Thermality of circular motion

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Quantum Foundations, Gravity, and Causal Order

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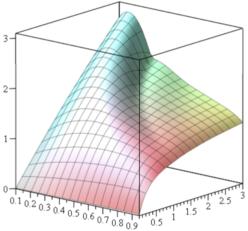
Biermann *et al.* PRD **102**, 085006 (2020) [arXiv:2007.09523]



Plan

1. **Unruh effect**
   - **Relativistic** spacetime and **analogue** spacetime
2. **"Quantum dot"**
   - Unruh-DeWitt
3. **Circular motion**
   - Wightman function: $3+1$ and $2+1$
4. **Results**
   - Ratio $T_{\text{circular}}/T_{\text{linear}}$
5. **Summary and outlook**

1. Unruh effect

Well established

- Uniformly **linearly** accelerated observer sees Minkowki vacuum as thermal, $T = \frac{a}{2\pi}$ Unruh 1976
- Weak coupling, long time, negligible switching effects
- Thermal: Observer/detector records detailed balance:

$$\boxed{\frac{P_\downarrow}{P_\uparrow} = e^{E_{\text{gap}}/T}}$$

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Uniform circular motion?

- Long time in finite size lab!
- Accelerator storage rings Bell and Leinaas 1983,...
- Analogue spacetime: BEC, ^{4}He,... Weinfurtner talk (Friday)

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Sense of "temperature" ?

Aims

Why now

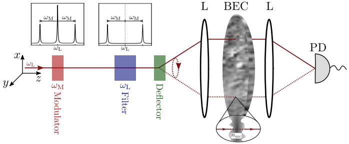
What today

What not today

Aims

Why now

- Analogue spacetime experiment proposal Gooding *et al.* 2020
  - Finite size lab
  - Time dilation $\leftrightarrow$ time-independent energy scaling



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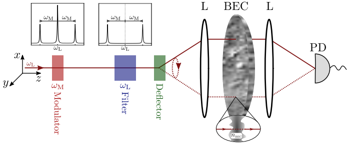

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- "**Quantum dot**"
- Weak coupling, long time, negligible switching effects
- **Sense of temperature**
  - **Relativistic** spacetime versus **analogue** spacetime
  - $3 + 1$ versus $2 + 1$

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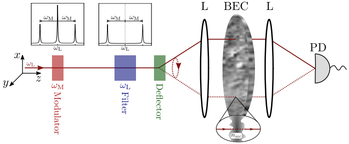

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  - $3+1$ versus $2+1$

What not today

- "Quantum dot" $\rightarrow$ actual experiment?

2. **"Quantum dot"** (relativistic)

Unruh(1976)-DeWitt(1979)

Quantum field

D	spacetime dimension
ϕ	real scalar field
$\|0\rangle$	Minkowski vacuum

Two-state detector (atom)

$\|0\rangle\rangle$	state with energy 0
$\|1\rangle\rangle$	state with energy E
$\mathsf{x}(\tau)$	detector worldline, τ proper time

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Interaction

$$H_{\text{int}}(\tau) = c\chi(\tau)\mu(\tau)\phi(\mathsf{x}(\tau))$$

c coupling constant

χ switching function, C_0^∞, real-valued

μ detector's monopole moment operator

Probability of transition

$$\|0\rangle\rangle \otimes |0\rangle \longrightarrow \|1\rangle\rangle \otimes |\text{anything}\rangle$$

in first-order perturbation theory:

$$P(E) = c^2 \underbrace{\left|\langle\langle 0\|\mu(0)\|1\rangle\rangle\right|^2}_{\substack{\text{detector internals only:}\\ \text{drop!}}} \times \underbrace{F_\chi(E)}_{\substack{\text{trajectory and } |0\rangle:\\ \text{response function}}}$$

$$F_\chi(E) = \int \mathrm{d}\tau' \mathrm{d}\tau''\, \mathrm{e}^{-iE(\tau'-\tau'')}\, \chi(\tau')\chi(\tau'')\, W(\tau', \tau'')$$

$$W(\tau', \tau'') = \langle 0|\phi\big(\mathsf{x}(\tau')\big)\phi\big(\mathsf{x}(\tau'')\big)|0\rangle$$

Wightman function (distribution)

