An overview of the ocean-atmosphere coupling

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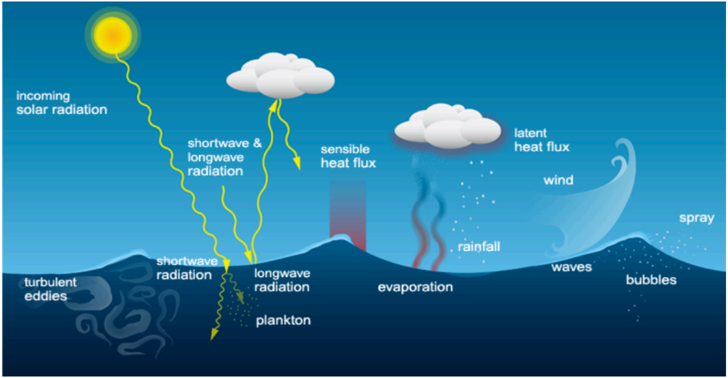
Air-sea interactions



- Significant amount of kinetic energy exchanged (fundamental difference with land/atmosphere coupling)
- Sea spray droplets can enhance the usual interfacial fluxes of heat and moisture but no consensus + measurements very difficult (too rudimentary to be included in CMs)
- The representation of surface waves-induced processes will become increasingly important

Content

1

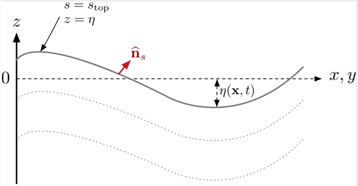
Interface conditions and usual assumptions

Oceanic component

Griffies S. & A. Adcroft : *Formulating the equations of ocean models*, AGU monographs, 2008

At the oceanic free-surface :

- Water and tracer penetrate through precipitation, evaporation, sea-ice melt
- Momentum exchange arises from stresses with atmosphere or ice.



- Ocean free-surface: surface of constant generalized vertical coordinate
- No overturns at the scales of interest $\rightarrow$ assume that breaking surface waves are filtered or averaged.
- Lorenz grid in the vertical

Surface kinematic boundary condition :

$$\rho \frac{d(z-\eta)}{dt} = -\rho_w q_w, \qquad z = \eta$$

Mass flux across the free-surface : $\rho_w q_w$ (q_w: fresh water flux)

Oceanic component (mass exchange)

Boussinesq assumption:

- Only density variations that matter are the ones which contribute to buoyancy variations
- Continuity equation $\rightarrow$ from mass to volume conservation
- The conservation laws are changed from mass to volume-integrated

Volume conservation for a water column

$$\partial_t \eta^{\mathrm{B}} = -\nabla \cdot \mathbf{U} + (\rho_w q_w)/\rho_0, \qquad \mathbf{U} = \int_{-H}^{\eta} \mathbf{u}\, dz$$

(ρ_0 = constant reference Boussinesq density)

Mass exchange due to precipitations and evaporation

$$\rho_w q_w = E - P_r - P_i$$

- P_r rain drops
- P_i ice (snow graupel))
- E evaporation

- Precipitated water = pure freshwater
- Salt particles remain in the ocean during evaporation

Oceanic component (momentum budget)

$$\frac{d\mathbf{u}}{dt} = -f\widehat{\mathbf{z}} \times \mathbf{u} - \rho_0^{-1} \nabla \cdot \boldsymbol{\Sigma_T}$$

with $\boldsymbol{\Sigma_T}$ the total stress tensor

$$\boldsymbol{\Sigma_T} = -p\mathbf{I} + \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$

- Continuity of stresses at the free boundary

$$\Sigma_T \widehat{\mathbf{n}}_{\mathrm{s}} = -p_{\mathrm{sfc}} \widehat{\mathbf{n}}_{\mathrm{s}} + \boldsymbol{\tau_s} \widehat{\mathbf{t}}_{\mathrm{s}}$$

Rain-Induced Momentum Flux **[e.g. Caldwell and Elliot (1971,1972)]**

$$\boldsymbol{\tau}_R = \rho_R \mathbf{u}_R P$$

Oceanic component (Conservative tracers for "heat content" and salt)

Conservation of "conservative temperature" (TEOS10 standard[1])

$$\frac{d\Theta}{dt} = -\nabla \cdot \boldsymbol{J_\Theta} - \frac{1}{\rho_0 c_p^{\rm oce}} \frac{\partial Q_s}{\partial z}, \qquad \Theta = h_0 / c_p^0, \qquad c_p^{\rm oce} \approx 3991.87 \text{ J kg}^{-1} \text{ K}^{-1}$$

Conservation of absolute salinity (g/kg)

$$\frac{dS_A}{dt} = -\nabla \cdot \boldsymbol{J_S}$$

Surface boundary conditions

$$\begin{aligned} Q_s(z=\eta,t) &= (1-\alpha_{\rm alb})\mathcal{F}_{\rm SW}^{\searrow} \\ J_\Theta^{(\rm sfc)} &= (\hat{\mathbf{z}} - \nabla_s z)\cdot \boldsymbol{J_\Theta} = \frac{Q_{\rm sen} + Q_{\rm lat} + (\mathcal{F}_{\rm LW}^{\searrow} - \mathcal{F}_{\rm LW}^{\nearrow})}{\rho_0 c_p^{\rm oce}} \\ J_S^{(\rm sfc)} &= (\hat{\mathbf{z}} - \nabla_s z)\cdot \boldsymbol{J_S} = 0 \qquad\qquad Q_{\rm lat} = \Lambda_v E \end{aligned}$$

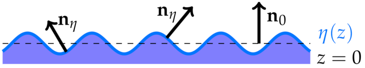
[1] Rich Pawlowicz, "What every oceanographer needs to know about TEOS-10", 2013

Atmospheric component (mass exchange through the surface)

From the atmospheric point of view

- Outflow of precipitating mass at the surface
- Evaporation flux of vapor from the surface

$\rightarrow$ change of surface pressure



$$\frac{\partial p_{\mathrm{sfc}}}{\partial t} = -g \int_{z_s}^{z_t} \nabla \cdot (\rho \mathbf{v}_h) \; \mathrm{dz} + g(\rho w)_{\mathrm{sfc}}$$

- ρ: total mass density
- $(\rho w)_{\mathrm{sfc}} = E - P_r - P_i$
- z_s ?

