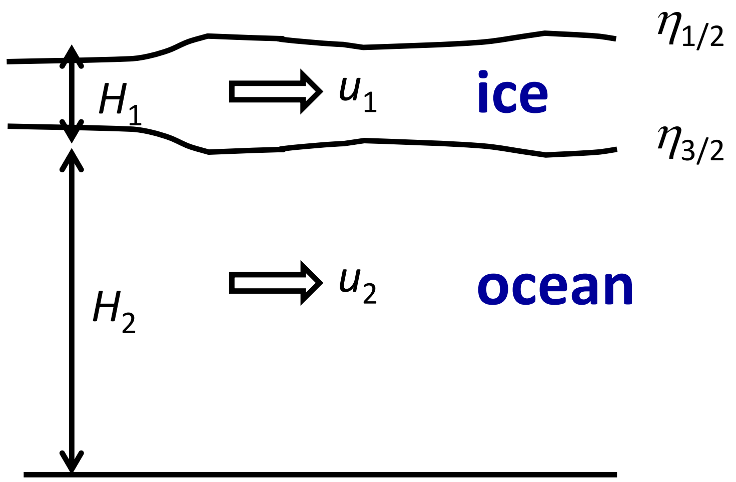
$$\frac{\partial u_2}{\partial t} = -g\frac{\rho_I}{\rho_o}\frac{\partial \eta_{1/2}}{\partial x} - g\frac{\rho_o - \rho_I}{\rho_o}\frac{\partial \eta_{3/2}}{\partial x}$$

$$= -(g - g')\frac{\partial}{\partial x}(h_1 + h_2) - g'\frac{\partial h_2}{\partial x}$$

$$= -(g - g')\frac{\partial h_1}{\partial x} - g\frac{\partial h_2}{\partial x}$$

$$\frac{\partial h_1}{\partial t} = -H_1\frac{\partial u_1}{\partial x}$$

$$\frac{\partial h_2}{\partial t} = -H_2\frac{\partial u_2}{\partial x}$$



A coupled gravity-wave toy model

From Hallberg, 2014

Sequential coupling of gravity waves only:

$$\frac{\partial h_1}{\partial t} = -H_1 \frac{\partial u_1}{\partial x} \qquad \frac{\partial u_1}{\partial t} = -g\frac{\partial h_1}{\partial x} - g\frac{\partial h_2^n}{\partial x}$$

$$\frac{\partial h_2}{\partial t} = -H_2 \frac{\partial u_2}{\partial x} \qquad \frac{\partial u_2}{\partial t} = -g\frac{\partial h_2}{\partial x} - (g-g')\frac{\partial h_1^{n+1}}{\partial x}$$

Sequential coupling:

Marginally stable if waves are treated analytically in each component.

$$\omega_1 \equiv \sqrt{gH_1}k \quad ; \quad \omega_2 \equiv \sqrt{gH_2}k$$

$$0 \le \omega_2 \Delta T <\sim 100$$

Concurrent (forward) coupling:

$$\frac{\partial h_1}{\partial t} = -H_1 \frac{\partial u_1}{\partial x} \qquad \frac{\partial u_1}{\partial t} = -g\frac{\partial h_1}{\partial x} - g\frac{\partial h_2^n}{\partial x}$$

$$\frac{\partial h_2}{\partial t} = -H_2 \frac{\partial u_2}{\partial x} \qquad \frac{\partial u_2}{\partial t} = -g\frac{\partial h_2}{\partial x} - (g-g')\frac{\partial h_1^{n}}{\partial x}$$

Concurrent forward coupling:

Unconditionally unstable, growth rate:

$$\approx \frac{(g-g')}{g\Delta T}\left[1-\cos(\omega_1\Delta T)\right]\left[1-\cos(\omega_2\Delta T)\right]$$

