High-Q Spectral Peaks and Nonstationarity in the Geomagnetic Field over the 400-4000 μHz Band

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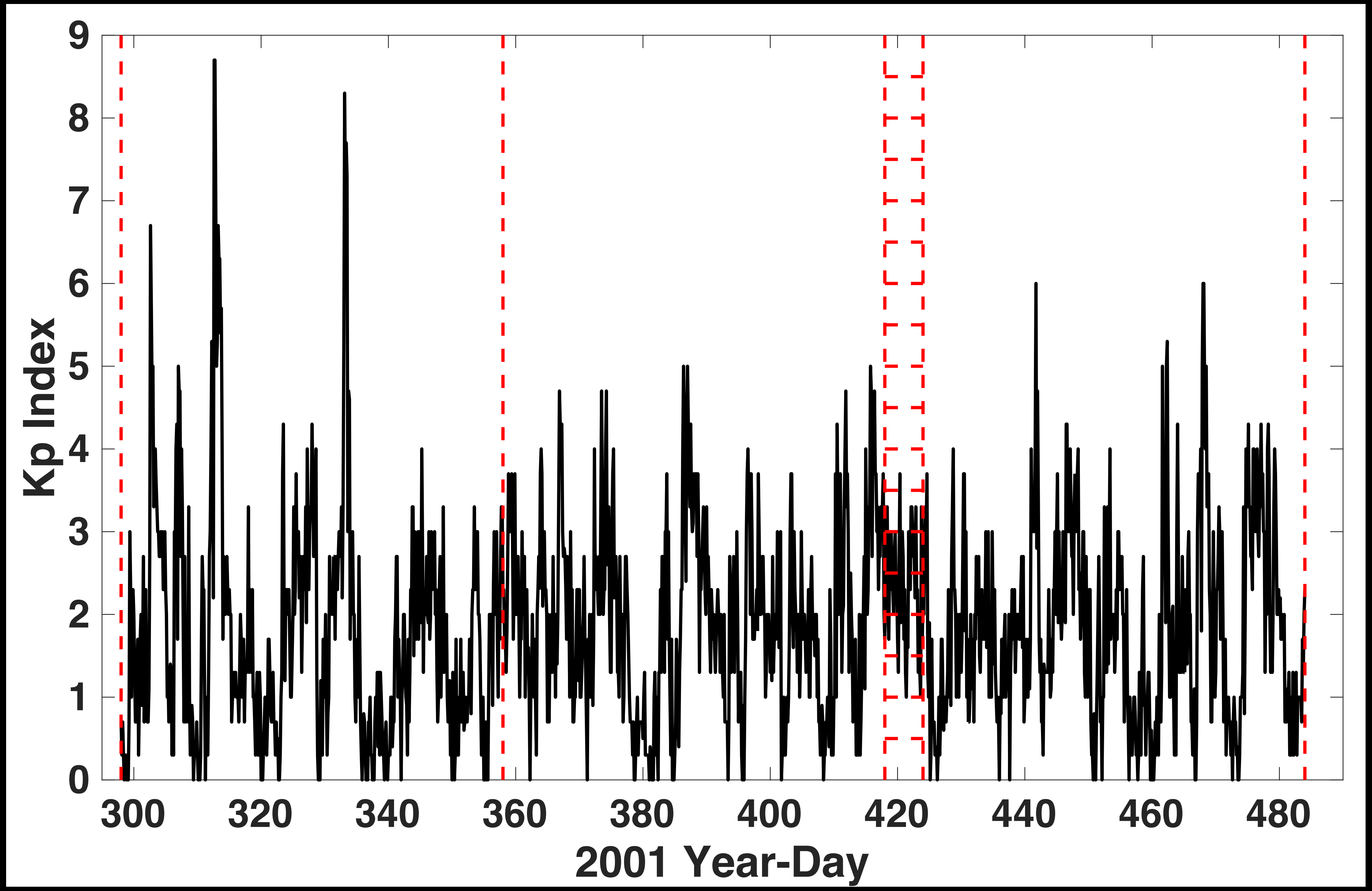
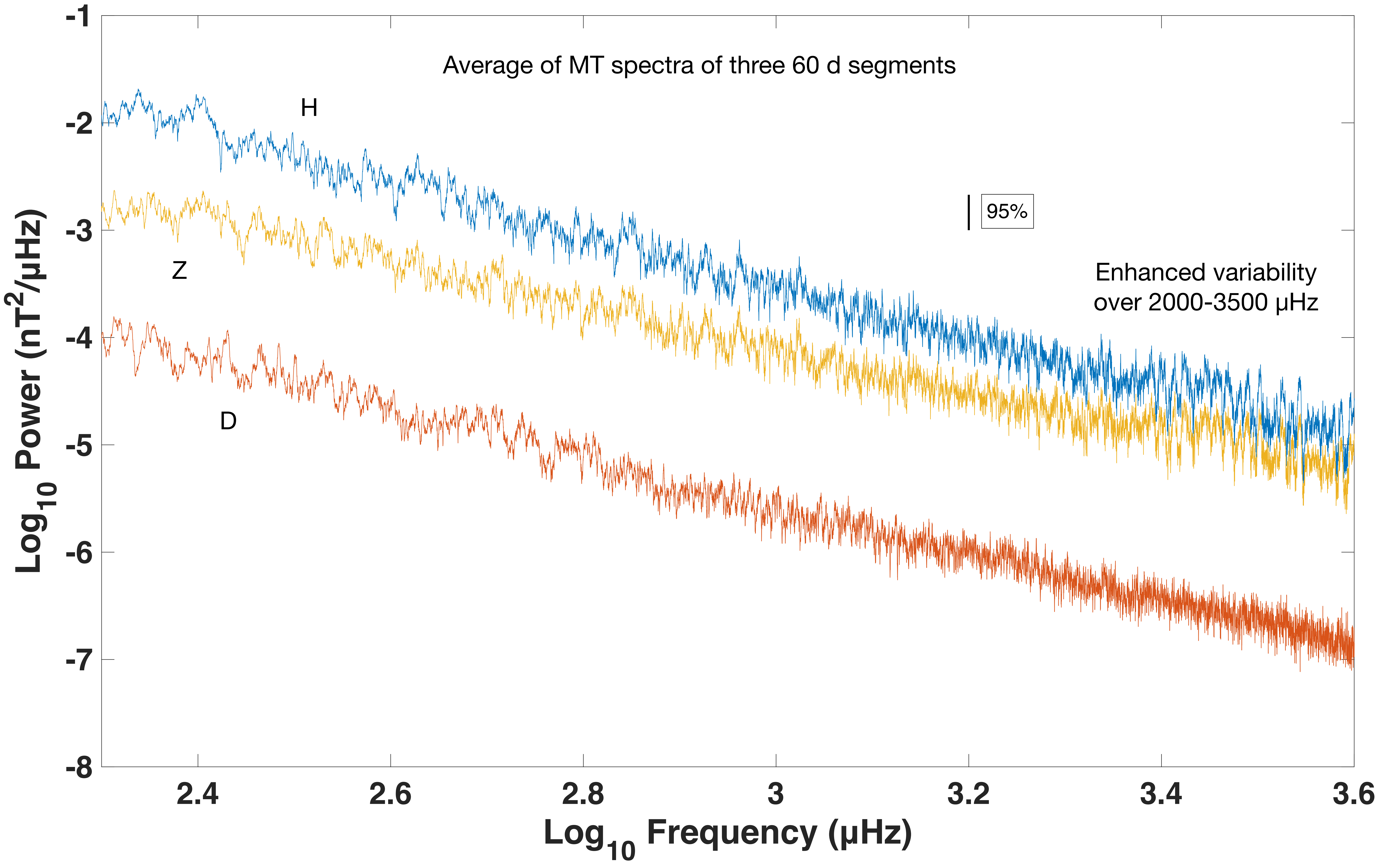
Data

- Honolulu Observatory standard 1 minute data from 4/2001-5/2002
- Data interval coincides with seafloor pressure data collected during same epoch
- Analysis uses data segments long enough for adequate frequency resolution but short on the time scale for nonstationarity—>60 days
- Data gaps leave only three 60 d intervals in time series
    - YD 298-358, 358-418, 424-484
- Solar cycle 23 peaked in late 2001, so geomagnetic activity level is moderate to high

Kp Index



Kp is a quasi-logarithmic measure of planetary geomagnetic activity
Kp under 5 is quiet, Kp=9 is extreme, rare event