Interface conditions

- Continuity of stresses at the free boundary (under the assumption of no surface waves)
- Mass exchange
- ”Heat” exchange

$$\rho_0 c_p^{\rm oce} \left(J_{\Theta_{\rm oce}}^{\rm (sfc)} + (1-\alpha_{\rm alb}) \mathcal{F}_{\rm SW}^{\searrow} \right) = \rho_{\rm atm} c_p^{\rm atm} \left(J_{\Theta_{\rm atm}}^{\rm (sfc)} + (1-\alpha_{\rm alb}) \mathcal{F}_{\rm SW}^{\searrow} \right)$$

- ”Subgrid” interface conditions

$$\theta_{\rm oce}(z=0^-) = \theta_{\rm atm}(z=0^+), \qquad \nu_{\rm oce}^{\theta} \partial_z \theta_{\rm oce}(z=0^-) = \nu_{\rm atm}^{\theta} \partial_z \theta_{\rm atm}(z=0^+)$$

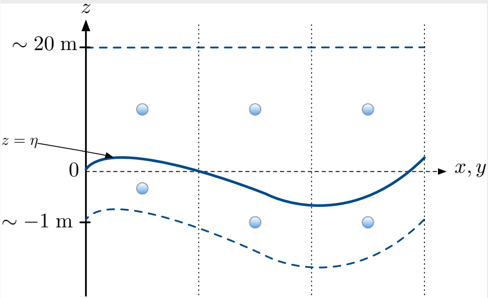
$$\mathbf{u}_{\rm oce} \widehat{\mathbf{t}}_{\rm oce}(z=0^-) = \mathbf{u}_{\rm atm} \widehat{\mathbf{t}}_{\rm atm}(z=0^+), \qquad \nu_{\rm oce} \nabla \mathbf{u}_{\rm oce} \cdot \widehat{\mathbf{n}}_{\rm oce}(z=0^-) = \nu_{\rm atm} \nabla \mathbf{u}_{\rm atm} \cdot \widehat{\mathbf{n}}_{\rm atm}(z=$$

2

Turbulent fluxes in the surface layer

- Representation of the transfer of heat, momentum and moisture between the surface and the lowest model level (i.e. within the surface layer)

$\rightarrow$ estimate of $\boldsymbol{\tau_s}$, Q_{sen}, Q_{lat}, E

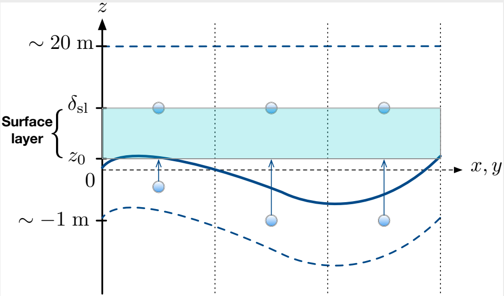


Assumptions

- Quantities at the first model level are interpreted in a FD sense
- First model vertical level is in the surface layer and above the roughness sublayer
- Effect of heterogeneity in free-surface elevation (?)

- Representation of the transfer of heat, momentum and moisture between the surface and the lowest model level (i.e. within the surface layer)

$\rightarrow$ estimate of $\boldsymbol{\tau_s}$, $Q_{\rm sen}$, $Q_{\rm lat}$, E



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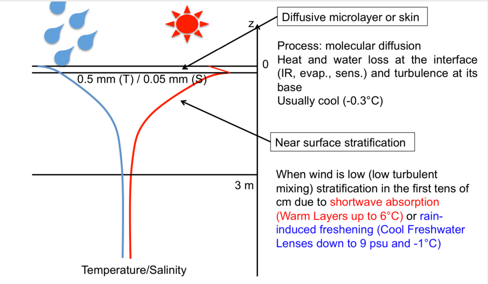

Assumptions

- Quantities at the first model level are interpreted in a FD sense
- First model vertical level is in the surface layer and above the roughness sublayer
- Effect of heterogeneity in free-surface elevation (?)
- Oceanic mean velocities are assumed constant in the oceanic surface layer
- Bulk vs skin sea surface temperature (?)

Monin-Obukhov similarity theory (aka surface-layer similarity)

Monin A.S., A.M. Obukhov : *Osnovnye zakonomernosti turbulentnogo peremesivanija v prizemnom sloe atmosfery*, Tr Geofiz Inst AN SSSR, 1954

$\Rightarrow$ **Semi-empirical Monin-Obukhov similarity theory** is the basis to derive turbulent quantities from the mean variables available from the models

$\rightarrow$ Generalization of the classical law of the wall to stratified conditions

Successive steps:

- Only mechanical and thermal forcing act on the turbulence (horiz. homogeneity + stationarity)

$$\mathcal{S}_{\rm m} = -(\overline{\mathbf{v}'w'}) \cdot \partial_z \overline{\mathbf{v}}$$

$$\mathcal{S}_{\rm t} = \frac{g}{\theta_v^{\rm ref}} (\overline{w'\theta_v'})$$

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Successive steps:

- Only mechanical and thermal forcing act on the turbulence (horiz. homogeneity + stationarity)
- Constant flux layer assumption
- Define fundamental turbulent parameters $z,\, u_\star,\, \theta_\star,\, q_\star$

$$u_\star = \frac{\|\boldsymbol{\tau_s}\|}{\rho_a}$$

$$\theta_\star = -\frac{(\overline{w'\theta'})_{\rm sl}}{u_\star}$$

$$q_\star = -\frac{(\overline{w'q'})_{\rm sl}}{u_\star}$$

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Successive steps:

- Only mechanical and thermal forcing act on the turbulence (horiz. homogeneity + stationarity)
- Constant flux layer assumption
- Define fundamental turbulent parameters z, $u_\star$, $\theta_\star$, $q_\star$
- Form dimensionless groups

$$\pi_\gamma = \frac{\kappa z}{\phi_\star}\partial_z\overline{\gamma}, \qquad \gamma = (\mathbf{v}, \theta, q)$$

$$\pi_L = \frac{\kappa g z}{\theta_v^{\rm ref} u_\star^3}(\overline{w'\theta_v'})_{\rm sl} = \frac{z}{L_{\rm ob}}$$

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Successive steps:

- Only mechanical and thermal forcing act on the turbulence (horiz. homogeneity + stationarity)
- Constant flux layer assumption
- Define fundamental turbulent parameters z, $u_\star$, $\theta_\star$, $q_\star$
- Form dimensionless groups
- Empirically define functional relationships

$$\pi_\gamma = \frac{\kappa z}{\phi_\star}\partial_z\overline{\gamma}, \qquad \gamma = (\mathbf{v}, \theta, q)$$

$$\pi_L = \frac{\kappa g z}{\theta_v^{\mathrm{ref}} u_\star^3}(\overline{w'\theta_v'})_{\mathrm{sl}} = \frac{z}{L_{\mathrm{ob}}}$$

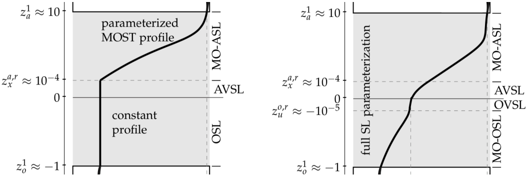
$$\begin{aligned} \pi_u &= \phi_m(\pi_L) \\ \pi_\theta &= \phi_s(\pi_L) \\ \pi_q &= \phi_s(\pi_L) \end{aligned}$$