- **Stationary** motion:

$$W(\tau', \tau'') = W(\tau' - \tau'', 0)$$

- Transition **rate** in the long time limit:

$$\frac{F_\chi(E)}{\Delta\tau} \xrightarrow[\Delta\tau\to\infty]{} F(E) = \int_{-\infty}^{\infty} \mathrm{d}s\, \mathrm{e}^{-iEs}\, W(s,0)$$

stationary response function

- **Temperature** via detailed balance:

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  - For uniform **linear** acceleration, $T_{\text{lin}} = \frac{a}{2\pi}$ Unruh 1976 (genuine KMS state)
  - For other uniform motions, T depends also on E Letaw 1981,. . . , Good *et al.* 2020

3. Circular motion

- **Metric**: $ds^2 = -dt^2 + (dx^1)^2 + \cdots + (dx^{D-1})^2$
- **Trajectory**: $\mathsf{x}(\tau) = \big(\gamma\tau, R\cos(\gamma\Omega\tau), R\sin(\gamma\Omega\tau), \cdots\big)$

  $R > 0$ radius, $0 < R\Omega < 1$ orbital velocity, $\gamma = 1/\sqrt{1-(R\Omega)^2}$
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- Relativistic spacetime: $\dfrac{T_{\mathrm{circ}}}{T_{\mathrm{lin}}}$ (for same proper acceleration)
- Analogue spacetime: similarly

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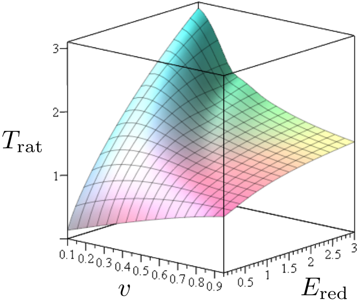
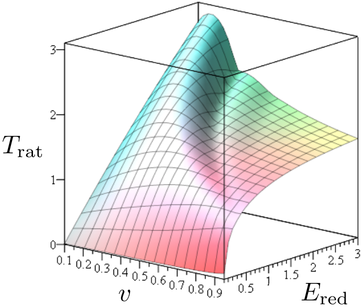


$3+1$ dimensions



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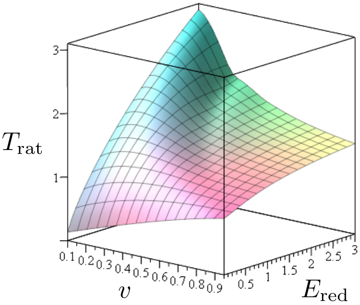

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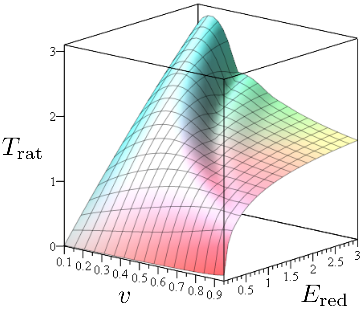

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Not constant!

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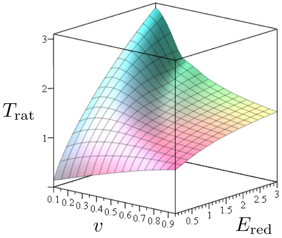

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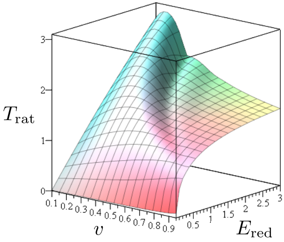

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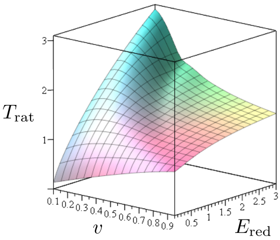
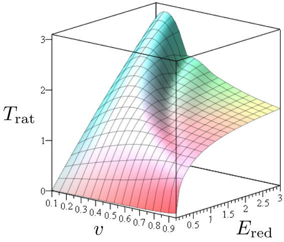


3 + 1 dimensions

$E_{\text{red}} \to 0$: **nonzero**



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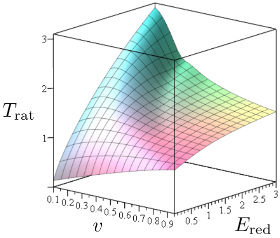

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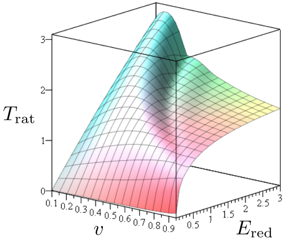