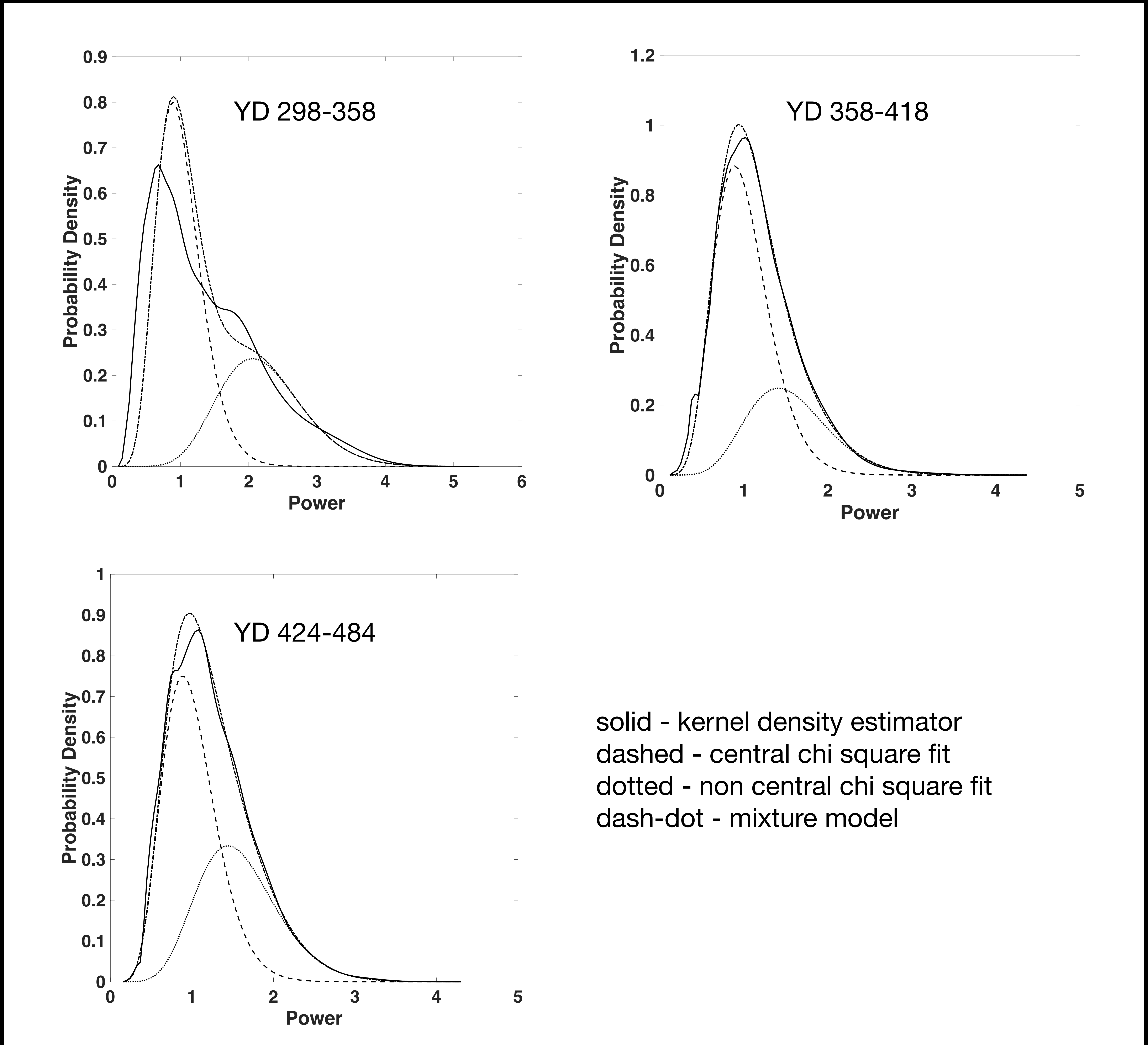


Average of MT spectra of three 60 d segments
H
Z
D
95%
Enhanced variability
over 2000-3500 μHz
-1
-2
-3
-4
-5
-6
-7
-8
2.4
2.6
2.8
3
3.2
3.4
3.6
Log10 Power (nT2/μHz)
Log10 Frequency (μHz)

Mixture Central/Noncentral Chi Square Fit

- Focus will be on H component
- $d(\varepsilon,\lambda) = \varepsilon\chi^2_{2K}(\lambda) + (1-\varepsilon)\chi^2_{2K}(0)$
- MLE implemented using nonlinear multivariable programming solver
- Fit to standardized spectra over 1000 μHz bands on 3 data sections

2000-3000 μHz



Average Fit to YD 358-418 and 424-484 Mixture Model

Frequency Band	ε
1000-2000 μHz	0.13
1500-2500 μHz	0.24
2000-3000 μHz	0.35
2500-3500 μHz	0.30
3000-4000 μHz	0.17

Nonstationary Spectra

$$x_t = \int_{-1/2}^{1/2} e^{i2\pi\xi t} dX(\xi)$$ where $E[dX(f)]=0$ and $X(f)$ is unobservable increment process

Stationary process has orthogonal increments

$E[dX(f_1)dX^*(f_2)] = S(f_1)\delta(f_1-f_2)df_1df_2$ —> distinct frequencies are uncorrelated

Nonstationary process has non-orthogonal increments

$E[dX(f_1)dX^*(f_2)] = S_L(f_1,f_2)df_1df_2$ —> distinct frequencies may be correlated

S_L is the Loève spectrum

Nonstationary process forced at a given frequency will result in power transfer to other unforced frequencies with concomitant high correlation.
Forced nonstationarity is inherently nonlinear.

Standardized Spectra

- High pass filter data segment using Butterworth 3 pole filter with 3 dB point at 46 μHz run forward and backward
- Compute MT spectrum utilizing AR filter pre-whitening
- Post-whiten by fitting and removing quadratic polynomial to log spectrum vs log frequency over 100-5000 μHz
- Plot on linear frequency vs linear power scale
- Assess significance using p-quantiles of mixture chi square model
- For 60 d segment with TBW = 5, resolution bandwidth is 1.9 μHz and Rayleigh resolution is 0.19 μHz

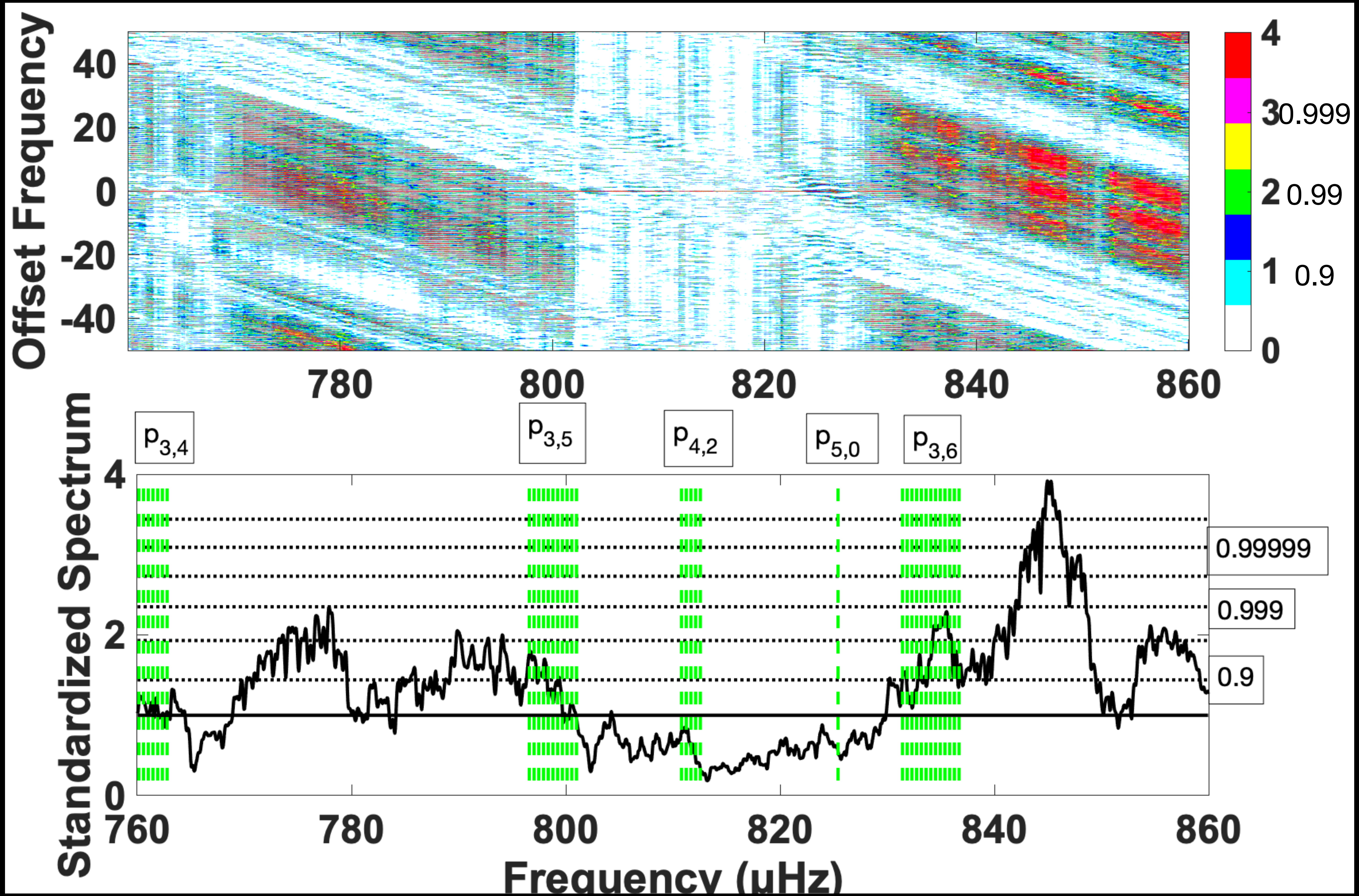
Frequency Offset Coherence

- Proxy for Loève spectrum
- Two independent variables
  - Ordinary frequency f
  - Offset frequency f’
- Stationary process offset coherence is zero except at f’=0
- Resolution bandwidth for ordinary frequency, Rayleigh resolution for offset frequency
- Contour plot of ordinary vs offset frequency, coherence scaled so that most coherent elements are emphasized

