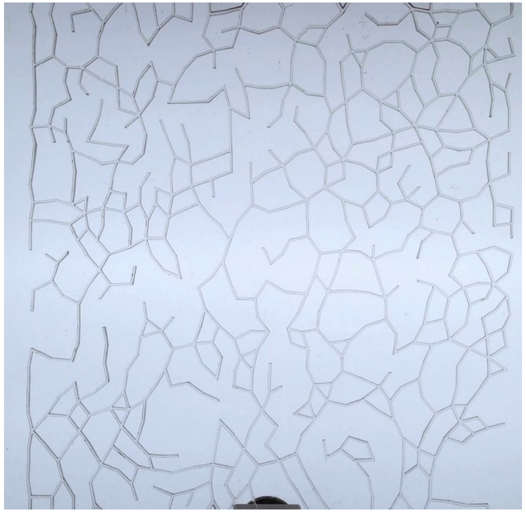
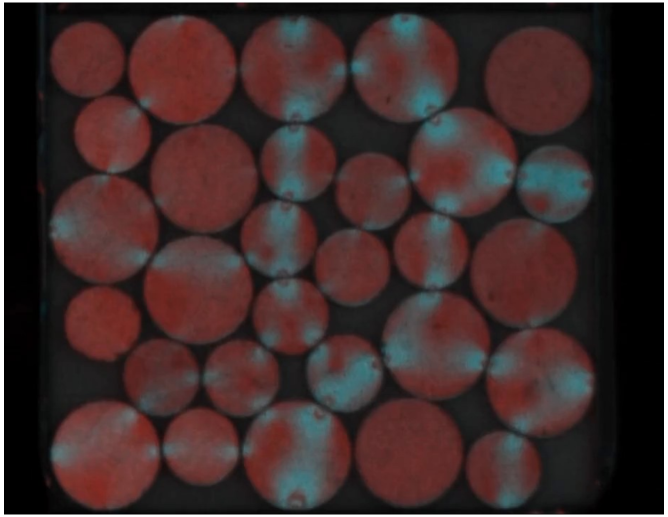
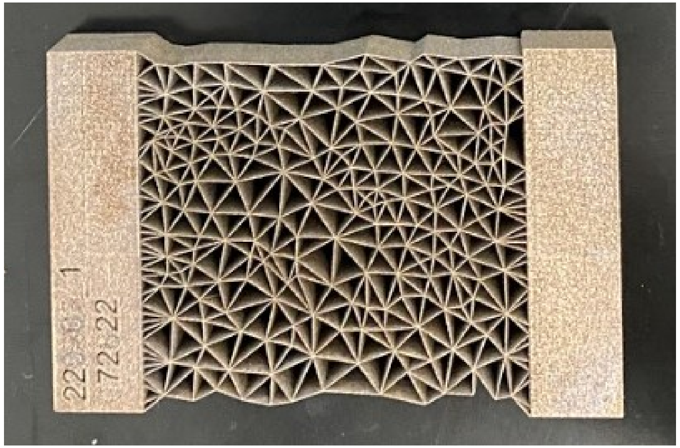
Building Networks

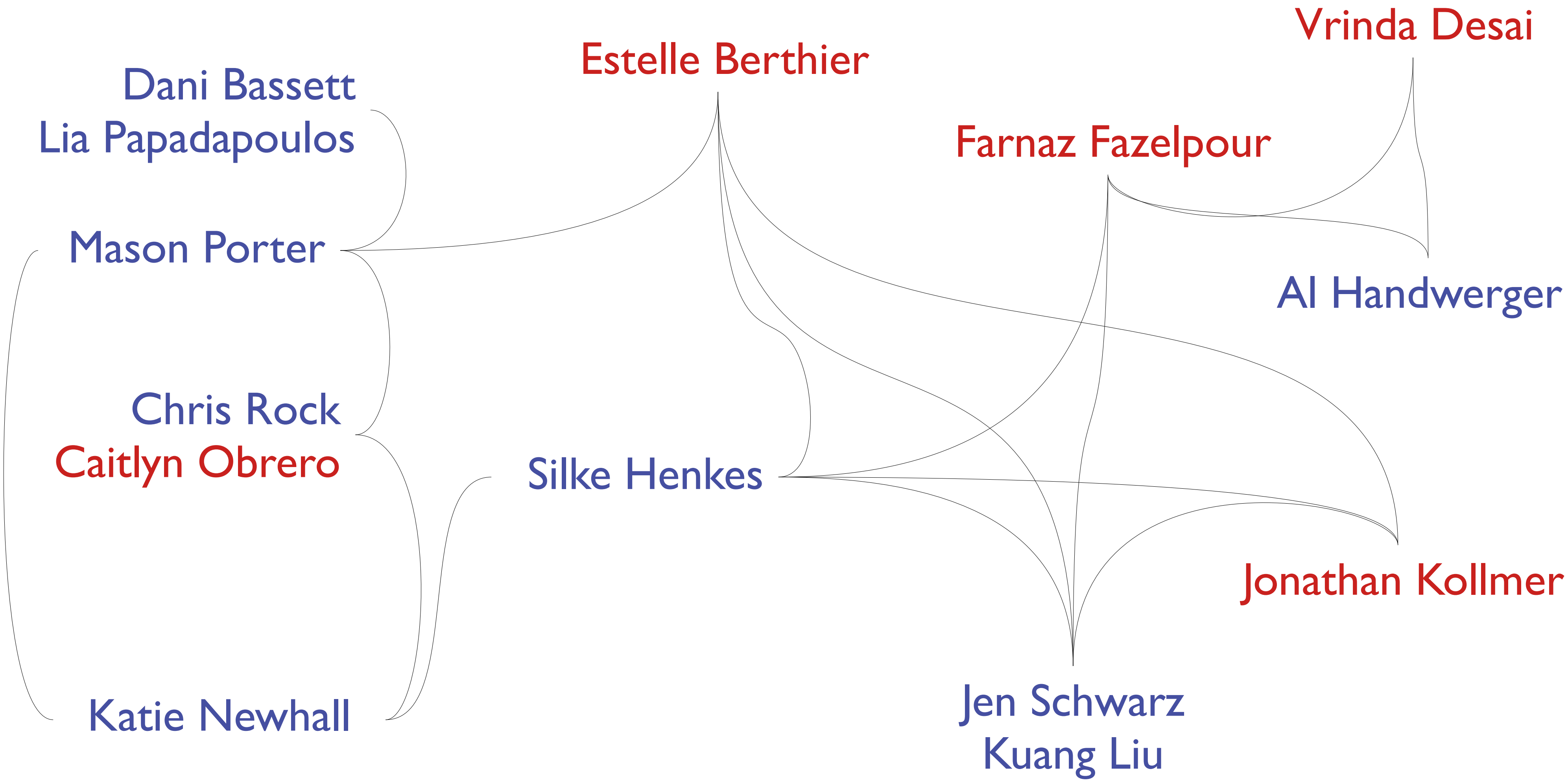




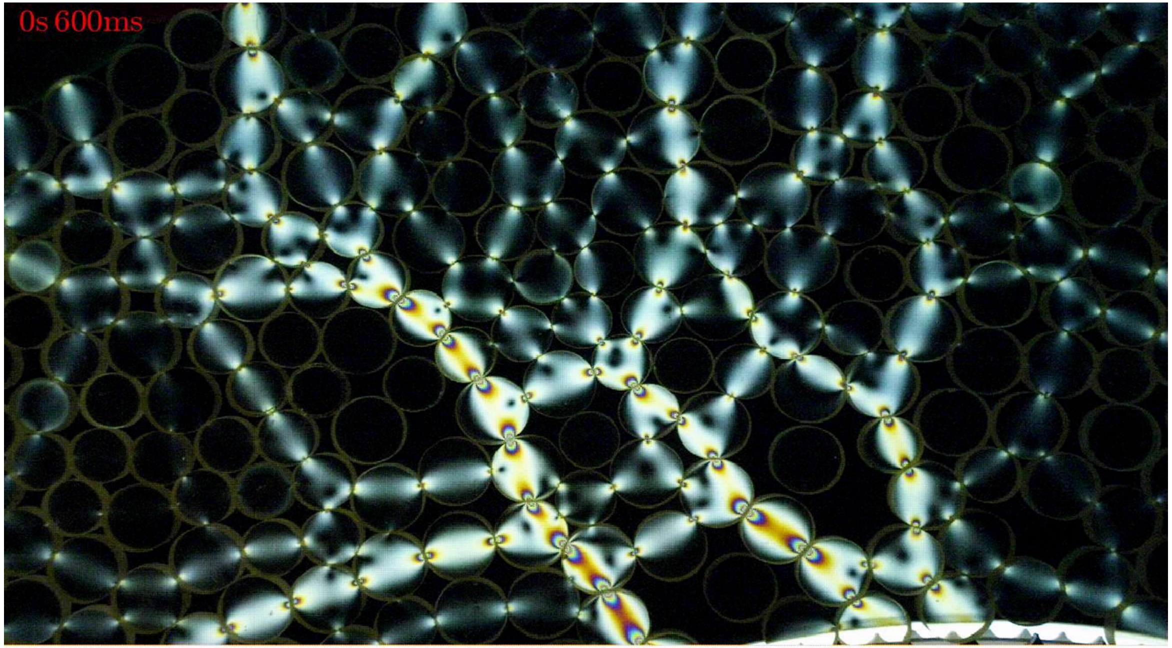
Karen Daniels
Department of Physics
NC State University

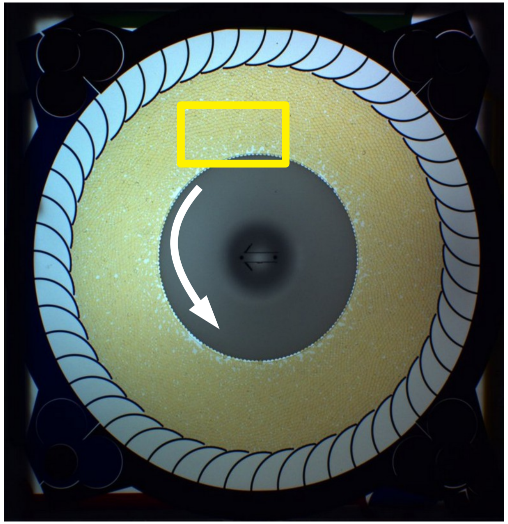


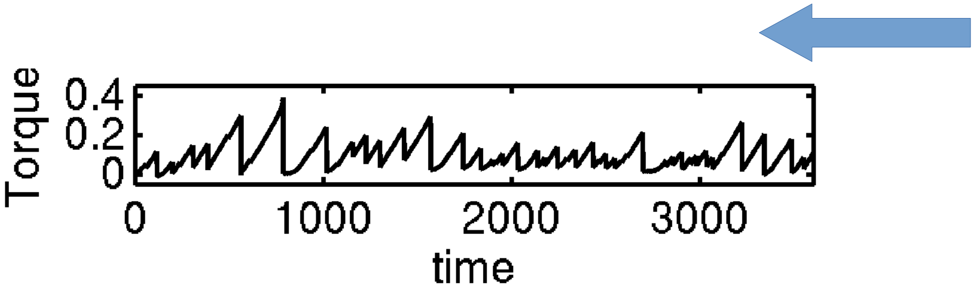
Dramatis Personnae



How do grains resist stresses?

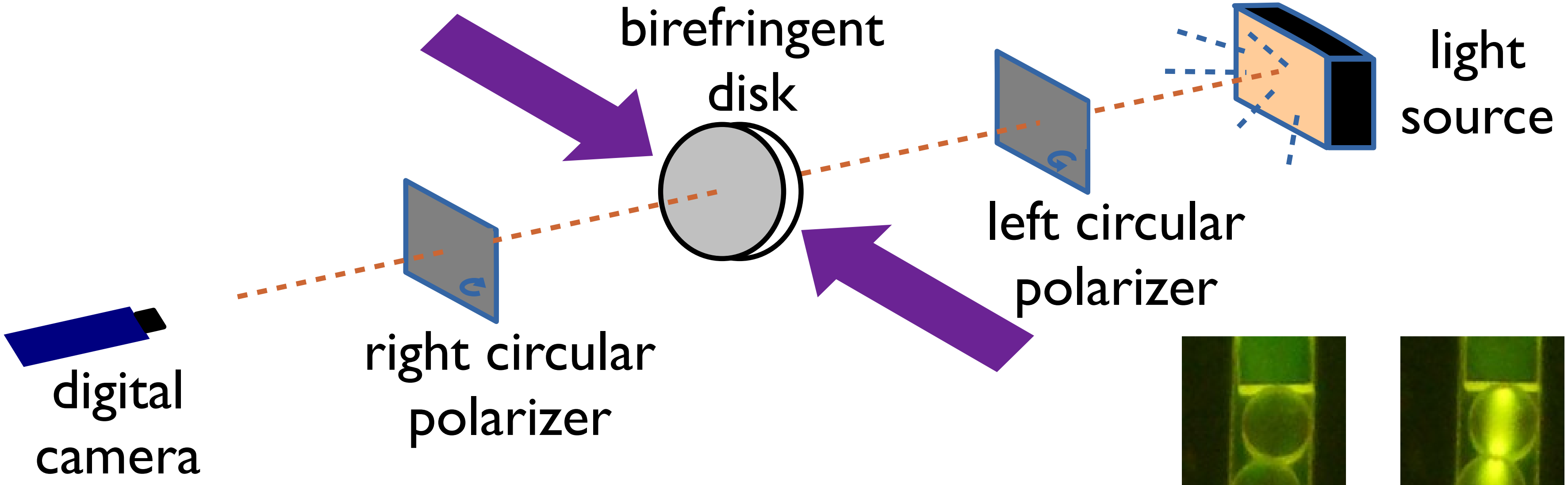


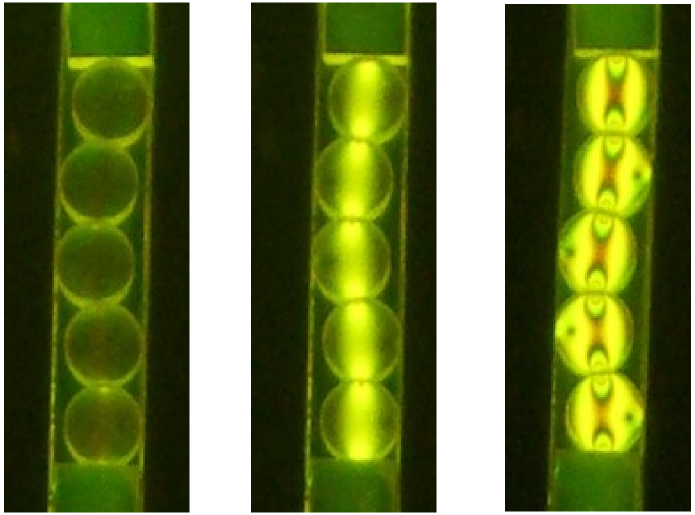




Estelle Berthier, Farnaz Fazelpour, Clayton Kirberger

Measuring Interparticle Contact Forces





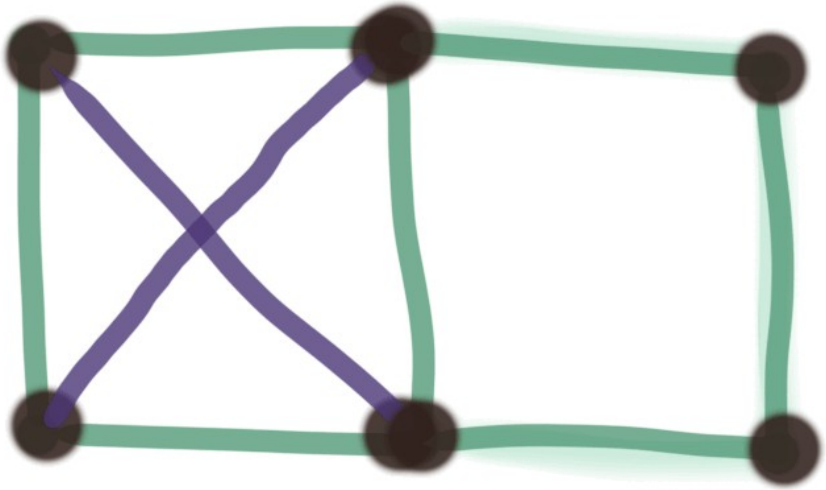
low medium high

Daniels, Kollmer, Puckett. *Rev. Sci. Inst.* (2017)
Abed-Zadeh, Barés, et al. *Granular Matter* (2019)

Rigidity

- the ability of a system to resist imposed stresses
- *caveat:* materials often contain rigid & floppy subregions

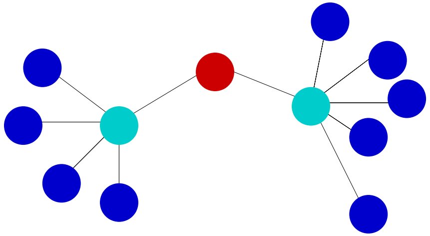
  … are system-wide averages still useful?
- where will failures occur?
- what sets failure criterion?



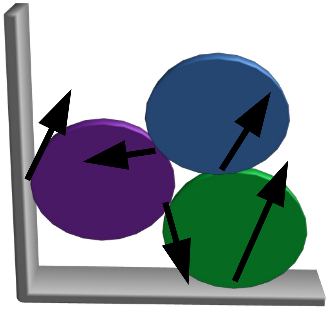
3 frameworks

less physics

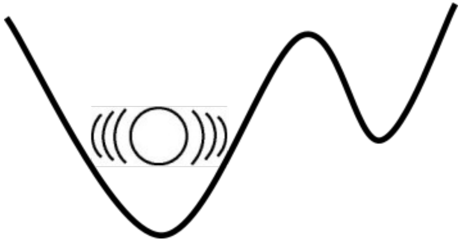
network science



constraint counting



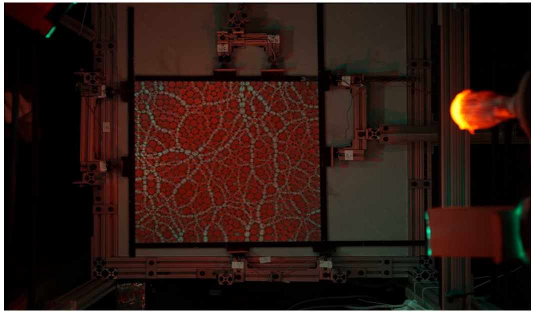
vibrational modes



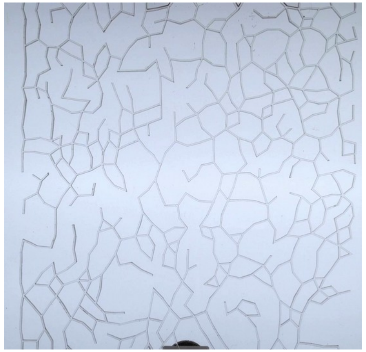
more physics

2 materials

frictional grains



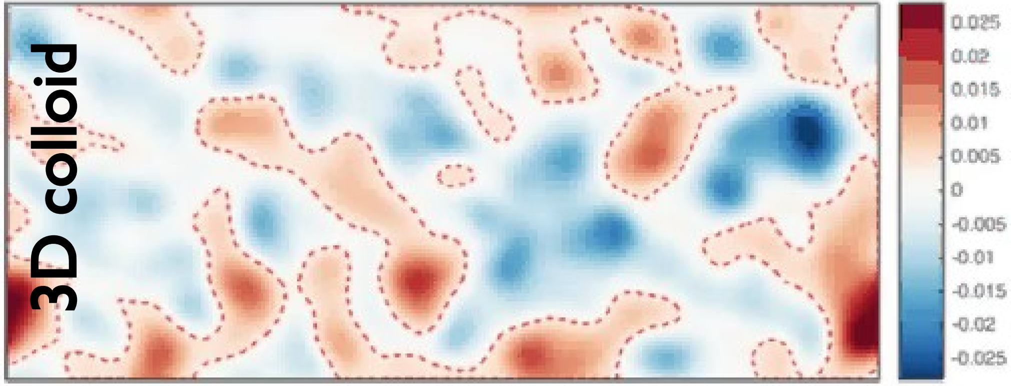
disordered lattices



Force chains

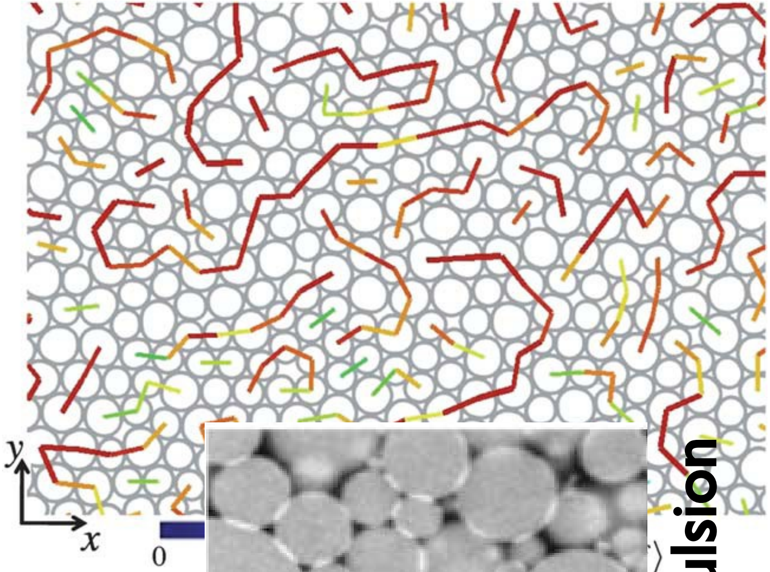
Lin, Bierbaum, Schall, Sethna, Cohen (2016)

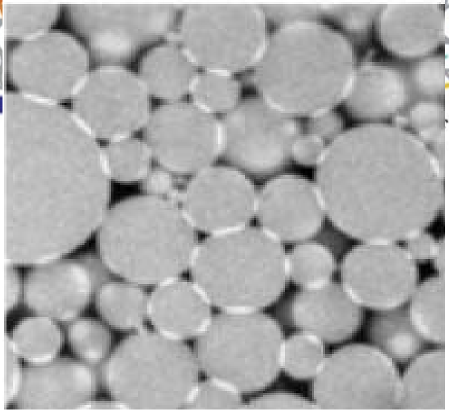
3D colloid



Desmond & Weeks. *Soft Matter* (2013)

