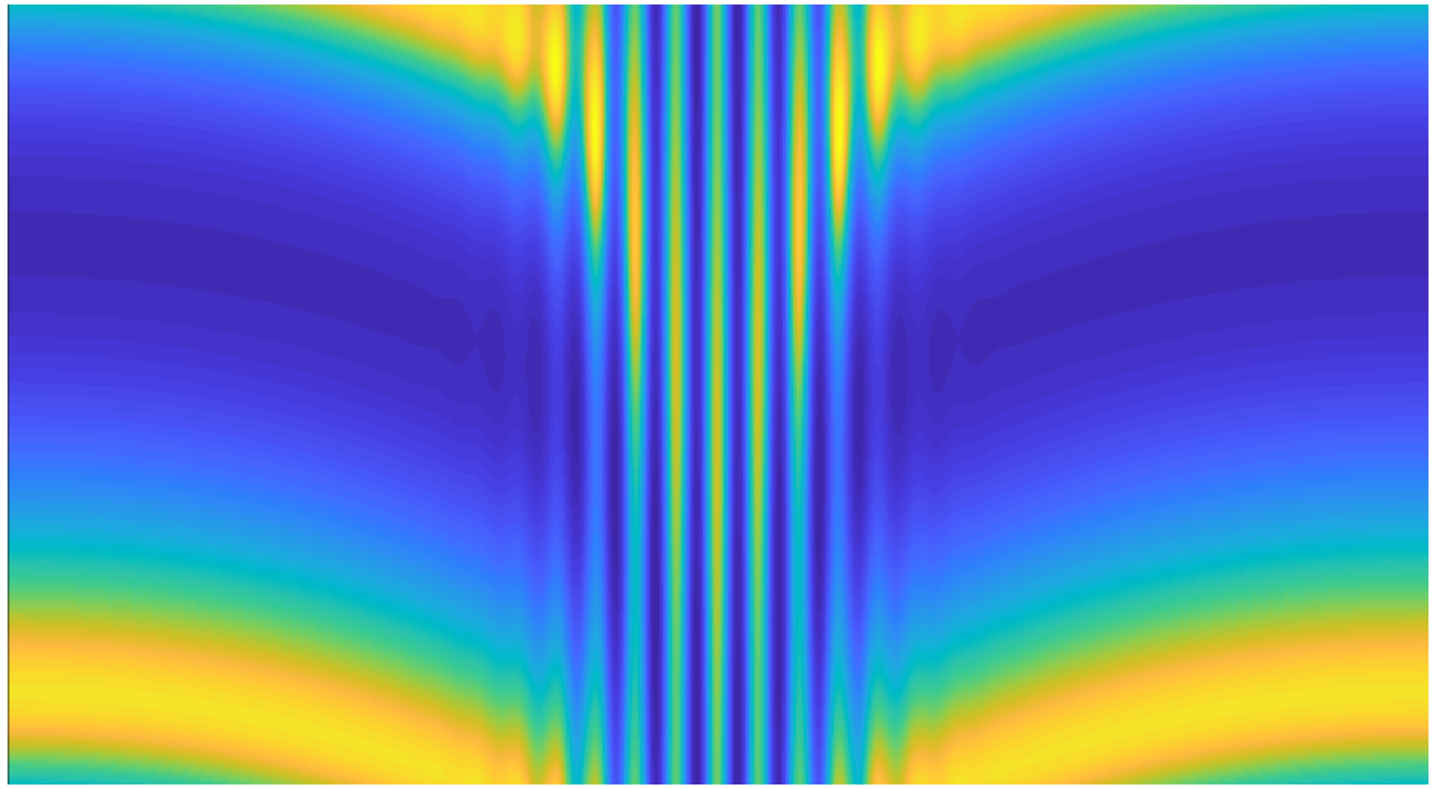


SNAKING OF CONTACT DEFECTS

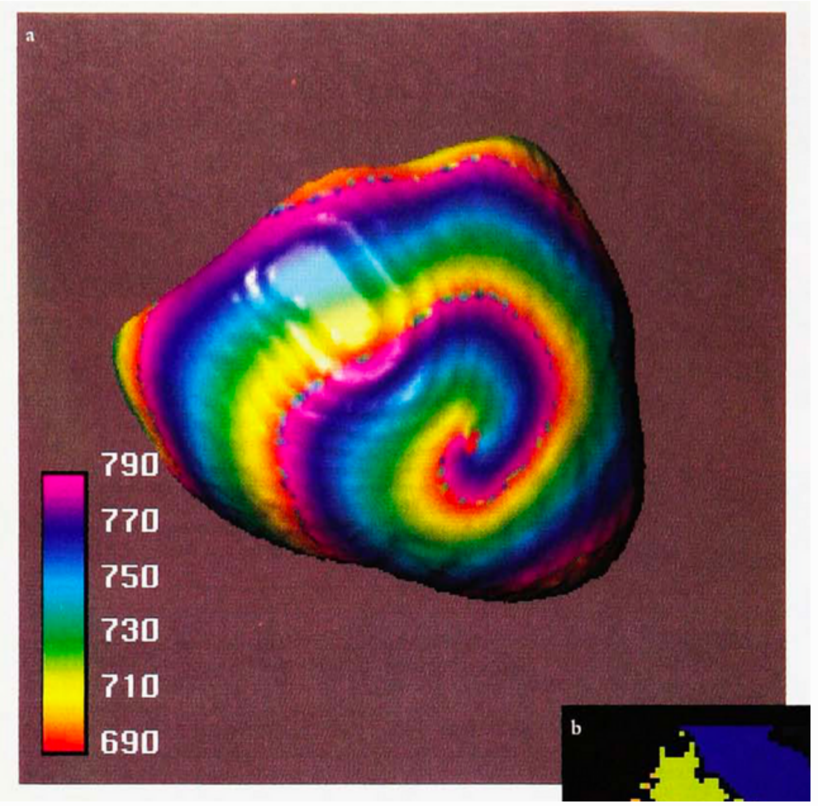
Timothy Roberts
Brown University

Joint work with

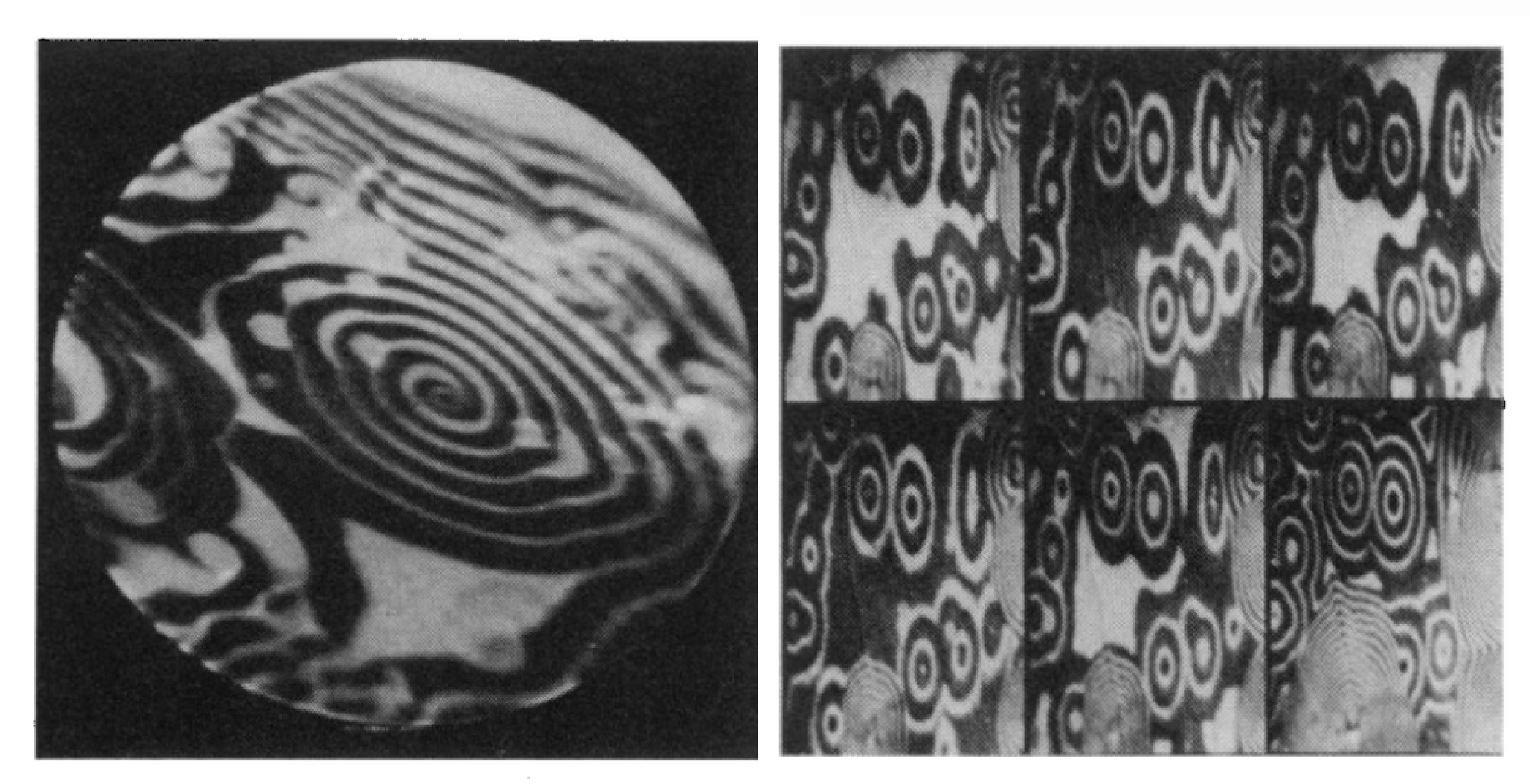
Bjorn Sandstede
Brown University

OUTLINE

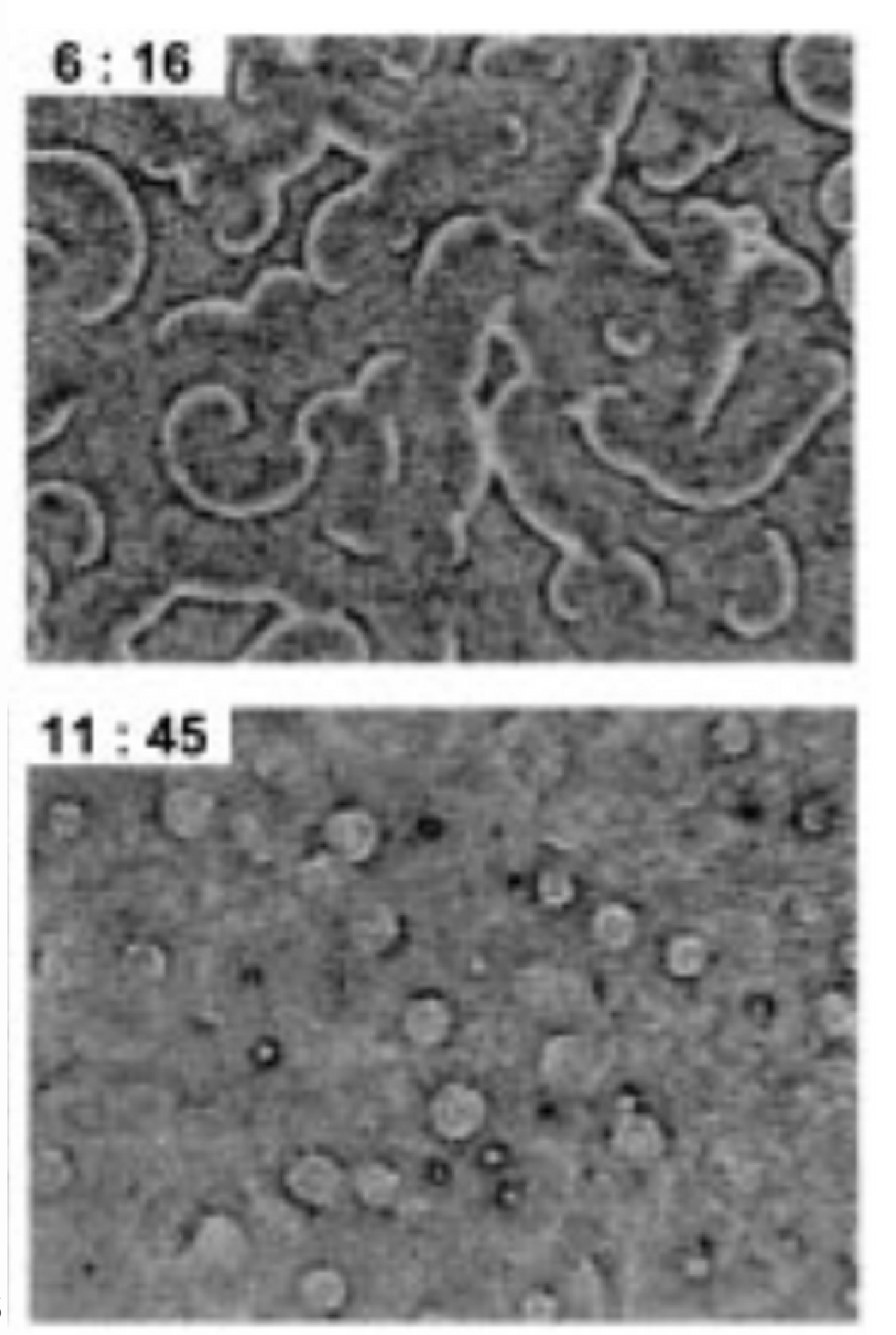
MOTIVATION



[Glass 1996] Electrochemical potentials of the heart.



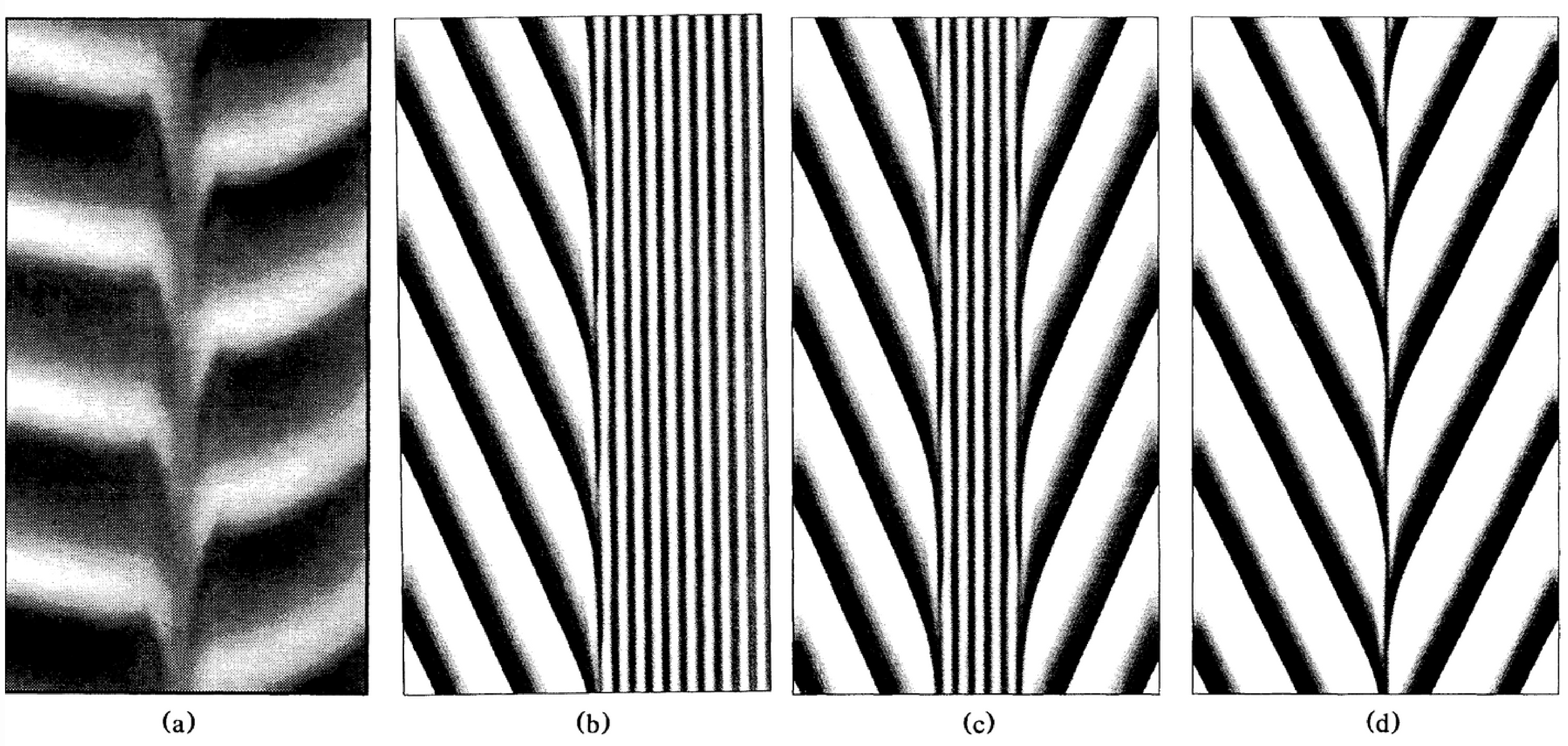