Solar Normal Modes

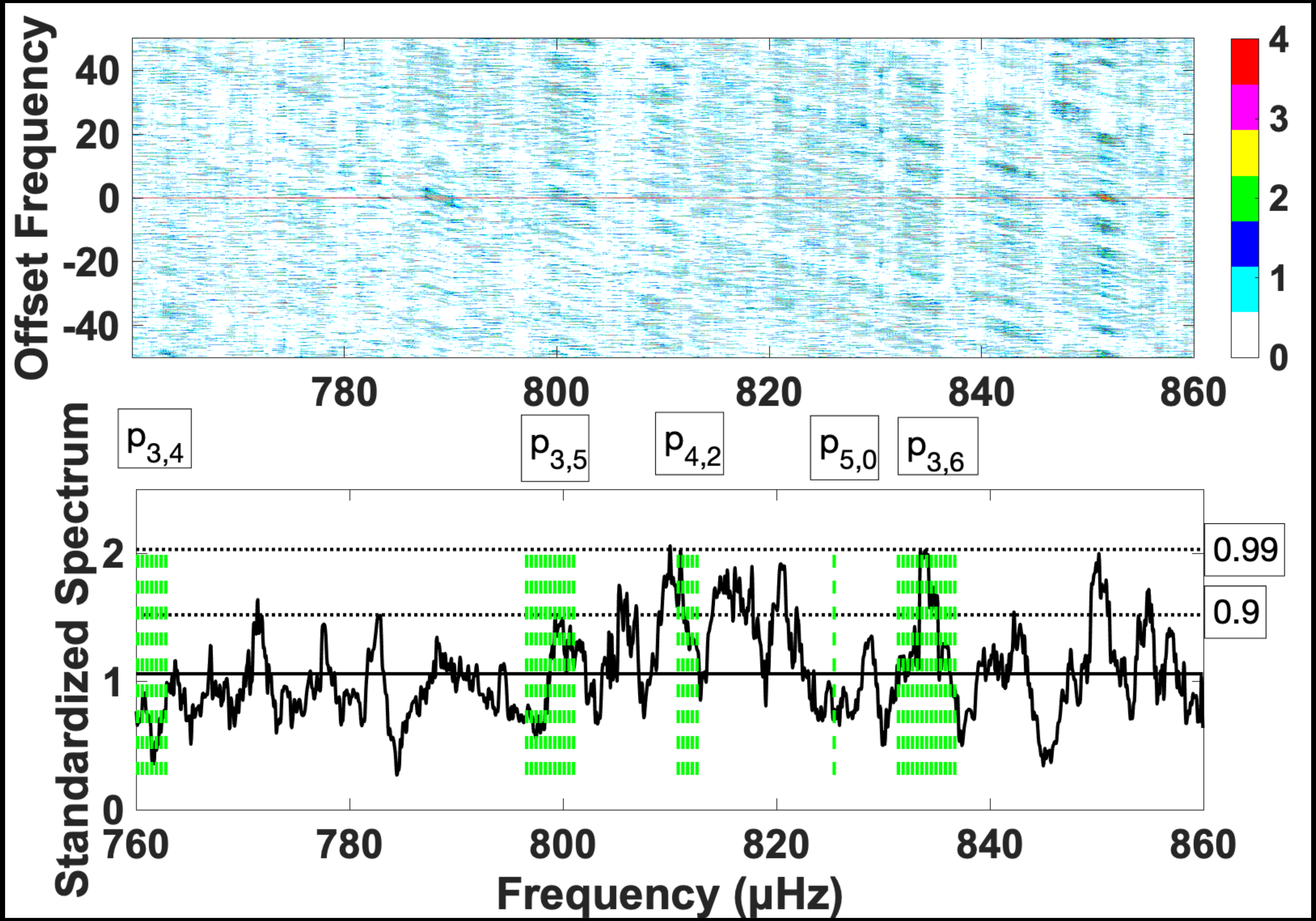
- Some internal fluctuations of Sun can be modeled as normal modes represented by a spherical harmonic expansion
- Quantum numbers n, l and m for radial zeros, latitudinal and longitudinal nodal lines
- Characterized by center frequency f and quality factor Q
- P-modes are solar acoustic standing waves over 250-5100 μHz
- Amplitudes are random due to origin in turbulence
- P-mode Qs are typically several thousand, so a given mode persists for a few months
- Cyclostationarity due to Earth rotation

YD 298-358 μHz



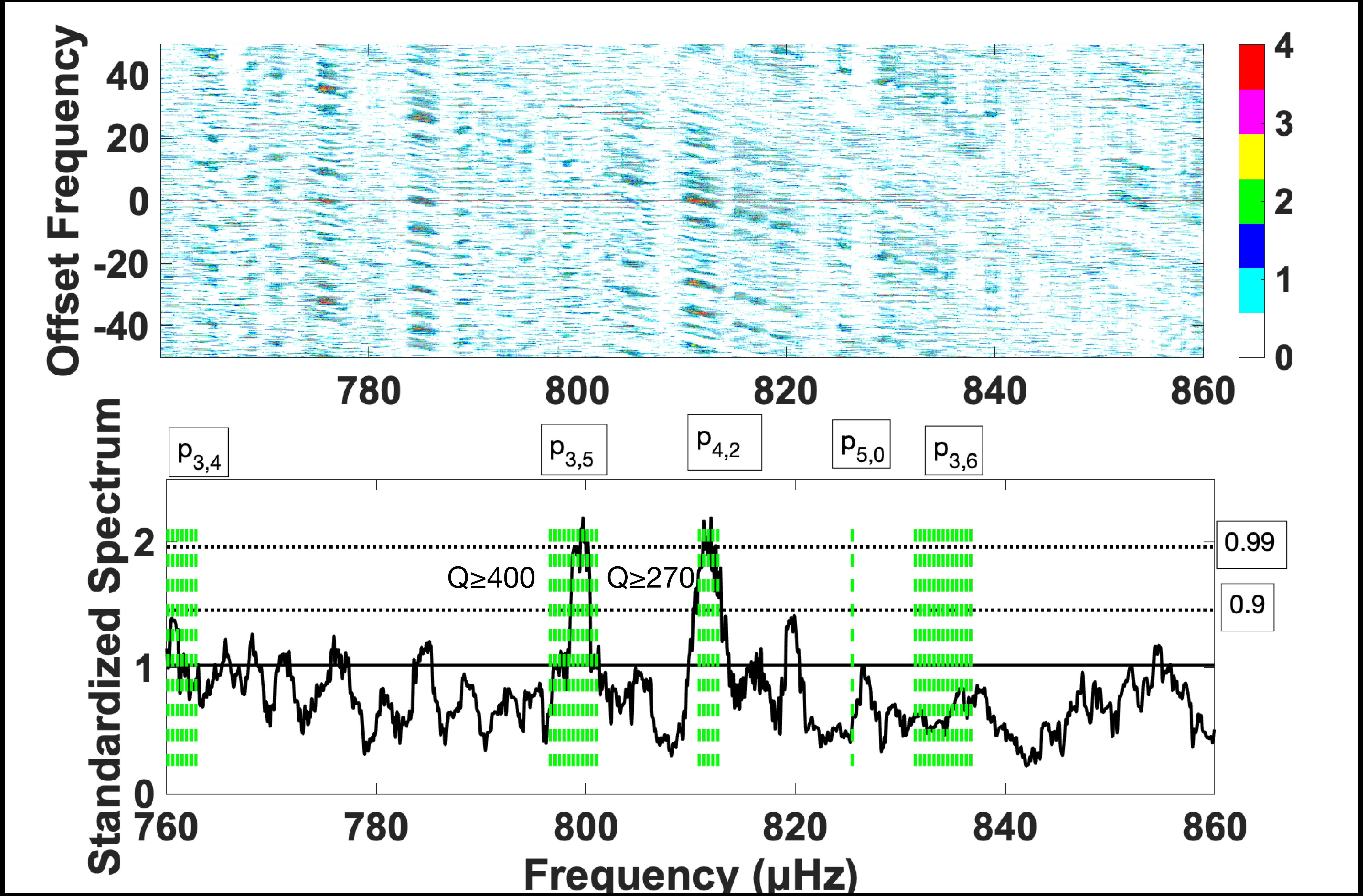
Trapezoidal block: if f_1 is coherent with $f_2 > f_1$, then the coherence will be high at positive offset frequency $f_2 - f_1$ and also at negative offset frequency $f_1 - f_2$ at frequency f_2