- Vertically integrate from $\mathbf{z_r} = (z_m, z_h, z_q)$ to $z \in]z_{\cdot}, \delta_{\rm sl}[$ to obtain

$$
\begin{aligned}
\mathbf{v}_{\rm atm}(z) &= \mathbf{v}^{\dagger}_{\rm oce}(z_m) - \frac{u_\star}{\kappa}\left[\ln\left(\frac{z}{z_m}\right) - \psi_m\left(\frac{z}{L_{\rm Ob}}\right) + \psi_m\left(\frac{z_m}{L_{\rm Ob}}\right)\right] e^{i\theta_\tau} \\
\theta_{\rm atm}(z) &= \theta^{\dagger}_{\rm oce}(z_h) - \frac{\theta_\star}{\kappa}\left[\ln\left(\frac{z}{z_h}\right) - \psi_s\left(\frac{z}{L_{\rm Ob}}\right) + \psi_s\left(\frac{z_h}{L_{\rm Ob}}\right)\right] \\
q_{\rm atm}(z) &= q_{\rm sat}(\theta^{\dagger}_{\rm oce}(z_h)) - \frac{q_\star}{\kappa}\left[\ln\left(\frac{z}{z_q}\right) - \psi_s\left(\frac{z}{L_{\rm Ob}}\right) + \psi_s\left(\frac{z_q}{L_{\rm Ob}}\right)\right]
\end{aligned}
$$

- Provided a proper treatment of viscous sublayers this amounts to extend the model mean vertical profiles to the surface to satisfy (same for θ)

$$
\mathbf{v}_{\rm atm}(0^+) = \mathbf{v}_{\rm oce}(0^-), \qquad \nu_{\rm atm}\partial_z \mathbf{v}_{\rm atm}(0^+) = \nu_{\rm oce}\partial_z \mathbf{v}_{\rm oce}(0^-)
$$



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q_{\rm atm}(z) &= q_{\rm sat}(\theta^\dagger_{\rm oce}(z_h)) - \frac{q_\star}{\kappa}\left[\ln\left(\frac{z}{z_q}\right) - \psi_s\left(\frac{z}{L_{\rm Ob}}\right) + \psi_s\left(\frac{z_q}{L_{\rm Ob}}\right)\right]
\end{aligned}
$$

▷ Usual form :

$$
\begin{aligned}
\boldsymbol{\tau_s} &= \rho_a C_D \|\mathbf{v}_{\rm atm}(z^1_{\rm atm}) - \mathbf{v}^\dagger_{\rm oce}(z_m)\| (\mathbf{v}_{\rm atm}(z^1_{\rm atm}) - \mathbf{v}^\dagger_{\rm oce}(z_m)) \\
Q_{\rm sen} &= \rho_a c_p^{\rm atm} C_H \|\mathbf{v}_{\rm atm}(z^1_{\rm atm}) - \mathbf{v}^\dagger_{\rm oce}(z_m)\| (\theta_{\rm atm}(z^1_{\rm atm}) - \theta^\dagger_{\rm oce}(z_h)) \\
E &= \rho_a C_E \|\mathbf{v}_{\rm atm}(z^1_{\rm atm}) - \mathbf{v}^\dagger_{\rm oce}(z_m)\| (q_{\rm atm}(z^1_{\rm atm}) - q_{\rm sat}(\theta^\dagger_{\rm oce}(z_h)))
\end{aligned}
$$

- Vertically integrate from $\mathbf{z_r} = (z_m, z_h, z_q)$ to $z \in]z., \delta_{\rm sl}[$ to obtain

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\begin{aligned}
\mathbf{v}_{\rm atm}(z) &= \mathbf{v}^\dagger_{\rm oce}(z_m) - \frac{u_\star}{\kappa}\left[\ln\left(\frac{z}{z_m}\right) - \psi_m\left(\frac{z}{L_{\rm Ob}}\right) + \psi_m\left(\frac{z_m}{L_{\rm Ob}}\right)\right] e^{i\theta_\tau} \\
\theta_{\rm atm}(z) &= \theta^\dagger_{\rm oce}(z_h) - \frac{\theta_\star}{\kappa}\left[\ln\left(\frac{z}{z_h}\right) - \psi_s\left(\frac{z}{L_{\rm Ob}}\right) + \psi_s\left(\frac{z_h}{L_{\rm Ob}}\right)\right] \\
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\end{aligned}
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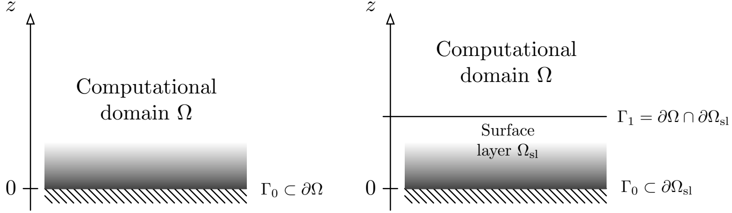
E &= \rho_a C_E \|\mathbf{v}_{\rm atm}(z^1_{\rm atm}) - \mathbf{v}^\dagger_{\rm oce}(z_m)\| (q_{\rm atm}(z^1_{\rm atm}) - q_{\rm sat}(\theta^\dagger_{\rm oce}(z_h)))
\end{aligned}
$$

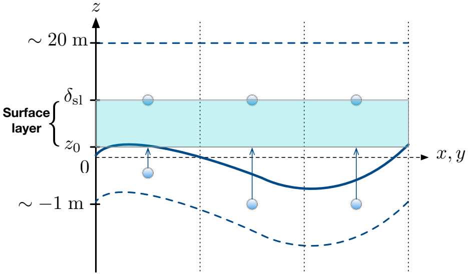
$\Rightarrow$ Historically ignored but strong impact on energetics from an oceanic perspective [**Dewar & Flierl, 1987; Duhaut & Straub, 2006; Renault et al. 2016**]

Side remarks

→ Usual practice in fluid-structure coupling community



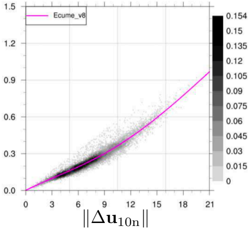
→ Treatment of boundary cell



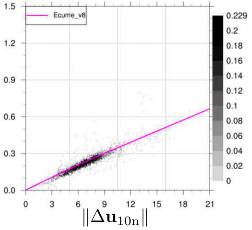
Relevant variables on which the turbulence should act ?