[Ertl 1991] Oxidation layers on platinum alloys.



[Lee et al. 1996]
Slime mould populations

We are interested in the existence, stability and interaction of spiral and target waves.

MOTIVATION



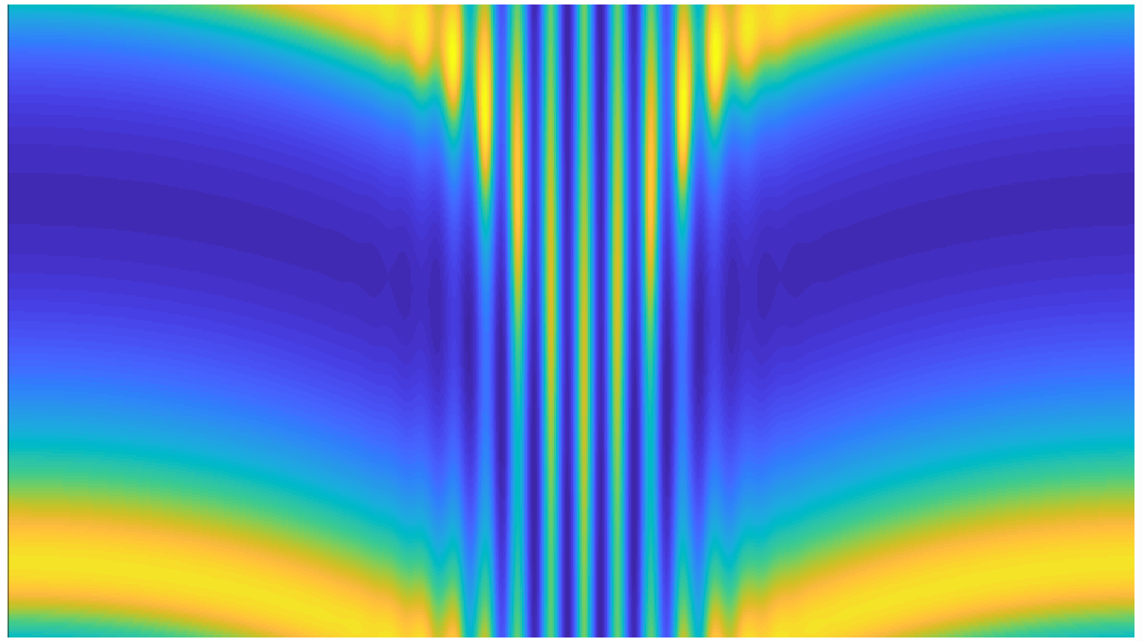
[Perraud et al. (1993)] (a) Experimental space-time diagram of the CIMA reaction. (b-d) Numerical results obtained from the Brusselator