Sequential (filtered) coupling:

$$\frac{\partial h_2}{\partial t} = -H_2 \frac{\partial u_2}{\partial x} \qquad \frac{\partial u_2}{\partial t} = -g\frac{\partial h_2}{\partial x} - (g-g')\frac{\partial h_1^{n}}{\partial x}$$

$$\frac{\partial h_1}{\partial t} = -H_1 \frac{\partial u_1}{\partial x} \qquad \frac{\partial u_1}{\partial t} = -g\frac{\partial h_1}{\partial x} - g\frac{\partial}{\partial x}\left(\frac{1}{\Delta T}\int_0^{\Delta T} h_2 dt\right)$$

Sequential filtered coupling:

Unconditionally unstable, growth rate:

$\approx \frac{1}{2}$ Concurrent growth rate for small $\omega_2\Delta T$

$\propto \frac{1}{\omega_2\Delta T}$, for large $\omega_2\Delta T$

Damping from an ice-pack can locally stabilize the instabilities.

Numerical Ice-Ocean Coupling Instabilities

From Hallberg, 2014

1. Lagged stress / inertial oscillation instability

$$\frac{\partial u}{\partial t} + ifu = \frac{c_d U}{H}\left(u_{Atm} - u^n\right)$$

$$u' = u - u_{Steady}$$

$$u'\left(t^{n+1}\right) = \left[e^{-if\Delta t} + i\frac{c_d U}{Hf}\left(1 - e^{-if\Delta t}\right)\right]u'\left(t^n\right) = Au'\left(t^n\right)$$

$$\|A\|^2 = 1 - 2\frac{c_d U}{Hf}\sin(f\Delta t) + 2\left(\frac{c_d U}{Hf}\right)^2\left(1 - \cos(f\Delta t)\right)$$

2. Thermal forcing instability

$$\frac{\partial \theta_1}{\partial t} = -\frac{\lambda}{H_1}(\theta_1 - \theta_2) \qquad \frac{\theta_1^{n+1} - \theta_1^n}{\Delta t} = -\frac{\lambda}{H_1}\left(\theta_1^{n+1} - \theta_2^n\right)$$

$$\frac{\partial \theta_2}{\partial t} = +\frac{\lambda}{H_2}(\theta_1 - \theta_2) \qquad \frac{\theta_2^{n+1} - \theta_2^n}{\Delta t} = +\frac{\lambda}{H_2}\left(\theta_1^{n+1} - \theta_2^n\right)$$

Eigenvalues:

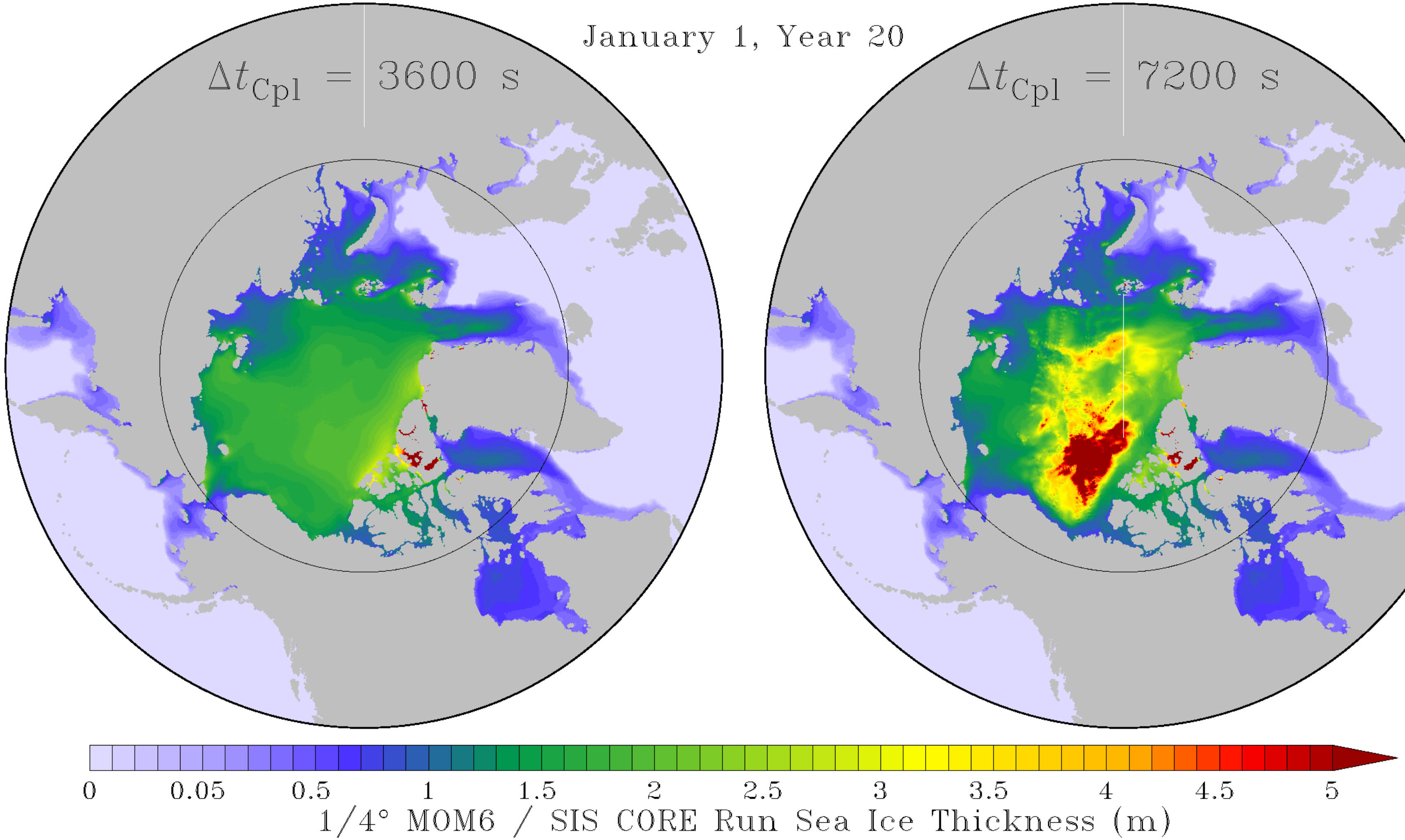
$$A_1 = \frac{1}{1 + \lambda\Delta t / H_1}$$

$$A_2 = 1 - \lambda\Delta t / H_2$$

3. Gravity wave instability
   - Sea-ice and icebergs participate in barotropic gravity waves
   - Stability analysis analogous to split-explicit ocean time stepping (e.g., Hallberg, J. Comp. Phys., 1997)
   - Instability growth rate proportional to the sea-ice external gravity wave CFL ratio based on the ***coupling time step***.