2D emulsion

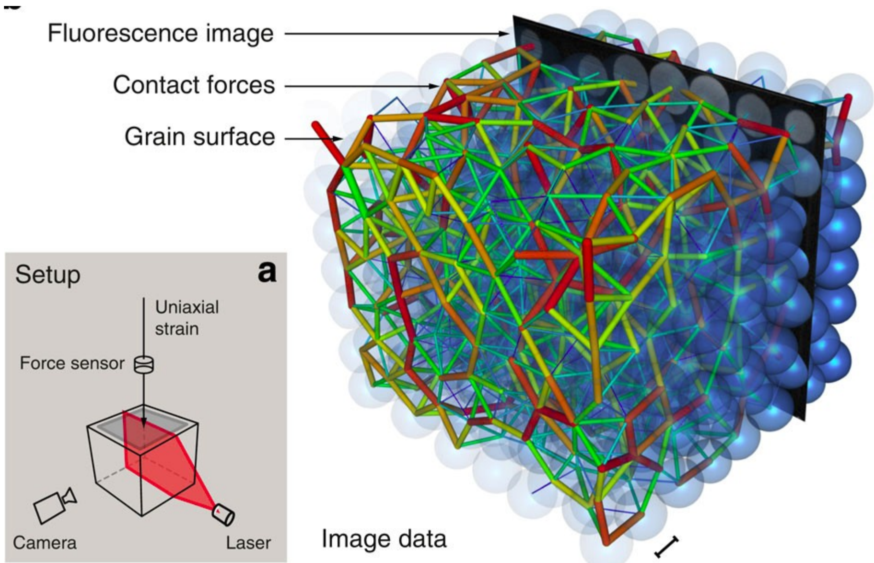




3D emulsion

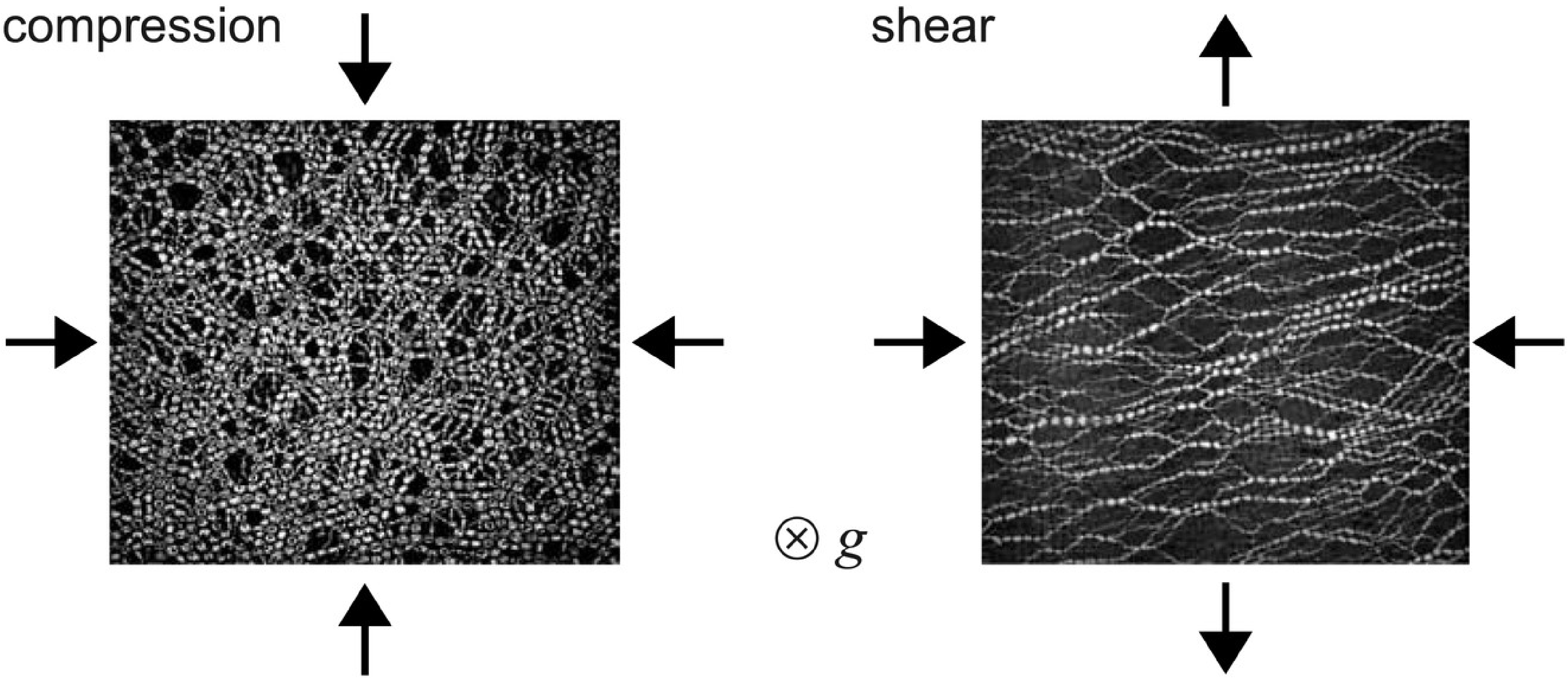
Brujic et al. Physica A (2003)

3D gel beads



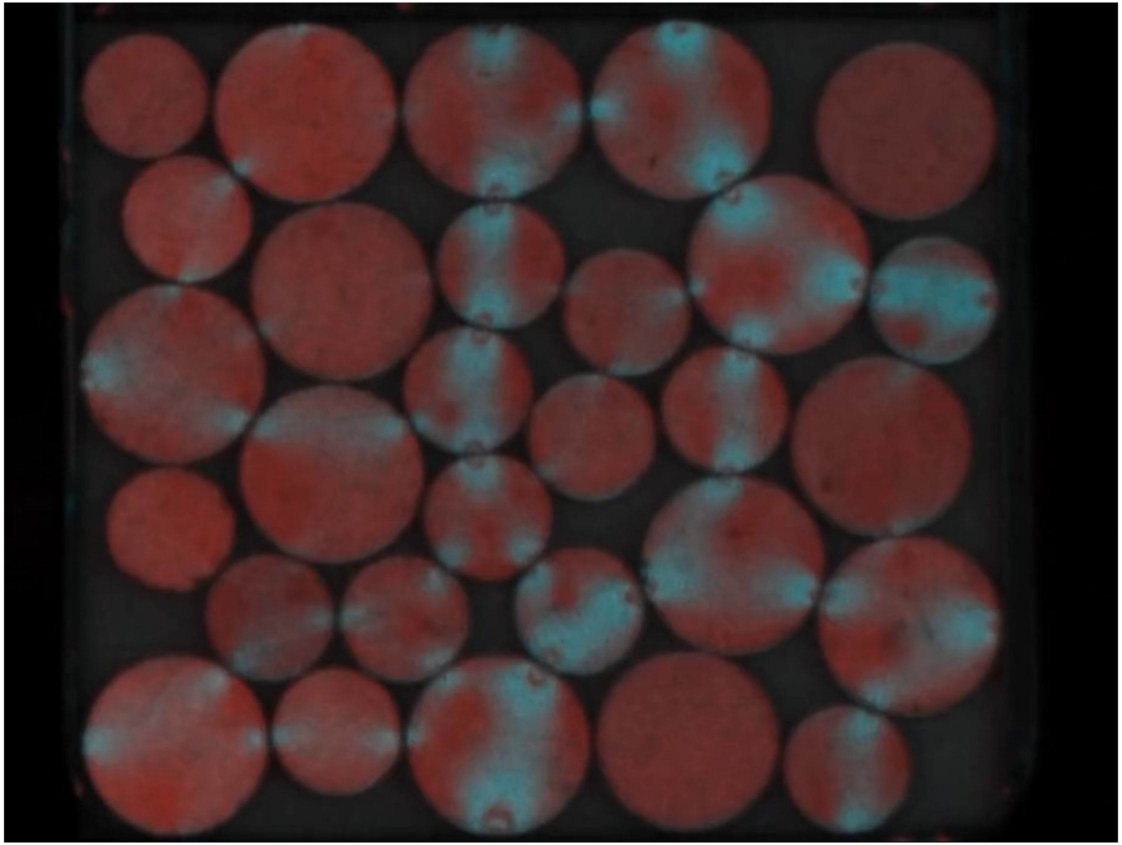
Brodu, Dijksman, Behringer. *Nat Comm.* (2015)

Force chains record history



Majmudar & Behringer *Nature* (2005)

Sensitivity to Small Changes

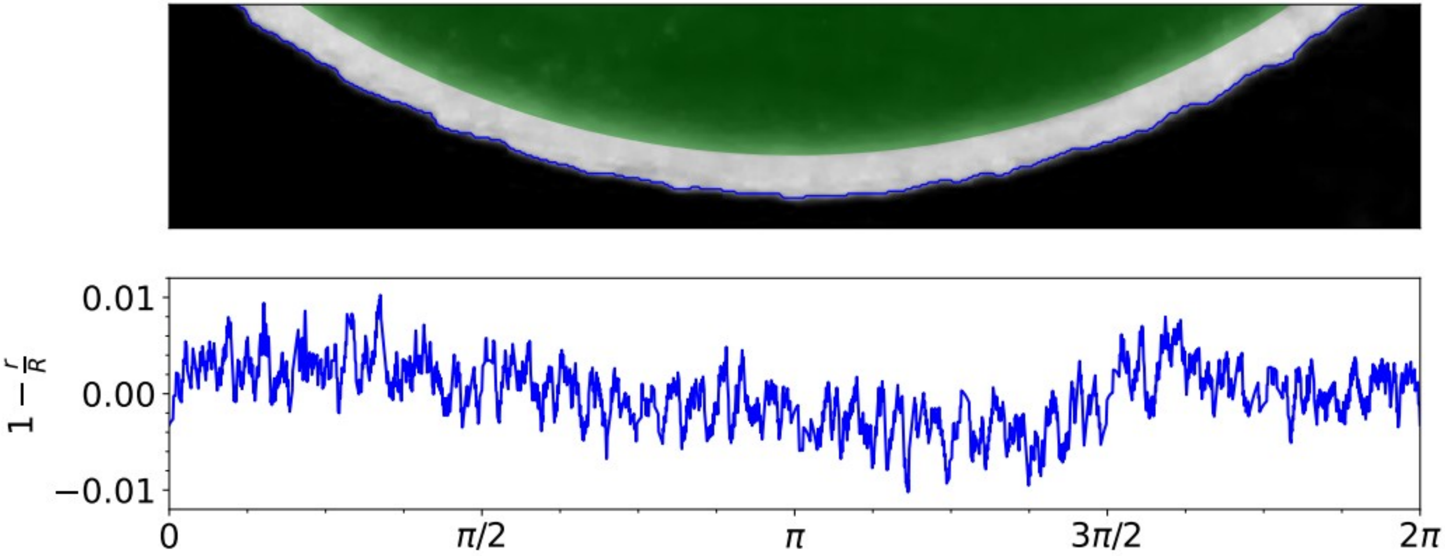


Jonthan Kollmer

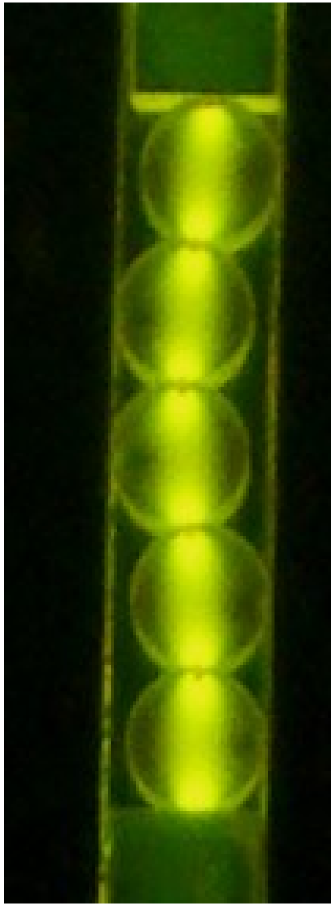
“movie” of images taken of the same, regenerated configuration

Kollmer & Daniels. *Soft Matter* (2019)

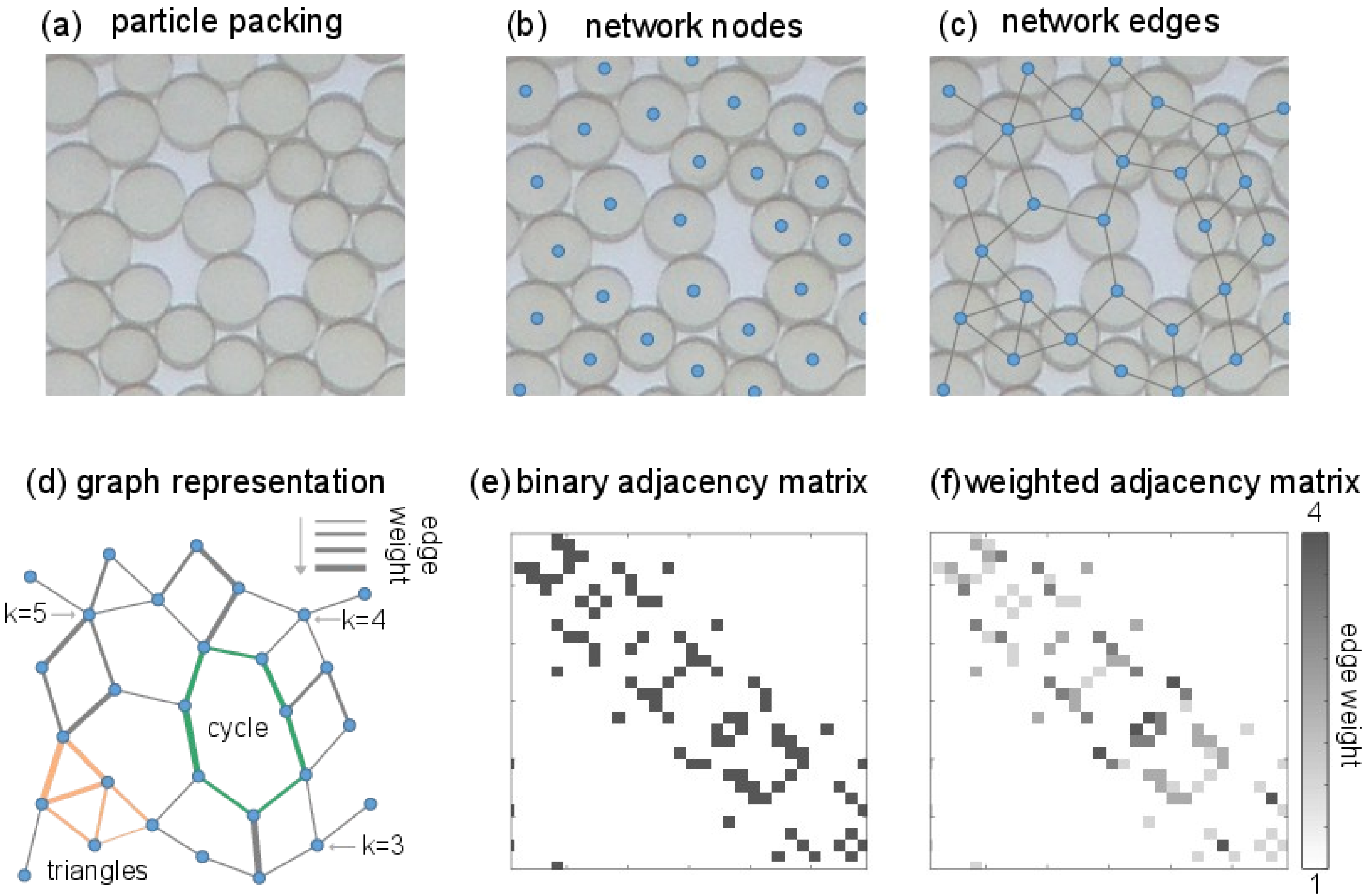
Real particles are rough



Kool, Charbonneau, Daniels, arXiv: 2205.06794



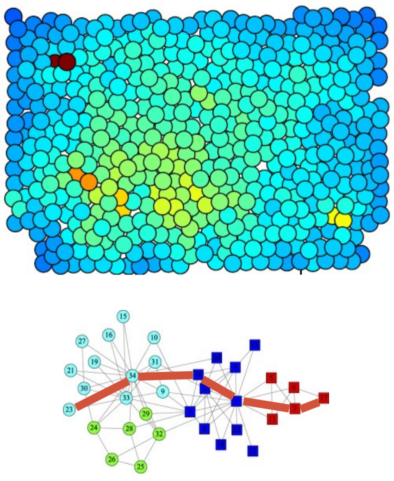
Configurations → Adjacency Matrix



Papadapoulous, Daniels, Porter, Bassett. *J. Complex Networks* (2018)

Network science metrics for different scales

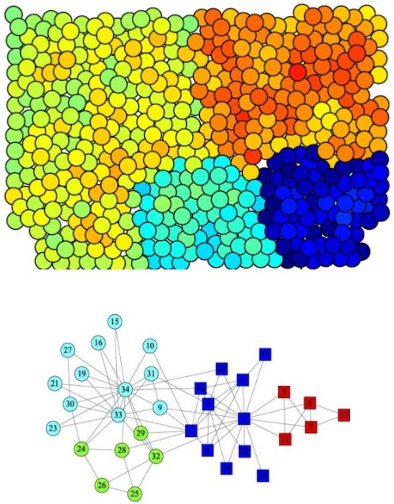
System



Global Efficiency

- ➢ Efficiency of global signal transmission

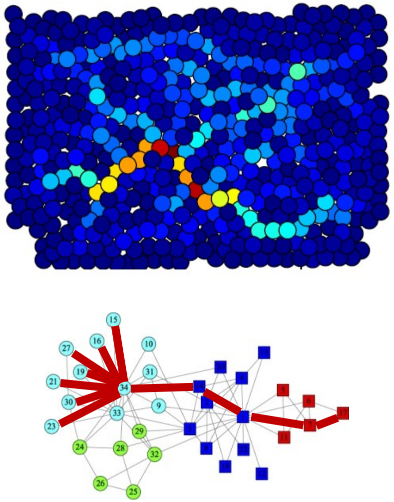
2D: Domains



Modularity

- ➢ Local geographic domains

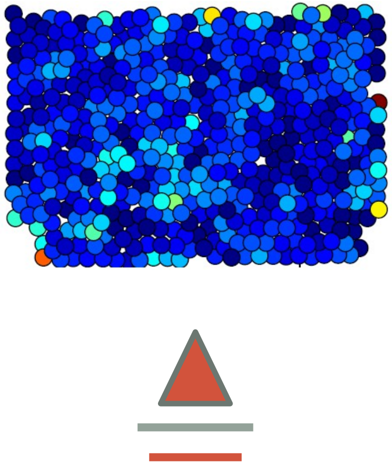
1D: Curves



Geodesic Node Betweenness

- ➢ Bottlenecks or centrality

0D: Particles

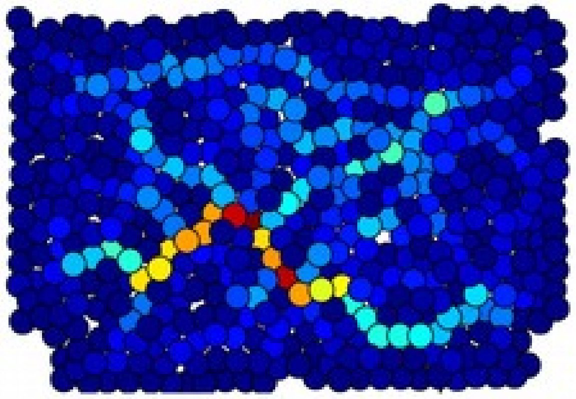


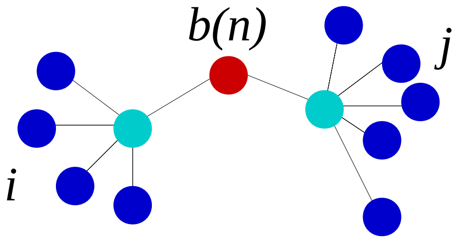
Clustering Coefficient

- ➢ Local loop structures

Betweenness Centrality

$$b(n) = \sum_{i \neq n \neq j} \frac{s_{ij}(n)}{s_{ij}}$$

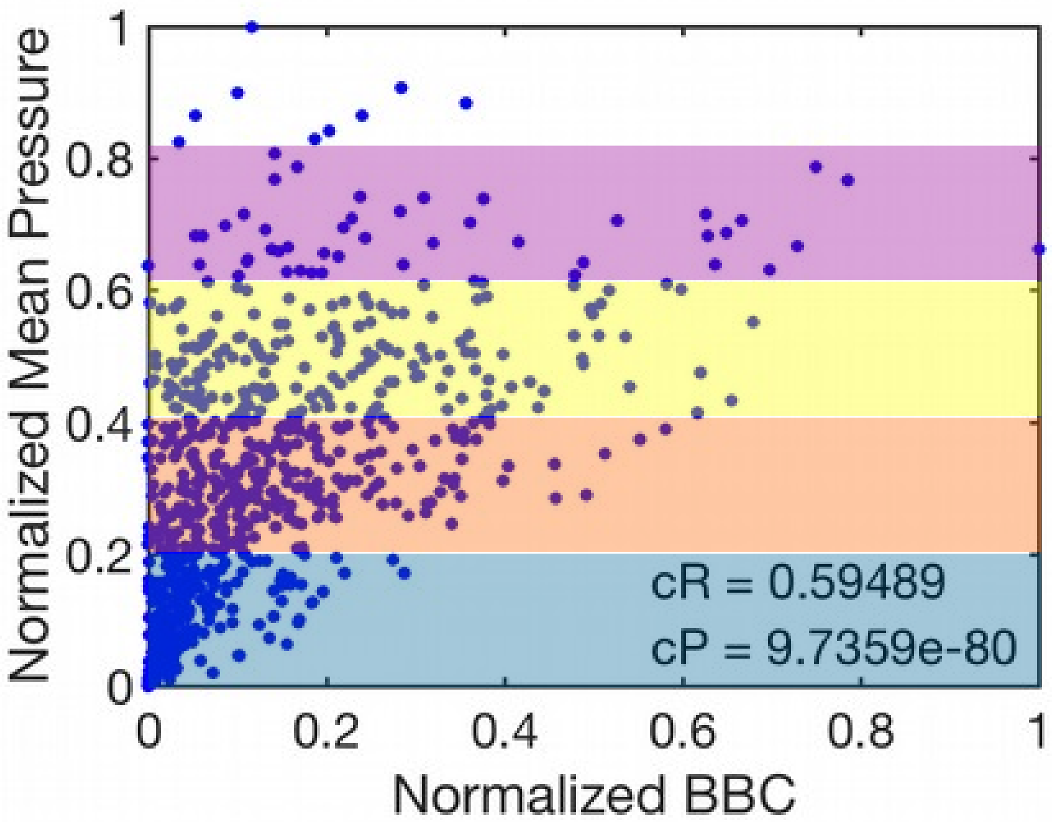




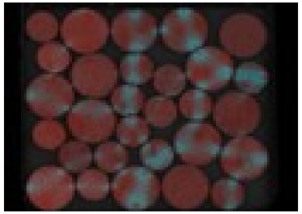
- s_{ij} = shortest path between particles i,j
- can be either # of hops or weighted
- $b(n)$ = fraction of total # of shortest paths that go through particles n
- high $b(n)$ ~ “airline hubs”

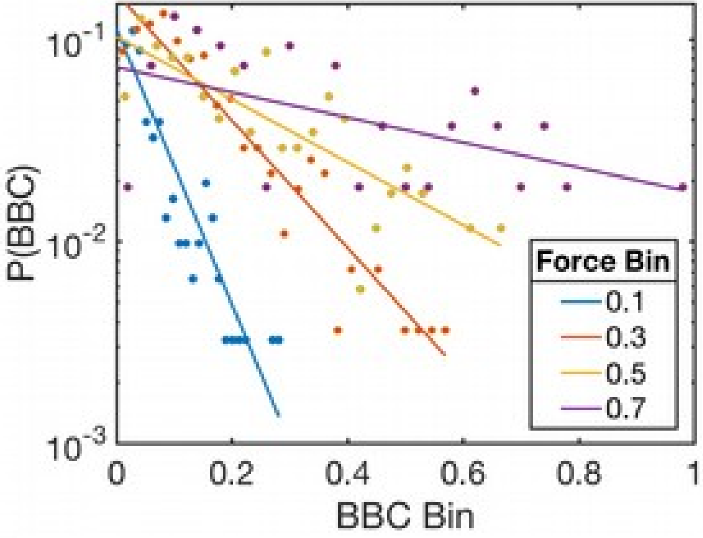
http://www.brain-connectivity-toolbox.net/