THE BRUSSELATOR

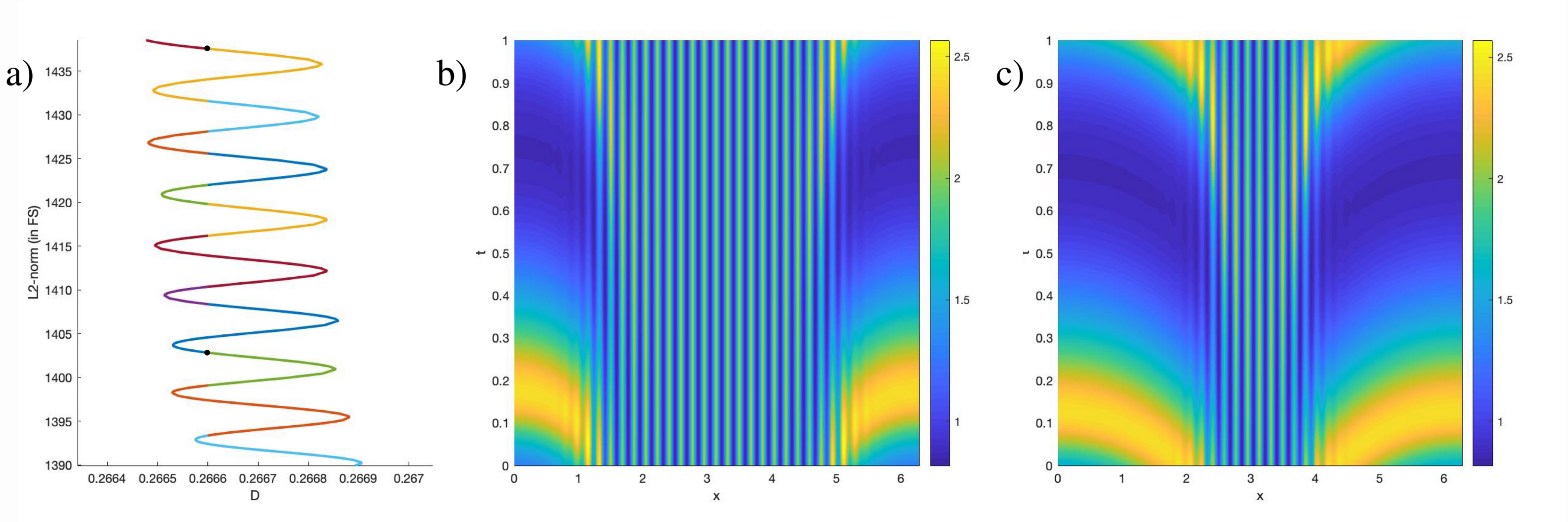
$$U_t = DU_{xx} + E - (B+1)U + VU^2$$
$$V_t = V_{xx} + BU - VU^2$$

"prototype of any system leading to dissipative structures... analagous to the harmonic oscillator as a prototype in classical or quantum mechanics."

[Auchmuty and Nicholis (1975)]



THE BRUSSELATOR

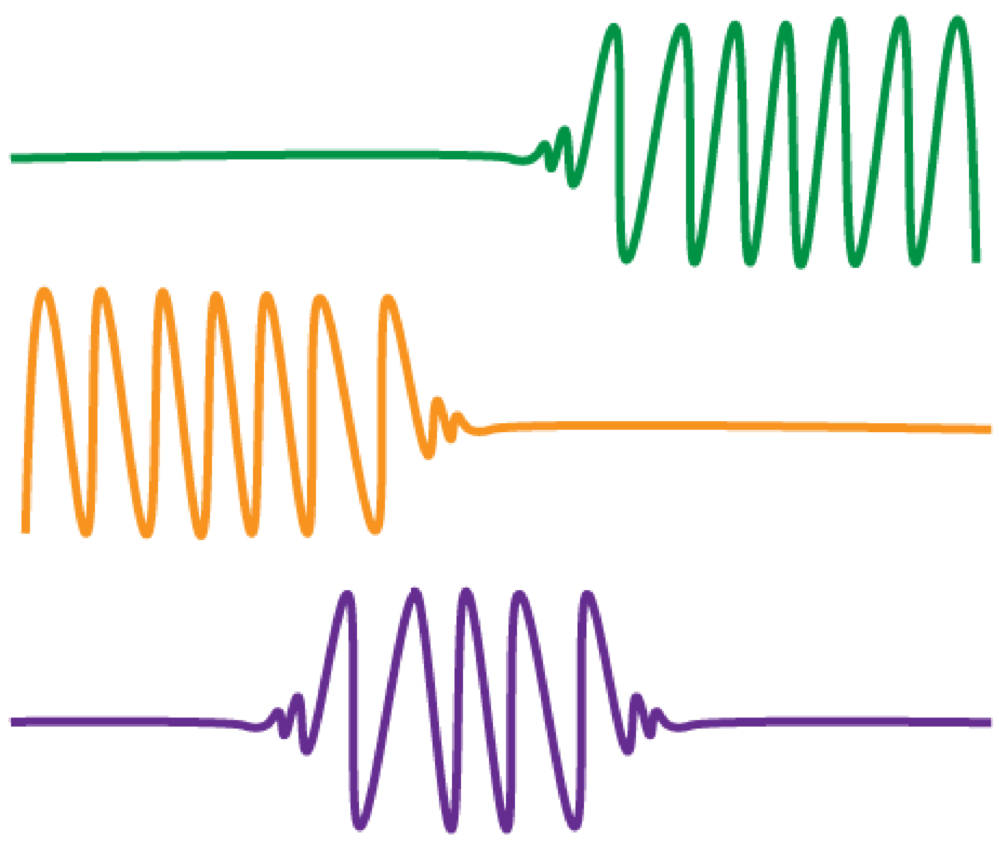


- Tzou et al (2013) noticed that contact defects exist and appear to snake

SWIFT-HOHENBERG

$$U_t = -(1+\partial_x^2)^2 U - \mu U + \nu U^2 - U^3$$ [Beck et al (2009)]

- Standing waves: time derivative zero
- 4-dim phase space



PDE Solution

Phase Space

SWIFT-HOHENBERG

$$U_t = -(1 + \partial_x^2)^2 U - \mu U + \nu U^2 - U^3$$

[Beck et al (2009)]

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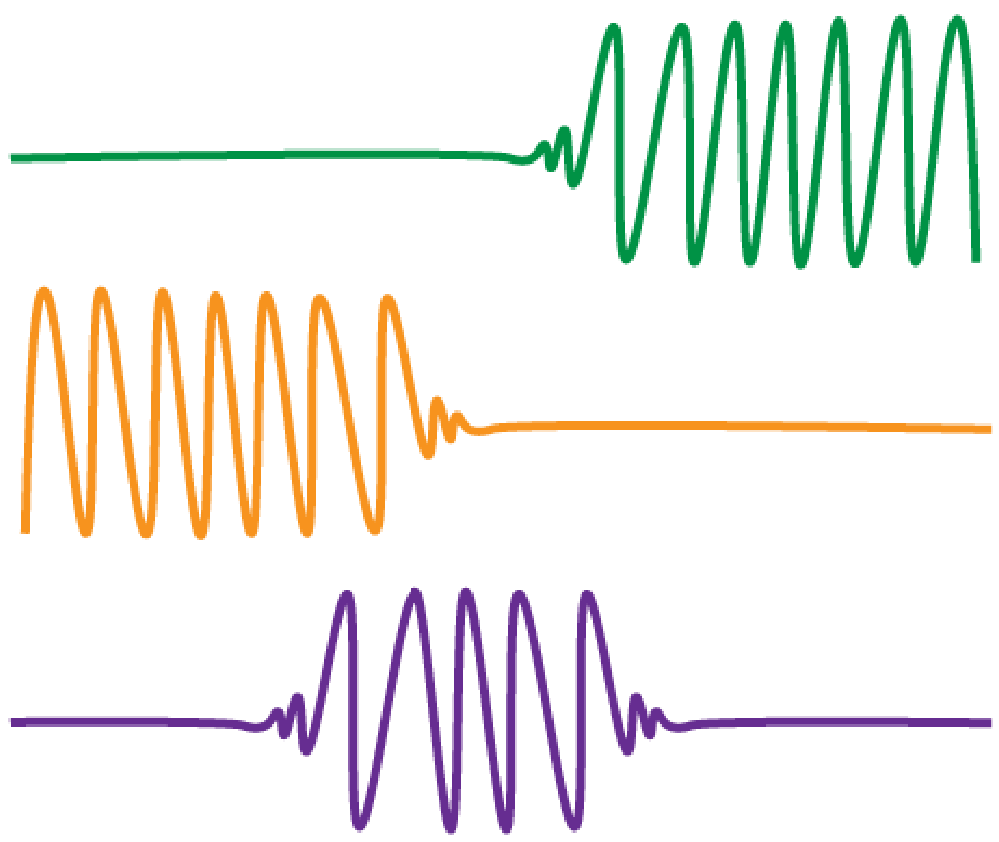
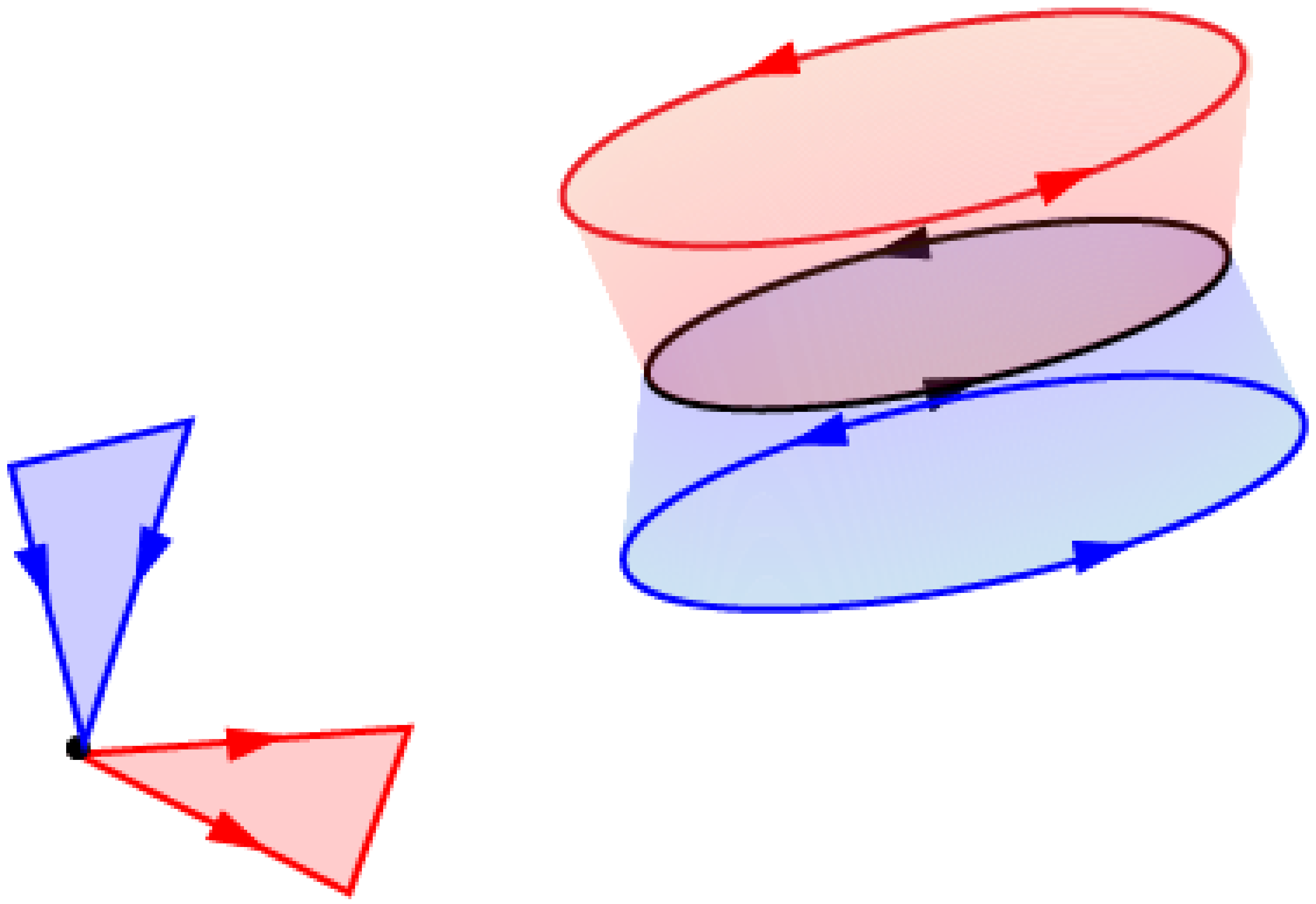