YD 358-418

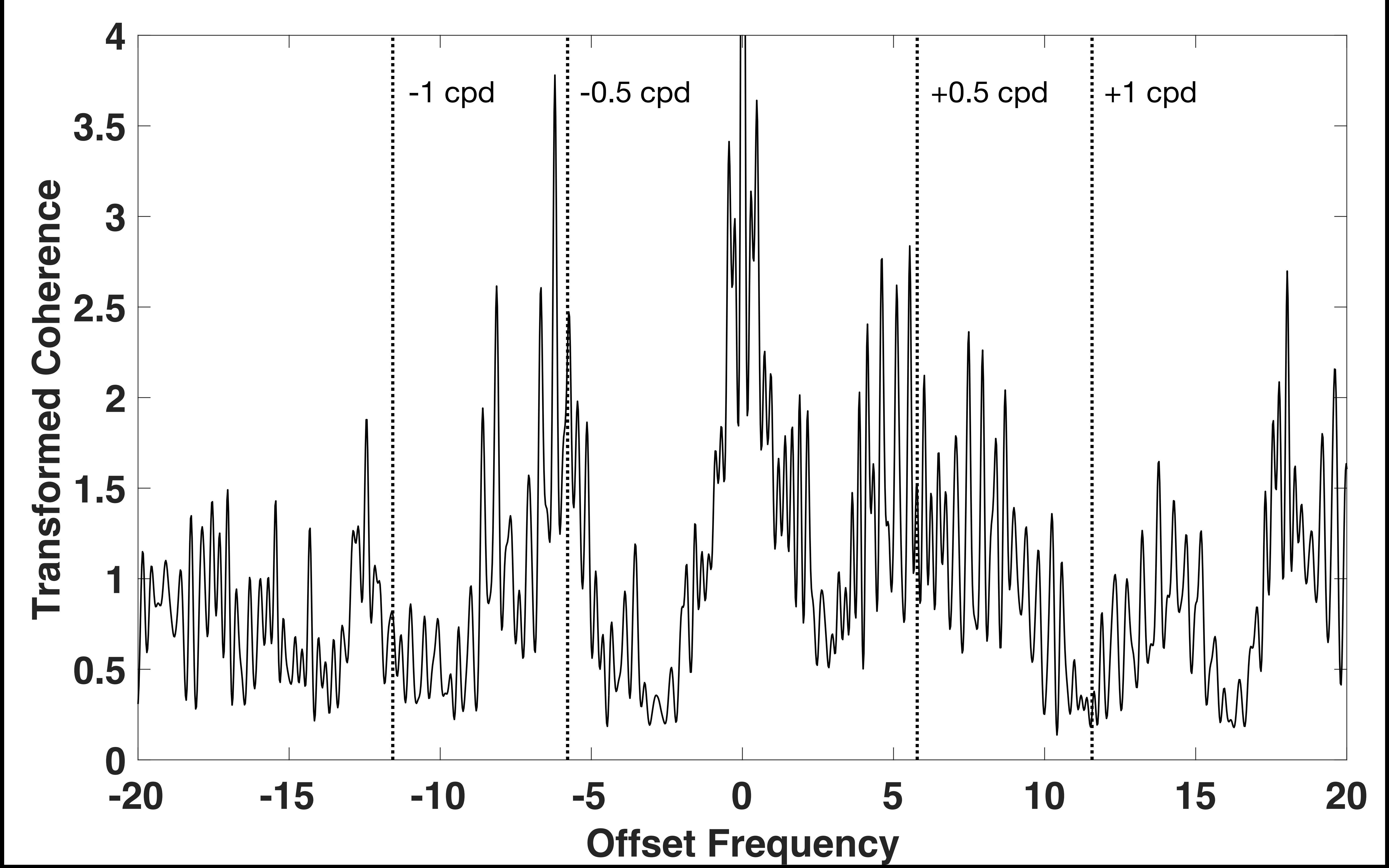


Offset Frequency
40
20
0
-20
-40
780
800
820
840
860
4
3
2
1
0
p3,4
p3,5
p4,2
p5,0
p3,6
0.99
0.9
Standardized Spectrum
2
1
0
760
780
800
820
840
860
Frequency (μHz)

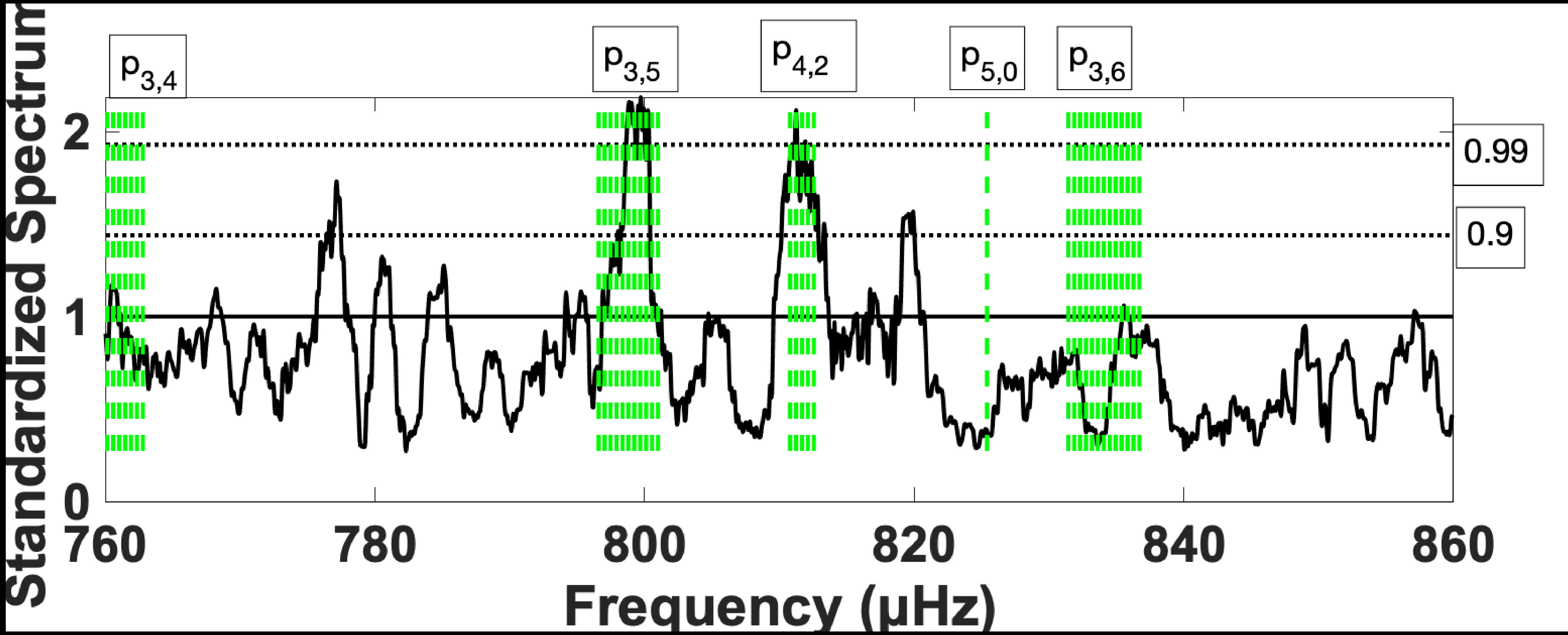
YD 424-484

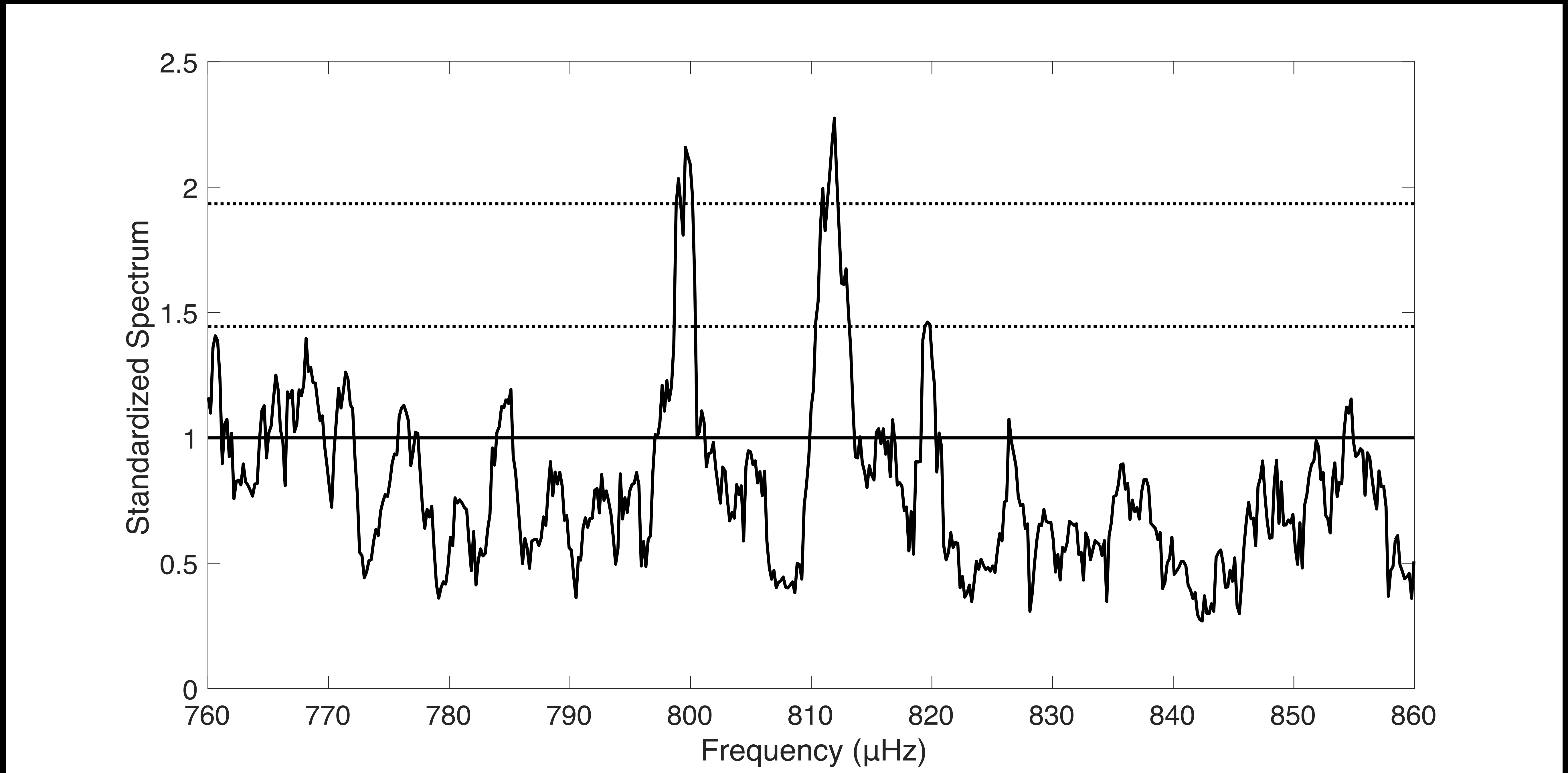


Offset Frequency
40
20
0
-20
-40
780
800
820
840
860
4
3
2
1
0
p3,4
p3,5
p4,2
p5,0
p3,6
Standardized Spectrum
Q≥400
Q≥270
0.99
0.9
760
Frequency (μHz)