Betweenness centrality predicts forces



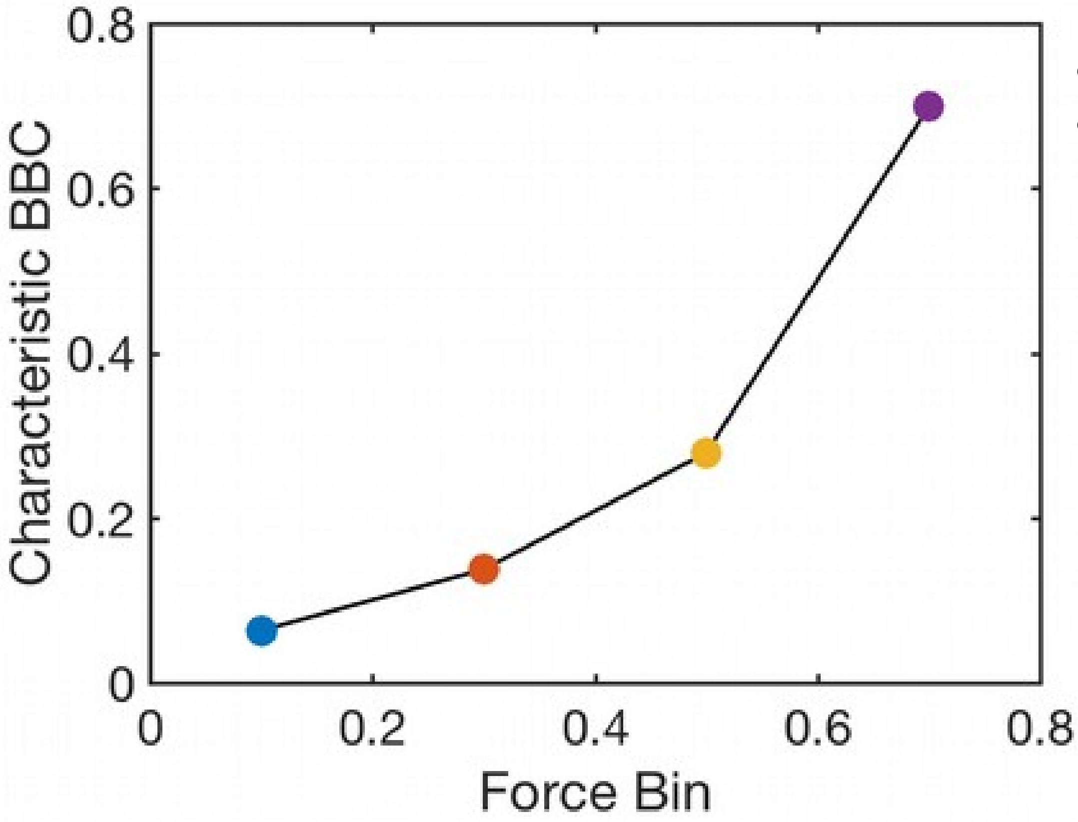
824 particles
80 cycles



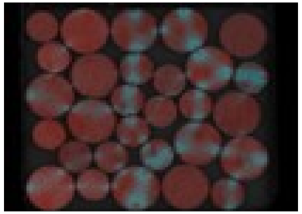


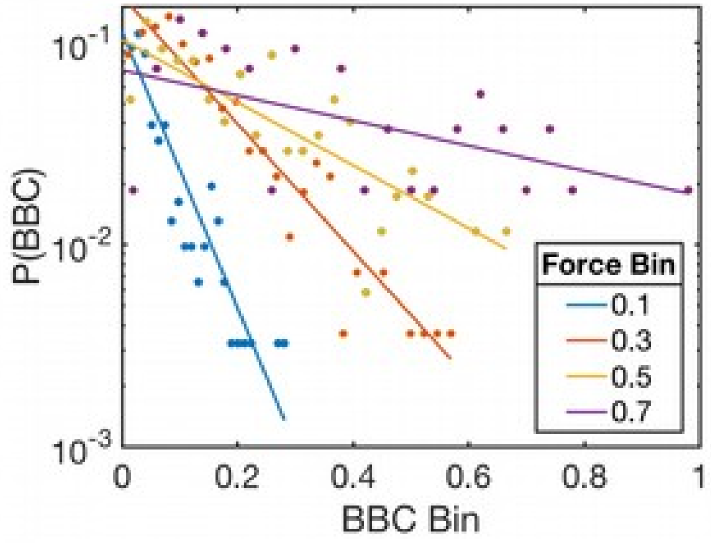
Kollmer & Daniels. *Soft Matter* (2019)

Betweenness centrality predicts forces

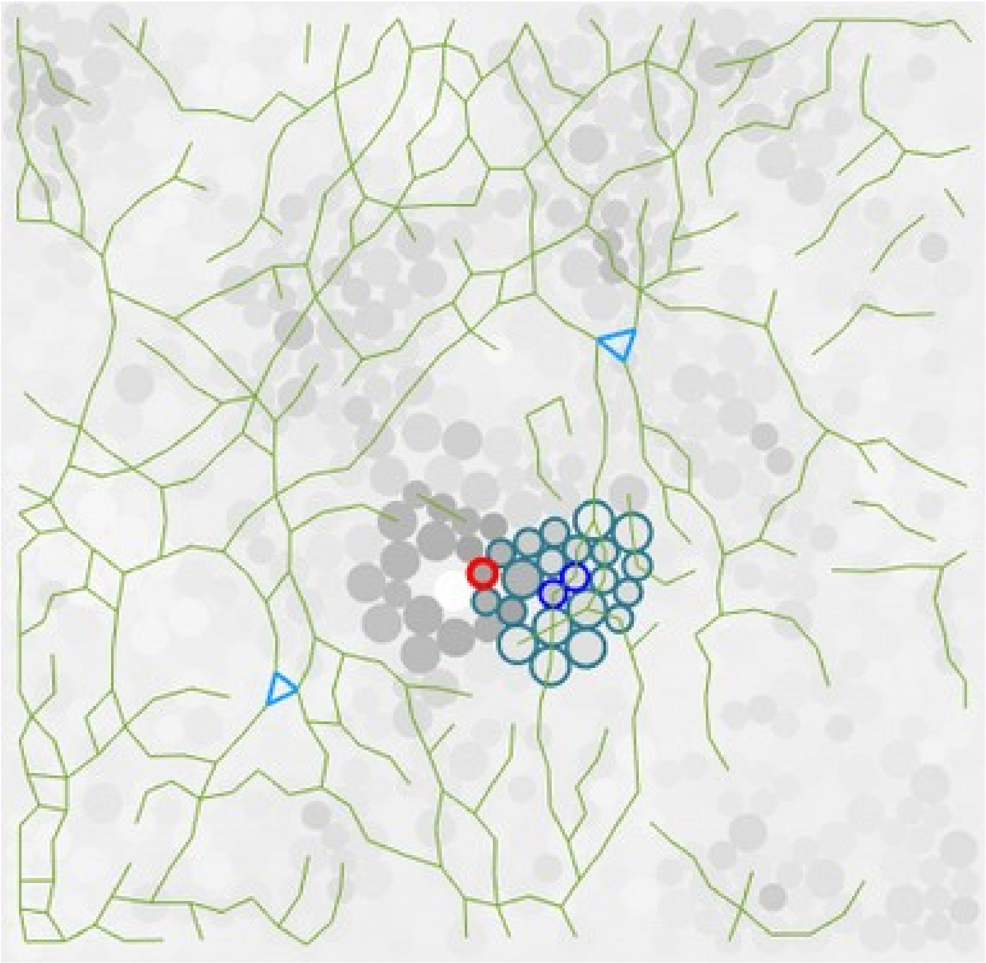


824 particles
80 cycles



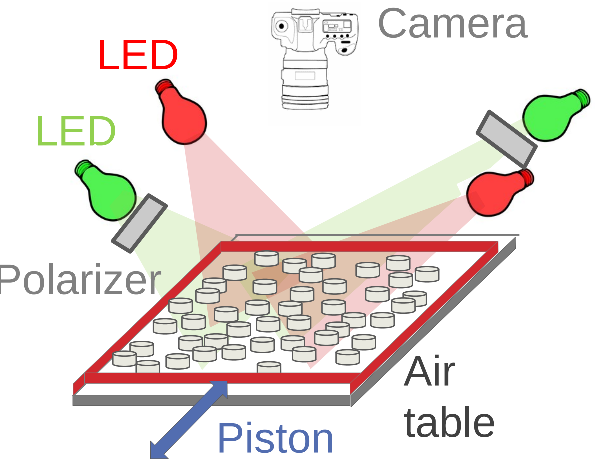


Kollmer & Daniels. *Soft Matter* (2019)



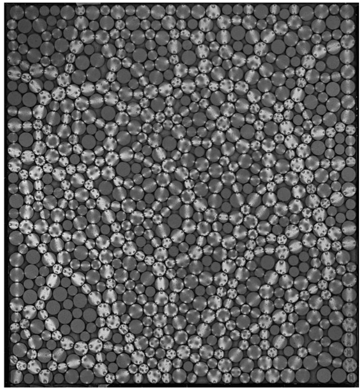
Does centrality forecast failure locations?

Simplify! → Disordered lattices

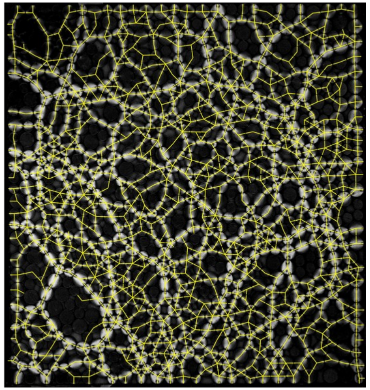


Estelle Berthier

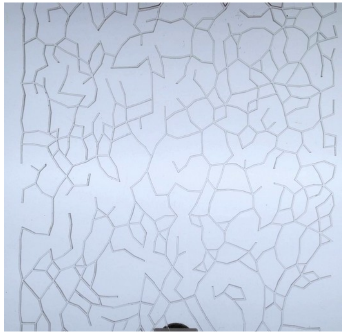
granular force chains



contact network



laser-cut lattice



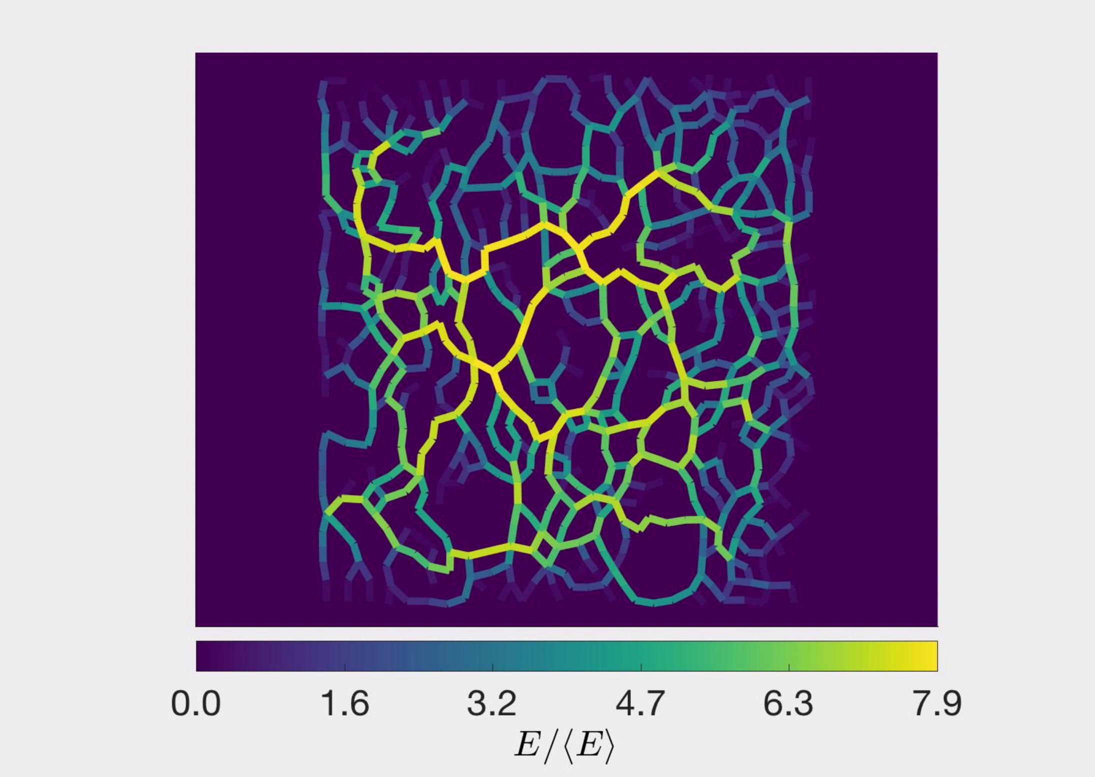
Berthier, Porter, Daniels. *PNAS* (2019)
Berthier, Kollmer, Henkes, Liu, Schwarz, Daniels. *Phys. Rev. Mat.* (2019)

Failure Locations & Betweenness

About 77% of failing edges have $E^{failed} > \langle E \rangle$

About 37% of the network edges have $E > \langle E \rangle$

Shared property of all networks for small damage event



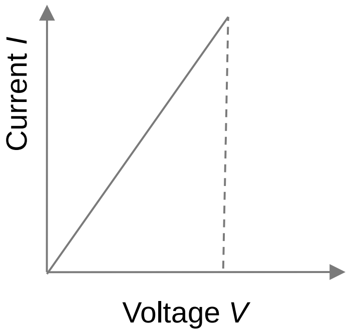
$E_{ij}/\langle \mathbf{E} \rangle$

Berthier, Porter, Daniels. *PNAS* (2019)

Better model → better prediction

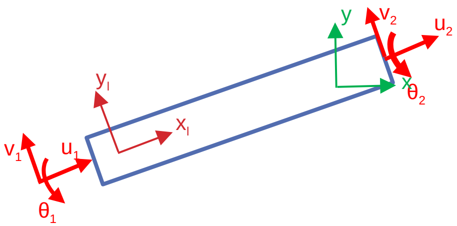
Fuse Model

Each beam → fuse
Fails at some current

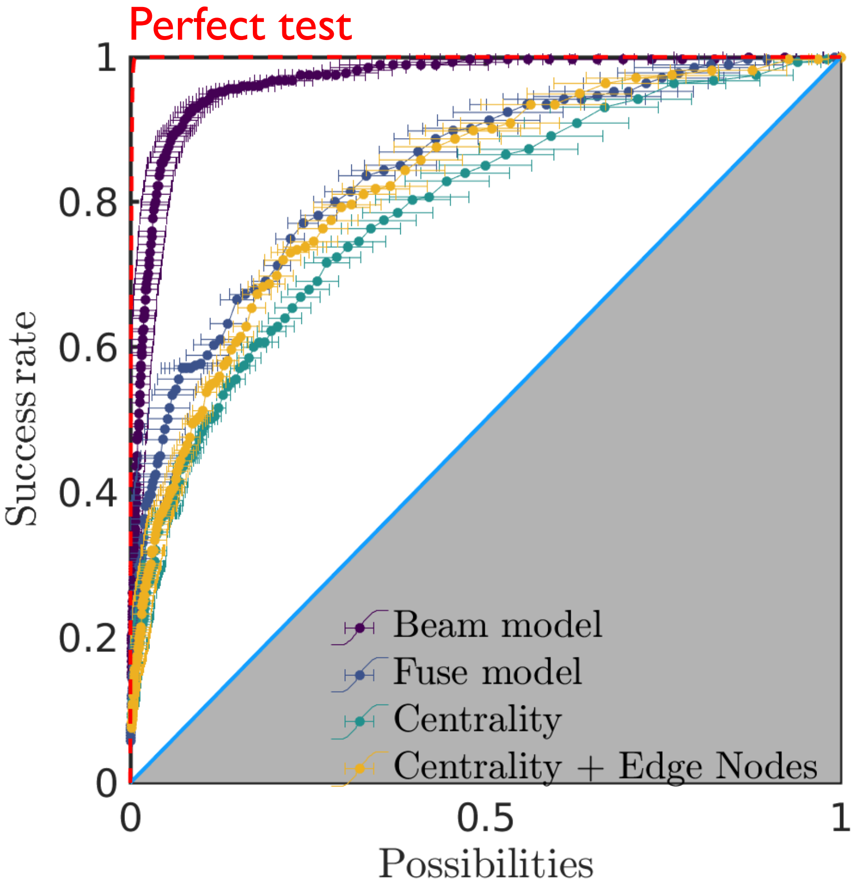


de Arcangelis et al.
J. Physique Lett. (1985)
Duxbury et al. *PRB* (1987)

Beams + Moments

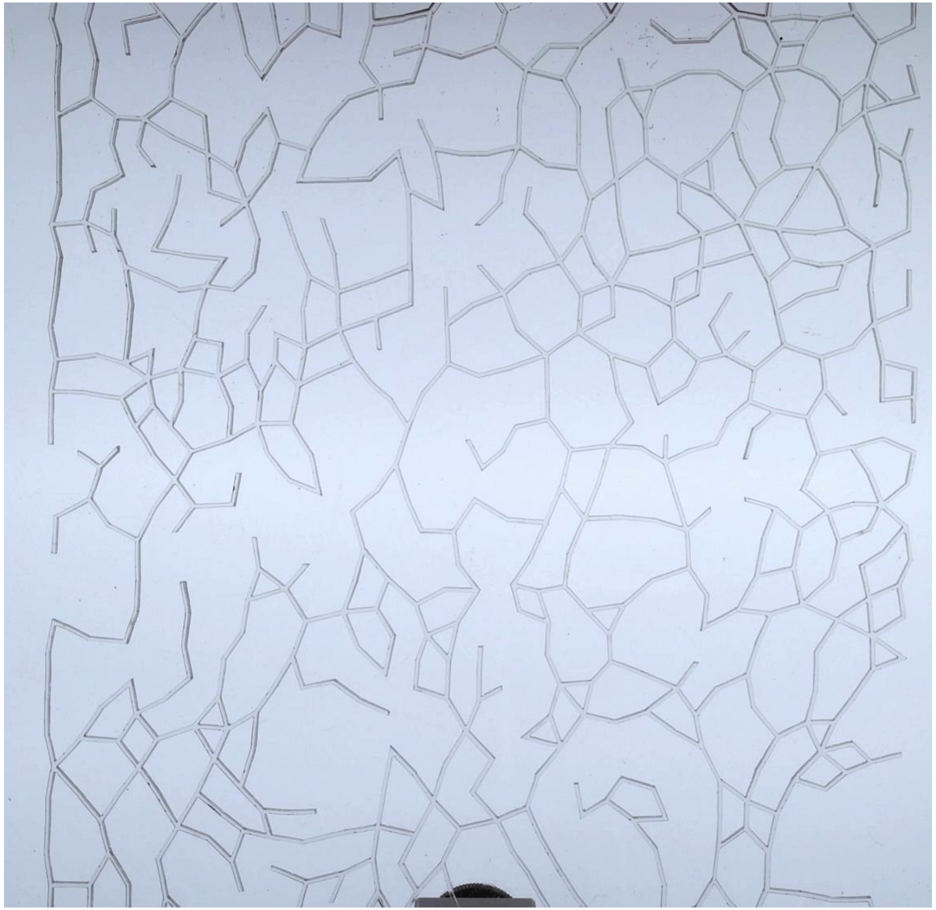


Nukala Zapperi, Alava, Šimunović. *PRE* (2008)



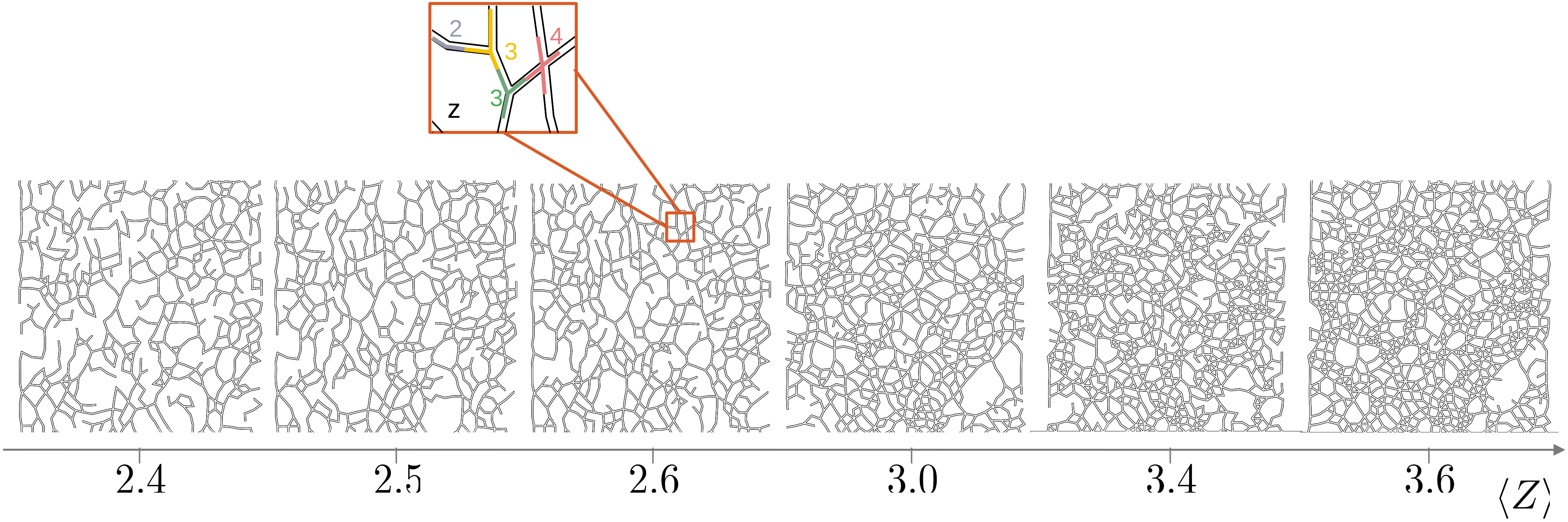
Berthier, Porter, Daniels. *PNAS* (2019)