PDE Solution



Phase Space

SWIFT-HOHENBERG

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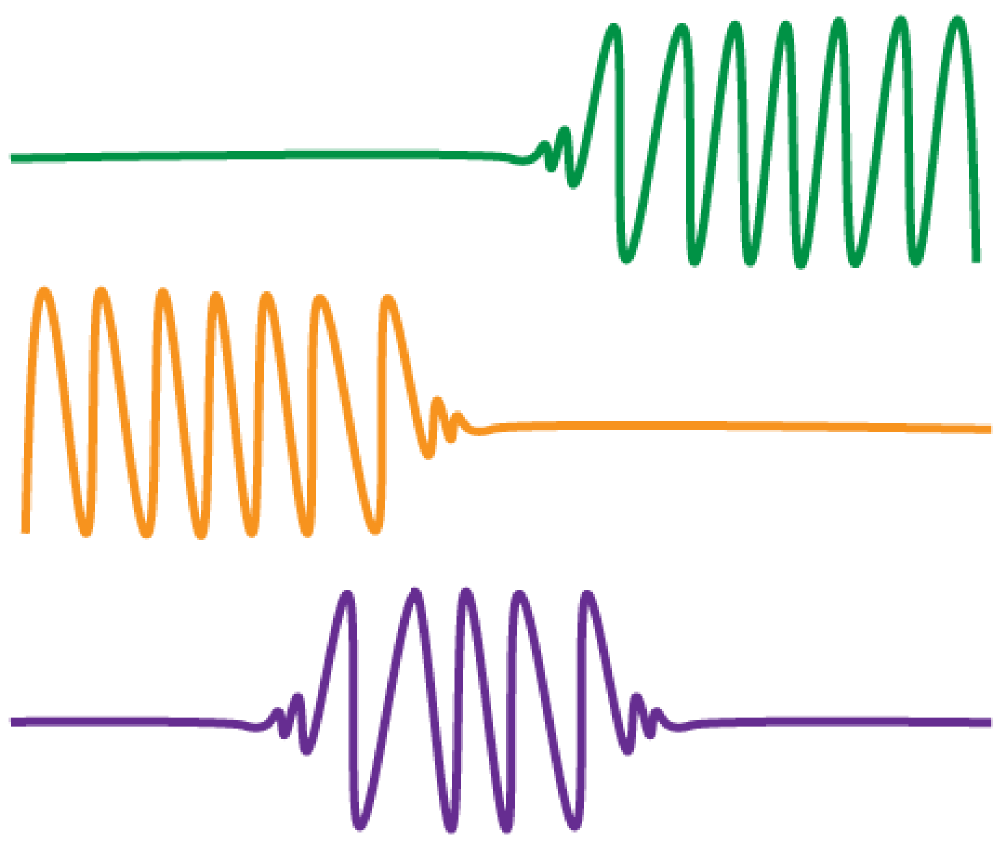
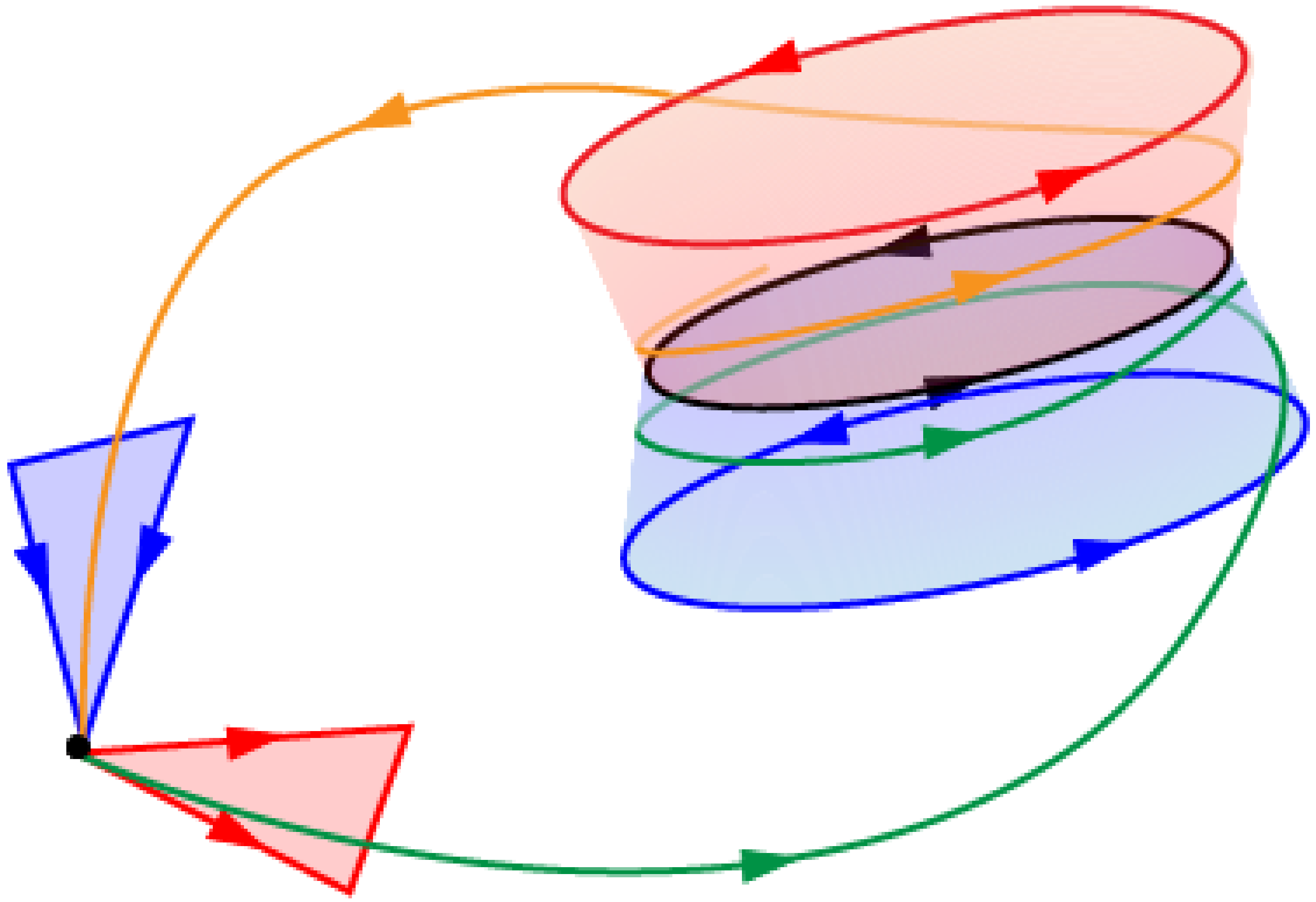