$$\frac{\sqrt{gH_{Ice}}\Delta T}{\Delta x} < O(1)$$

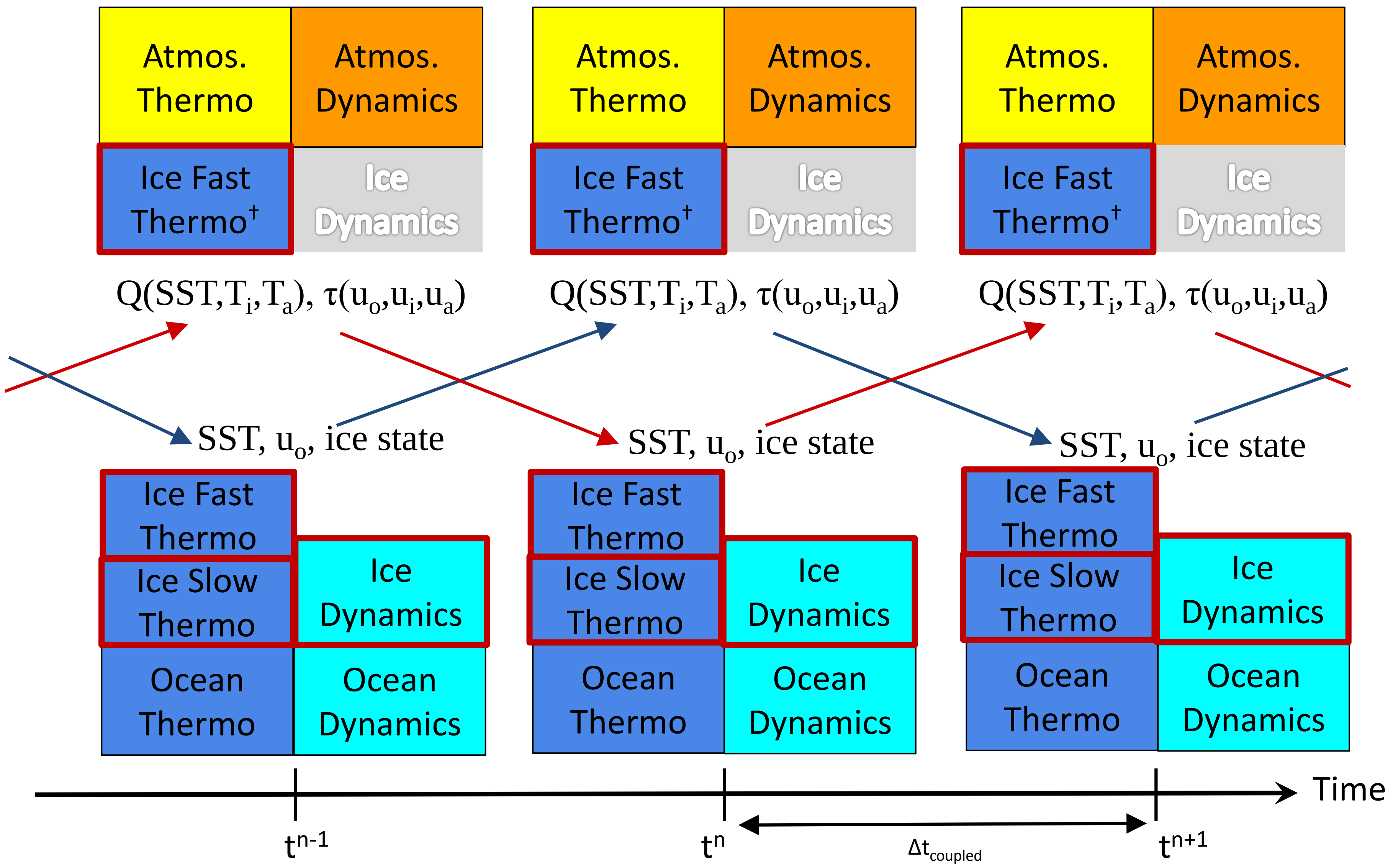
Lagged Stress-Inertial Coupling Instability in Sea-Ice Thickness