Lattice Fracture



Continuous Cast Acrylic
Thickness = 3.17 mm
Beam width = 1.5 mm

Vary mean coordination number *z*

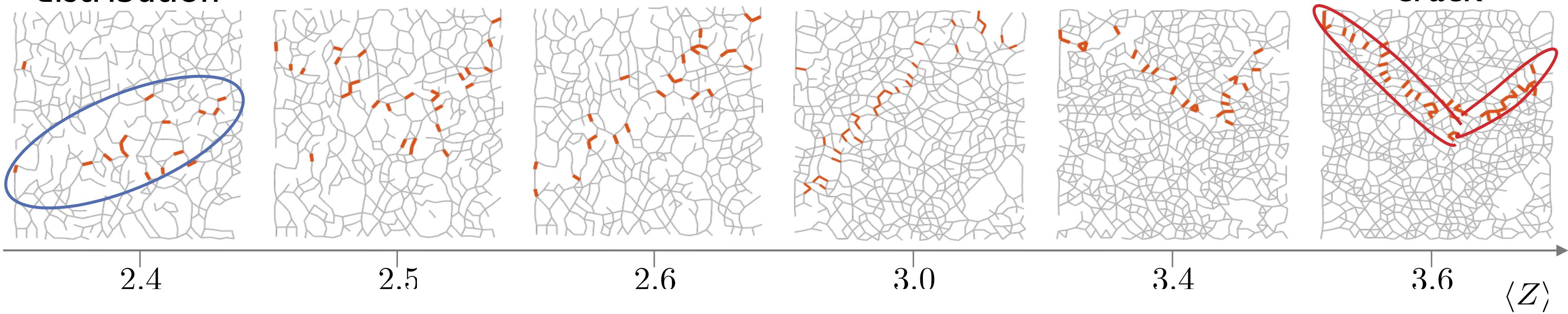


Berthier, Kollmer, Henkes, Liu, Schwarz, Daniels. *Phys. Rev. Mat.* (2019)

Connectivity controls failure mode

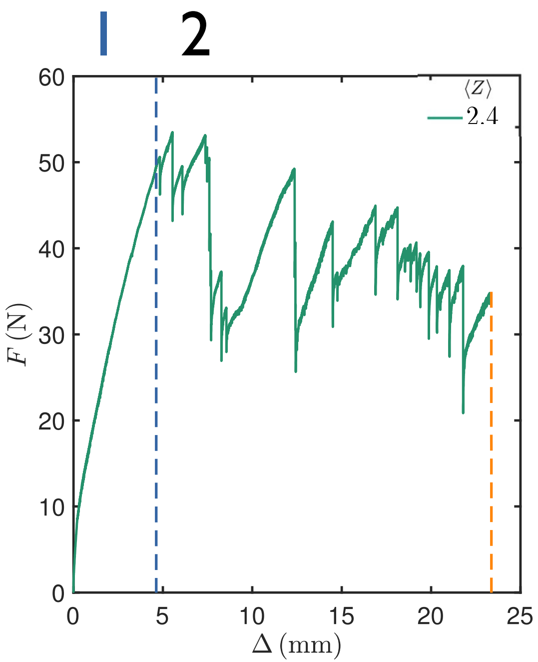
ductile-like:
broad
distribution

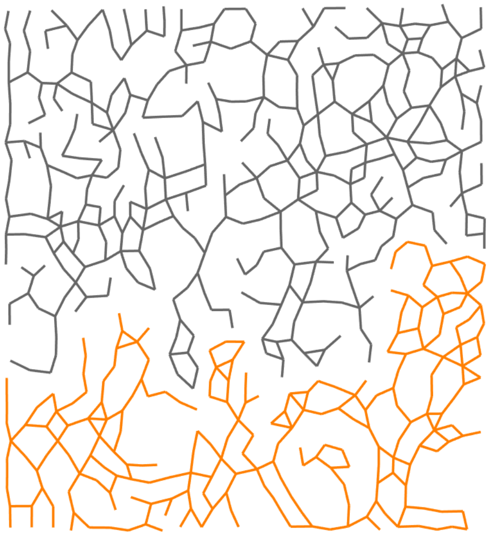
brittle-like:
narrow
crack



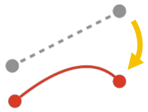
Berthier, Kollmer, Henkes, Liu, Schwarz, Daniels. *Phys. Rev. Mat.* (2019)

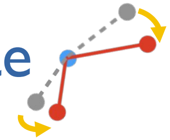
Low-*z* response & failure



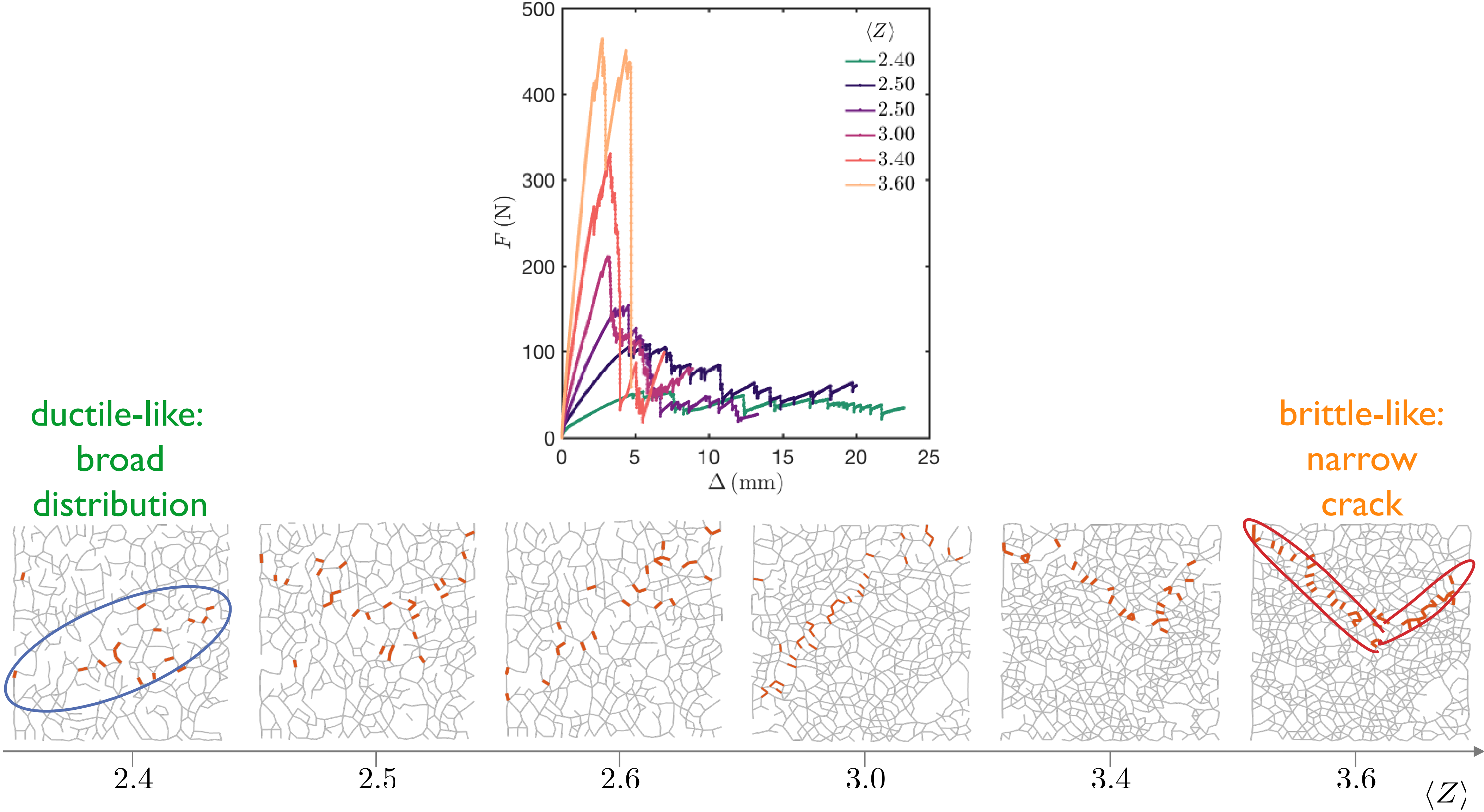


- Phase 1: Elastic response
  - Beams compress & stretch
  - Intersections rotate
- Phase 2: Successive Failures
  - Progressive damage
  - Distributed damage
- End result: spanning crack





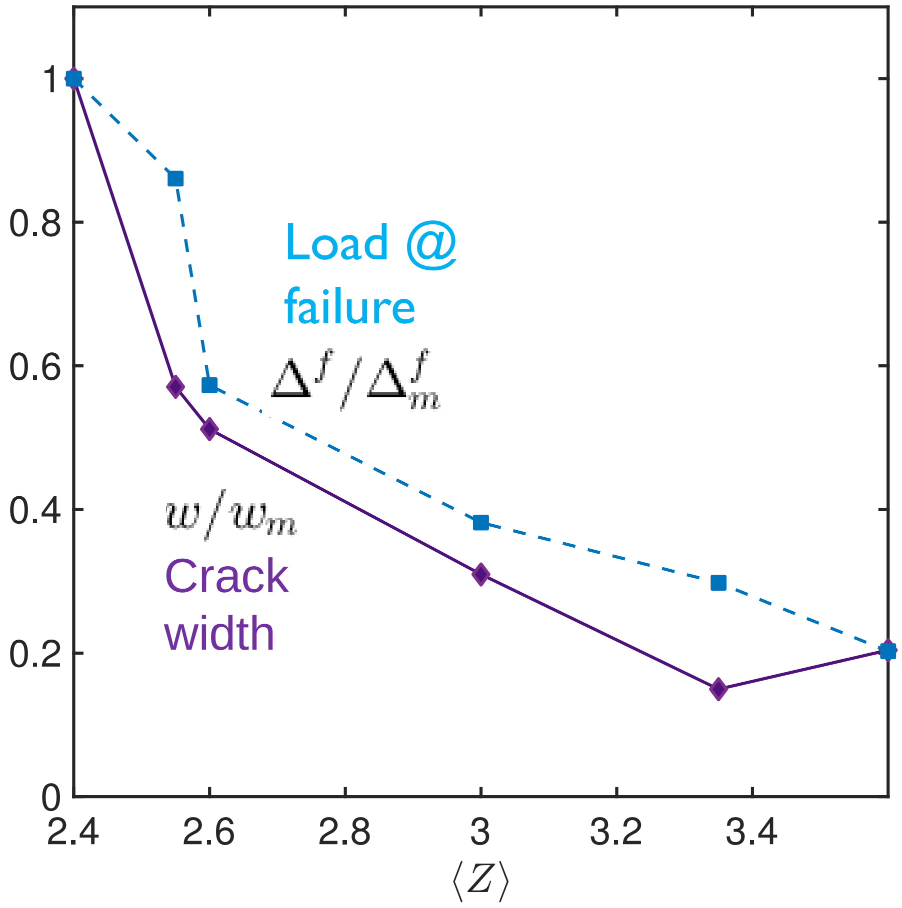
Berthier, Kollmer, Henkes, Liu, Schwarz, Daniels. *Phys. Rev. Mat.* (2019)



500
400
300
200
100
0
F (N)
0
5
10
15
20
25
Δ (mm)
⟨Z⟩
2.40
2.50
2.50
3.00
3.40
3.60
ductile-like:
broad
distribution
brittle-like:
narrow
crack
2.4
2.5
2.6
3.0
3.4
3.6
⟨Z⟩

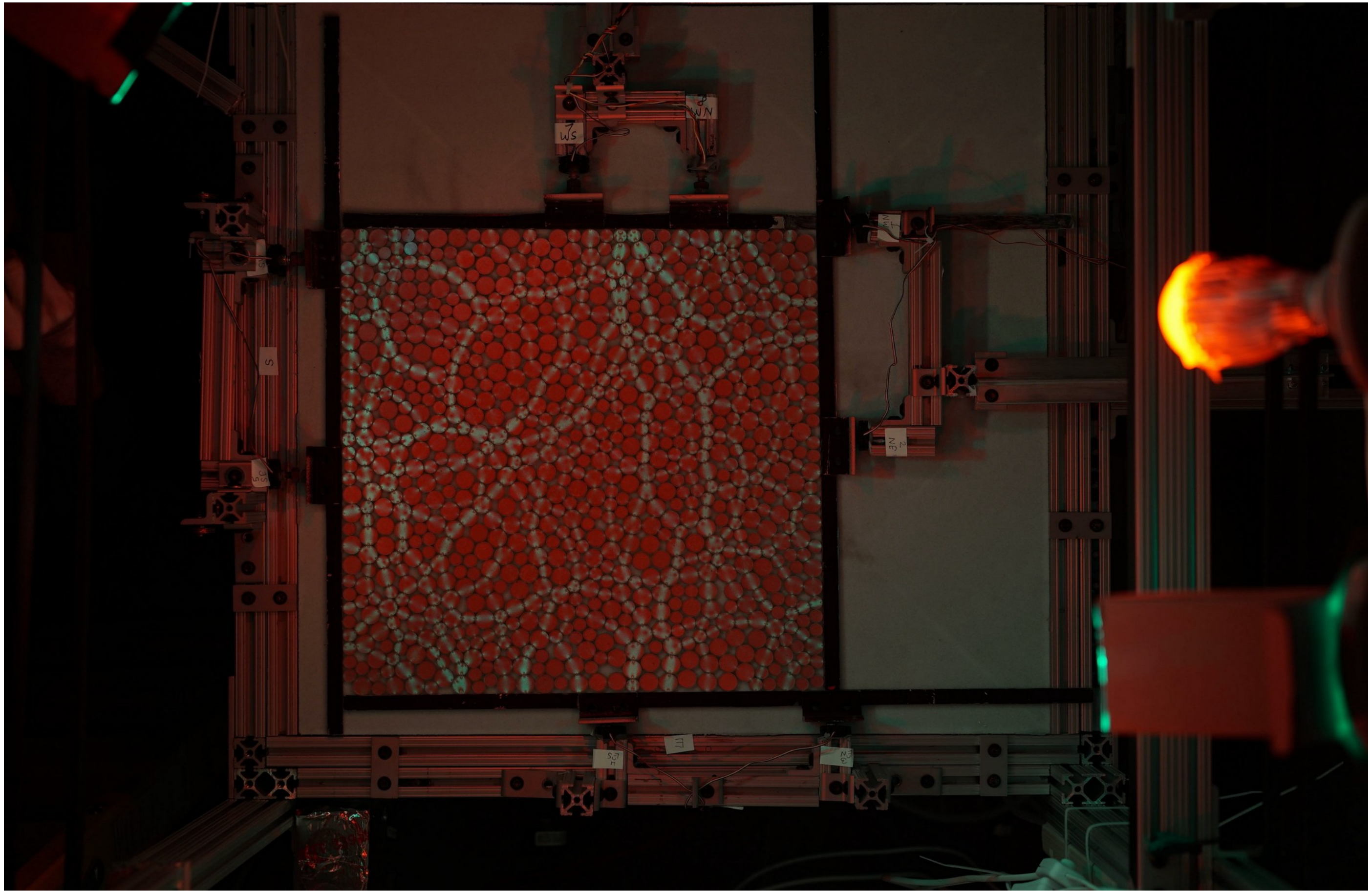
Changes in behavior with <z>

Ductile-like:
- Diffuse
- Progressive
- High deformation

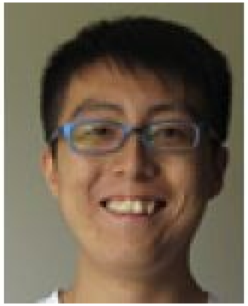


Brittle-like:
- Localized
- Catastrophic
- Low deformation

Rigidity in granular experiments

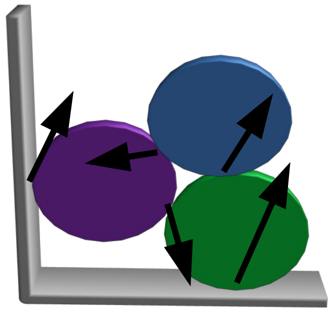


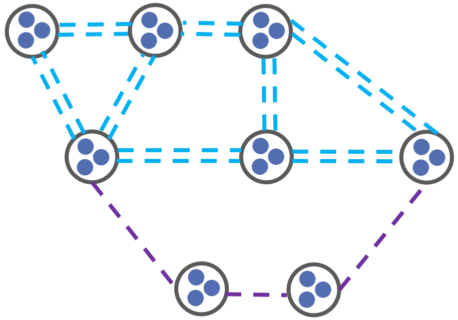
Jonthan Kollmer

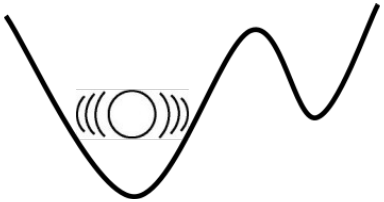


Kuang Liu

Constraint Counting ::: Vibrational Modes







$$\delta \ddot{r}^{i}_{\alpha\beta} = -D^{ij}_{\alpha\beta}\,\delta r^{j}_{\beta} + \text{dissipation}(\delta \dot{r}) + O(\delta r^2),$$

$$D^{ij}_{\alpha,\beta} = \frac{1}{\sqrt{m_{i,\alpha} m_{j,\beta}}} \frac{\partial^2 V_{ij}}{\partial r_{i,\alpha} \partial r_{j,\beta}}.$$

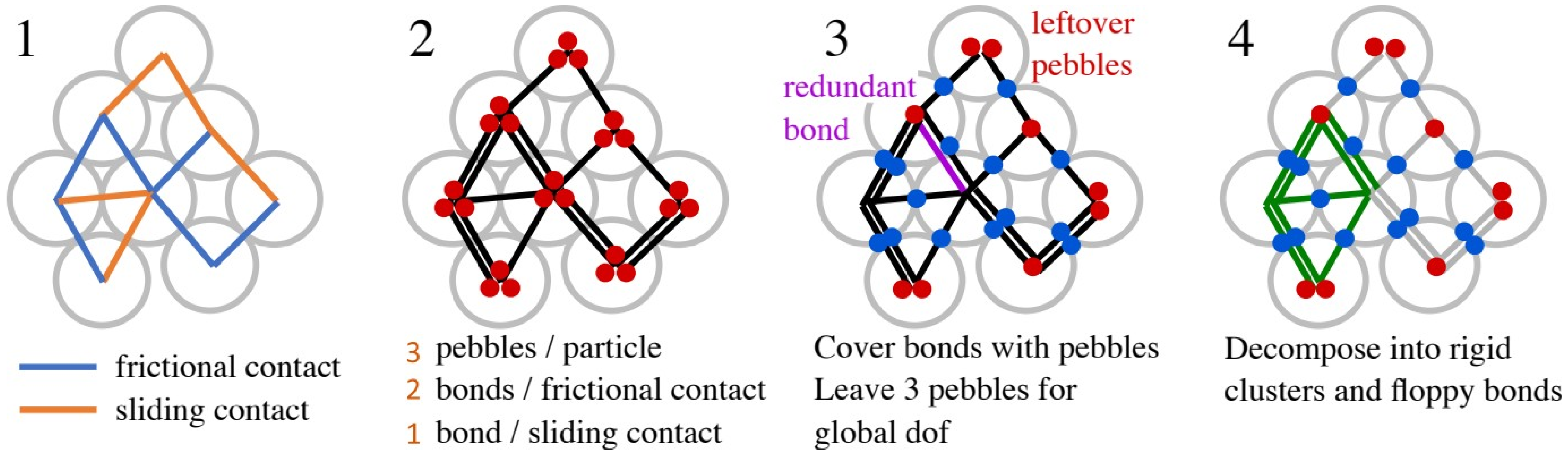
less physics ⟷ more physics

- torque and force balance
- degrees of freedom
- look for clusters where constraints are satisfied

- consider (frictional, dissipative) particles as being in energy wells
- look for regions of low-frequency modes

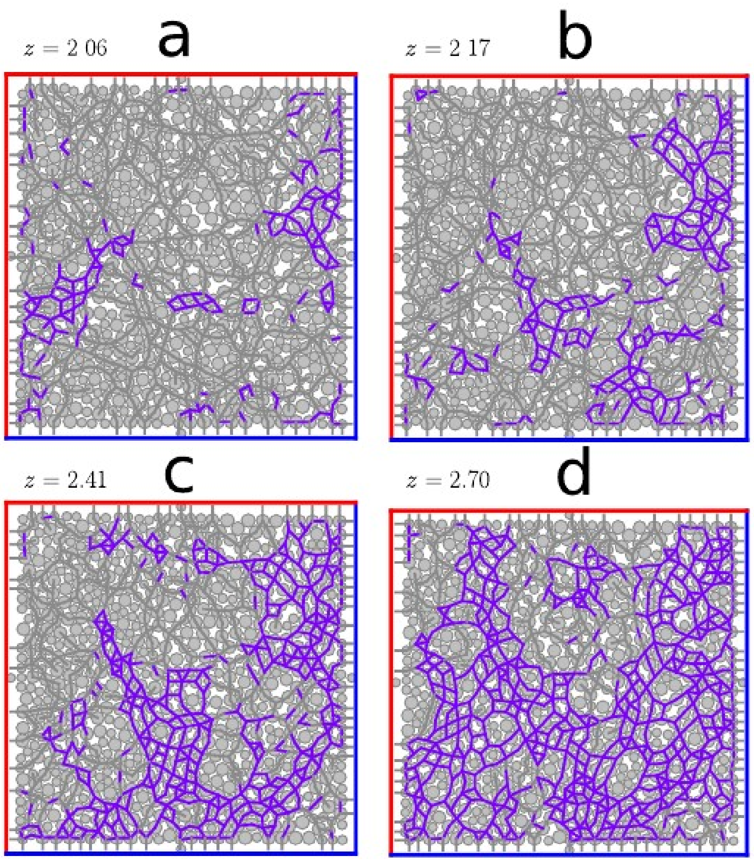
Liu, Kollmer, Daniels, Schwarz, Henkes *PRL* (2021)

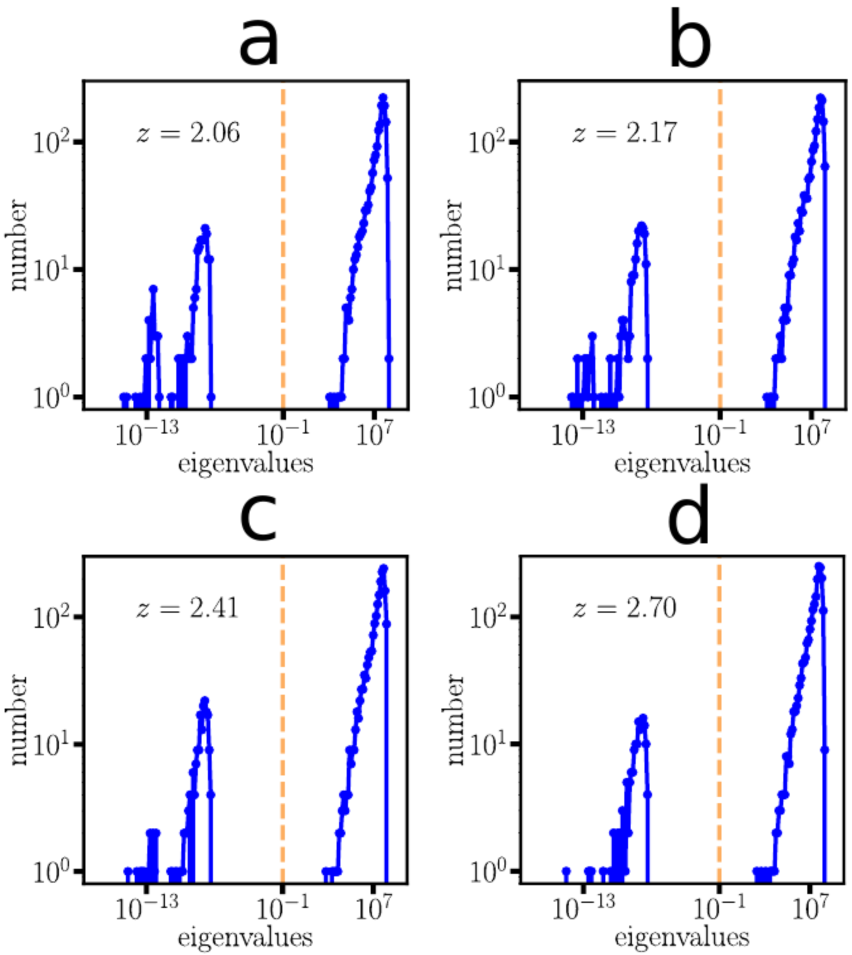
Pebble game reveals rigid clusters



Jacobs & Thorpe. *PRL* (1995)
Henkes, Quint, Fily, Schwarz. *PRL.* (2016)