PDE Solution



Phase Space

SWIFT-HOHENBERG

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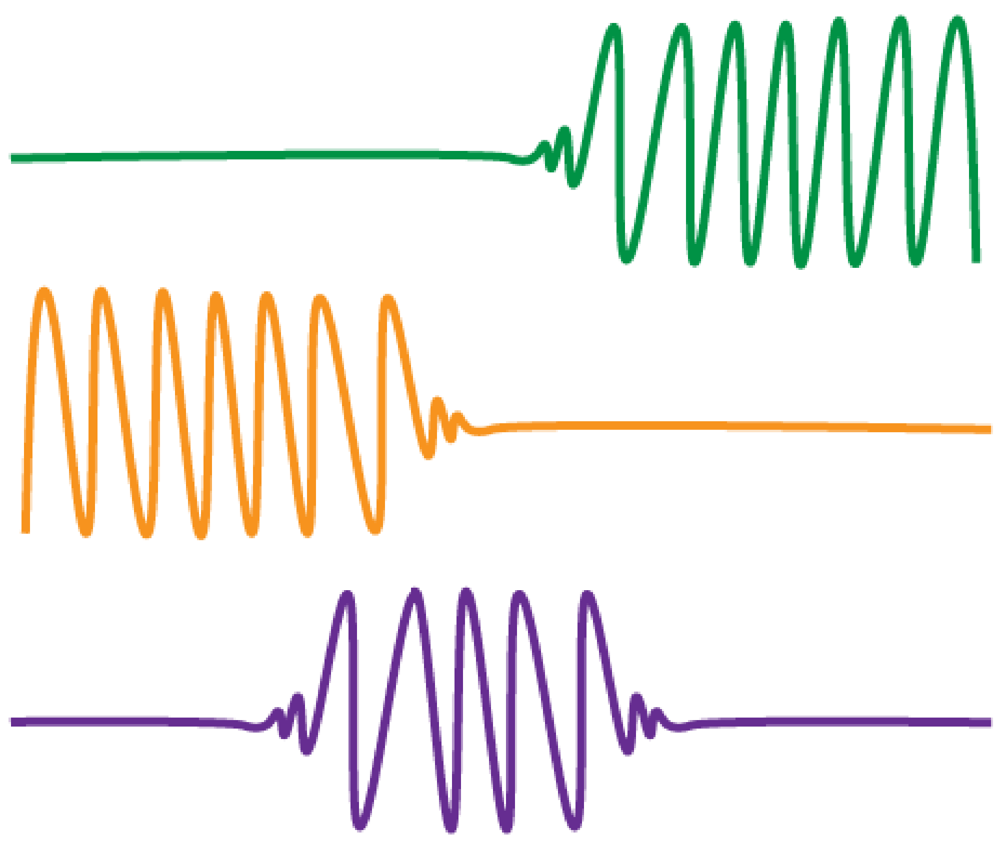
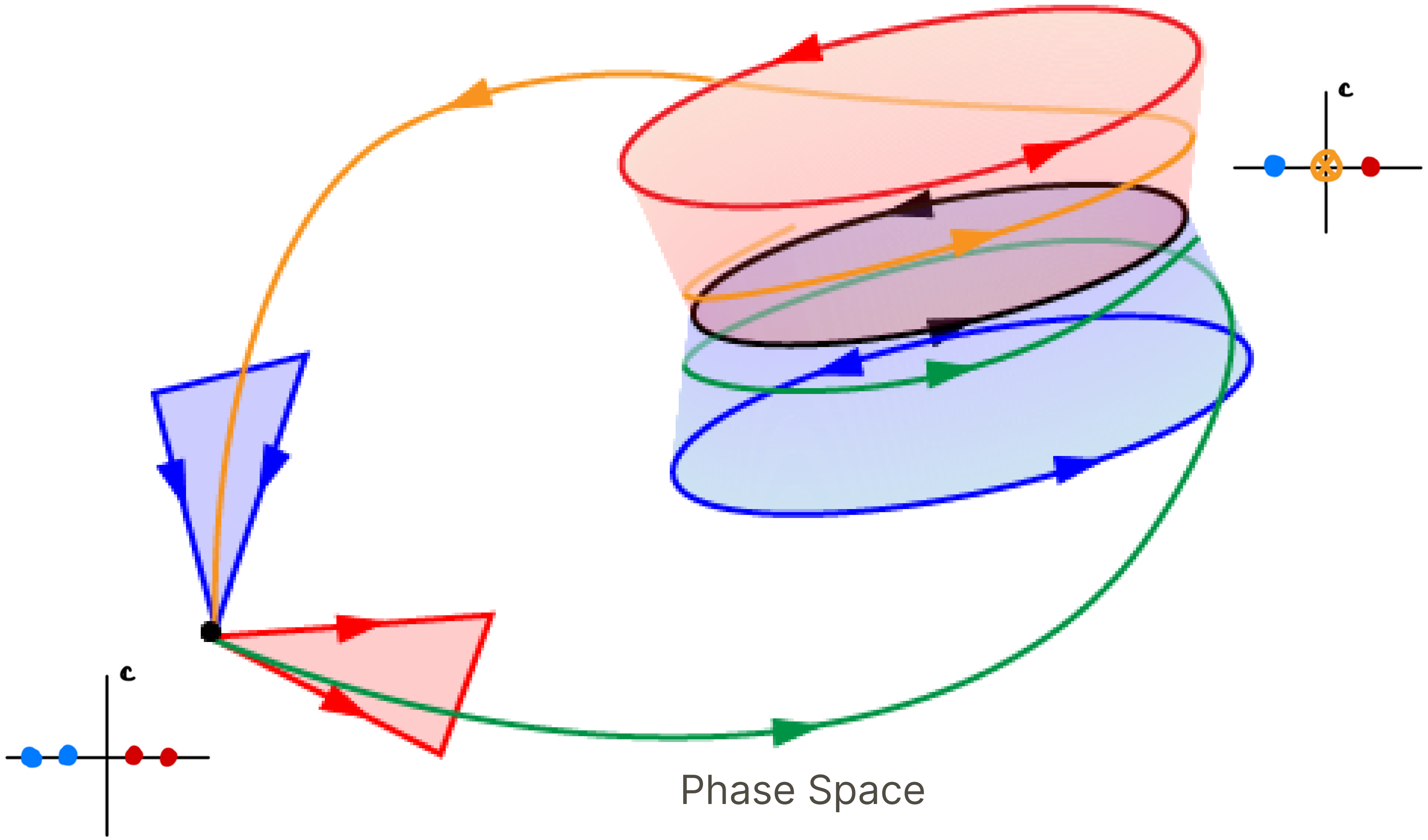
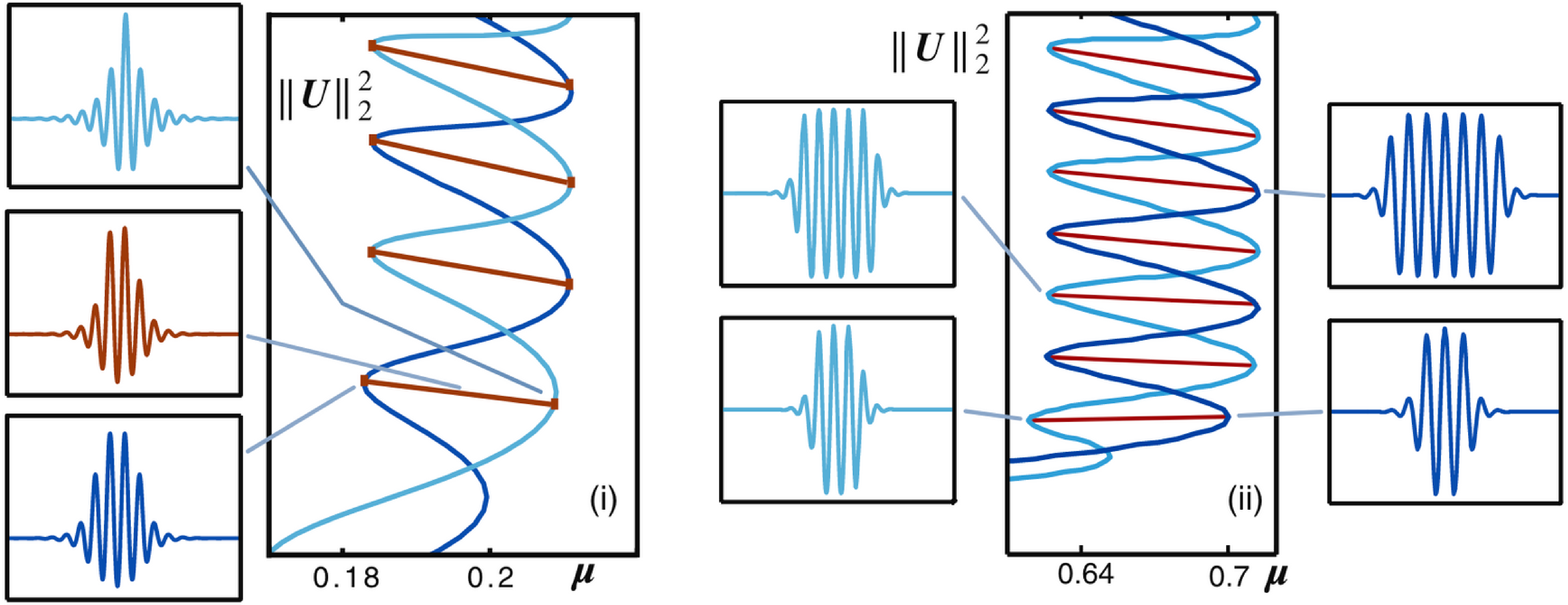


PDE Solution



Phase Space

SWIFT-HOHENBERG



[Beck et al (2009)]

Snakes of symmetric solutions joined by "rungs" of asymmetric solutions.