Marquet P. & S. Belamari : ***On new bulk formulas based on moist-air entropy*****, WGNE Blue-Book, 2017**

$$P_u = \left(\frac{C_{dn}}{\sqrt{C_{dn}}}\right)\|\Delta \mathbf{u}_{10n}\|$$



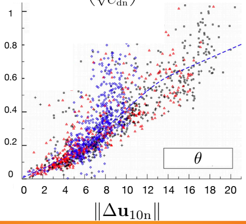
$$P_q = \left(\frac{C_{en}}{\sqrt{C_{dn}}}\right)\|\Delta \mathbf{u}_{10n}\|$$



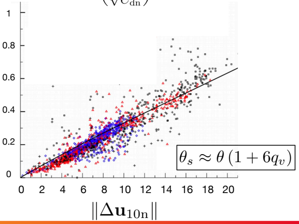
Moderate scattering for the "speed scales equivalent parameters" P_u & P_q

$$\overline{w'\theta'_s} \approx e^{\Lambda \overline{q_v}}\left(\overline{w'\theta'}\right) + \Lambda \overline{\theta}_s \left(\overline{w'q'_v}\right)$$

$$P_\theta = \left(\frac{C_{hn}}{\sqrt{C_{dn}}}\right)\|\Delta \mathbf{u}_{10n}\|$$



$$P_s = \left(\frac{C_{sn}}{\sqrt{C_{dn}}}\right)\|\Delta \mathbf{u}_{10n}\|$$



General comments

Add-on components to the theory :

- Effect of gustiness on the surface fluxes [**e.g. Redelsperger et al., 2000**]
- Effect of surface waves on the roughness lengths

- Even under ideal conditions, the theory has an accuracy of only about 10–20% [**Foken, 2006**]
- Bill Large (2006) "Surface fluxes for practitioners of global ocean data assimilation"
  - *For hourly fluxes on a spatial scale of* $10\ \mathrm{km}$*, there is at least a factor of* 2 *uncertainty due to transfer coefficient variability on these scales*
  - *Sign can be uncertain on time scales less than about* $10\ \mathrm{min.}$
  - *Annual averaging is probably required before the uncertainty in bulk fluxes is minimized*

$\Rightarrow$ Internal time-scale Δt_{blk} to keep uncertainty on turbulent flux estimates at a "reasonable level"

$\rightarrow$ LES codes are also based on the MO theory

$\rightarrow$ no notion of surface-layer resolving model (?)

3

Non-conformity in time (algorithmic perspective)

Preliminaries

Keyes and coauthors : *Multiphysics simulations: Challenges and opportunities*, Int. J. High Perform. Comput. Appl., 2013

Coupled problems can be solved by using

1. **Monolithic methods**: a single model representing all components to be coupled is defined

   $\rightarrow$ *not tractable when considering two individual models developed independently with distinct numerical techniques*

2. **Partitioned/Split methods**: the full problem is split into smaller problems solved independently with boundary exchanges through their interface

   $\rightarrow$ *This type of approach can give rise to various sources of 'splitting' errors*

A numerical coupling method can be referred to as

1. **Tightly (a.k.a. strongly) coupled**: state variables across different models are synchronized at all times
2. **Loosely coupled**: state variables are shifted by one time-step or a sequence of time-steps

Various alternatives

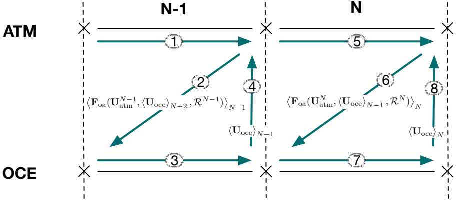
- ▷ Concurrent coupling vs sequential coupling (aka parallel vs multiplicative)
- ▷ Global-in-time vs local-in-time
- ▷ Computation of surface fluxes in coupler or in atmospheric component ?

Some criteria to choose an appropriate coupling algorithm

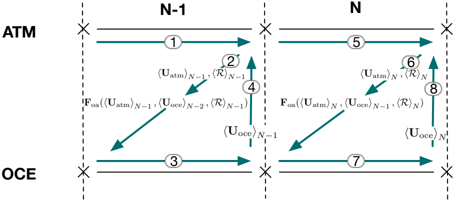
- Practical aspects
  - Computational efficiency
  - Minimal modification to existing codes
- Numerical & physical aspects
  - Numerical stability and consistency
  - Conservation properties
    (in a weak or strong sense $\sim$ local-in-time vs global-in-time)
  - Consistent with underlying assumptions of physical parameterizations
    (e.g. $\Delta t_{\rm blk}$, $\Delta t_{\rm rad}$, PBL scheme, ...)
  - Effect of missing processes (diurnal cycle, surface waves, ...)

Standard partitioned time-stepping methods in CMs

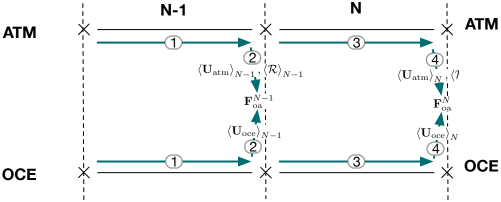
Global-in-time (sequential)



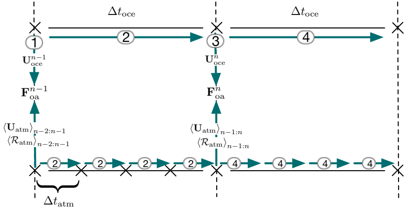
Global-in-time (sequential v2)



Global-in-time (Concurrent)



Local-in-time (Concurrent)



Coupling diffusion equations ($\Delta t_{\mathrm{oce}} = \Delta t_{\mathrm{atm}}$)

Concurrent local-in-time approach (a.k.a explicit flux coupling)

$$\text{ATM Model} \left\{ \begin{array}{rcl} u_{\mathrm{atm}}^{n+1} & = & u_{\mathrm{atm}}^{n} + \Delta t_{\mathrm{atm}} \partial_z \left(K_z^{\mathrm{a}} \partial_z u_{\mathrm{atm}}^{n+1} \right) + \mathcal{F}^{\mathrm{a}} \\ \rho_{\mathrm{a}} K_z^{\mathrm{a}} \partial_z u_{\mathrm{atm}}^{n+1}(z=0) & = & F_{\mathrm{oa}}(u_{\mathrm{atm}}^{n}(z^{+}), u_{\mathrm{oce}}^{n}(z^{-})) \end{array} \right.$$

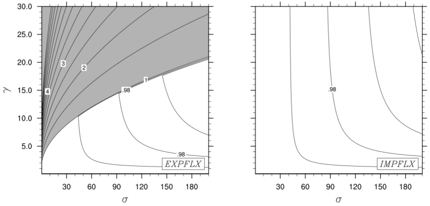
$$\text{OCE Model} \left\{ \begin{array}{rcl} u_{\mathrm{oce}}^{n+1} & = & u_{\mathrm{o}}^{n} + \Delta t_{\mathrm{oce}} \partial_z \left(K_z^{\mathrm{o}} \partial_z u_{\mathrm{oce}}^{n+1} \right) + \mathcal{F}^{\mathrm{o}} \\ \rho_{\mathrm{o}} K_z^{\mathrm{o}} \partial_z u_{\mathrm{o}}^{n+1}(z=0) & = & \rho_{\mathrm{a}} K_z^{\mathrm{a}} \partial_z u_{\mathrm{atm}}^{n+1}(z=0) \end{array} \right.$$

$\rightarrow$ Numerical stability ?