Sequentially coupled data-driven ice-ocean model

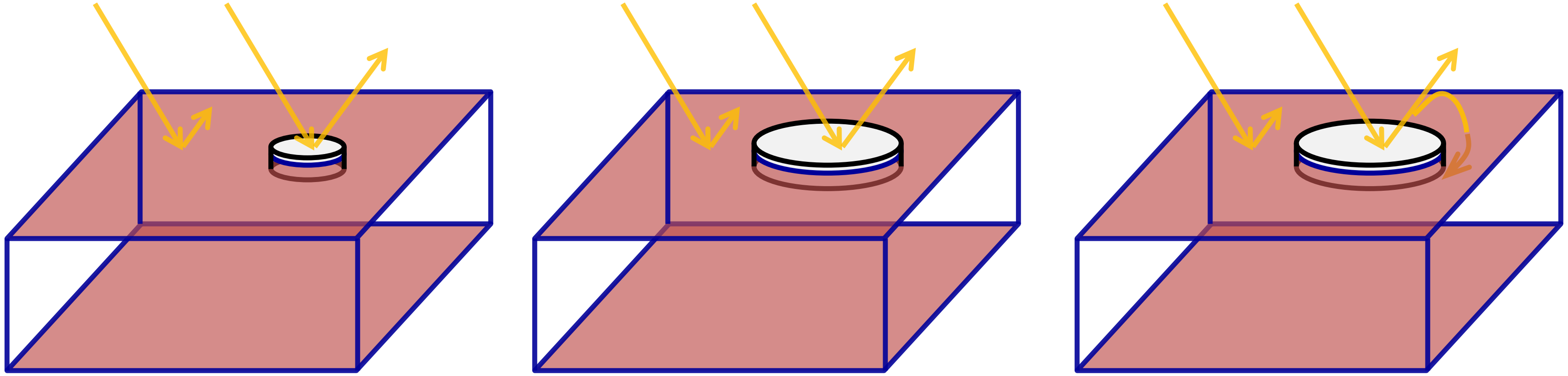
Hallberg (2014, *Clivar Exchanges*)

Concurrent/Embedded Ice Coupling



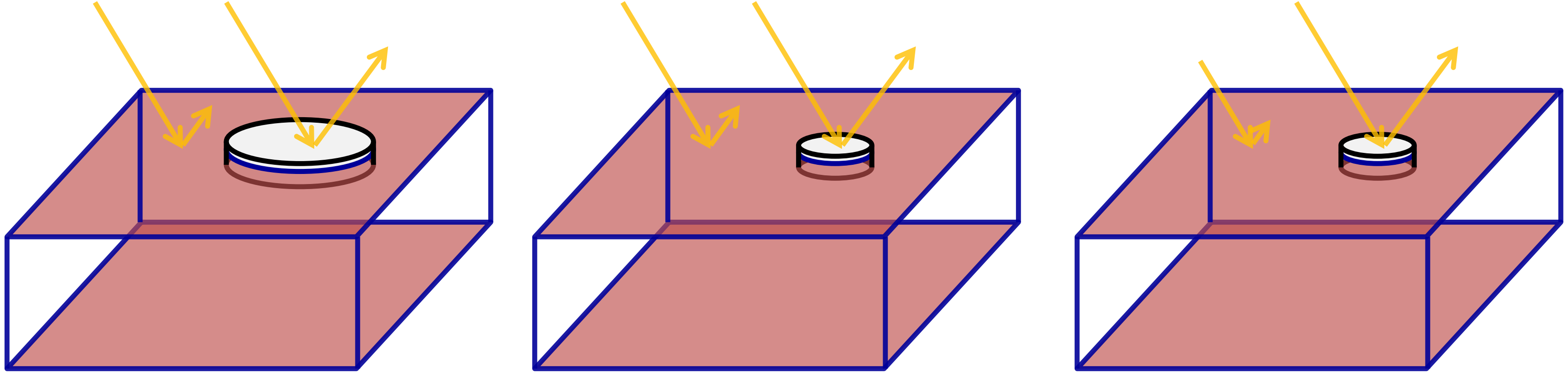
Conservatively Recalculating Solar Heating

Increasing sea-ice area or albedo ➔ Apply excess reflected shortwave to ocean



Previous ice state | Current ice state | Shortwave applied to current ice state

Decreasing ice area or albedo ➔ Reduce incident shortwave to ocean



Previous ice state | Current ice state | Shortwave applied to current ice state

From Hallberg

Summary

- Ocean models need to conserve heat, salt and volume or mass
  - Other moments not as critical (at least for now)?
- The energy budget for mixing work on the ocean is critical
- Spurious heat uptake is understood but still an issue
  - Compensating errors (spurious mixing – inefficient eddies)
- Sea-ice is really part of the ocean
  - Challenges when treating sea-ice as an independent component