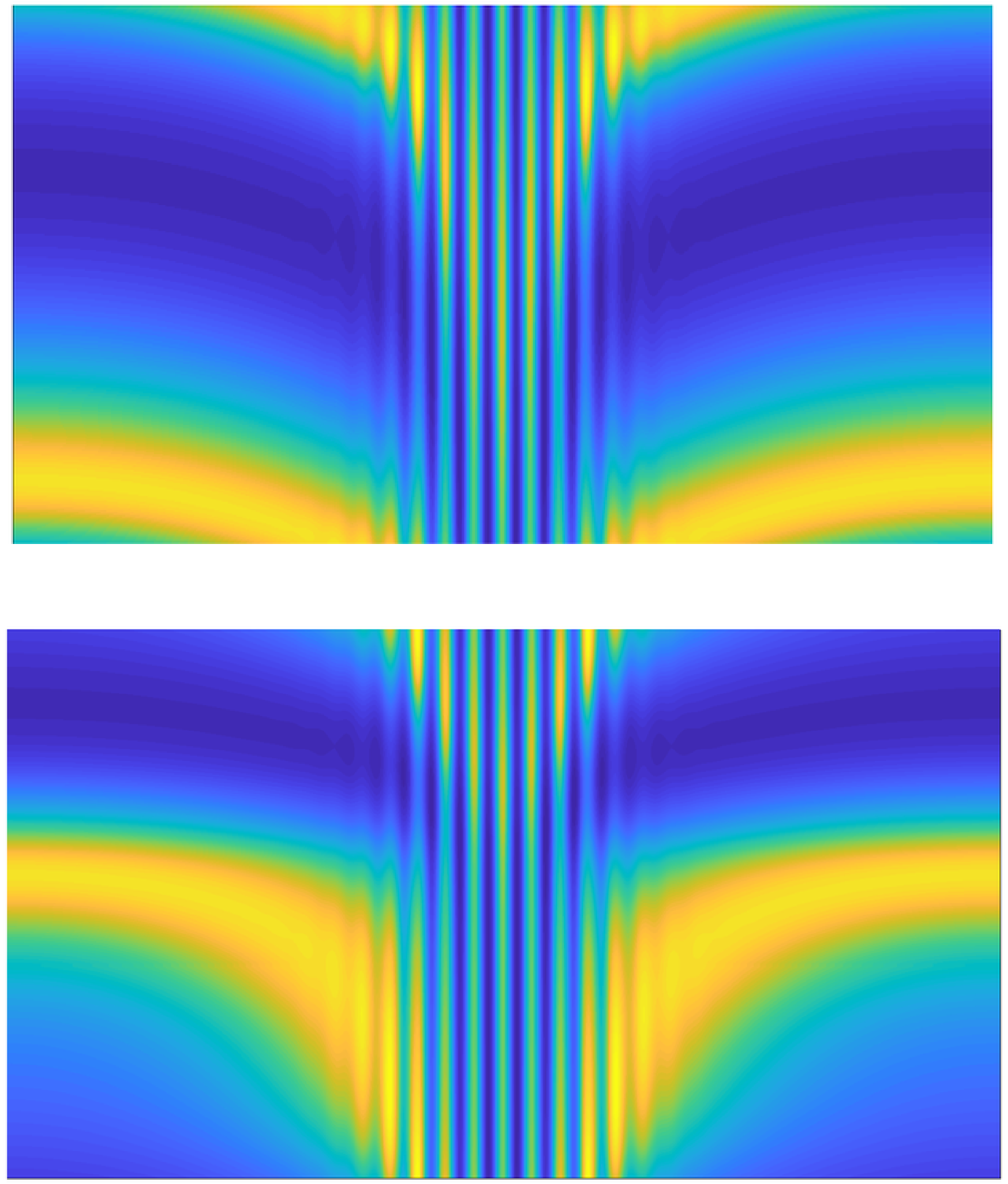
INFINITE DIMENSIONS

$$u_t = Du_{xx} + f(u)$$

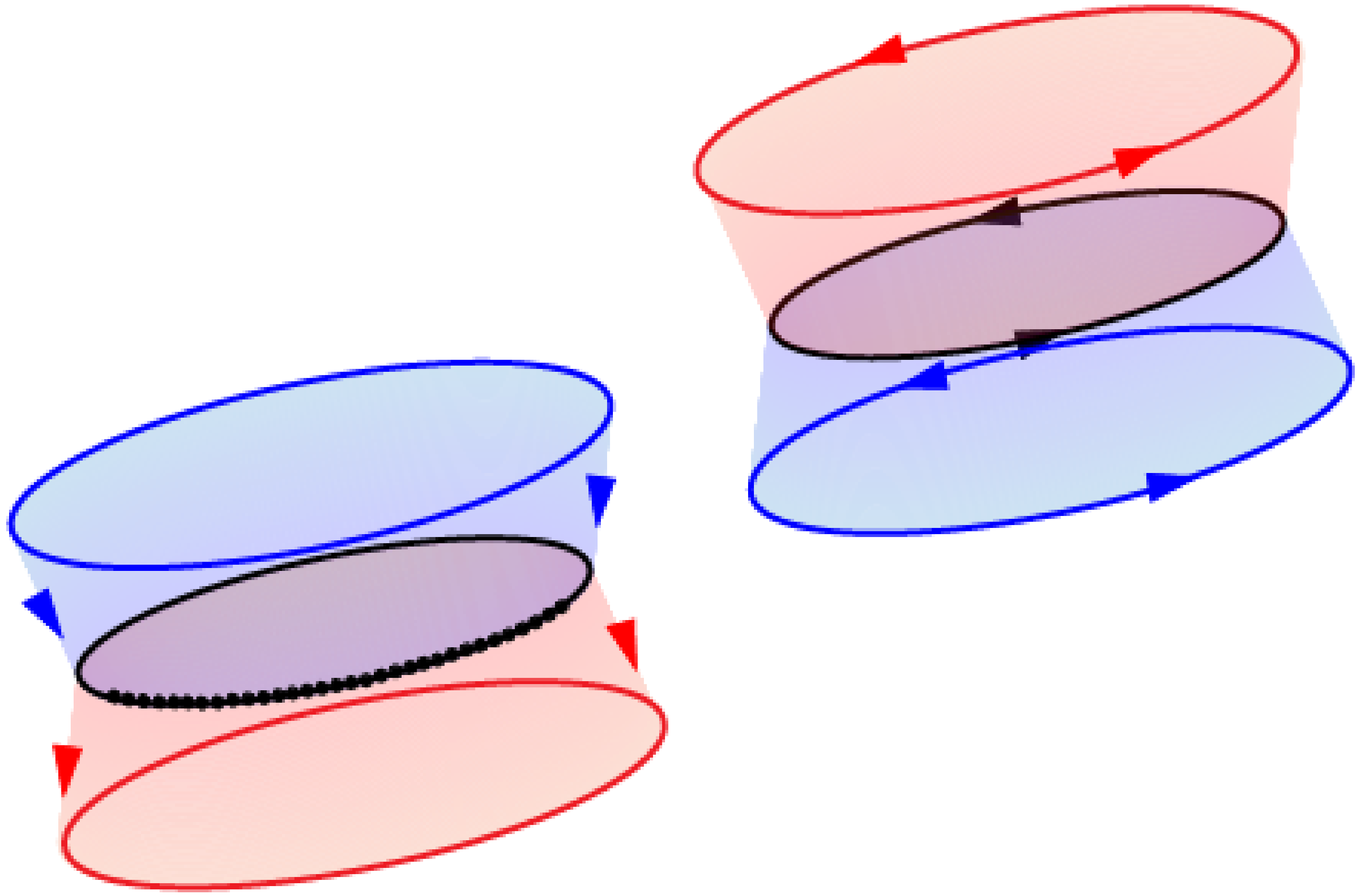
$$u_x = v$$
$$v_x = D^{-1}(u_t - f(u))$$

- The Brusselator can be written as a first-order equation
  - **We cannot remove the time derivatives.**
  - Our phase space is the space of periodic functions.
- We still have geometry!