Numerical stability of the coupling with surface layer

Beljaars A., E. Dutra, G. Balsamo, F. Lemarié : *On the numerical stability of surface-atmosphere coupling in weather and climate models*, Geosci. model dev., 2017

- Matrix stability analysis : $\boldsymbol{A}\boldsymbol{T}^{n+1} = \boldsymbol{B}\boldsymbol{T}^{n}, \qquad \boldsymbol{M} = \boldsymbol{A}^{-1}\boldsymbol{B}$



$$\sigma = \frac{K_z \Delta t}{\Delta z^2}$$

$$\gamma = \frac{C_D \|\delta u\| \Delta t}{\Delta z}$$

Figure : Spectral radius of the matrix $\boldsymbol{M}$ with respect to the dimensionless coefficients σ and γ

- Empirical stability constraint for explicit flux coupling : $\gamma \leq 2 + \sqrt{\sigma}^{1.1}$
- Can be an issue depending on the numerics used for other terms
  $\rightarrow$ e.g. problematic for IFS because of large integration time steps compared to the physical timescale of the problem and the use of explicit flux coupling for modularity
- Local-in-time concurrent coupling stable if surface-layer/atmosphere (resp. surface-layer/ocean) coupling is stable [Lemarié et al., 2015]

Coupling diffusion equations ($\Delta t_{\rm oce} = \Delta t_{\rm atm}$)

Concurrent local-in-time approach (a.k.a explicit flux coupling)

ATM Model
$$\left\{ \begin{array}{rcl} u_{\rm atm}^{n+1} &=& u_{\rm atm}^{n} + \Delta t_{\rm atm} \partial_z \left(K_z^{\rm a} \partial_z u_{\rm atm}^{n+1} \right) + \mathcal{F}^{\rm a} \\ \rho_{\rm a} K_z^{\rm a} \partial_z u_{\rm atm}^{n+1} (z=0) &=& F_{\rm oa}(u_{\rm atm}^{n}(z^+), u_{\rm oce}^{n}(z^-)) \end{array} \right.$$

OCE Model
$$\left\{ \begin{array}{rcl} u_{\rm oce}^{n+1} &=& u_{\rm o}^{n} + \Delta t_{\rm oce} \partial_z \left(K_z^{\rm o} \partial_z u_{\rm oce}^{n+1} \right) + \mathcal{F}^{\rm o} \\ \rho_{\rm o} K_z^{\rm o} \partial_z u_{\rm o}^{n+1} (z=0) &=& \rho_{\rm a} K_z^{\rm a} \partial_z u_{\rm atm}^{n+1} (z=0) \end{array} \right.$$

Sequential local-in-time approach (a.k.a implicit flux coupling)

ATM Model
$$\left\{ \begin{array}{rcl} u_{\rm atm}^{n+1} &=& u_{\rm atm}^{n} + \Delta t_{\rm atm} \partial_z \left(K_z^{\rm a} \partial_z u_{\rm atm}^{n+1} \right) + \mathcal{F}^{\rm a} \\ \rho_{\rm a} K_z^{\rm a} \partial_z u_{\rm atm}^{n+1} (z=0) &=& F_{\rm oa}(u_{\rm atm}^{n+1}(z^+), u_{\rm oce}^{n}(z^-)) \end{array} \right.$$

OCE Model
$$\left\{ \begin{array}{rcl} u_{\rm oce}^{n+1} &=& u_{\rm o}^{n} + \Delta t_{\rm oce} \partial_z \left(K_z^{\rm o} \partial_z u_{\rm oce}^{n+1} \right) + \mathcal{F}^{\rm o} \\ \rho_{\rm o} K_z^{\rm o} \partial_z u_{\rm o}^{n+1} (z=0) &=& \rho_{\rm a} K_z^{\rm a} \partial_z u_{\rm atm}^{n+1} (z=0) \end{array} \right.$$

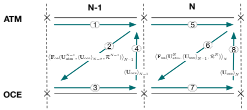
Alternative : monolithic approach based on a linearization of fluxes as in land surface / atmosphere coupling **[e.g. Ryder et al., 2016]**

Local-in-time methods

- lagged coupling $\rightarrow$ loosely coupled partitioned scheme
- Relevance of instantaneous air-sea fluxes ?
- Very frequent exchanges/remapping of interface data between models

\+ Ability to represent processes related to diurnal cycle

Coupling diffusion equations ($\Delta t_{\mathrm{oce}} \neq \Delta t_{\mathrm{atm}}$)



Sequential global-in-time approach (a.k.a asynchronous coupling)

For $n = 0, \mathrm{Nsteps}_{\mathrm{atm}} - 1$

ATM model
$$\begin{cases} u_{\mathrm{atm}}^{n+1} = u_{\mathrm{atm}}^{n} + \Delta t_{\mathrm{atm}} \partial_z \left(K_z^{\mathrm{a}} \partial_z u_{\mathrm{a}}^{n+1} \right) + \mathcal{F}^{\mathrm{a}} \\ \rho_{\mathrm{a}} K_z^{\mathrm{a}} \partial_z u_{\mathrm{atm}}^{n+1}(z=0) = F_{\mathrm{oa}}(u_{\mathrm{atm}}^{n+1}, I_{\Delta t_{\mathrm{oce}}}^{\Delta t_{\mathrm{atm}}} \{u_{\mathrm{oce}}^{\star}\}(t_{n+1})) \end{cases}$$

For $m = 0, \mathrm{Nsteps}_{\mathrm{oce}} - 1$

OCE model
$$\begin{cases} u_{\mathrm{o}}^{m+1} = u_{\mathrm{o}}^{m} + \Delta t_{\mathrm{o}} \partial_z \left(K_z^{\mathrm{o}} \partial_z u_{\mathrm{oce}}^{m+1} \right) + \mathcal{F}^{\mathrm{o}} \\ \rho_{\mathrm{o}} K_z^{\mathrm{o}} \partial_z u_{\mathrm{oce}}^{m+1}(z=0) = \mathcal{G}_{\Delta t_{\mathrm{atm}}}^{\Delta t_{\mathrm{oce}}} \{\rho_{\mathrm{a}} K_z^{\mathrm{a}} \partial_z u_{\mathrm{atm}}\}(t_{m+1}) \end{cases}$$

$\mathcal{G}_{\Delta t_{\mathrm{atm}}}^{\Delta t_{\mathrm{oce}}}$ and $\mathcal{I}_{\Delta t_{\mathrm{oce}}}^{\Delta t_{\mathrm{atm}}}$: intergrid transfer operators (only $\mathcal{G}$ has to be conservative)

Current practice: $\mathcal{G}_{\Delta t_{\mathrm{atm}}}^{\Delta t_{\mathrm{oce}}} \left\{ f|_{[0,T]} \right\}(t_i) = \dfrac{1}{T} \displaystyle\int_0^T f(t)dt, \qquad \forall t_i \in]0,T]$