Vibrational modes: set a threshold

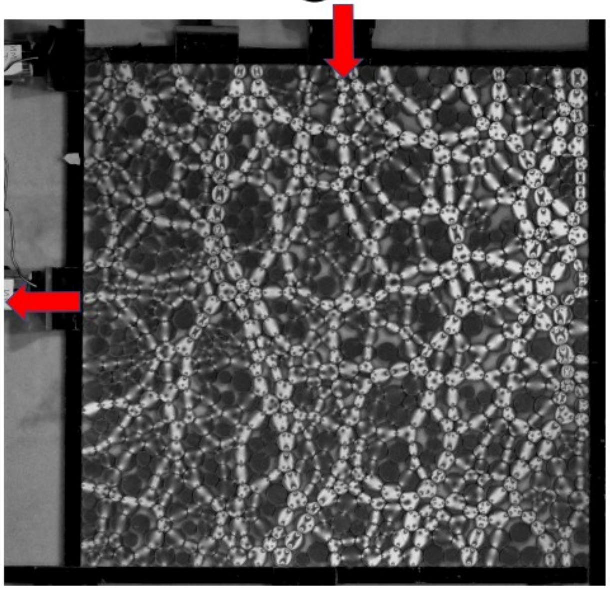




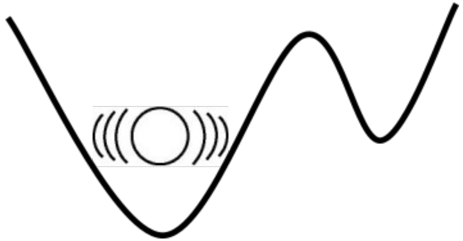
Liu, Kollmer, Daniels, Schwarz, Henkes *PRL* (2021)

Force Chains

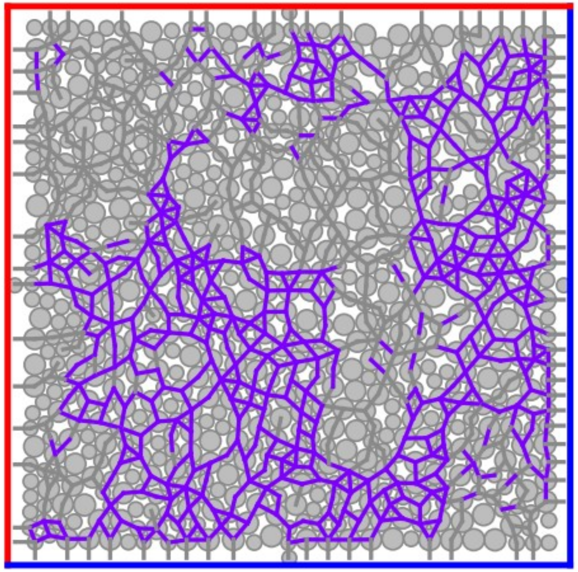
b



Vibrational

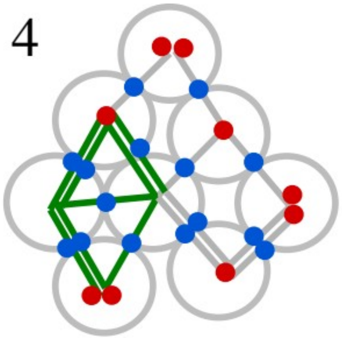


c

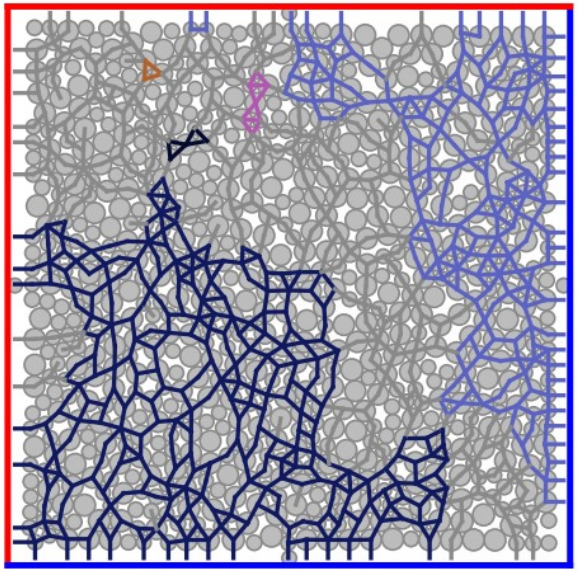


Constraints

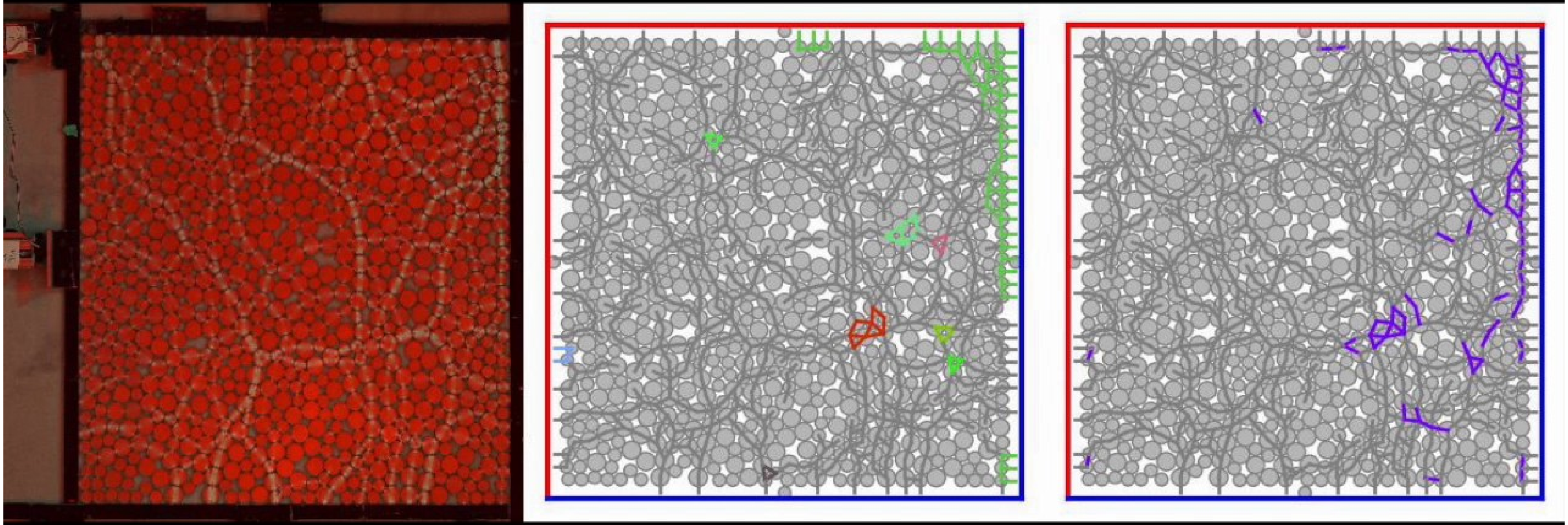
4



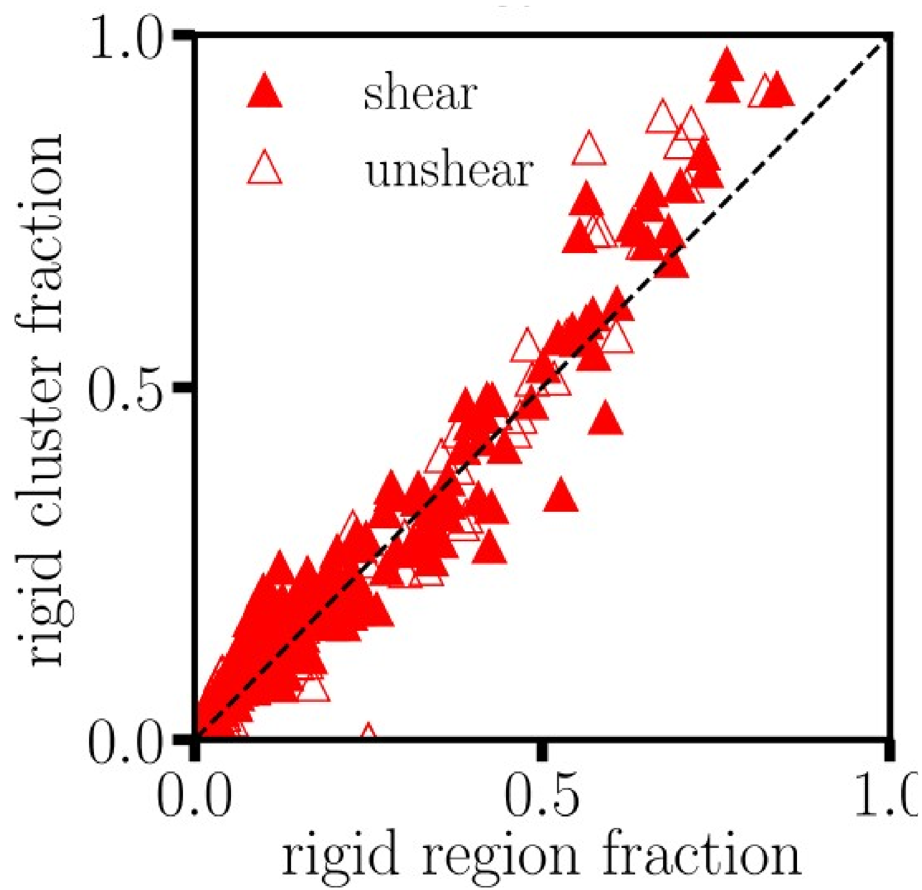
d



Force Chains Constraints Vibrational

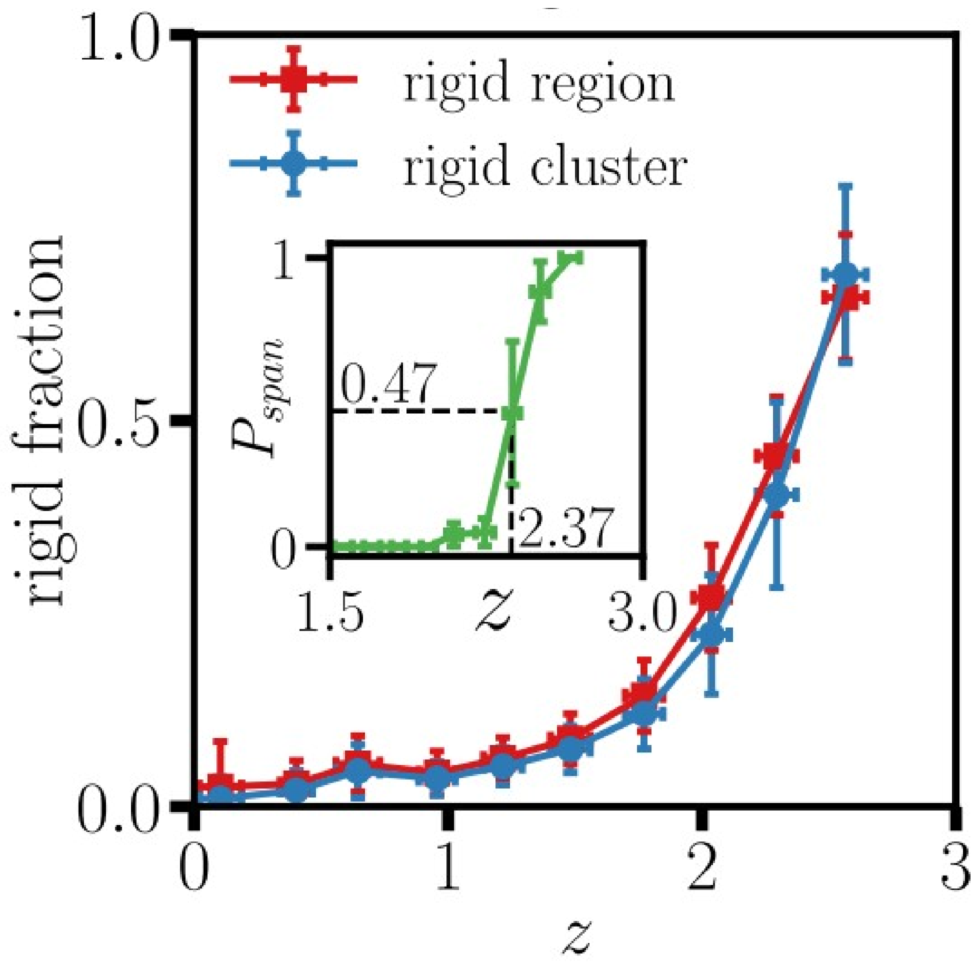


2 frameworks tell the same story



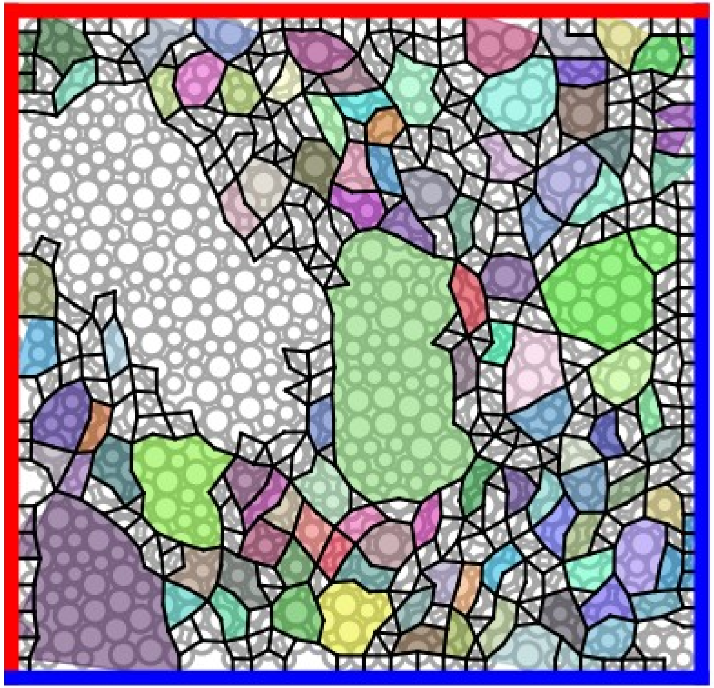
Liu, Kollmer, Daniels, Schwarz, Henkes *PRL* (2021)

Threshold for global rigidity: $z_c \sim 2.4$

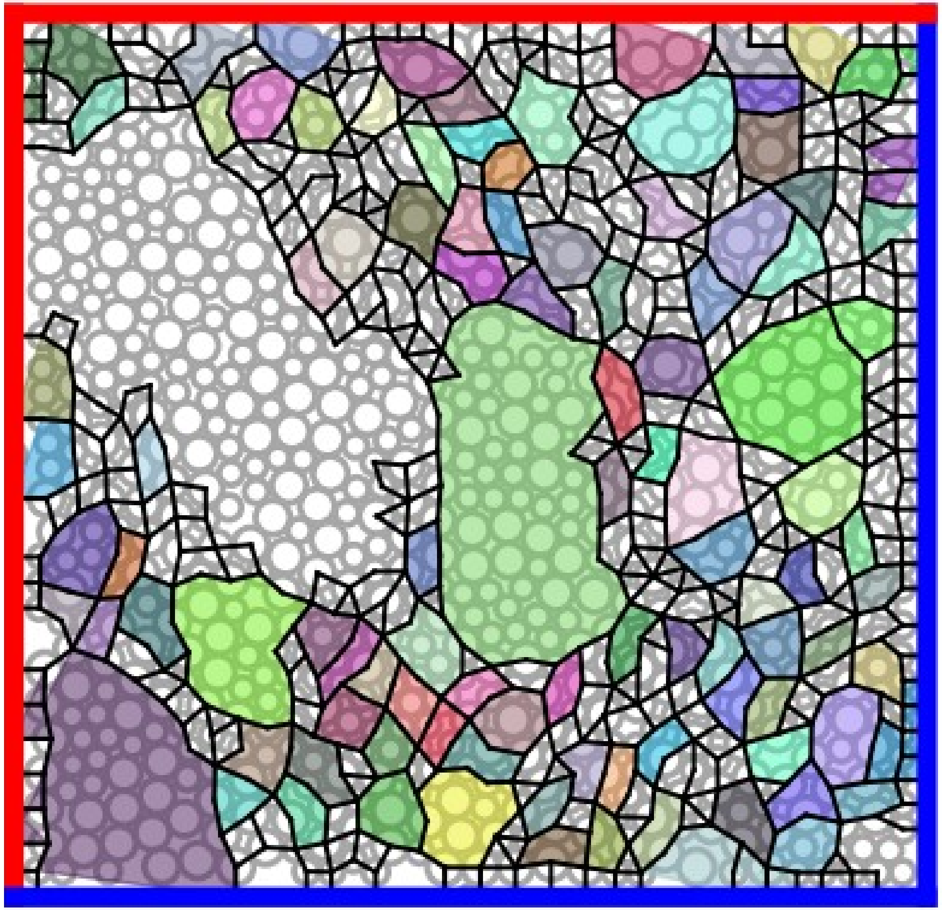


- 50% probability of finding a system-spanning cluster
- no tossing out of rattlers

Characterizing floppy regions



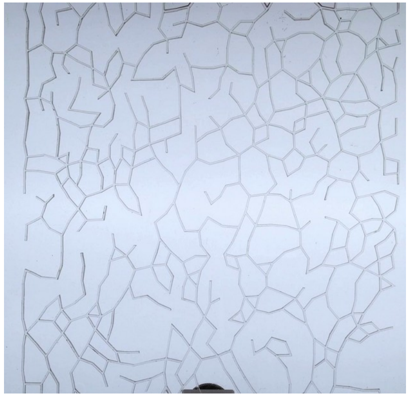
Liu, Kollmer, Daniels, Schwarz, Henkes *PRL* (2021)

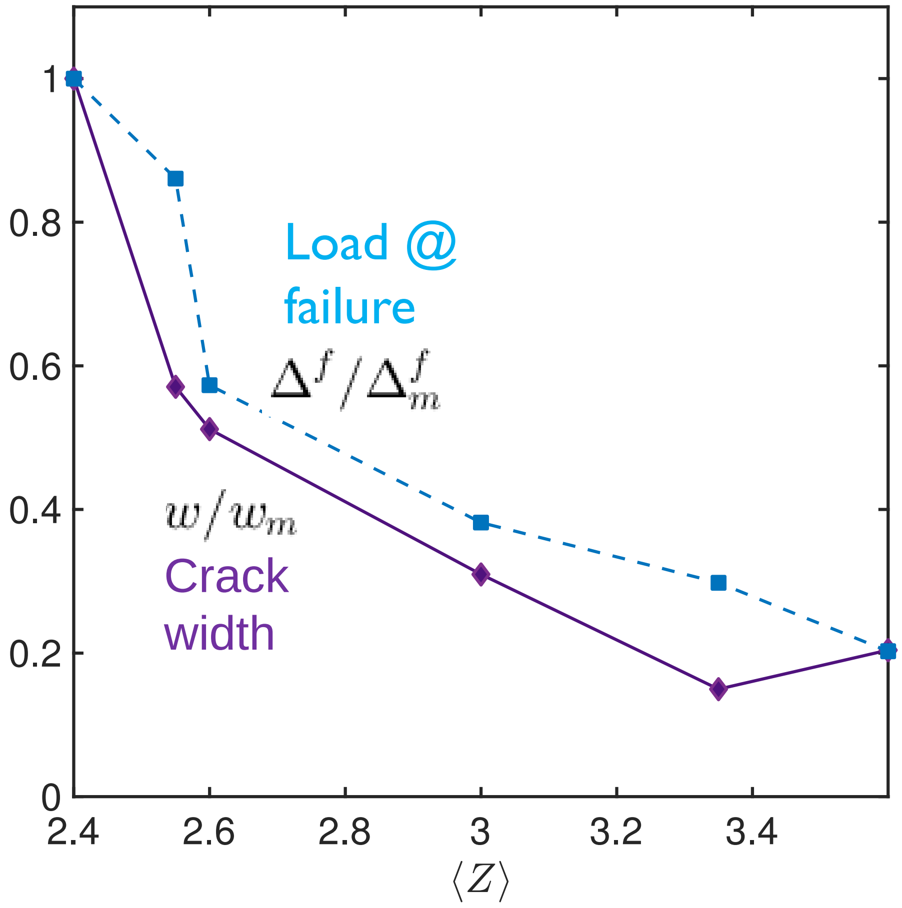


Do floppy regions forecast failure locations?