INFINITE DIMENSIONS

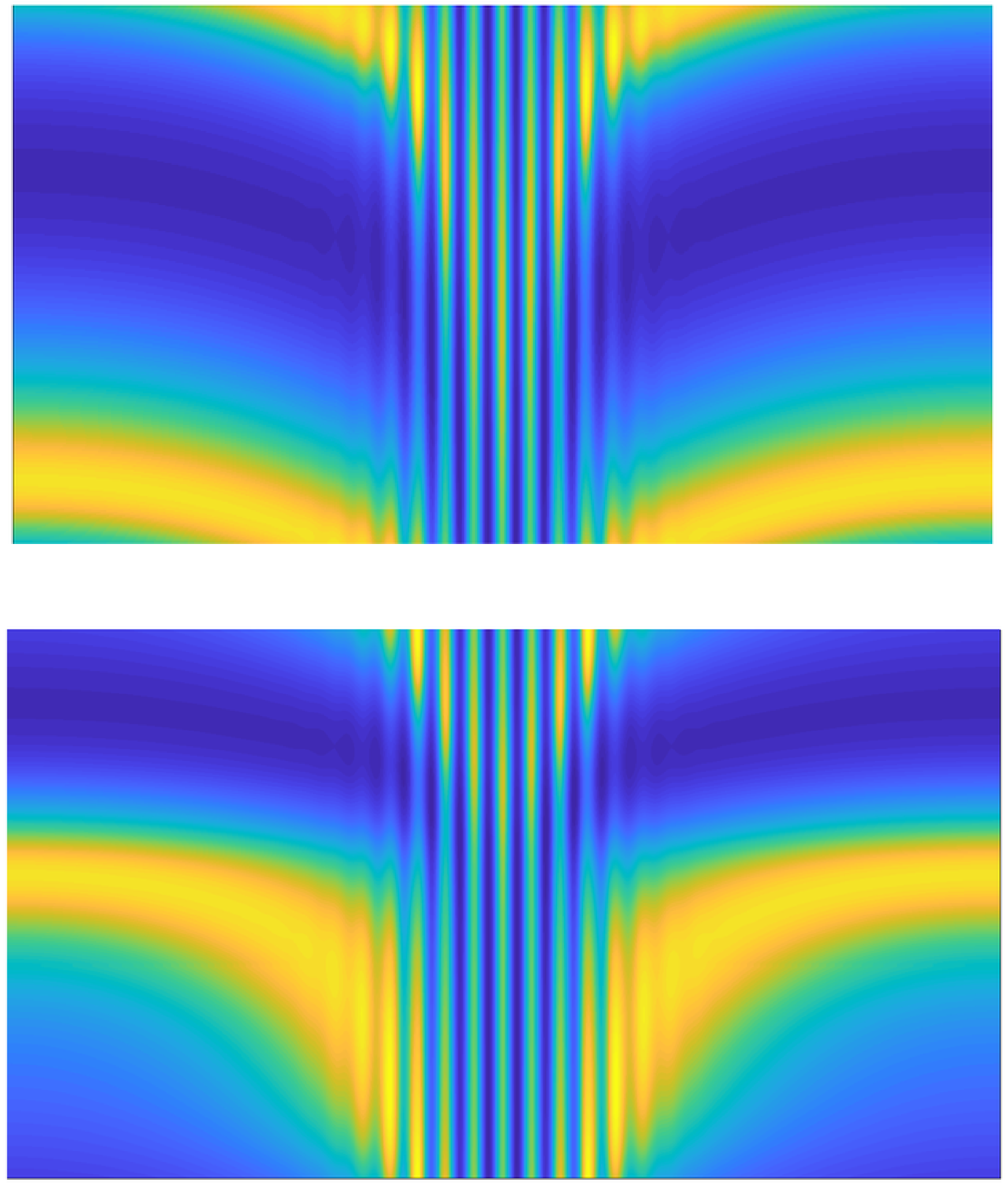


PDE Solution

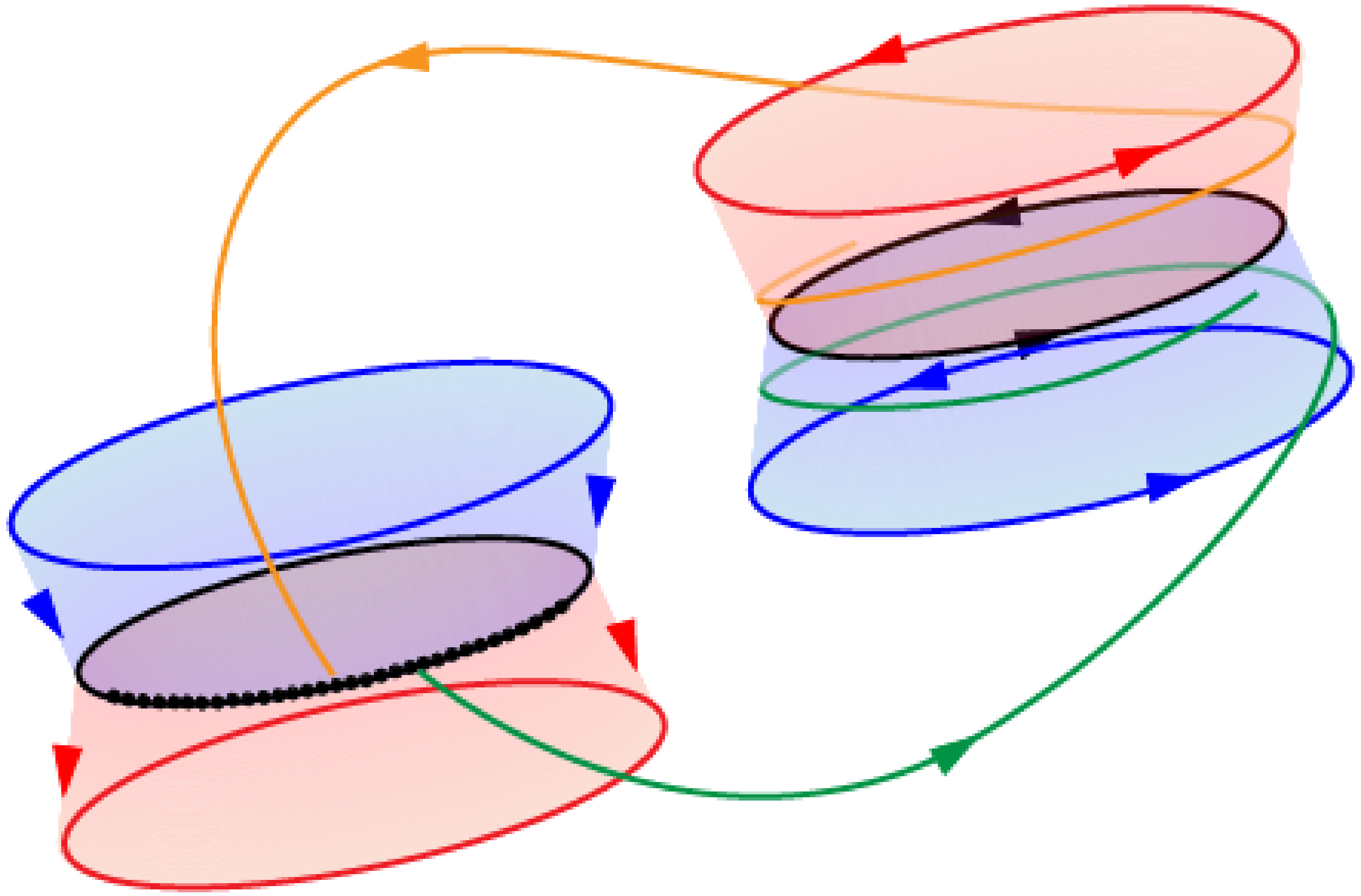


Phase Space

INFINITE DIMENSIONS

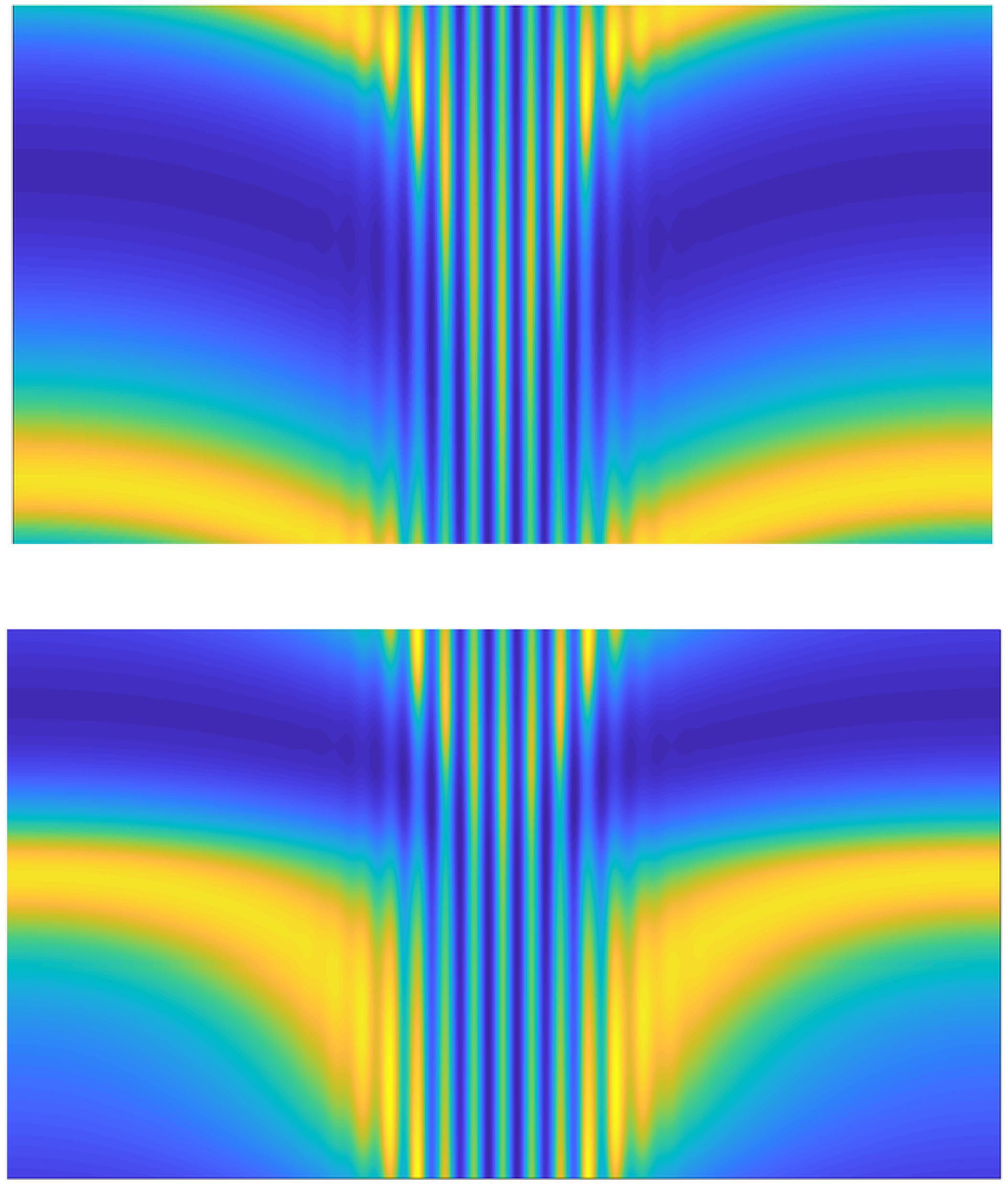


PDE Solution

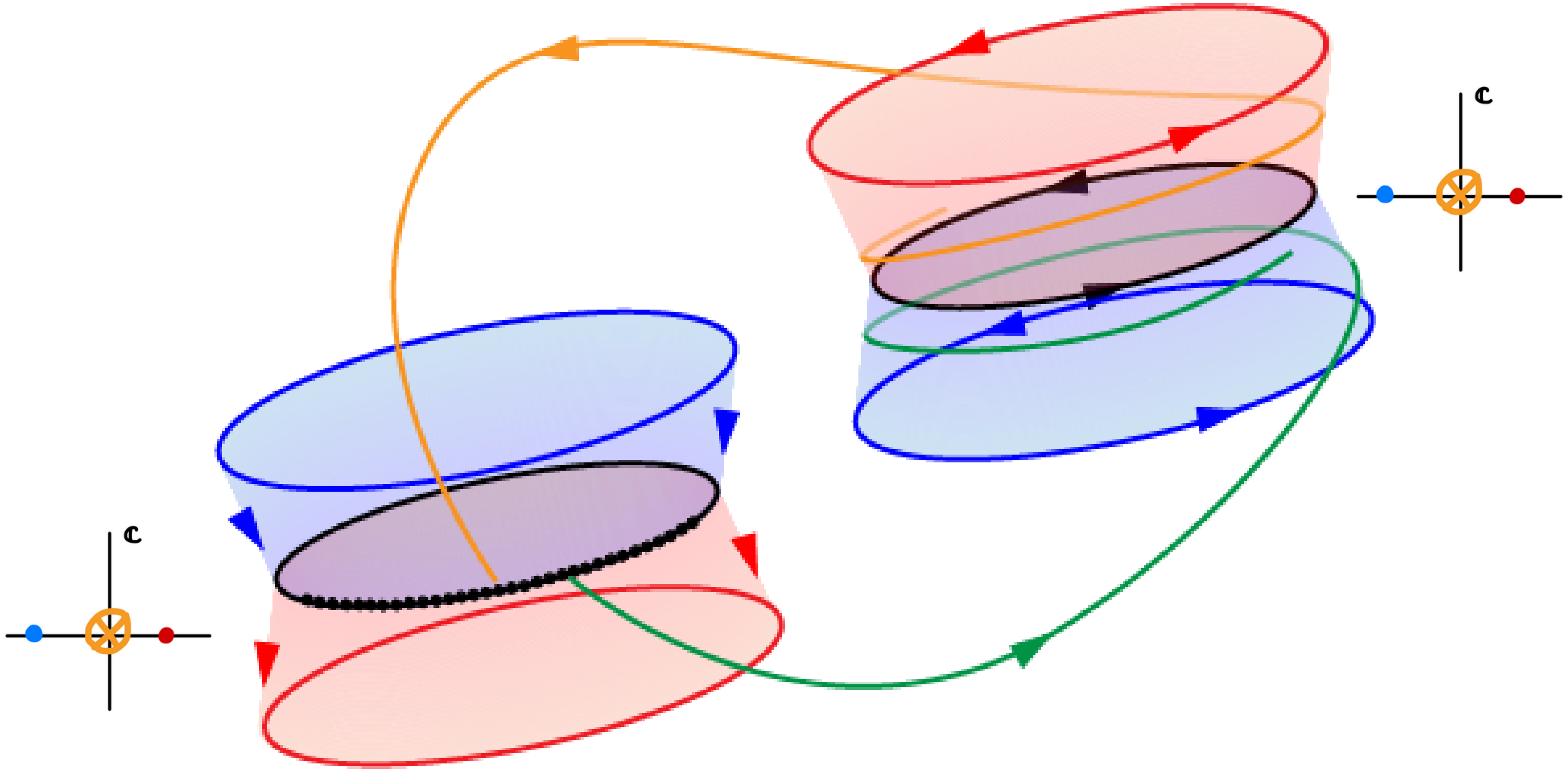


Phase Space

INFINITE DIMENSIONS

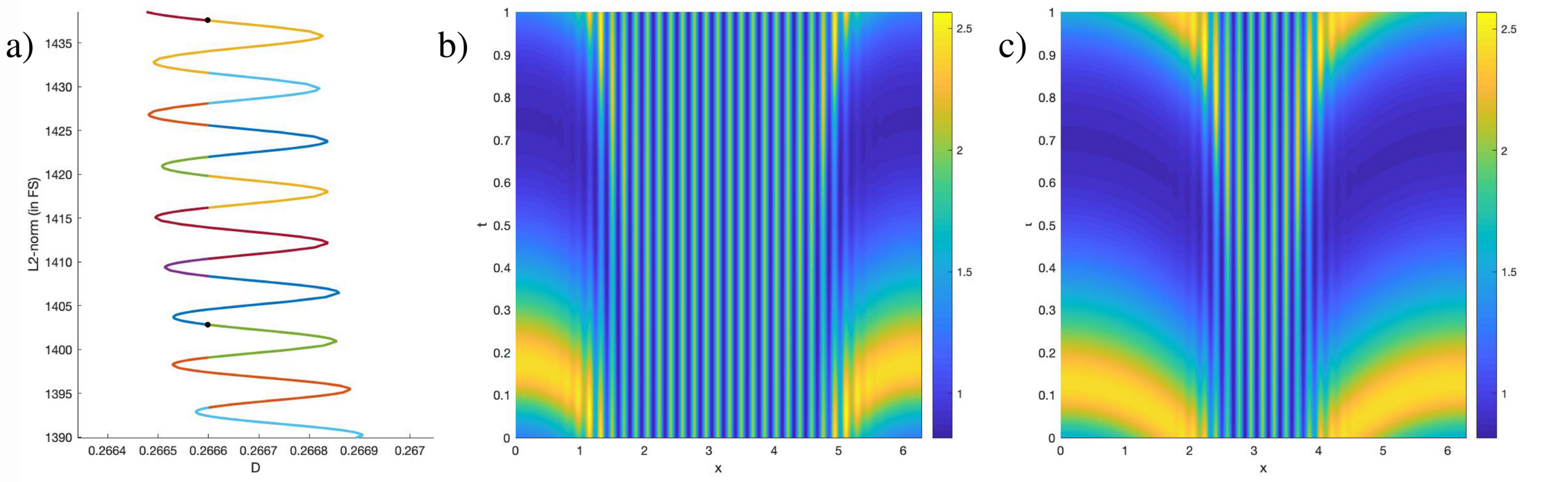


PDE Solution



Phase Space

THE BRUSSELATOR

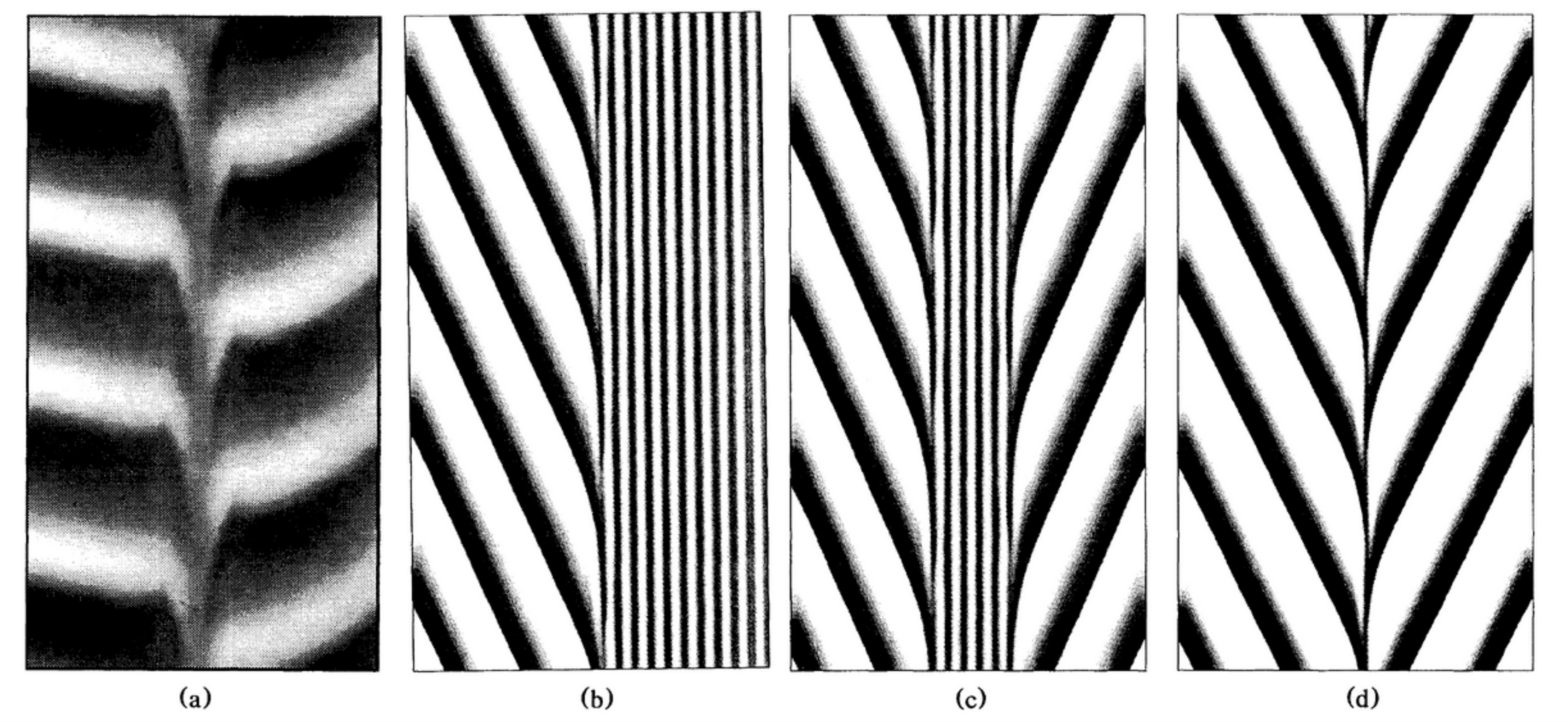


Prediction:

- Asymmetric solutions exists in 2-parameter families.