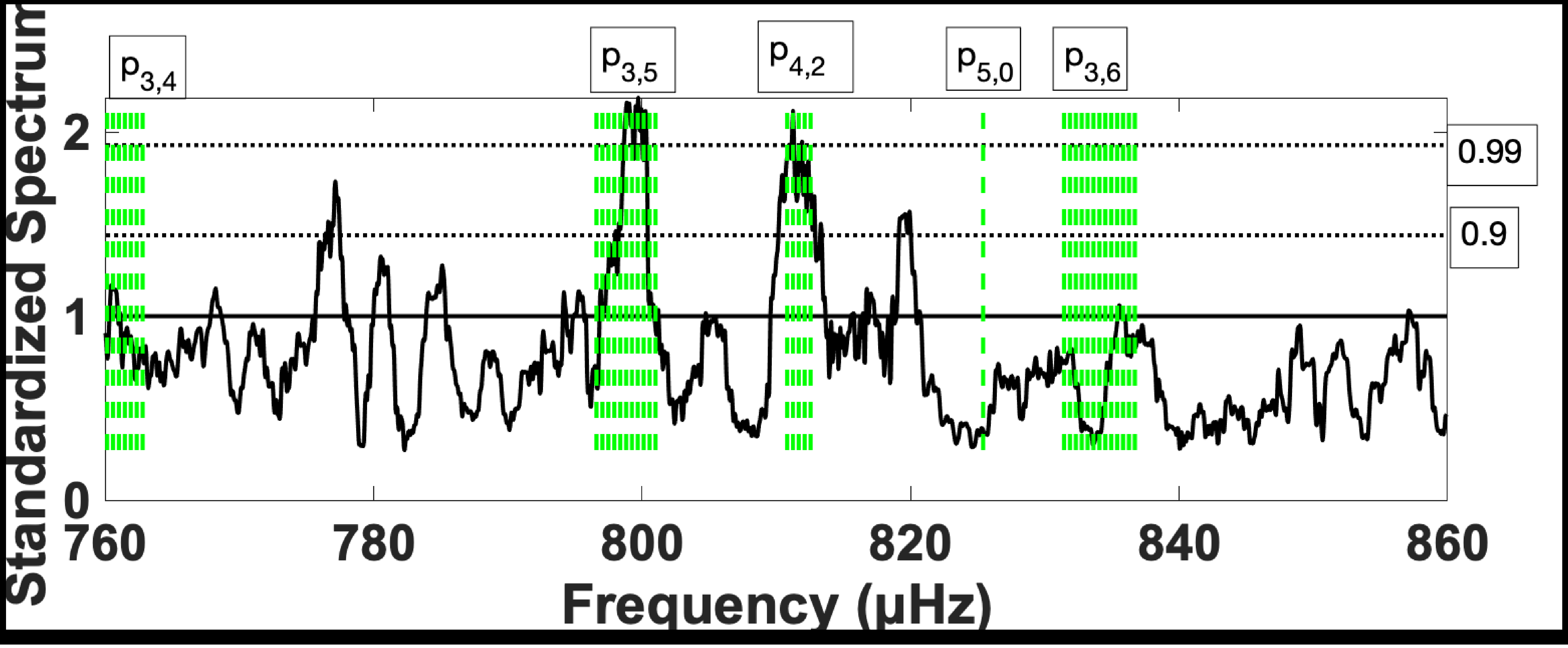


Average offset coherence over ordinary frequency across 811.5 μHz peak

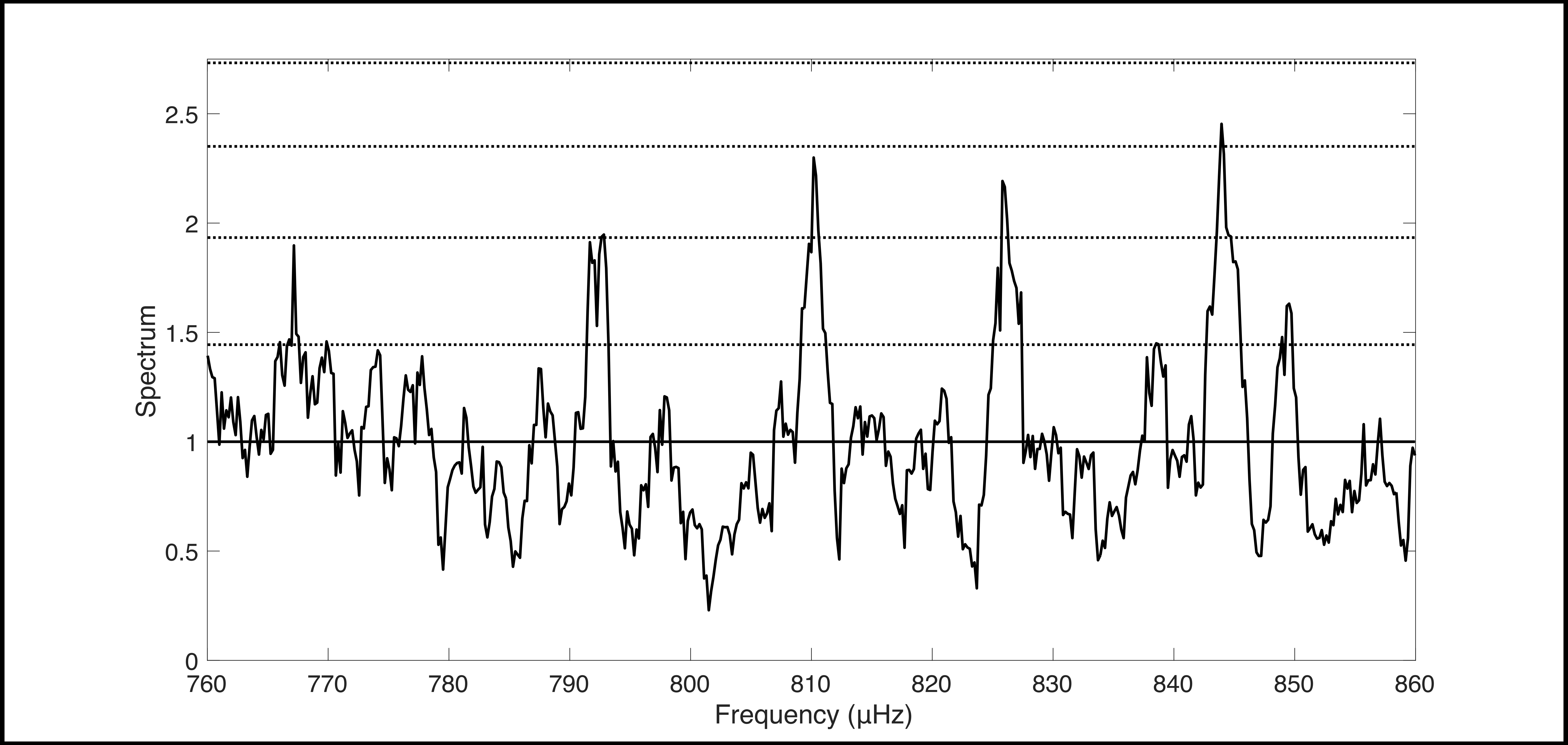




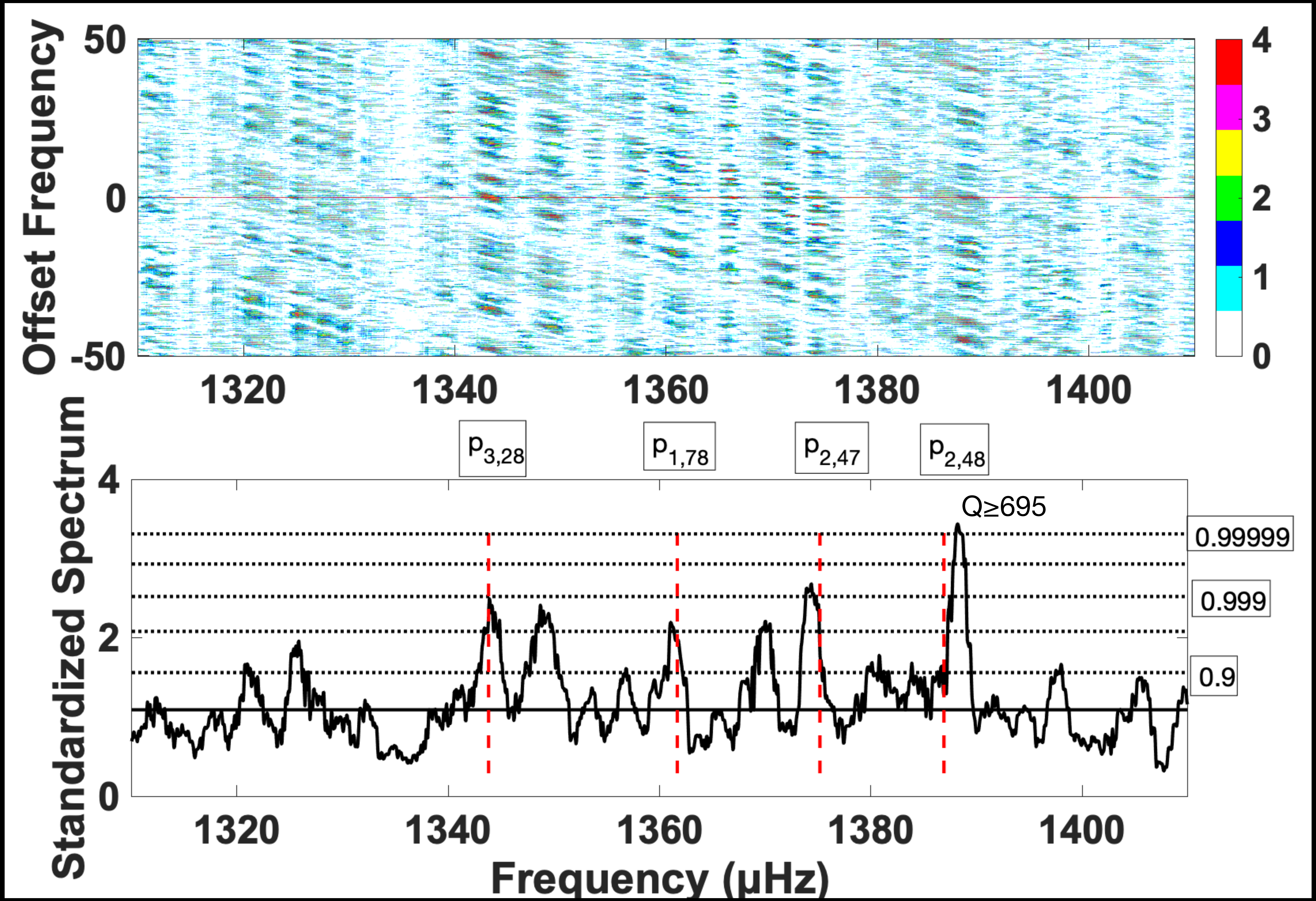
Periodogram of data using high pass filtering, pre-whitening and post-whitening