Time-dependent, coupled, Ekman boundary layer model

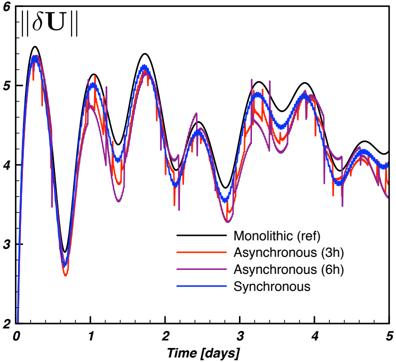
Complex horizontal velocity in the x-y plane : $\mathbf{U}_j = u_j + iv_j \ (j = 1, 2)$

$$\left\{ \begin{array}{rcll} \partial_t \mathbf{U}_j + if(\mathbf{U}_j - \mathbf{U}_{g,j}) & = & \partial_z \left(\nu_j(z) \partial_z \mathbf{U}_j \right), & \text{in } \Omega_j \times [0,T] \\ \mathbf{U}_j(z = H_j, t) & = & \mathbf{U}_{g,j}(z = H_j, t) & t \in [0,T] \\ \nu_2 \partial_z \mathbf{U}_2(z=0,t) & = & C_D \|\delta \mathbf{U}\| (\mathbf{U}_2(0^+,t) - \mathbf{U}_1(0^-,t)) & t \in [0,T] \\ \nu_1 \partial_z \mathbf{U}_1(z=0,t) & = & (\rho_2/\rho_1) \{ \nu_2 \partial_z \mathbf{U}_2(z=0,t) \} & t \in [0,T] \end{array} \right.$$

$\rightarrow$ Euler backward for diffusion and forward-backward for Coriolis

$\Omega_1 =]-50 \text{ m}; 0[$, $\Omega_2 =]0; 500 \text{ m}[$,
$\nu_1 = 0.05 \text{ m s}^{-1}$, $\nu_2 = 0.1 \text{ m s}^{-1}$,
$\Delta t_1 = \Delta t_2 = 600 \text{ s}$, $f = 10^{-4} \text{s}^{-1}$
$\rho_1 = 1000 \text{ kg m}^{-3}$, $\rho_2 = 1 \text{ kg m}^{-3}$
$C_D \|\delta \mathbf{U}\| = \alpha_1 + \alpha_2 \|\delta \mathbf{U}\| + \alpha_3 \|\delta \mathbf{U}\|^2$
α_j from (Large, 2005)
$u_{g,1} = v_{g,1} = v_{g,2} = 0 \text{ m s}^{-1}$,
$u_{g,2} = 10 \text{ m s}^{-1}$
$\mathbf{U}_1(-H_1, t) = \mathbf{U}_{g,1}(-H_1, t) + f_1(t)$
$\mathbf{U}_2(H_2, t) = \mathbf{U}_{g,2}(H_2, t) + f_2(t)$

$\Rightarrow$ Spurious high frequencies



Global-in-time (sequential or concurrent)

- + Both models forced by the exact same mean flux on a given time window
- + Models communicate only once per time window
- + More consistent with the underlying assumptions of some physical parameterizations
- + More freedom to select the relevant temporal scales to be exchanges between models through the $\mathcal{G}_{\Delta t_{\rm atm}}^{\Delta t_{\rm oce}}$ and $\mathcal{I}_{\Delta t_{\rm atm}}^{\Delta t_{\rm oce}}$ operators
- - Shifted retroaction $\rightarrow$ loosely coupled solution
- - Current choice for $\mathcal{G}_{\Delta t_{\rm atm}}^{\Delta t_{\rm oce}}$ $\Rightarrow$ discontinuity in forcing fields between two successive time windows

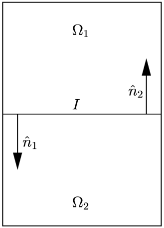
- Asynchronous coupling = one single iteration of a global-in-time Schwarz algorithm

Tight coupling

- [**Keyes et al. (2013)**]: *" Using an approach that ignores strong couplings between components gives a false sense of completion"*

  If the iterations actually fail to converge, using only one iteration won't reveal this fact but in this case numerical results would be questionable

$$
\begin{cases}
\partial_t u_1 - \nabla \cdot (\nu_1 \nabla u_1) & = & f_1, & x \in \Omega_1 \\
\partial_t u_2 - \nabla \cdot (\nu_2 \nabla u_2) & = & f_2, & x \in \Omega_2 \\
u_1 & = & u_2, & x \in \Gamma \\
\nu_1 \nabla u_1 & = & \nu_2 \nabla u_2, & x \in \Gamma
\end{cases}
$$

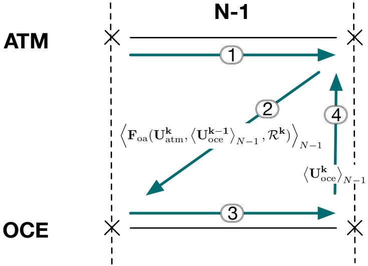


Convergence rate : $R = \sqrt{\nu_2/\nu_1}$ [**e.g. Lemarié et al., 2013**]

- **BUT:** tight coupling between components require smoothness

- *Easy way to try it :* Schwarz algorithms (global-in-time for multiphysics problems; only require "perfect" restartibility of numerical models)

Global-in-time domain decomposition based on Schwarz method



Repeat steps 1,2,3 and 4 iteratively until « convergence »

Really ? Does it converge ? If yes, what's the impact on the physics ?

Connecting model stability and model uncertainty

Connors J. M. and B. Ganis : *Stability of algorithms for a two domain natural convection problem and observed model uncertainty*. Comput. Geosci., 2011

- Simplified equation sets
- Numerical experiments with uniformly distributed noise of 10% in C_D and C_H
- 3 different coupling methods
  1. TWN $\leftrightarrow$ Schwarz
  2. TWP-GA $\leftrightarrow$ Concurrent local-in-time
  3. OWP-GA $\leftrightarrow$ Concurrent local-in-time (no currents)

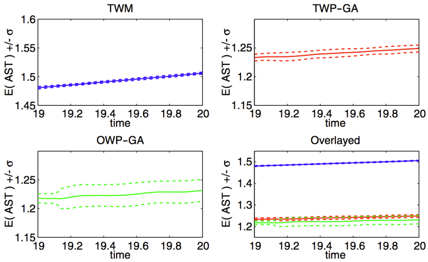


Fig. 14 Expectation of AST (*solid line*) and adding or subtracting one standard deviation (*dotted lines*)

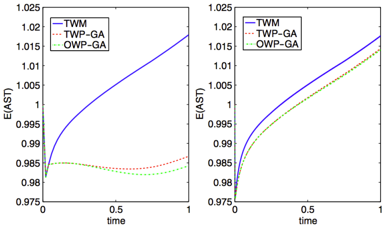


Fig. 12 Expectation of AST using $\Delta t = 1/50$ (*left*) and $\Delta t = 1/1{,}000$ (*right*) for $t \in [0, 1]$