Recall: ductile vs. brittle behavior

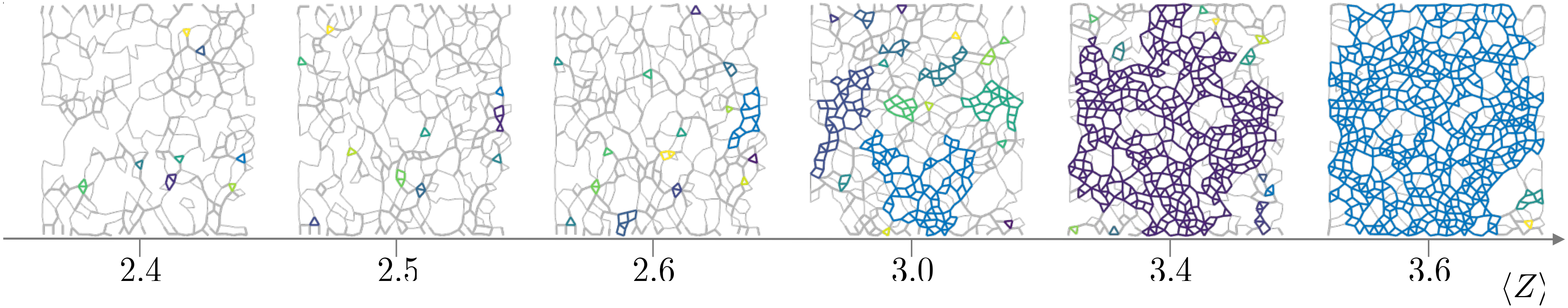
Ductile-like:
- Diffuse
- Progressive
- High deformation

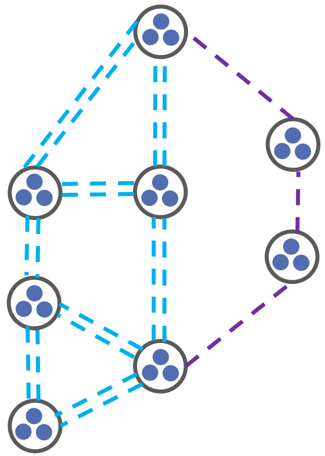




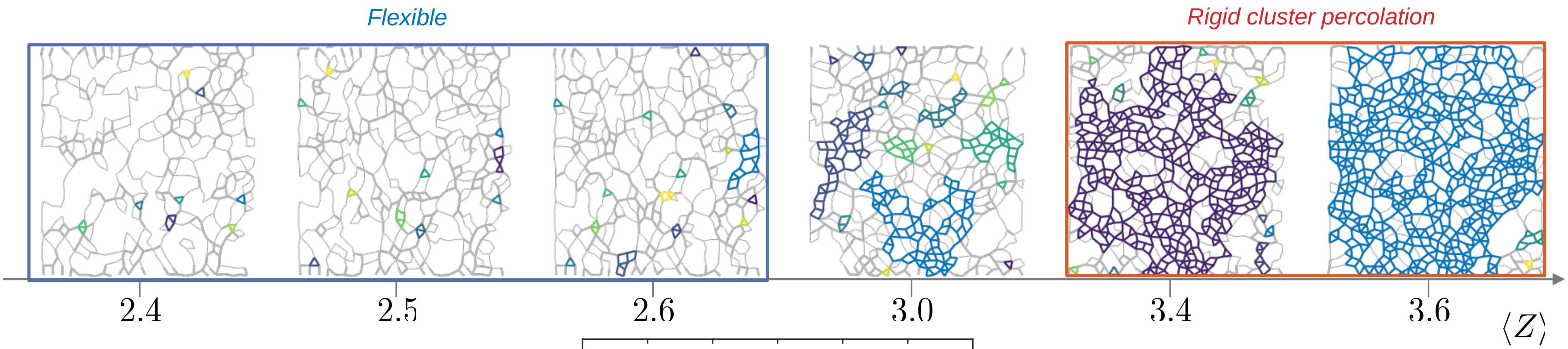
Brittle-like:
- Localized
- Catastrophic
- Low deformation

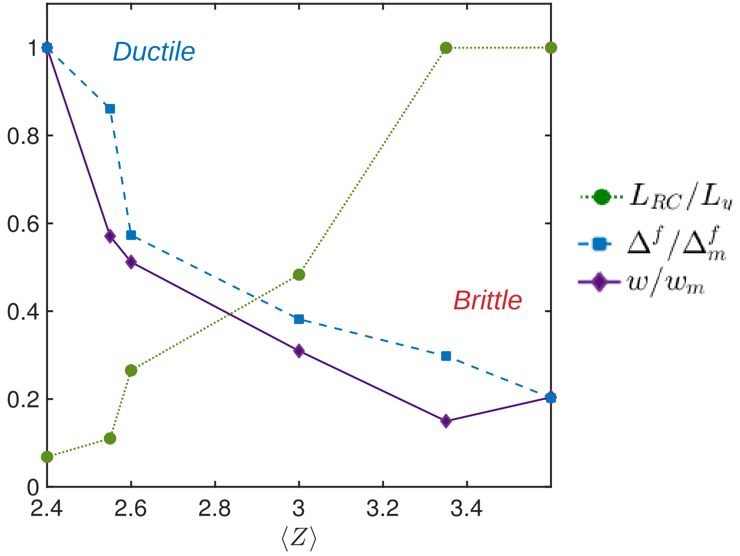
Identify rigid clusters via pebble game



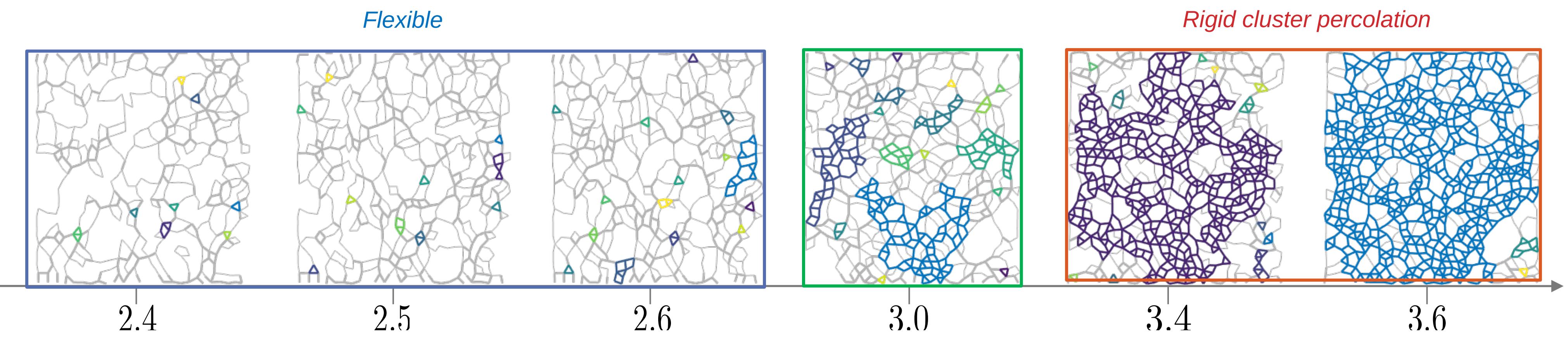


Compare to failure dynamics

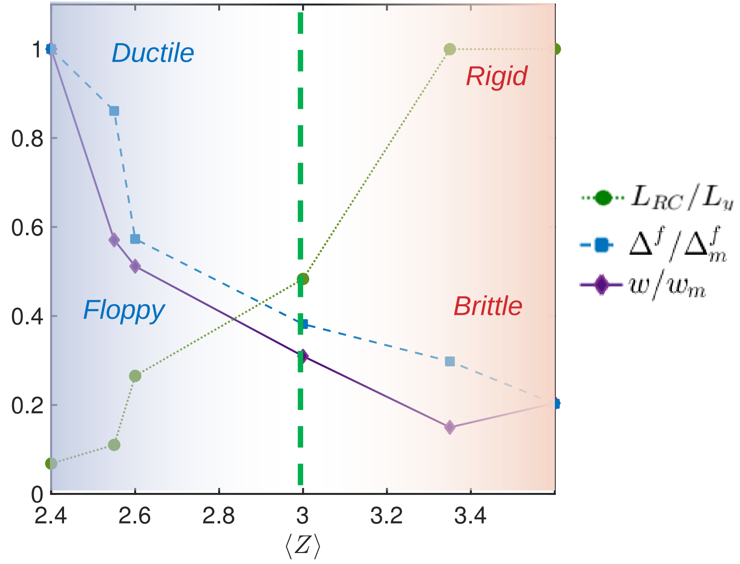




Lattice failure transition point



Rigidity percolation associated to the failure transition



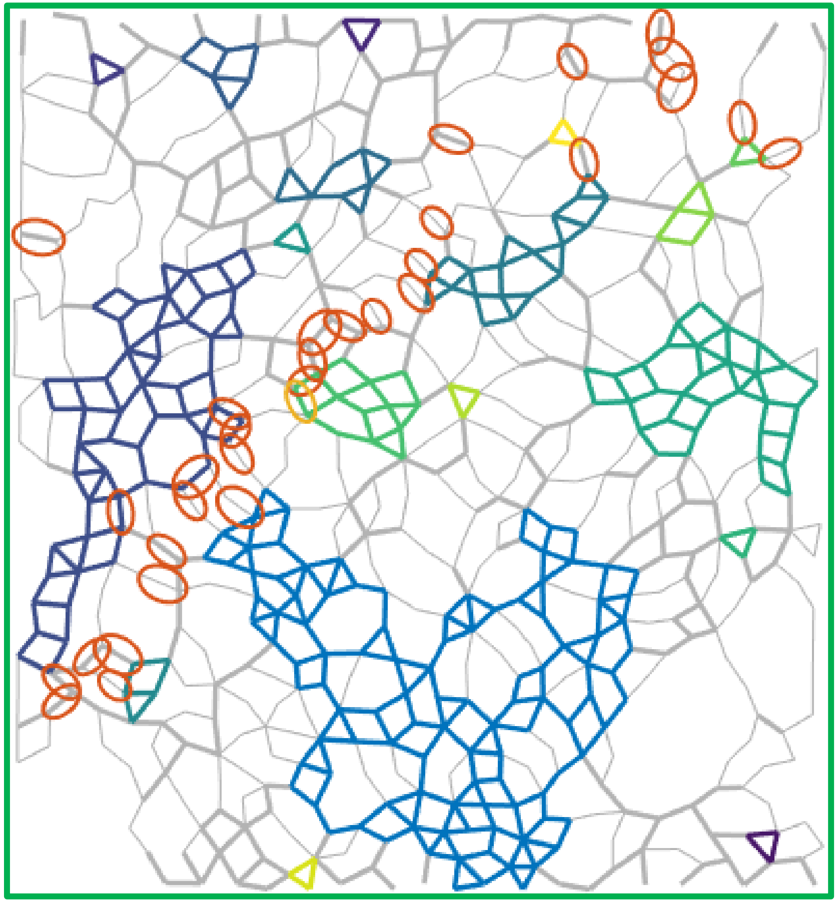
Transition point of lattice failure close to

$$Z_{\text{iso}}^{\mu=\infty}$$

Berthier, Kollmer, Henkes, Liu, Schwarz, Daniels. *Phys. Rev. Mat.* (2019)

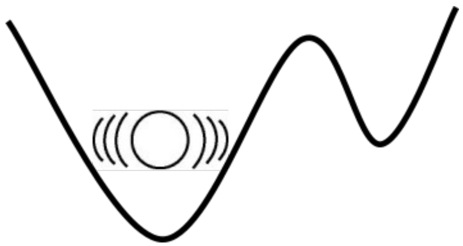
Most failures occur on flexible bonds

<z> = 3.0

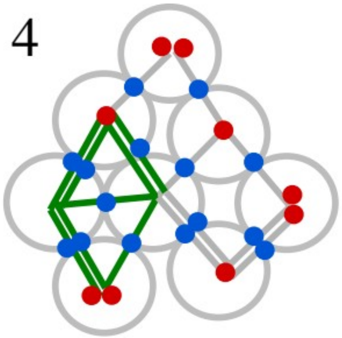


Force Chains

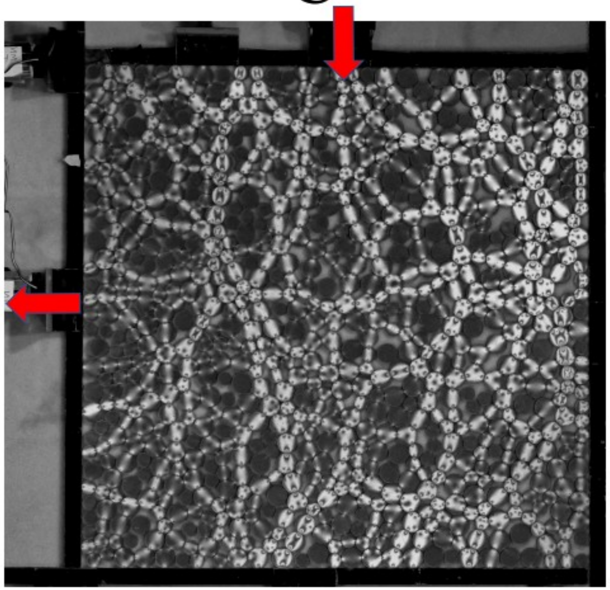
Vibrational



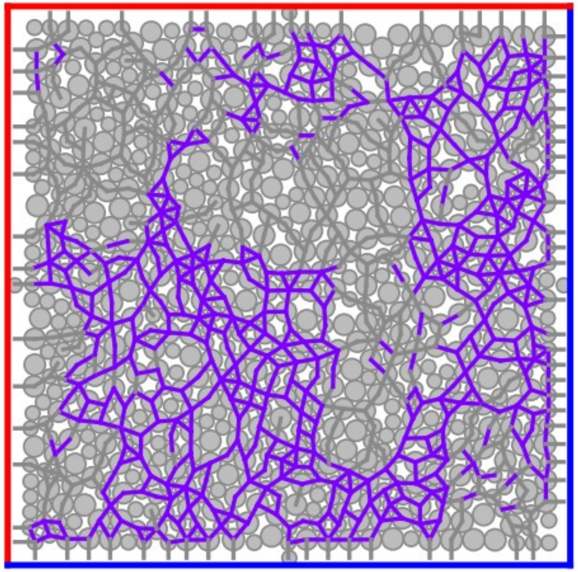
Constraints



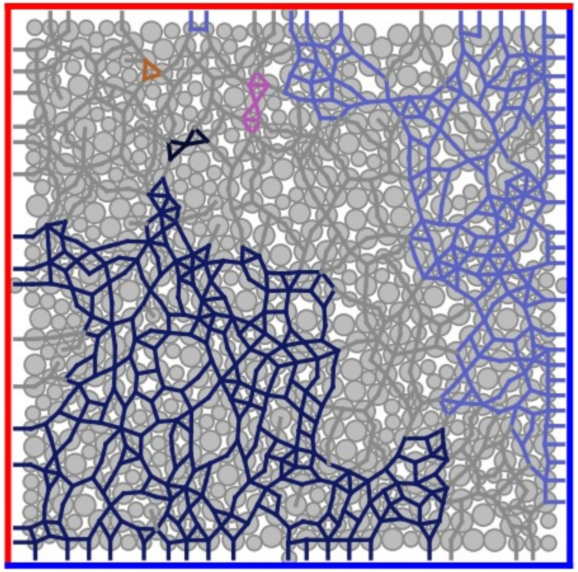
b



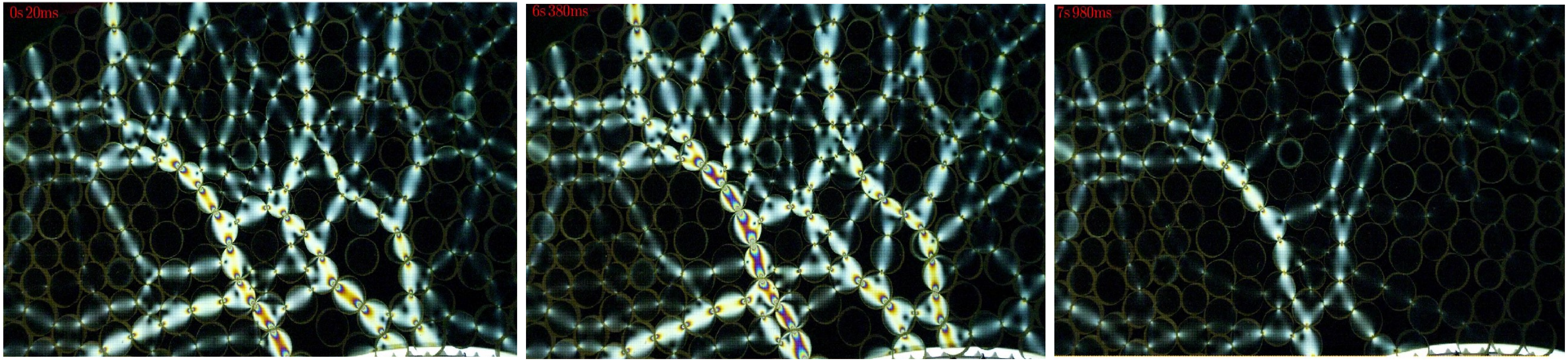
c



d

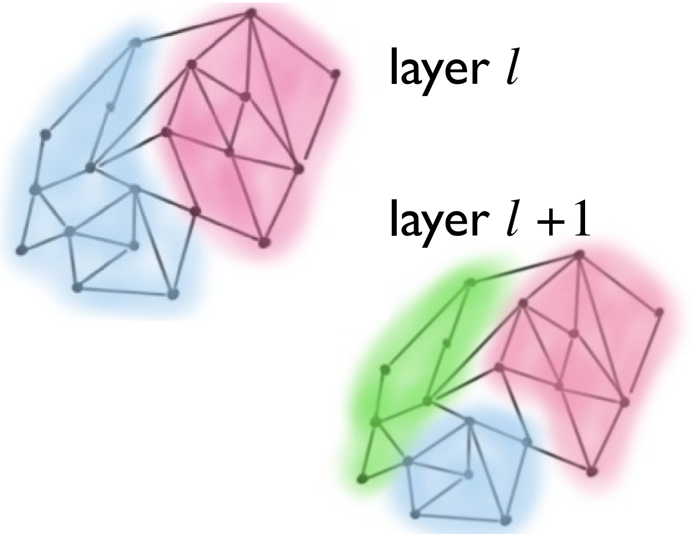


Forecasting loss of rigidity



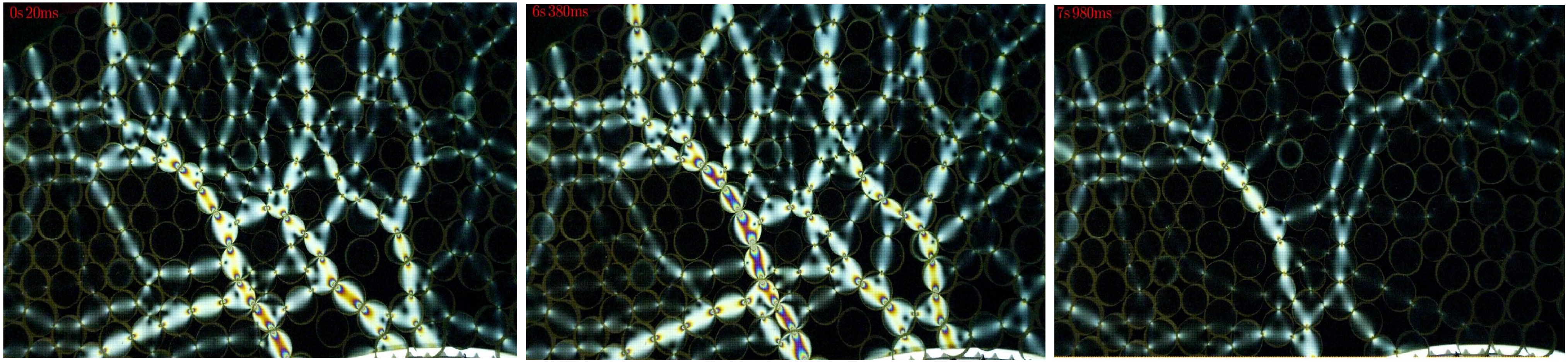
- Multilayer community detection
- GenLouvain modularity maximization

$$Q = \frac{1}{2\mu} \sum_{ij\ell m} [(A_{ij\ell} - \gamma P_{ij\ell})\delta_{\ell m} + \omega_{j\ell m}\delta_{ij}]\delta(c_{i\ell}, c_{jm})$$



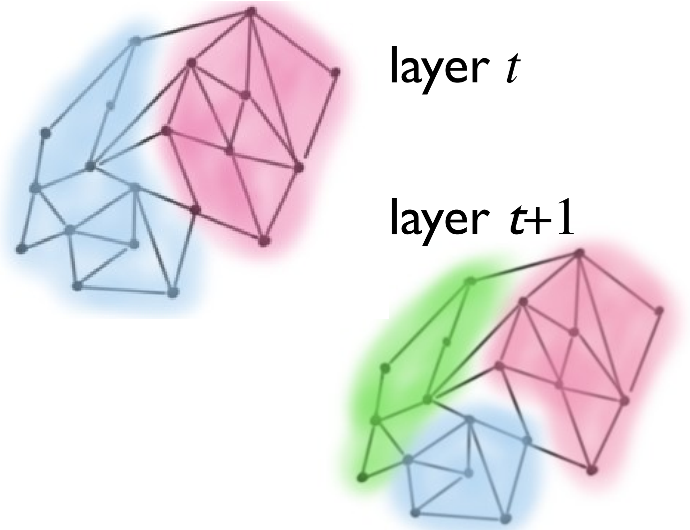
Mucha, Richardson, Porter, Onnela, *Science* (2010)
http://netwiki.amath.unc.edu

Forecasting loss of rigidity

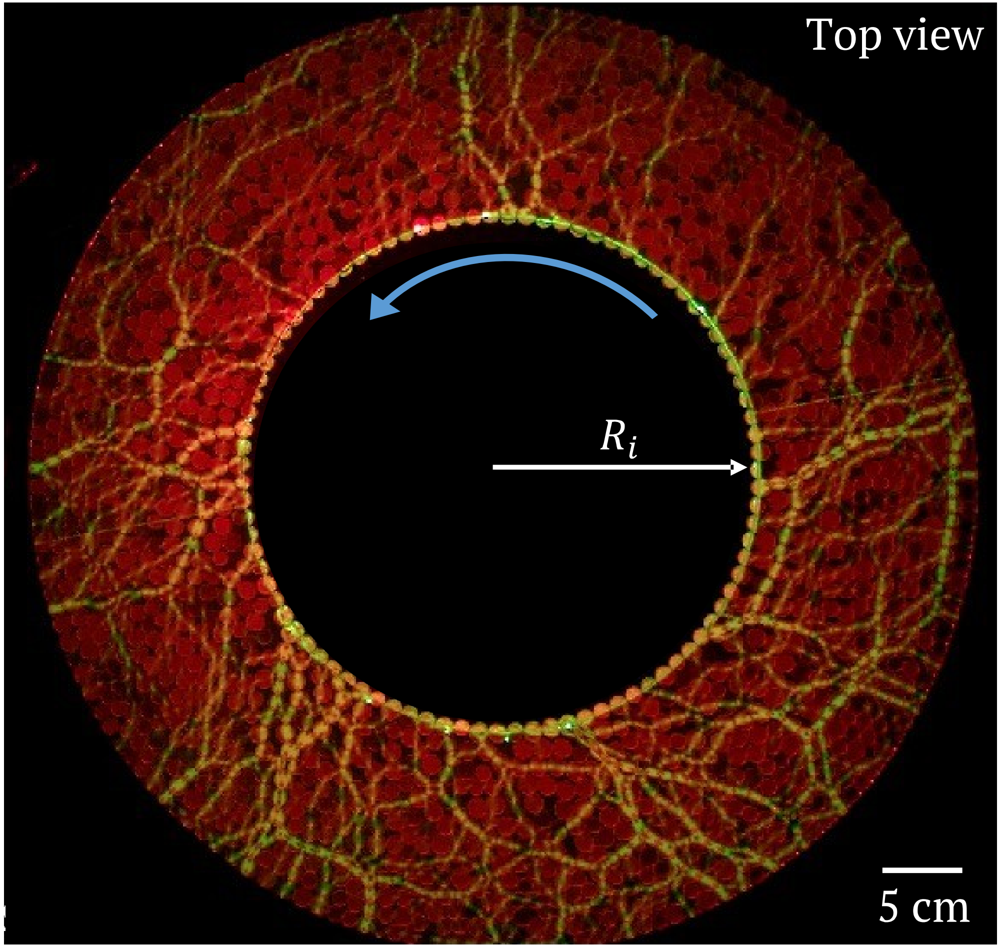


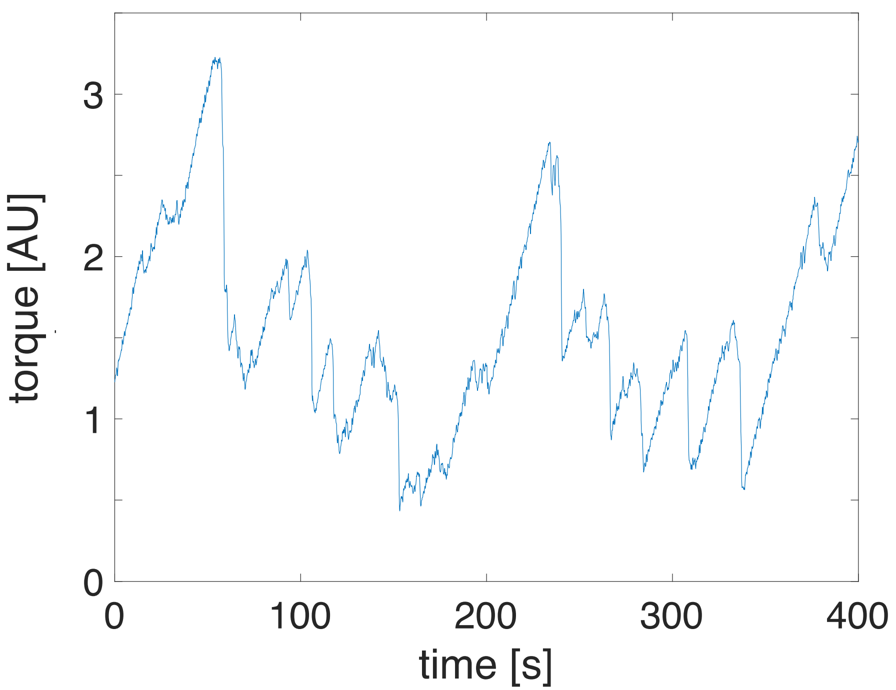
Farnaz Fazelpour

- Size: number of particles in community
- Strength: average interparticle for in community
- Volatility: how much communities change from layer to layer