Raw Periodogram

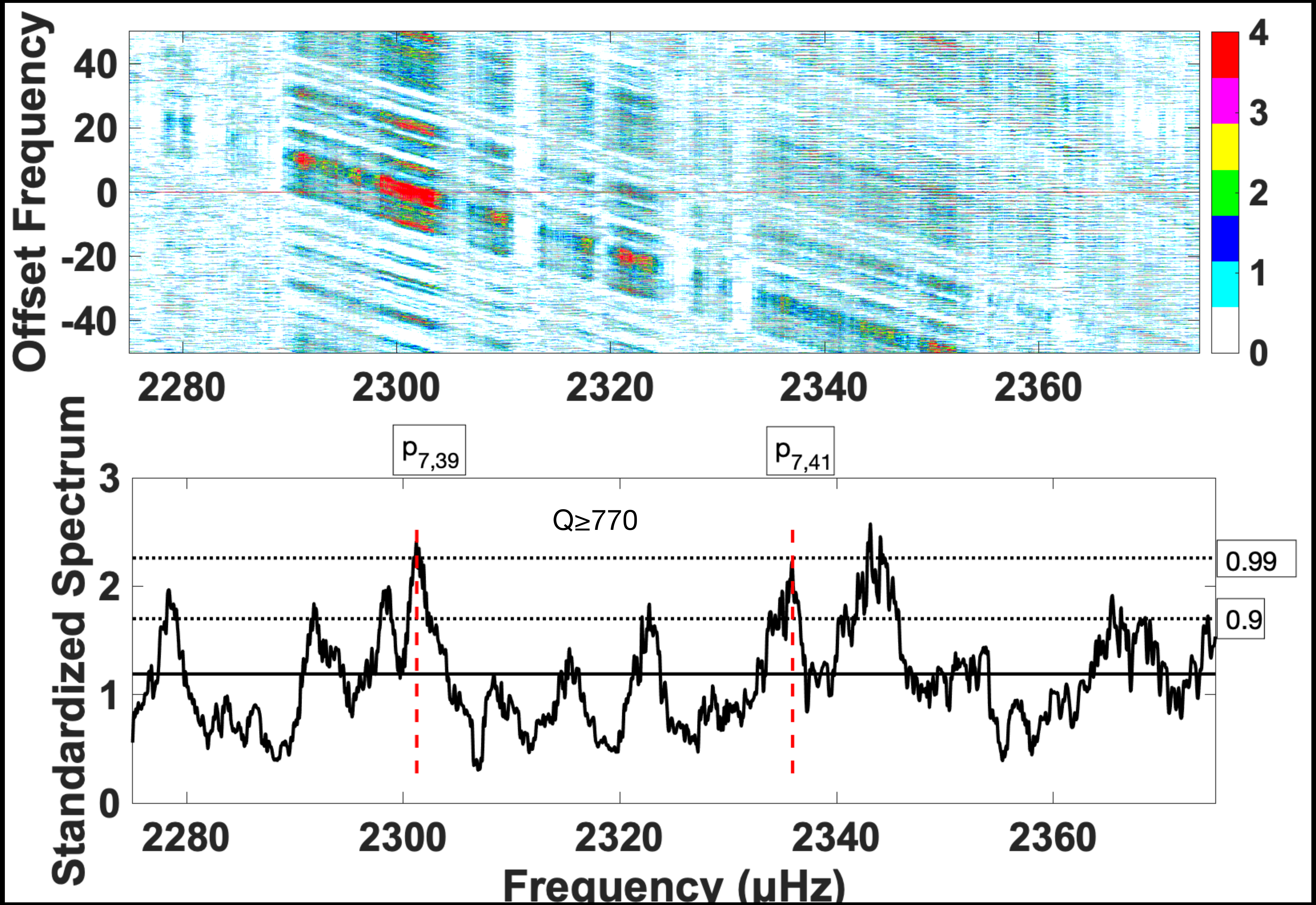


YD 424-484



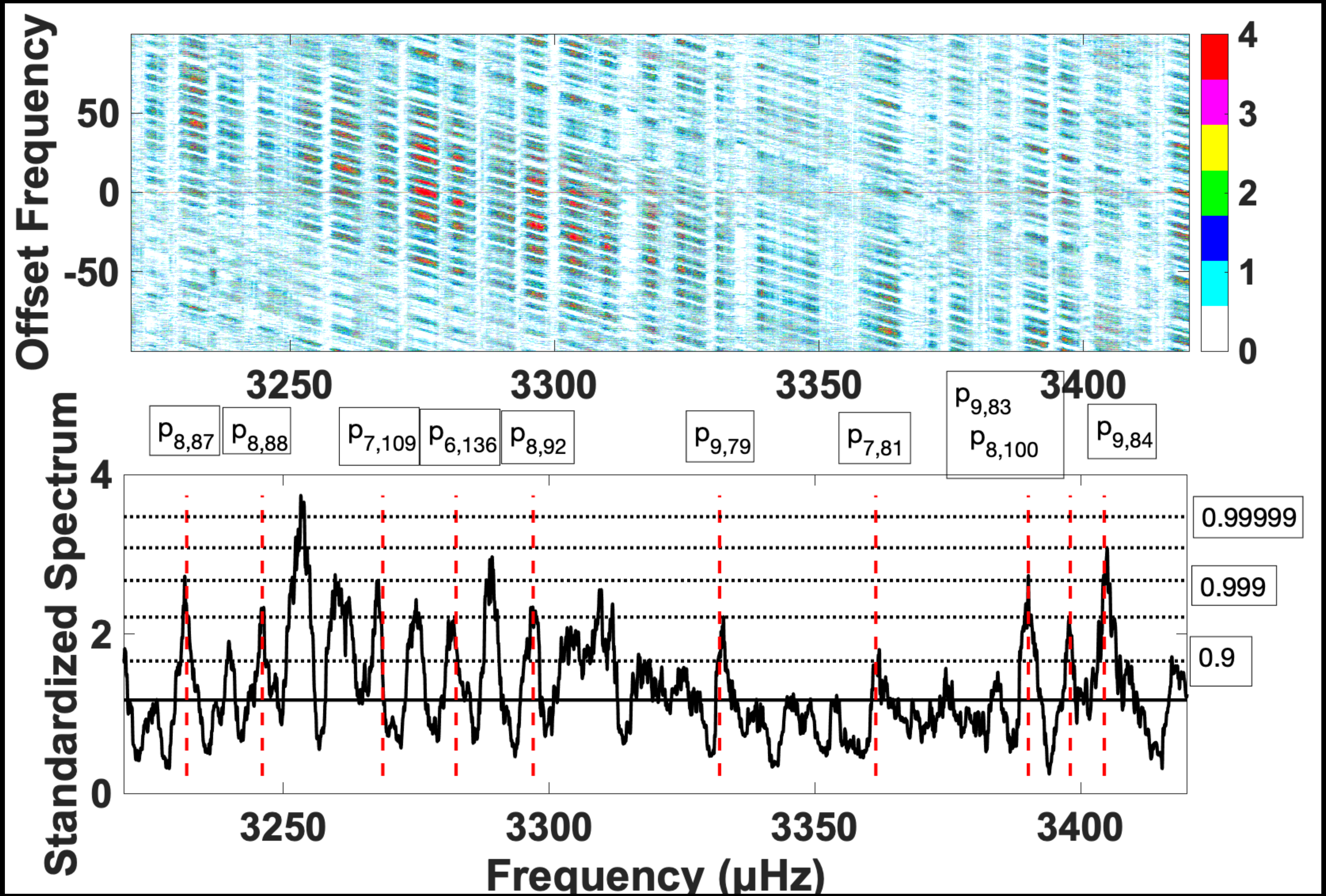
Offset Frequency
50
0
-50
1320
1340
1360
1380
1400
4
3
2
1
0
p3,28
p1,78
p2,47
p2,48
Q≥695
0.99999
0.999
0.9
Standardized Spectrum
Frequency (μHz)