AST = Average Surface Temperature

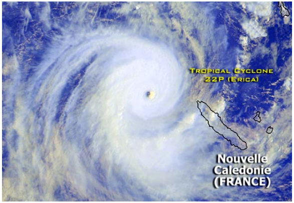
Impact on realistic numerical simulations

Lemarié F., P. Marchesiello, L. Debreu, and E. Blayo: *Sensitivity of ocean-atmosphere coupled models to the coupling method : example of tropical cyclone Erica*. Report RR-8651, 2014

- Numerical codes :
  - ▷ **WRF** (NCAR), Compressible Euler
  - ▷ **ROMS** (UCLA, IRD, INRIA), Primitive equations

- Test-case : simulation of tropical cyclone Erica (New Caledonia, 03/2003)

$\Delta x_{\mathrm{atm}} = \Delta x_{\mathrm{oce}} = 25$ km, $\Delta t_{\mathrm{atm}} = 120$ s, $\Delta t_{\mathrm{oce}} = 2400$ s

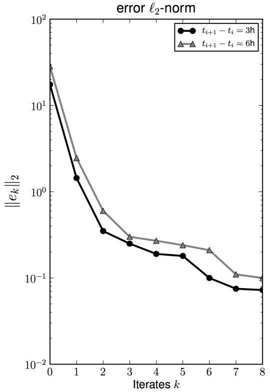


- Ensemble approach : perturbation of initial conditions and coupling frequency (3h vs 6h + linear reconstruction of surface fluxes)

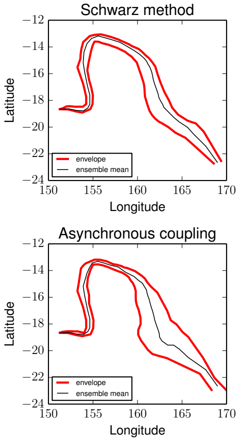
Two ensembles: asynchronous method *vs* Schwarz method

Impact on realistic numerical simulations

Iterative process and ensemble spread



$$\|e_k\|_2 = \sqrt{\sum_{n=1}^{n_x \times n_y} |\mathrm{SST}_n^k - \mathrm{SST}_n^{k=9}|^2}$$

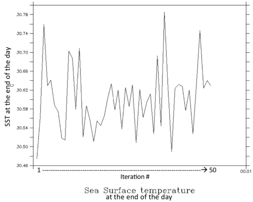


	asynchronous	Schwarz
Trajectory	125 km	92 km
Intensity	6.2 m s^{-1}	4.1 m s^{-1}

The COCOA project

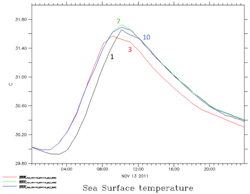
COmprehensive Coupling approach for the Ocean and the Atmosphere

single column version of CNRM-CM



$\Delta t_{\mathrm{cpl}} = 86400$ s

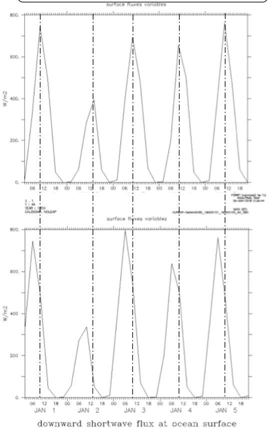
No convergence



$\Delta t_{\mathrm{cpl}} = 7200$ s

IPSL-CM

Shortwave flux seen by the ocean model



1 iteration

50 iterations

Partial conclusion

- May not be needed to dig too much into advanced partitioned methods as most models integrate the vertical physics implicitly (e.g. no problem with order reduction)
- Different OA coupled models seem to converge when using iterative coupling methods
- Additional work on the impact of the $\mathcal{G}_{\Delta t_{\rm atm}}^{\Delta t_{\rm oce}}$ and $\mathcal{I}_{\Delta t_{\rm atm}}^{\Delta t_{\rm oce}}$ operators would be worthwhile ($\rightarrow$ smoother transition between coupling periods)
- Huge gap to fill between the theory and practical applications.
  Analysis of Schwarz waveform relaxation for the coupled Ekman boundary layer problem with continuously variable coefficients (Théry, Lemarié & Blayo, 2019, SIAM SISC, submitted)
- ⚠Literature claiming non-conservation of energy in OA models

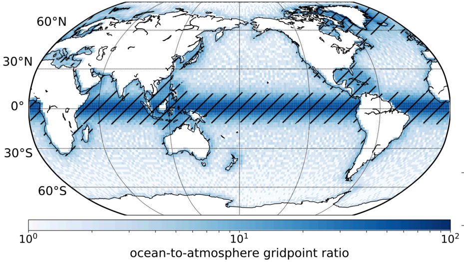
ATM Model
$$\left\{\begin{array}{rcl} u_{\rm atm}^{n+1} &=& u_{\rm atm}^{n} + \Delta t_{\rm atm}\partial_z\left(K_z^{\rm a}\partial_z u_{\rm atm}^{n+1}\right) + \mathcal{F}^{\rm a} \\ \rho_{\rm a} K_z^{\rm a}\partial_z u_{\rm atm}^{n+1}(z=0) &=& F_{\rm oa}(u_{\rm atm}^{n+1}(z^+), u_{\rm oce}^{n}(z^-)) \end{array}\right.$$

OCE Model
$$\left\{\begin{array}{rcl} u_{\rm oce}^{n+1} &=& u_{\rm o}^{n} + \Delta t_{\rm oce}\partial_z\left(K_z^{\rm o}\partial_z u_{\rm oce}^{n+1}\right) + \mathcal{F}^{\rm o} \\ \rho_{\rm o} K_z^{\rm o}\partial_z u_{\rm o}^{n+1}(z=0) &=& F_{\rm oa}(u_{\rm atm}^{n}(z^+), u_{\rm oce}^{n+1}(z^-)) \end{array}\right.$$

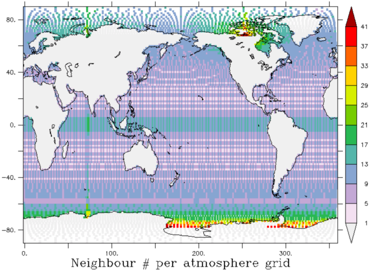
4

Non-conformity in space

Ocean to atmosphere resolution ratio



AWI-CM (Echam6-FESOM)



CNRM-CM (NEMO-Arpege)

- Large local differences in resolution
- The atmosphere must ”integrate” different subgrid surface fractions (tiles)