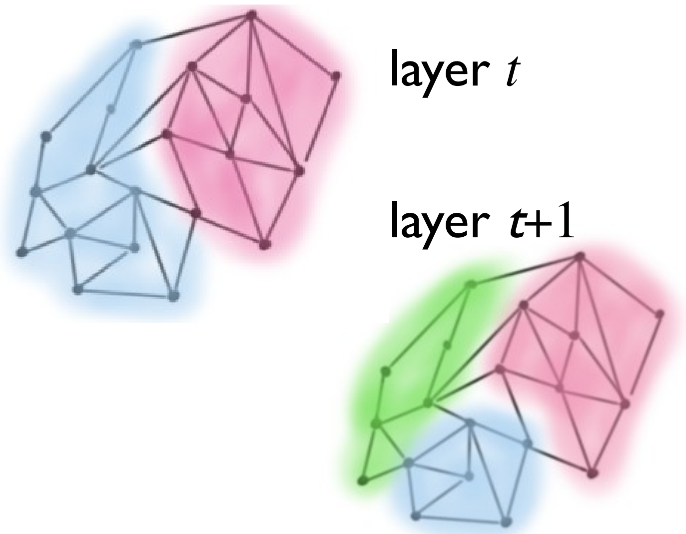


Examine a series of stick-slip failures

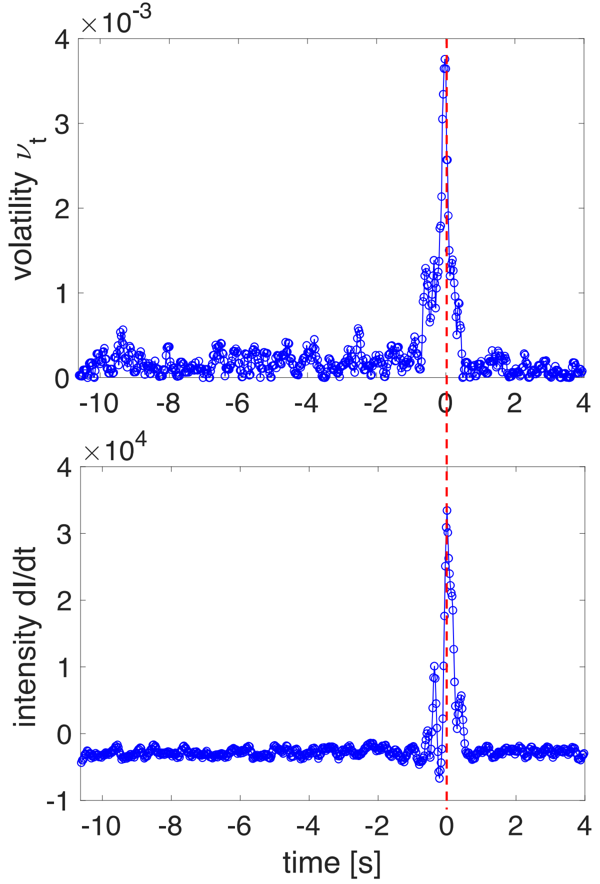




Volatility changes slightly before image intensity changes



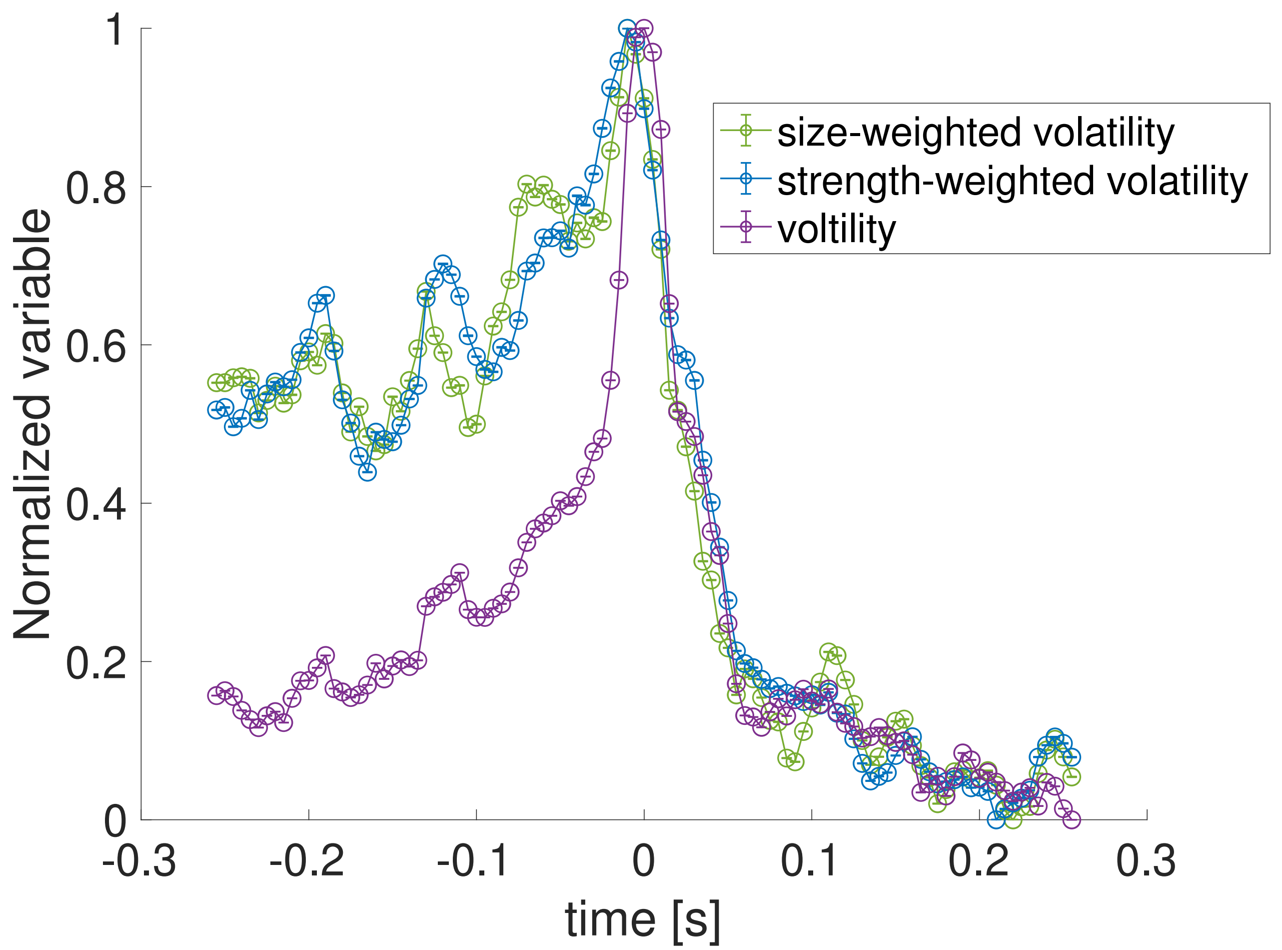
$$\upsilon_{tc} = \frac{|c_t \Delta c_{t+1}|}{|c_t \cup c_{t+1}|}$$

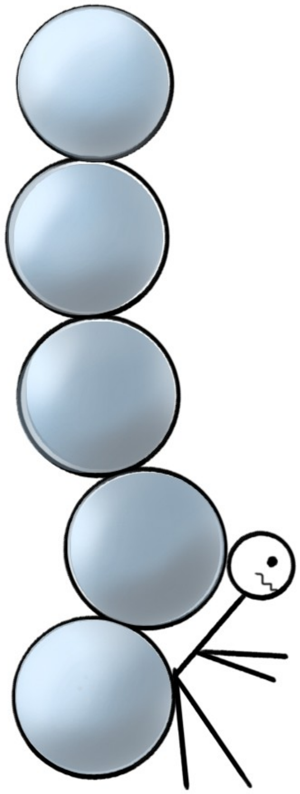


$|c_t \Delta c_{t+1}|$: distinct nodes

$|c_t \cup c_{t+1}|$: total nodes

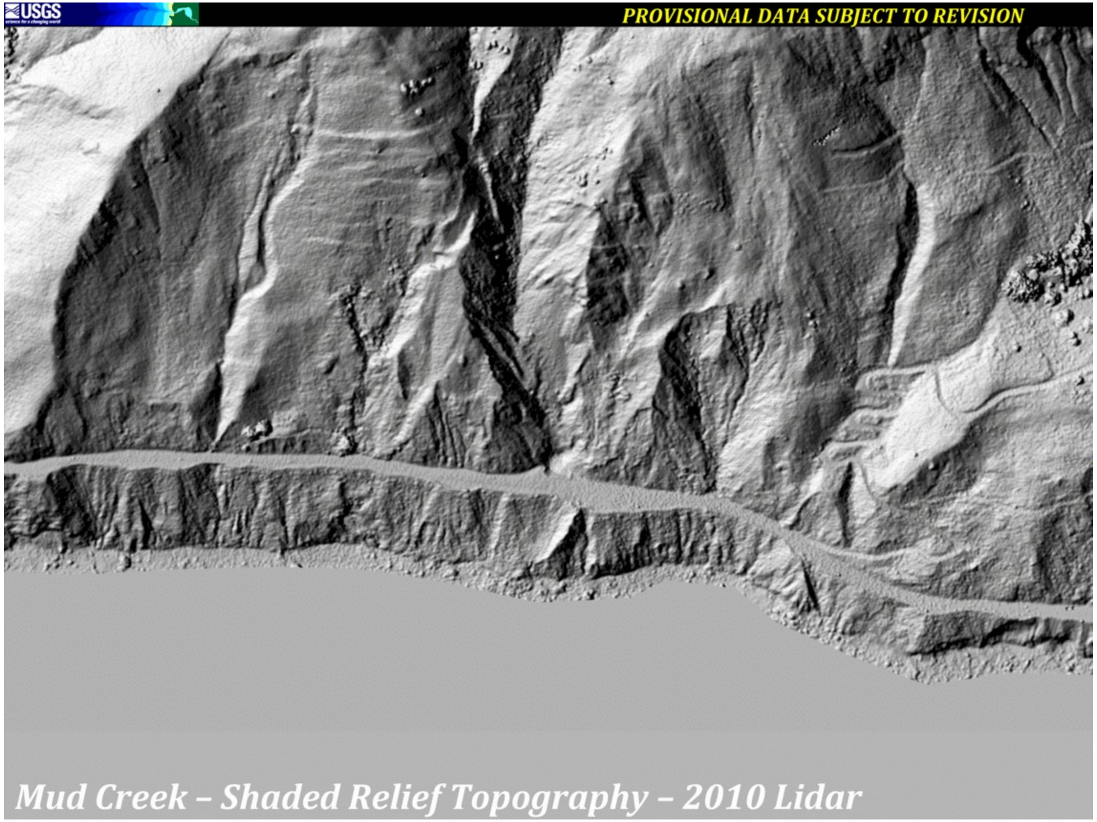
Weak chains matter





How about for real landslides?

Handwerger et al. *Scientific Reports* (2019)

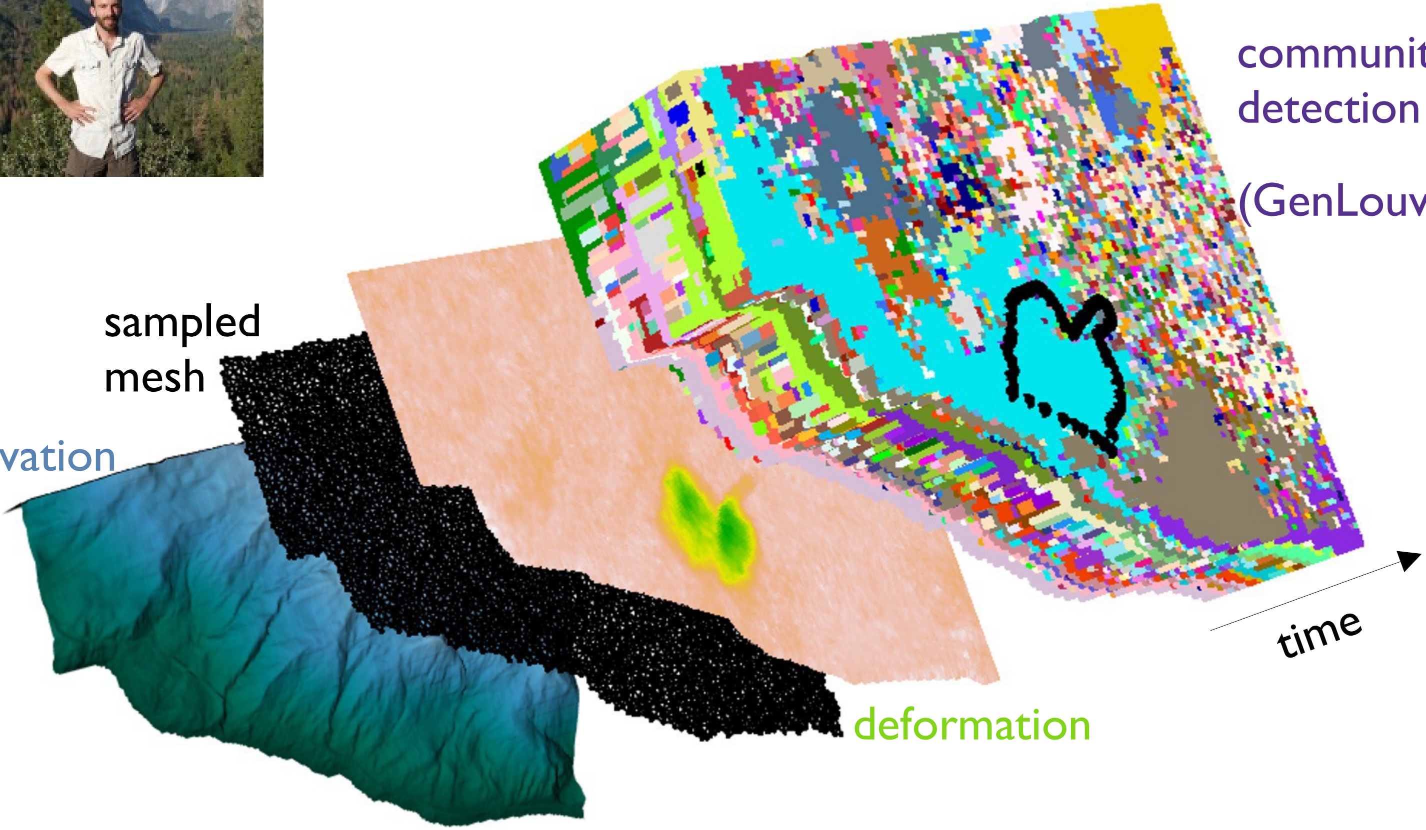


https://www.usgs.gov/media/images/mud-creek-shaded-relief-topography-2010-2017

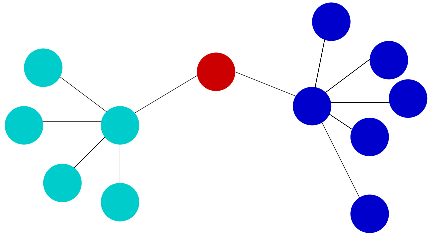
Forecasting loss of rigidity

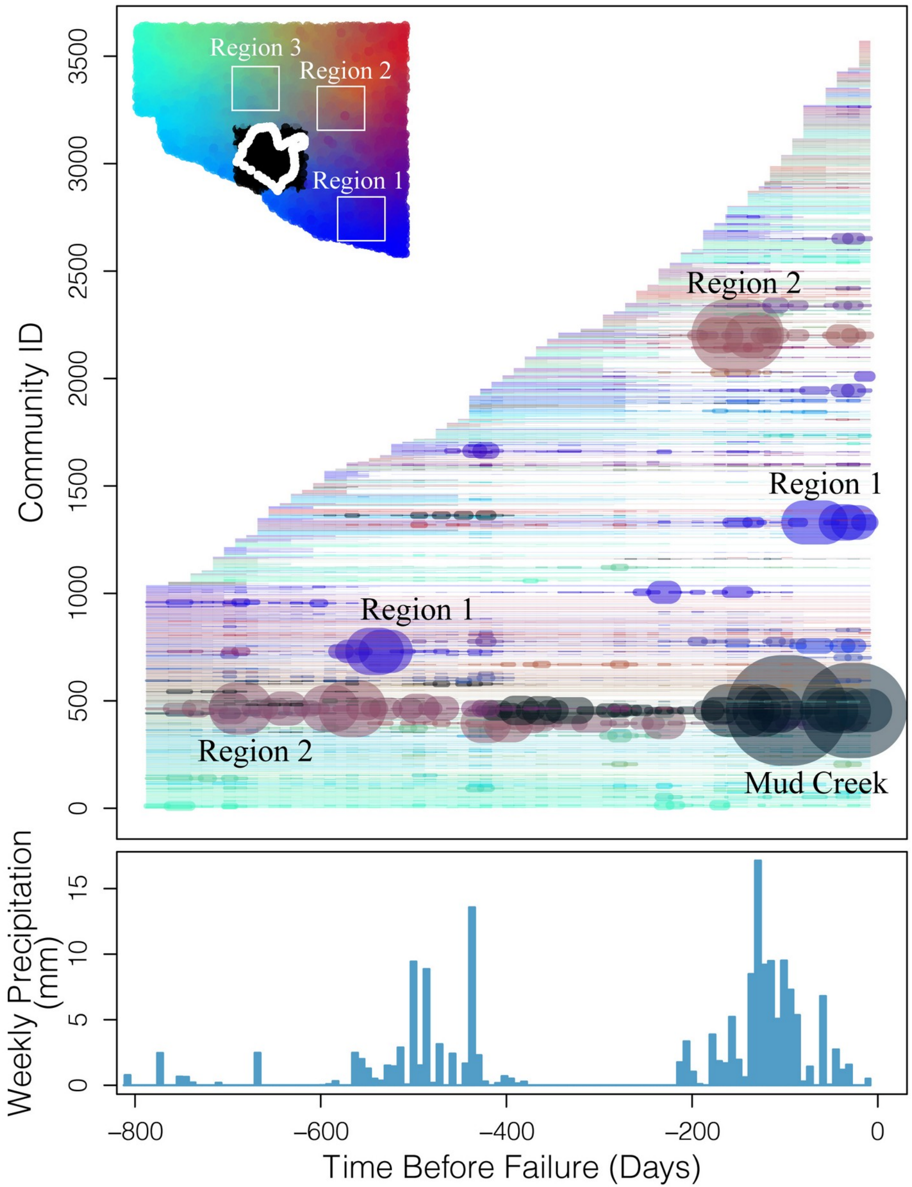
Vrinda Desai

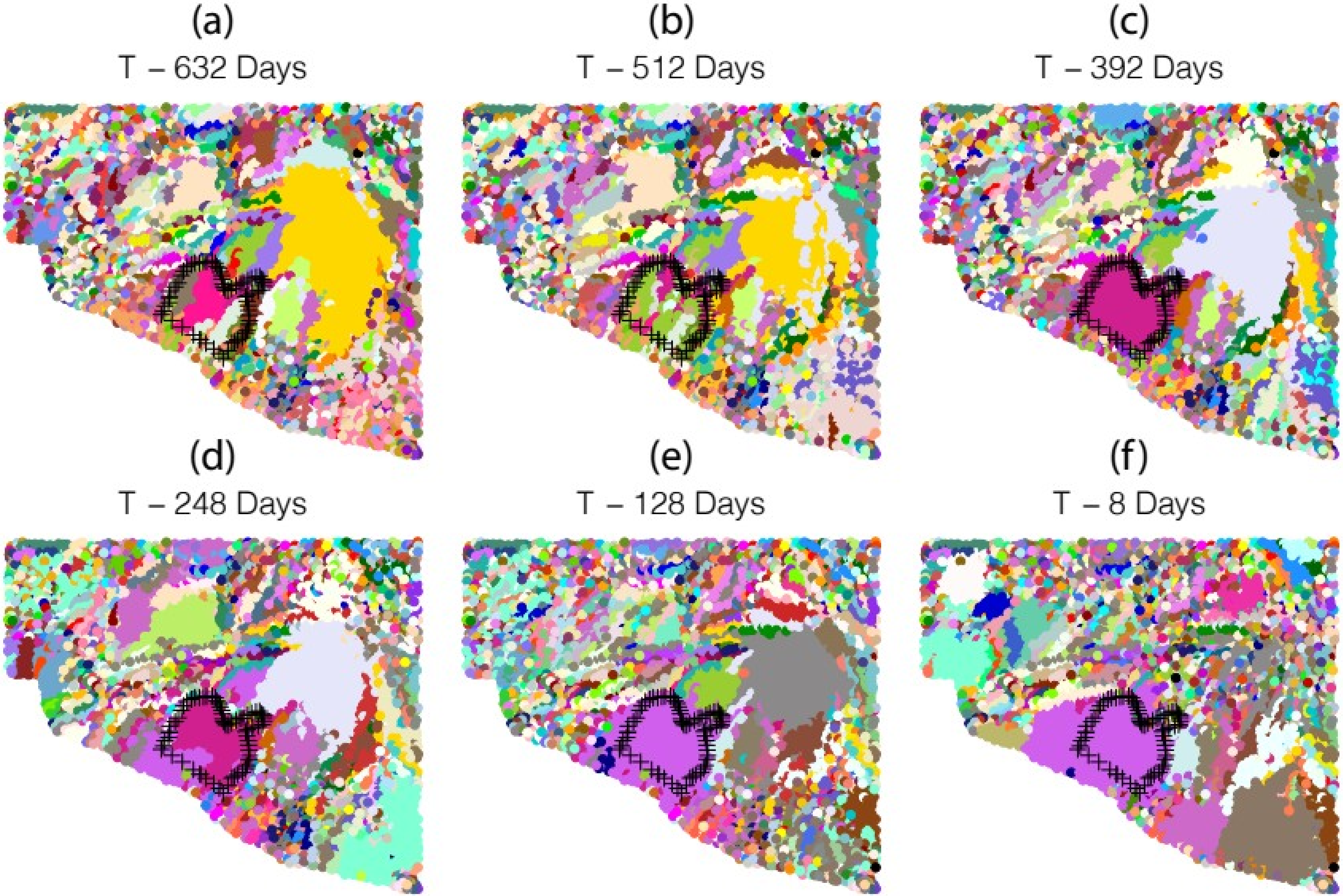
Al Handwerger (JPL)



Which locations have reliable community detection?