YD 358-418

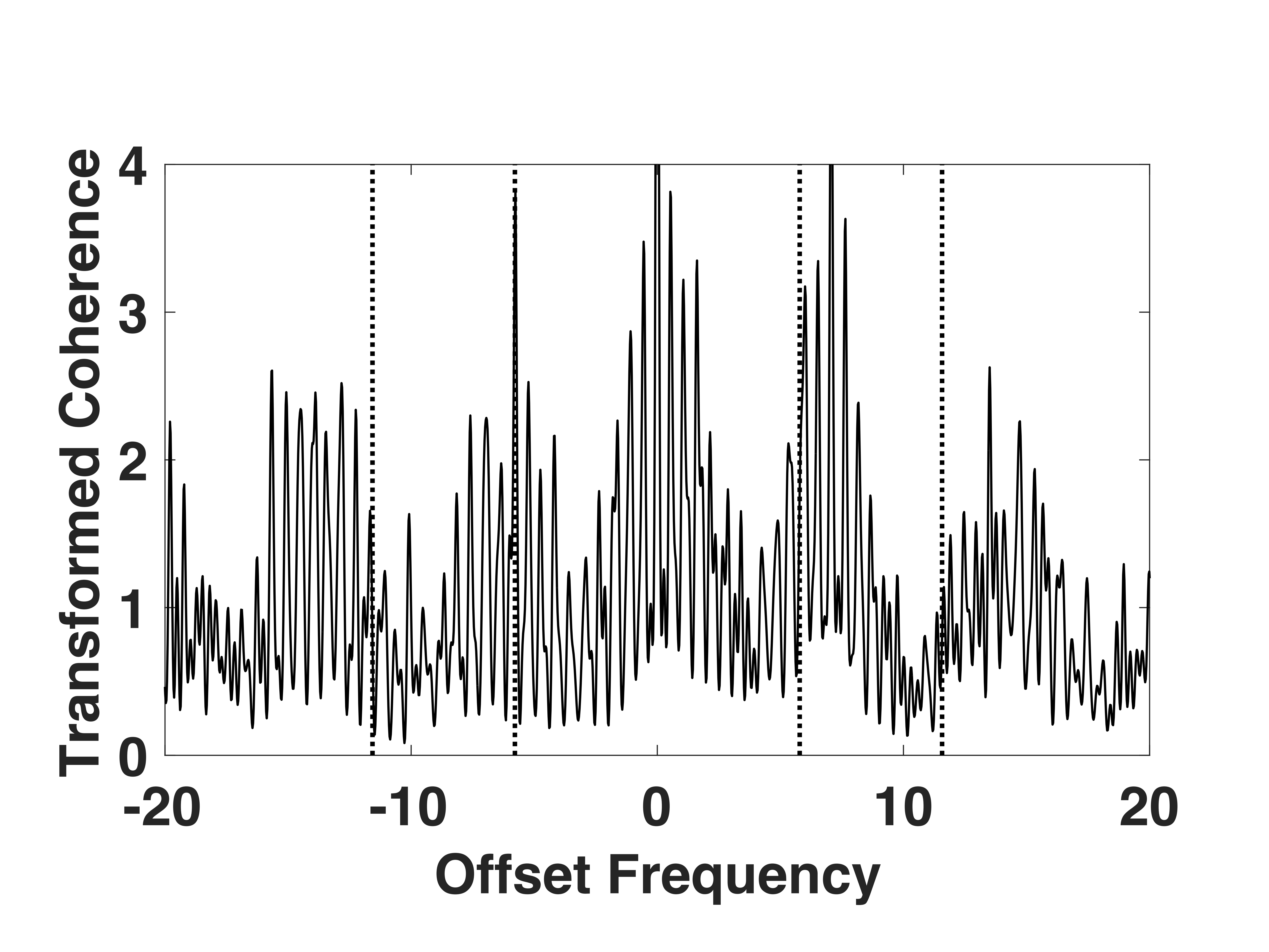


Offset Frequency
40
20
0
-20
-40
4
3
2
1
0
2280
2300
2320
2340
2360
p7,39
p7,41
Q≥770
0.99
0.9
Standardized Spectrum
3
2
1
0
Frequency (µHz)

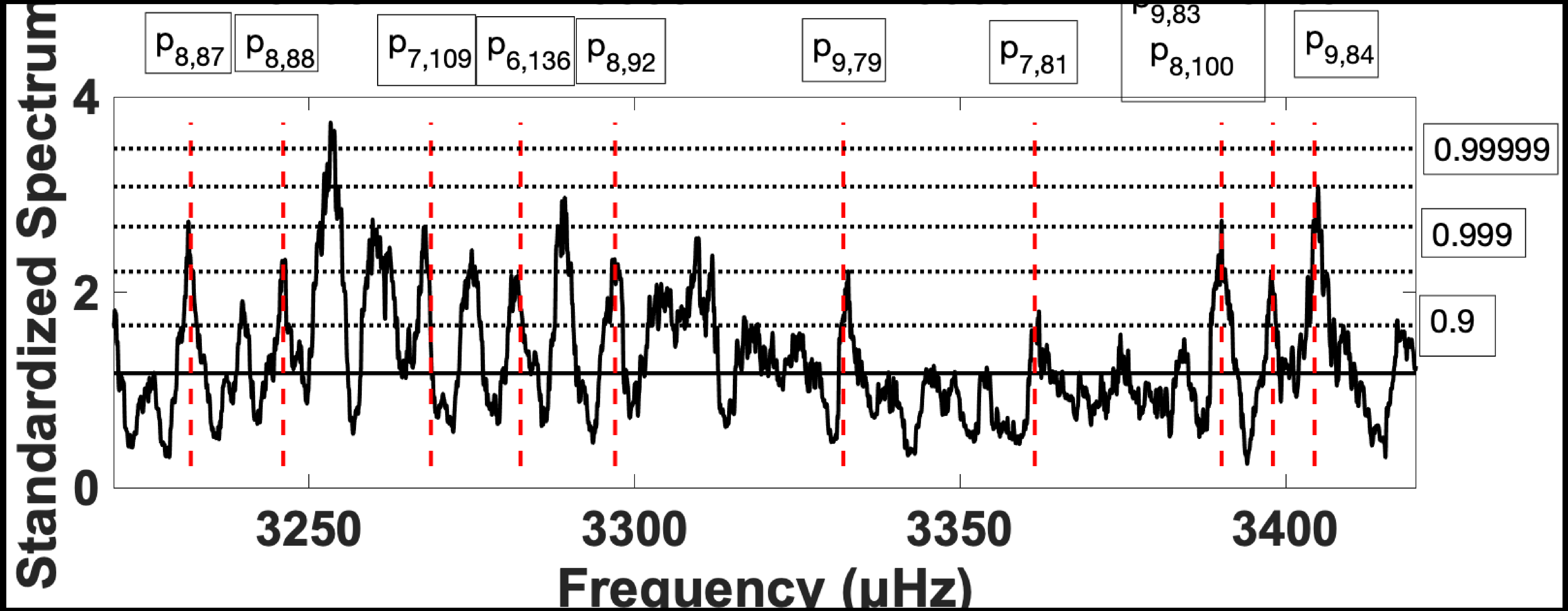
YD 358-418



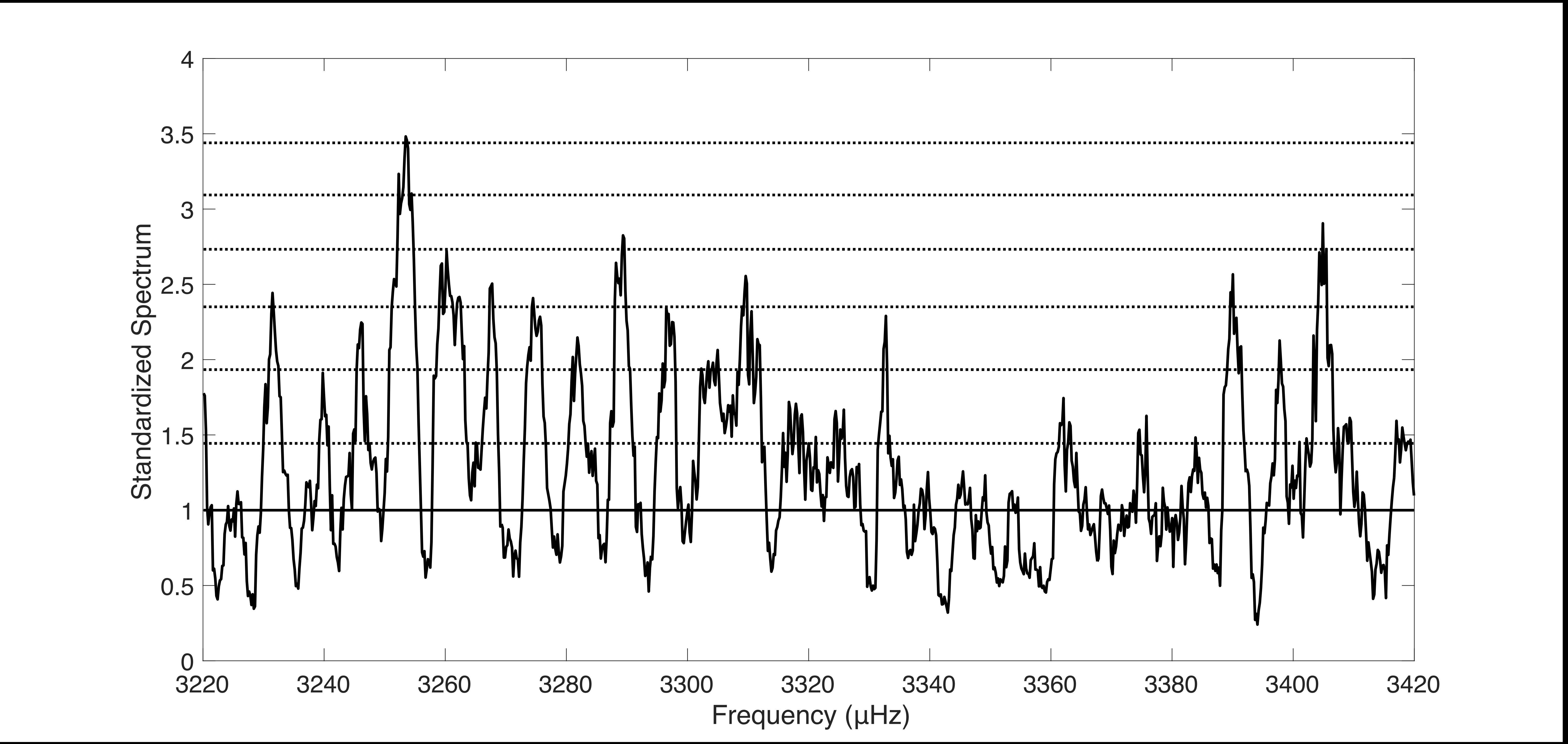
Offset Frequency
50
0
-50
4
3
2
1
0
3250
3300
3350
3400
p8,87
p8,88
p7,109
p6,136
p8,92
p9,79
p7,81
p9,83
p8,100
p9,84
Standardized Spectrum
0.99999
0.999
0.9
Frequency (μHz)