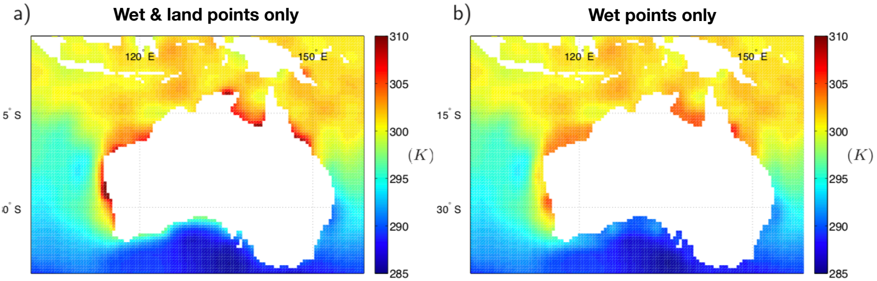
  Grid-box surface fluxes are calculated separately for the different tiles

$$K_\phi \partial_z \phi \big|_{\rm sfc} = \sum_{i-1}^{N_T} F_i C_{\phi_i} \| \mathbf{v}_{\rm atm}(z^1_{\rm atm}) - \mathbf{v}_{\rm oce}(0^+) \| (\phi(z^1_{\rm atm}) - \phi_{\rm sfc})$$

  each tile (except the open sea) is treated via a surface energy balance
- Loss of small-scale information

Additional difficulty

- ▷ Interpolation stencils should be exclusive



- ▷ Ocean/sea-ice and Ocean-land (wetting & drying) boundaries vary with time
  - No wetting & drying in current generation of OGCMs
  - Remapping weights are computed once for all with a predefined sea-ice cover.

Some current practices

1. **Coupler grid = Atm. dynamics grid (= Atm. physics grid)**

   Compute fluxes on the atmospheric computational grid using bilinear/bicubic remapping (e.g. SCRIP package within MCT)

2. **Coupler grid = exchange grid $\neq$ Atm. dynamics grid** [**Balaji et al., 2005**]

   Exchange grid = juxtaposition of the land-surface, ocean, sea-ice, glacial ice computational grids

3. **Coupler grid = exchange grid = Atm. physics grid $\neq$ Atm. dynamics grid** [**Vintzileos & Sadourny, 1995**]

   ”Delocalized physics”

Exchange grid

Balaji V. et al. : ***The exchange grid: a mechanism for data exchange between earth system components on independent grids*****, 2005**

Surface fluxes are computed on the exchange grid

"Delocalized" physics

Vintzileos & Sadourny : *A general interface between an atmospheric general circulation model and underlying ocean and land surface models: delocalized physics scheme*, MWR, 1995

$\rightarrow$ Atmospheric single column physics is computed on a juxtaposition of the land-surface, ocean, sea-ice, glacial ice computational grids

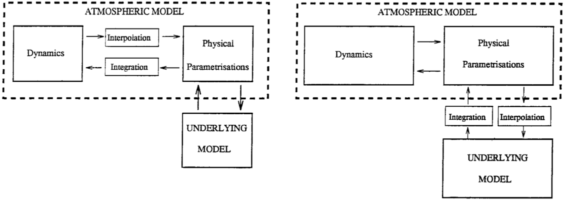
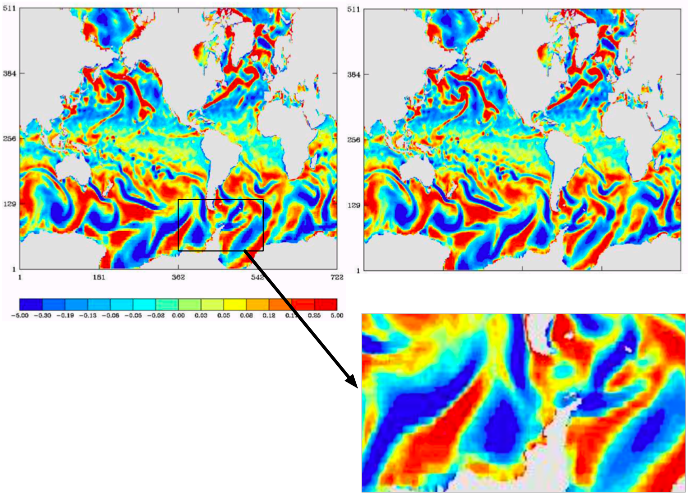


FIG. 1. (a) Schematic diagram of the delocalized physics method. The two basic parts of the AGCM, namely, physical parameterizations and dynamics, are interfaced through the interpolation–integration scheme, the physics being directly coupled to the underlying model. (b) The usual coupling approach. The interface is now defined between the physical parameterizations of the AGCM and the underlying model.

- Robustness to "under-resolved" scales on the atmospheric dynamics grid ?
- Scale-awareness of the physics package ?
  (Challenging already to find a robust parameterization set for uniform resolution)

Loss of momentum conservation

$$\vec{\nabla} \times \vec{U}_{10} = \frac{1}{R_{terre}\ \cos\phi}\left[\frac{\partial\ v_{10}}{\partial\lambda} - \frac{\partial}{\partial\phi}(u_{10}\ \cos\phi)\right]\vec{e_z}$$



5

Concluding remarks

Coupling technologies used in CMs (Courtesy of S. Valcke)

Coupling of codes :

- Exchange and transform information at the code interface
- Manage the execution and synchronization of the codes
- Applicable to existing and independently developed codes

Softwares

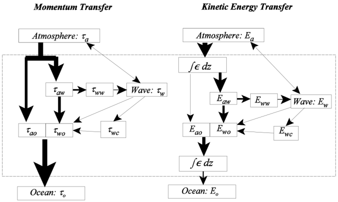
- ESMF (Earth System Modeling Framework) +NUOPS layer **[Hill et al., 2004]**
  - Single executable
  - Can be run sequentially, concurrently, in mixed mode
- CESM/cpl7 **[Craig et al., 2012]**
  - Developed by NCAR, uses MCT for data regridding and exchange
  - Single executable
- FMS (Flexible Modeling System)
  - Single executable
  - Serial or concurrent execution of components
  - Exchange grid
- OASIS3-MCT **[Craig et al., 2017]**
  - Developed by Cerfacs, uses MCT for data regridding and exchange
  - used by $\approx$35 climate modelling groups world-wide

Comments on surface waves & momentum exchange

- Surface waves are characterized by small $(1 - 100\ \mathrm{m})$ and fast $(1 - 10\ \mathrm{s})$ scales $\Rightarrow$ Wave-induced processes often ignored in CMs

$\rightarrow$ But surface waves
- roughen the surface and affect the mechanical coupling
- modify the structure of the surface boundary layers in both fluids
- are responsible for a momentum transport

- Usual assumption: all of the surface wind stress goes into direct forcing of the surface currents



- MO similarity theory only account for the influence of mechanical and thermal forcing on the turbulence
  - $\rightarrow$ additional scaling parameters required to describe the wave boundary layers
  - $\rightarrow$ MO not applicable where the flow is influenced by ocean waves

Summary

- Interface conditions
  - Clarify the mass exchange interface condition
  - Does the atmosphere care about the evolution of the oceanic free-surface ?
- Air-sea flux estimates
  - Which alternative to the MO similarity theory ?
- Non-conformity in time
  - Global-in-time approach may offer more flexibility (even for high-frequency coupling)
- Non-conformity in space
  - Cleaner treatment of sea-ice ?

▷ Need for more systematic benchmarking of coupled models under simplified settings (!)