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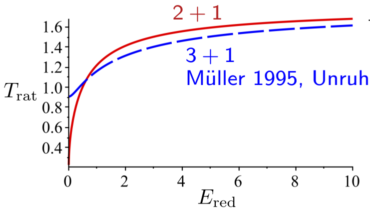
$v \to 1$:



$\xrightarrow[E_{\text{red}}\to\infty]{} \frac{\pi}{\sqrt{3}} \approx 1.8$

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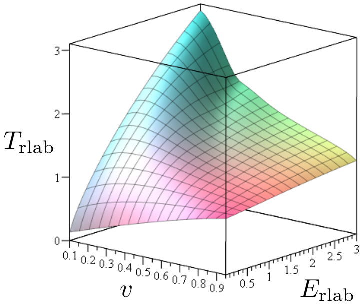
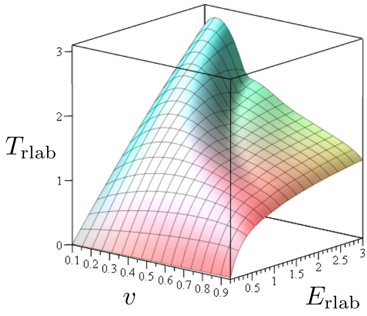


$3 + 1$ dimensions



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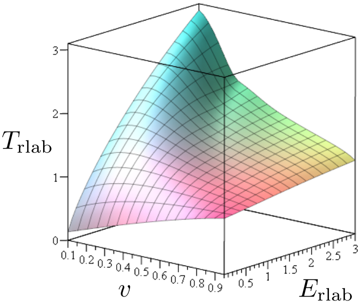

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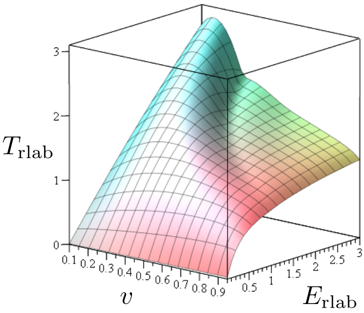

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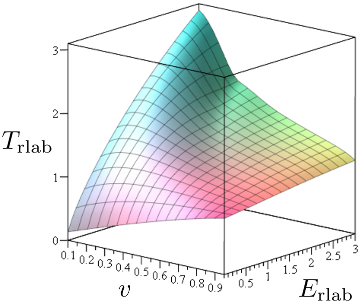

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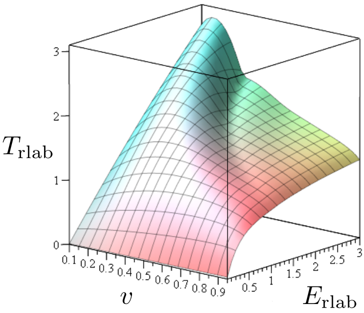

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$v \to 1$: $\quad T_{\text{rlab}} \to \dfrac{\pi}{2\sqrt{3}} \approx 0.9$

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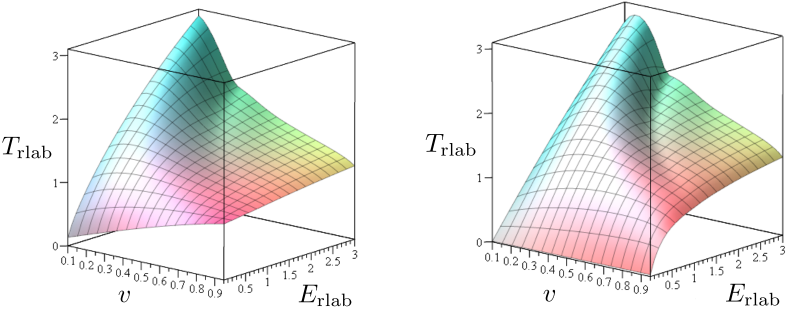

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Suppressed at $v \to 1$

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- "**Quantum dot**" in **circular motion**; massless scalar
- Relativistic and analogue
- Unruh temperature T_{circ} via detailed balance

Outcomes

- $3+1$: $T_{\text{circ}}/T_{\text{lin}}$ of order unity (relativistic and analogue)
- $2+1$: $T_{\text{circ}}/T_{\text{lin}} \ll 1$ for:
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Fun fact: $2+1$ time-time correlations are purely imaginary (!?!)