(a)
T – 632 Days
(b)
T – 512 Days
(c)
T – 392 Days
(d)
T – 248 Days
(e)
T – 128 Days
(f)
T – 8 Days

Community Persistence Π

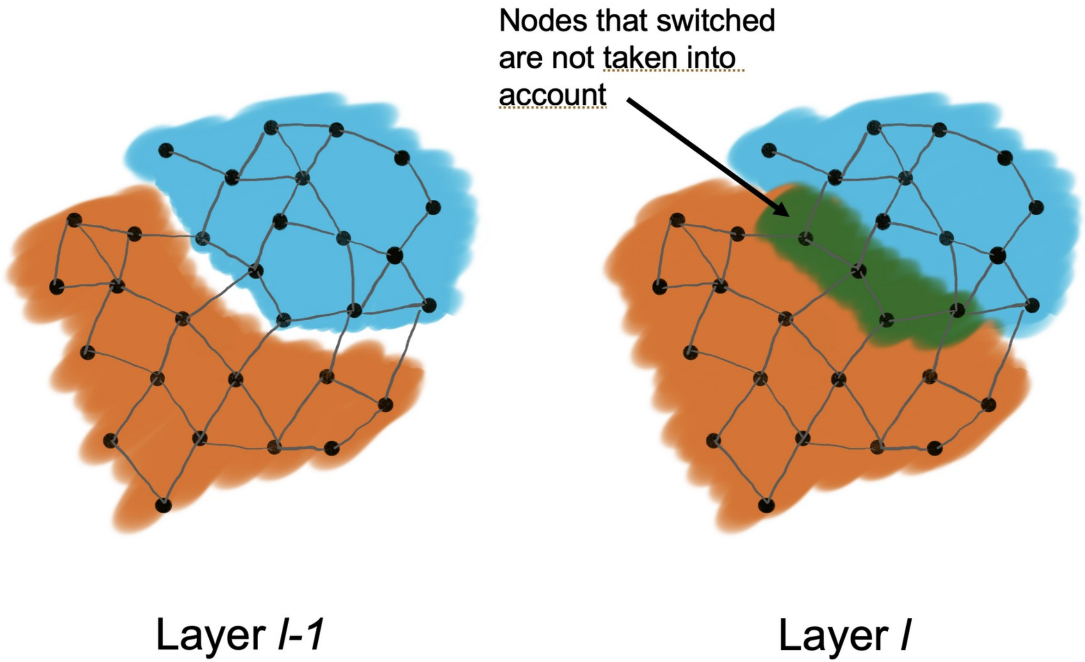
A measure based on the stability of nodal composition for each community in relation to community size for each layer *l*

$$\Pi = \frac{1}{N} \sum_{c} \frac{|c_{l-1} \cap c_l|}{n_{c,l}}$$

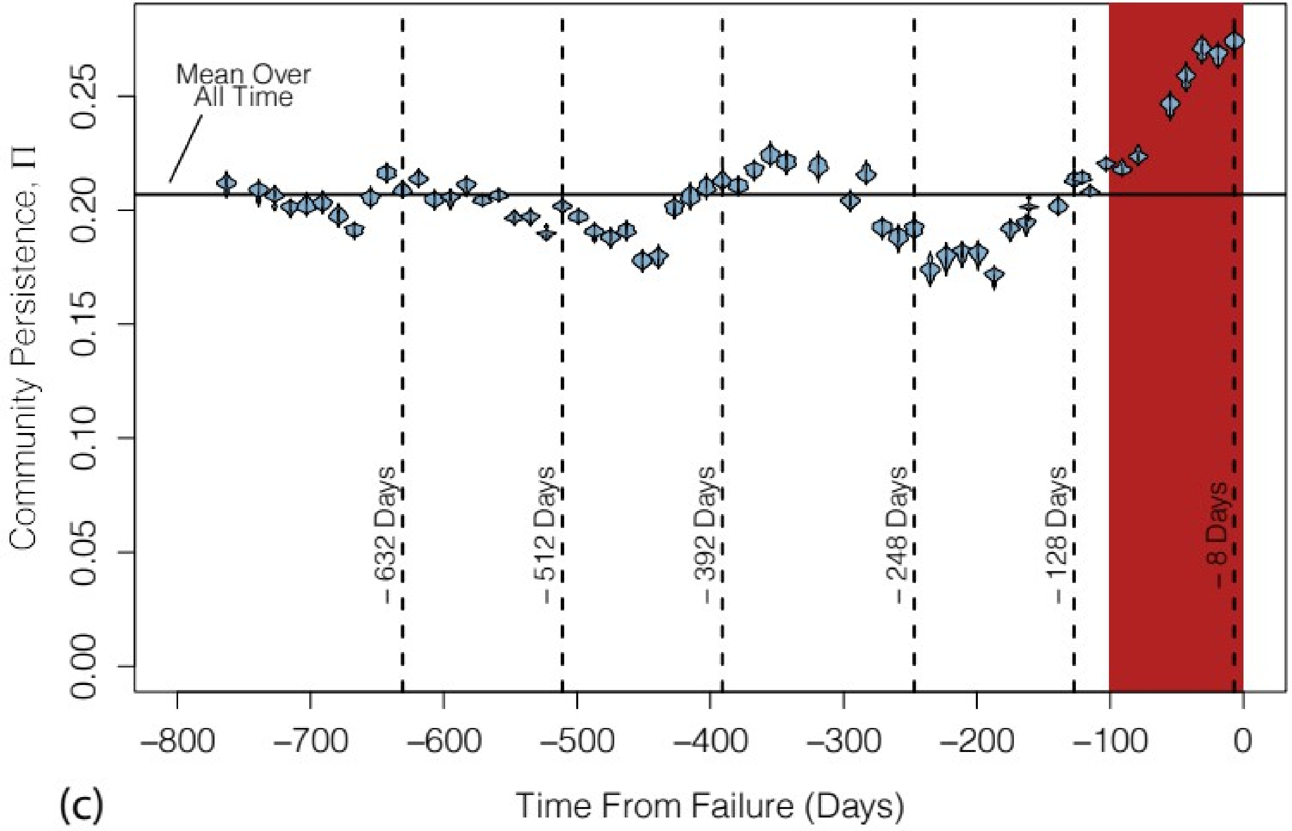
N : Total number of nodes

$n_{c,l}$: Number of nodes in community *c* at layer *l*

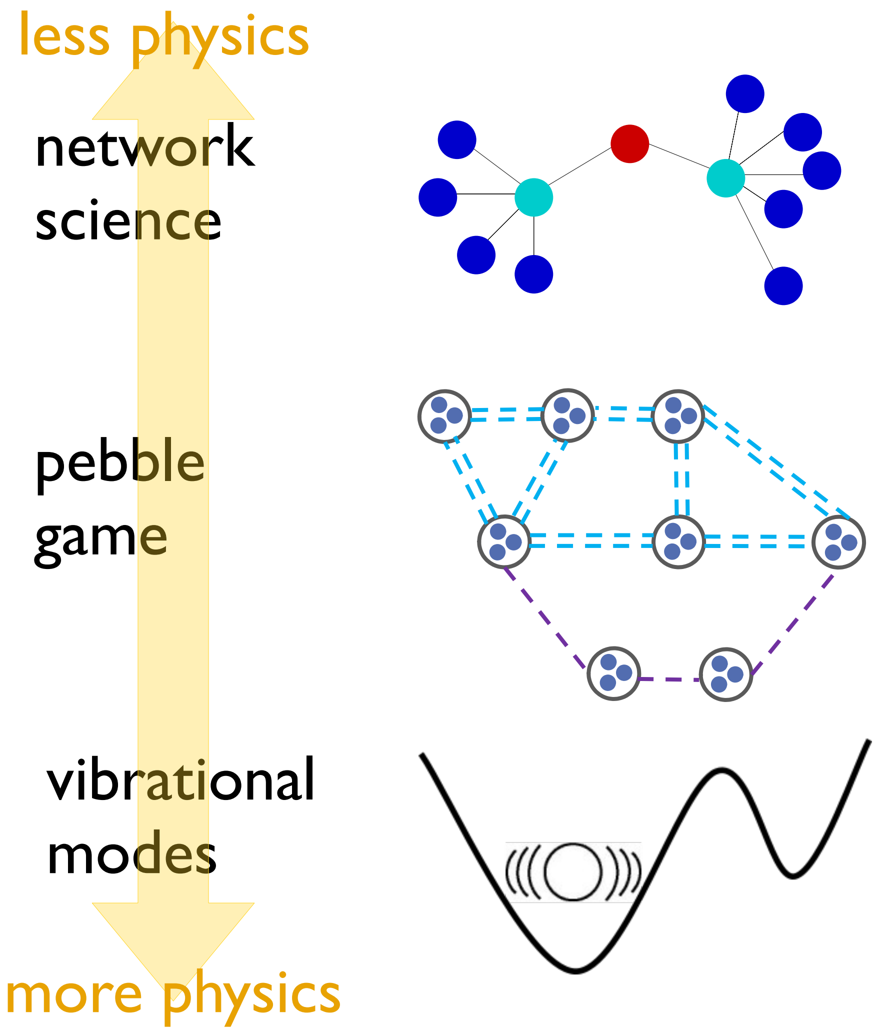
$|c_{l-1} \cap c_l|$: number of nodes present in community *c* in both layers *l* and *l-1*



Increased community persistence forecasts failure



General Conclusions

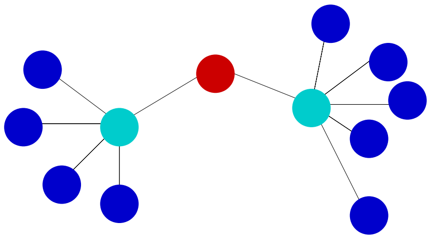


- more physics gives you better predictions and a better understanding …
- but simple models are surprisingly effective
- sometimes topology is a strong control

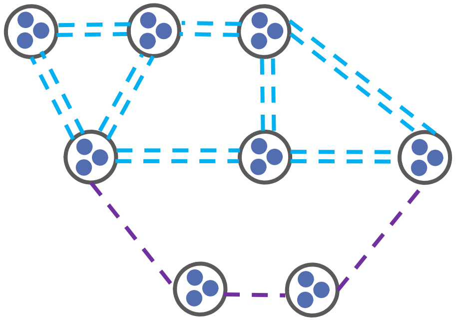
Conclusions

less physics

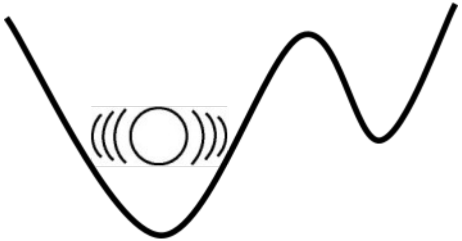
network science



pebble game



vibrational modes



more physics

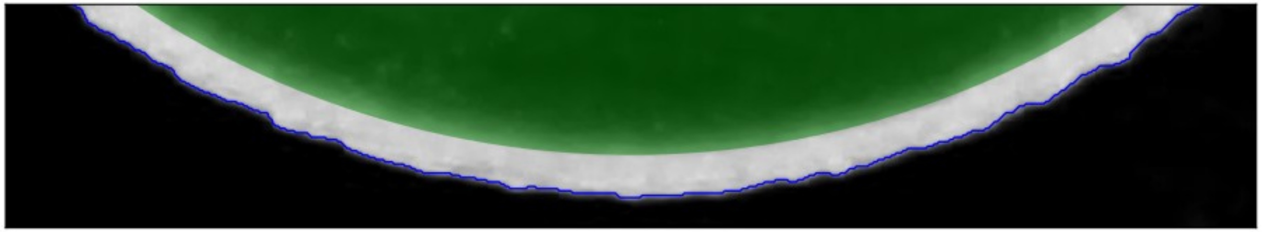
- centrality identifies likely force chain locations, lattice failure locations
- communities-detected change character ahead of failure
- granular packings: pebble game identifies same rigid areas as vibrational modes
- floppy areas may be more prone to failures (at least for some lattices)

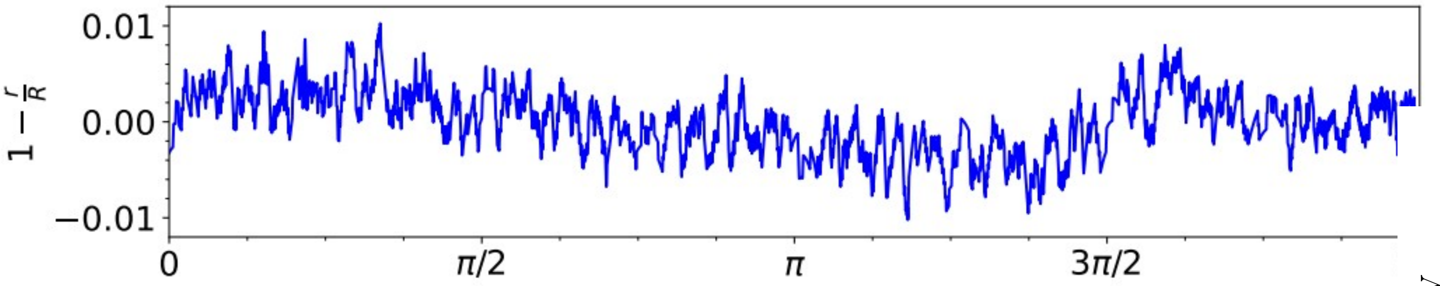
Open Science Tools

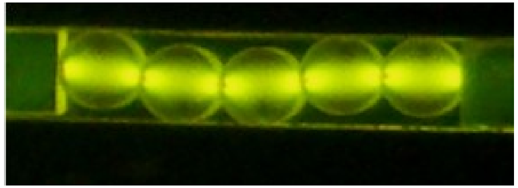
- Data from our papers: http://datadryad.org
- Photoelastic Granular Solver: Jonathan Kollmer
  github.com/jekollmer/PEGS
- Rigidity Toolbox: Silke Henkes
  https://github.com/silkehenkes/RigidLibrary
- NetWiki: Mason Porter, Peter Mucha
  http://netwiki.amath.unc.edu/
- Brain Connectivity Toolbox: Mikail Rubinov, Olaf Sporns
  http://www.brain-connectivity-toolbox.net/

Real particles

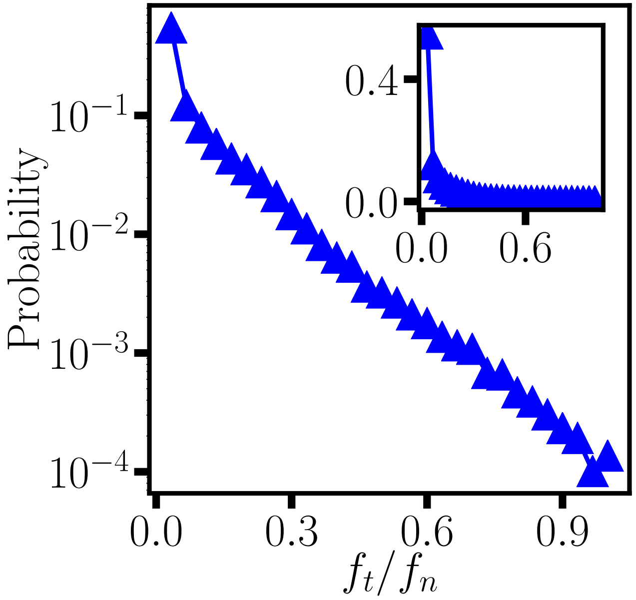
Kool, Charbonneau, Daniels, arXiv
Liu, Kollmer, Daniels, Schwarz, Henkes *PRL* (2021)



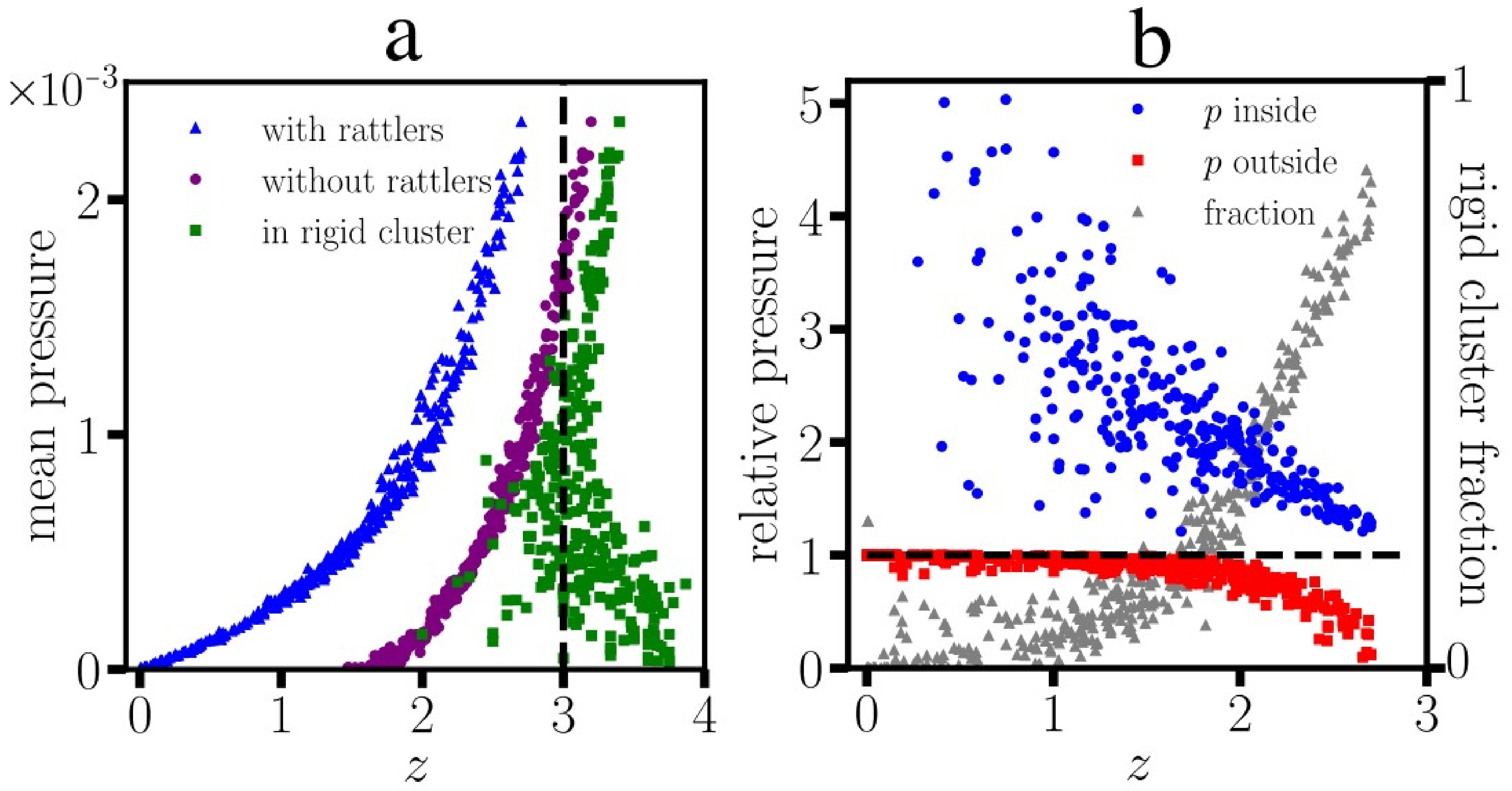




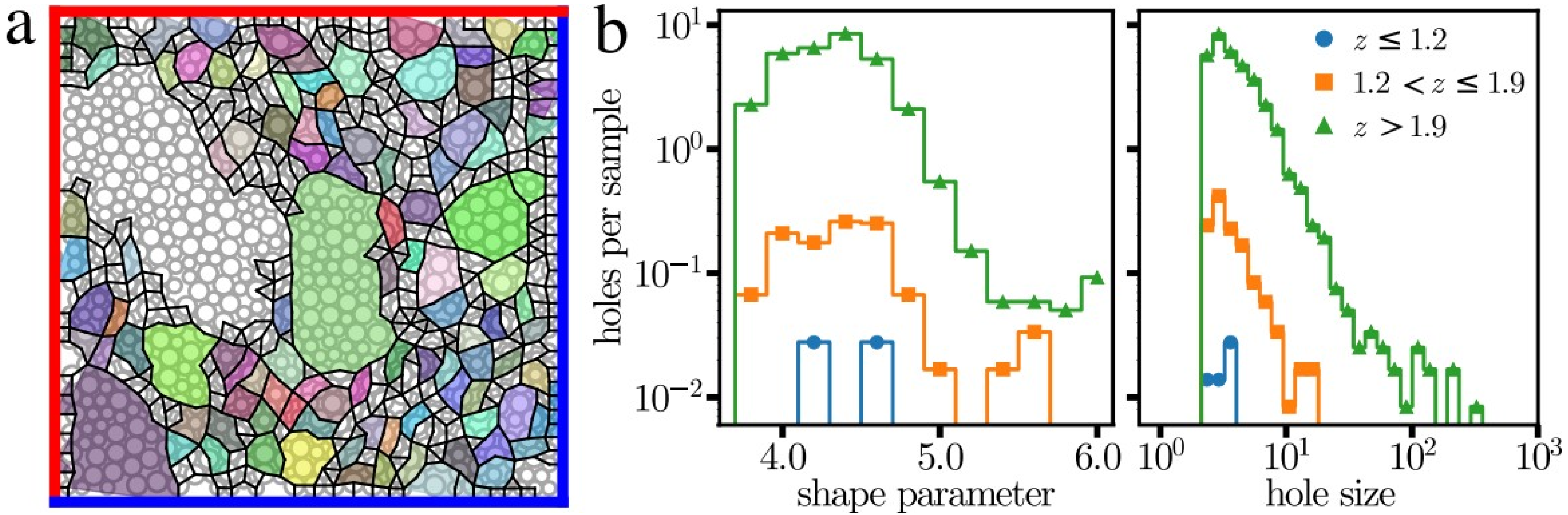
- are rough
- deform elastically
- are dissipative
- might not have μ = constant



Rigid vs. floppy clusters



Characterizing floppy regions



Liu, Kollmer, Daniels, Schwarz, Henkes *PRL* (2021)