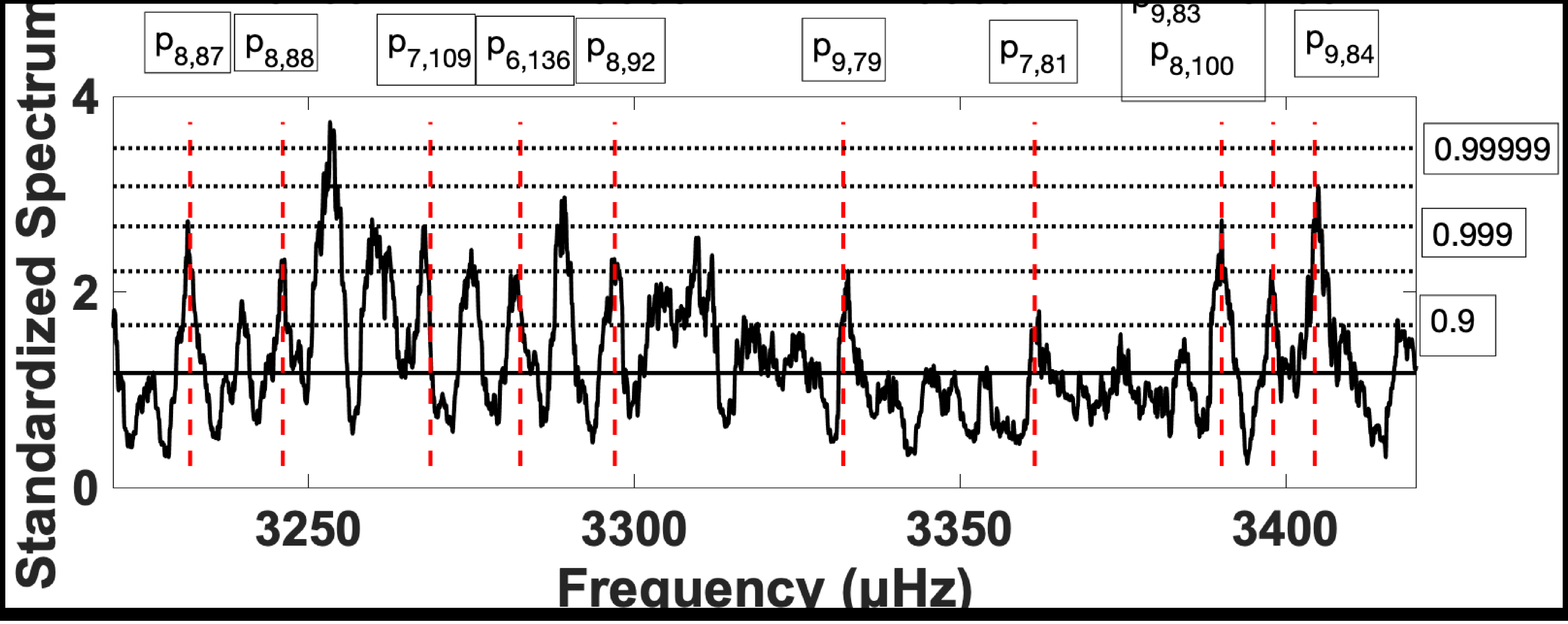


Average offset coherence over ordinary frequency across 3390 μHz peak

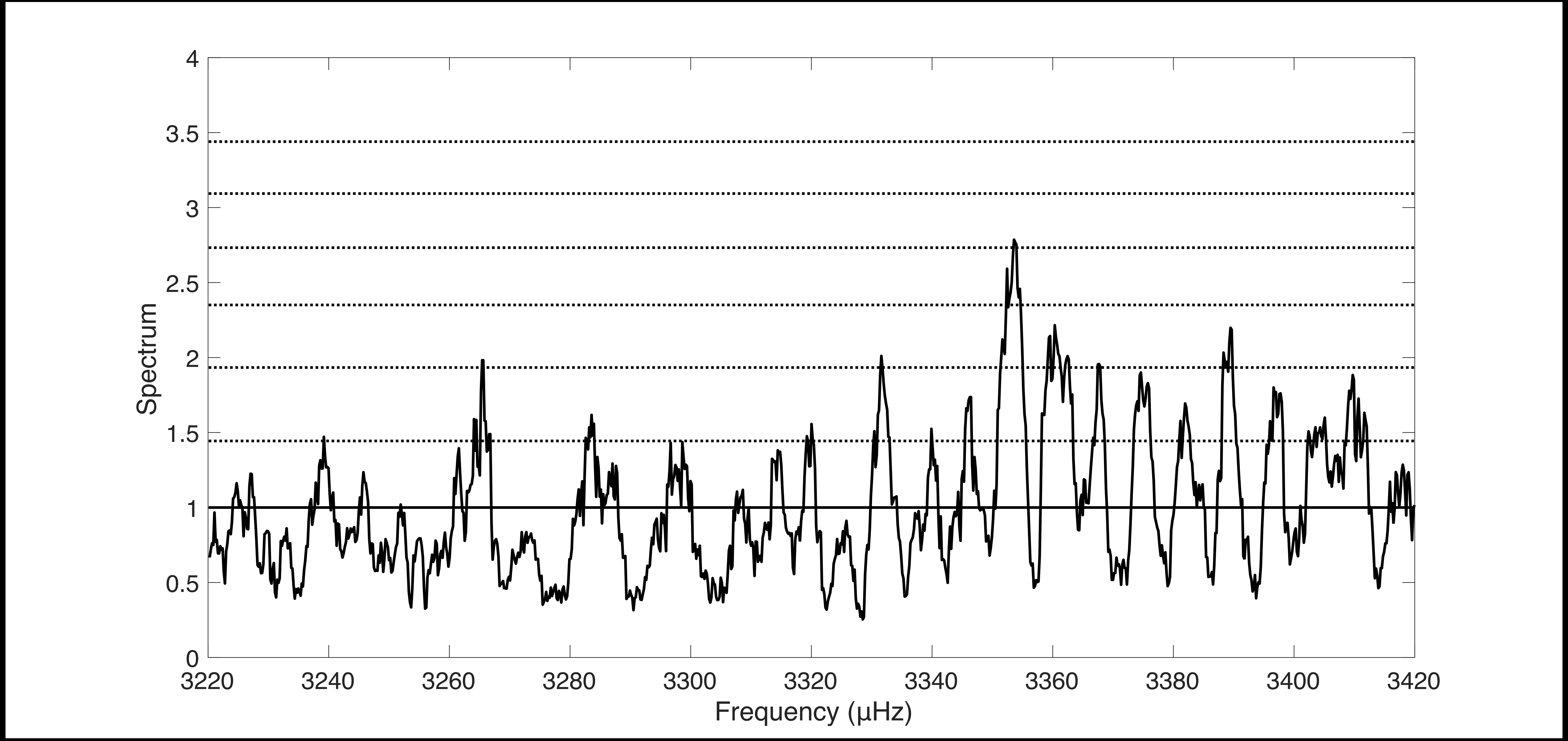


Periodogram of data using high pass filtering, pre-whitening and post-whitening





Raw Periodogram



Conclusions

- A substantial non-central fraction (up to 35%) is detectable over 2000-4000 μHz
- Narrowband, high Q, non-stationary peaks are pervasive in these geomagnetic data
- Quasi-two day mode is apparent in some instances, but not diurnal cyclostationarity
- Solar normal modes are detectable with medium degree, but rotational splitting appears to be confined to low values
- Non-stationarity is pervasive, with forcing at modal frequencies resulting in transfer of power to non-modal frequencies