- They travel at small speeds and have variable wavenumber and phase shifted background states.

FUTURE DIRECTIONS

- Numerics: Can we verify our predictions hold?
- How do the asymmetric solutions connect?
- Source Defect case
- Spectrum and Stability



[Perraud et al (1993)]

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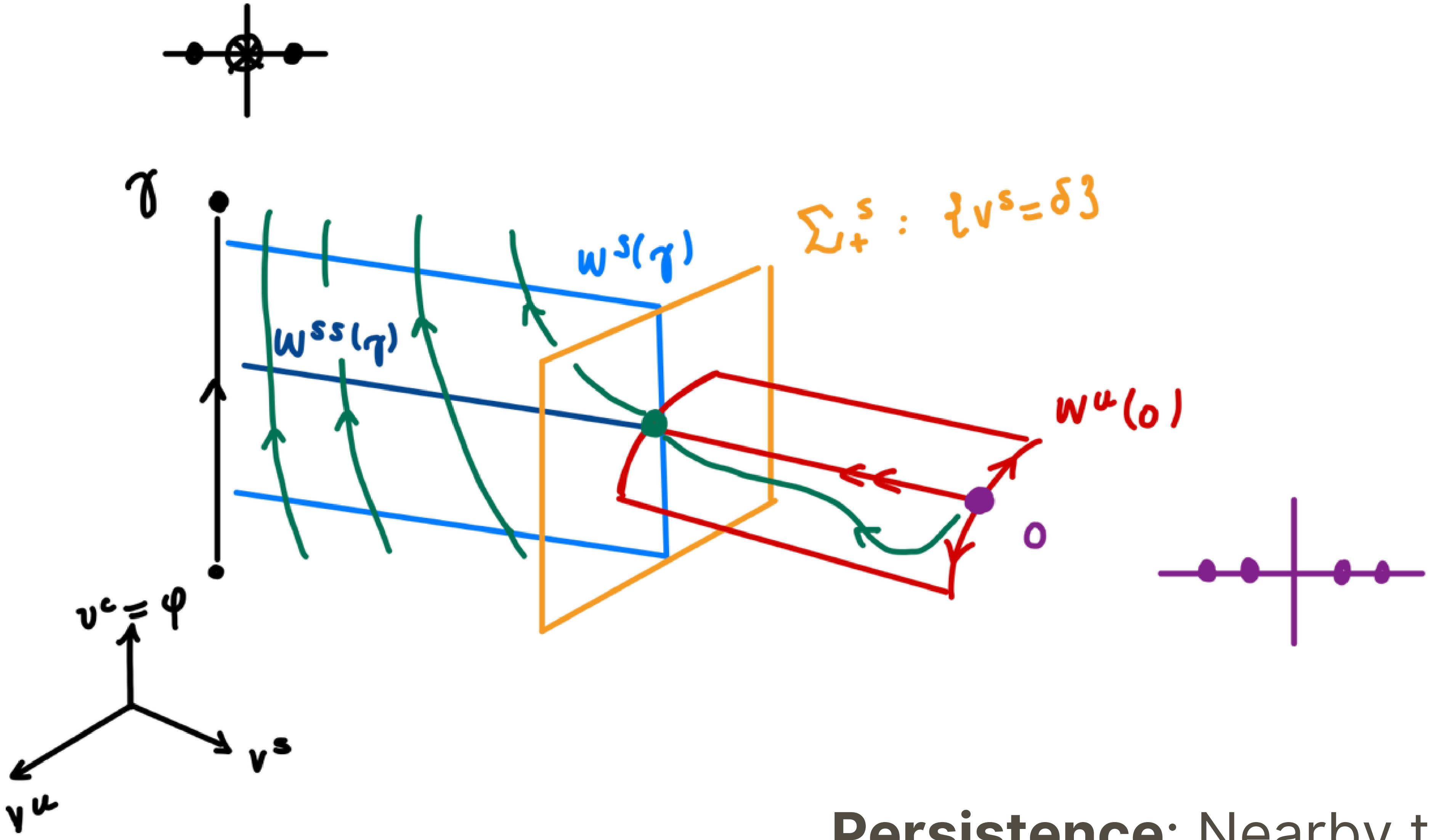
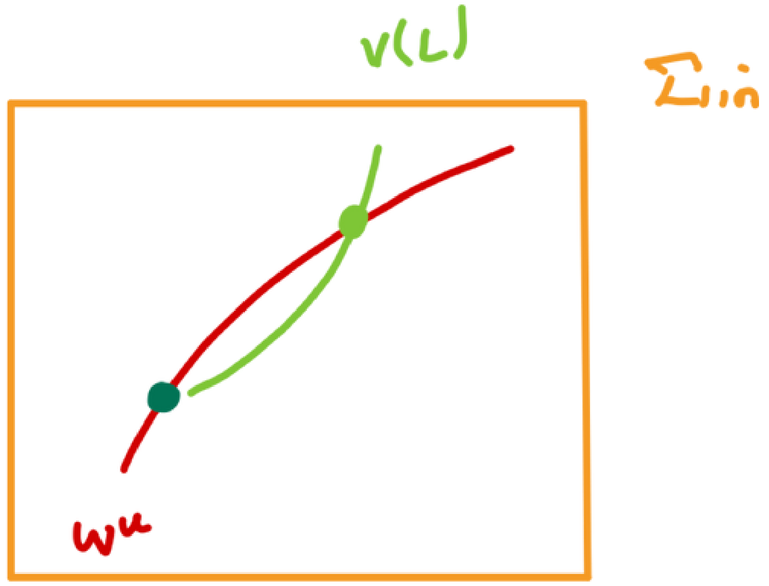
SWIFT-HOHENBERG: EQUILIBRIA





Persistence: Nearby the fronts - we find defects.

SWIFT-HOHENBERG: SNAKING