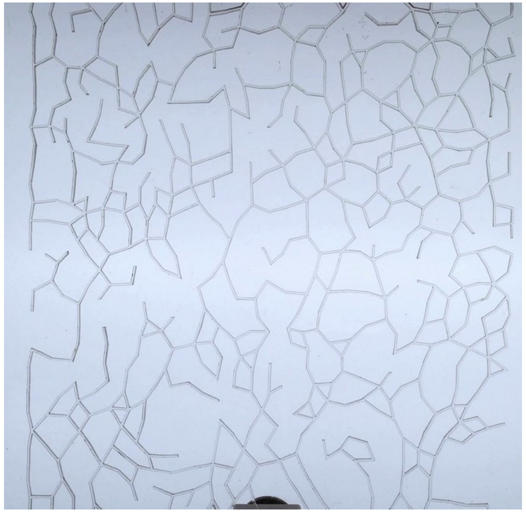
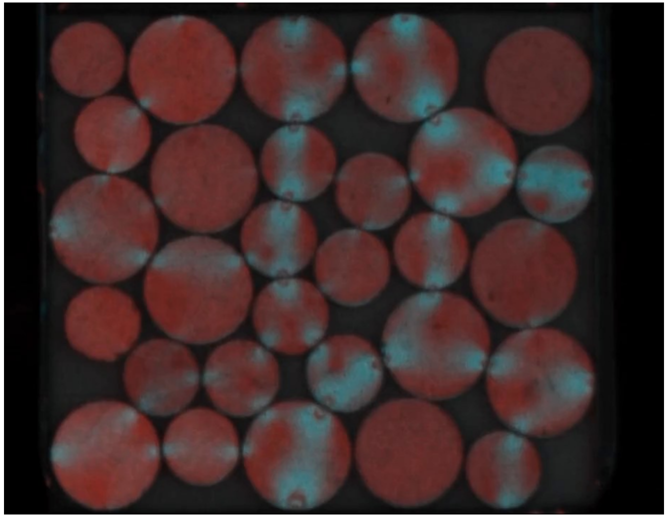
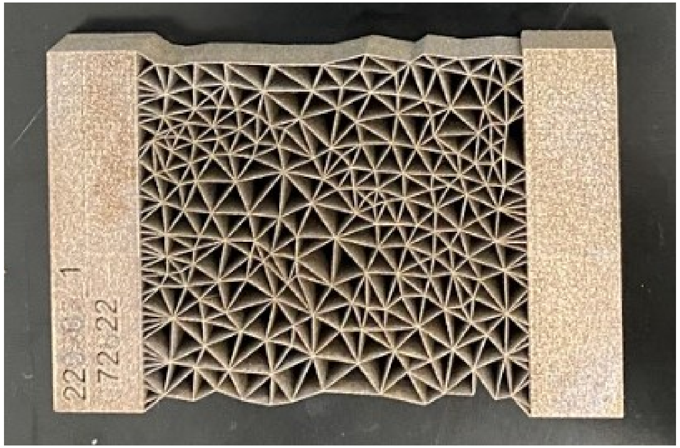
# Building Networks

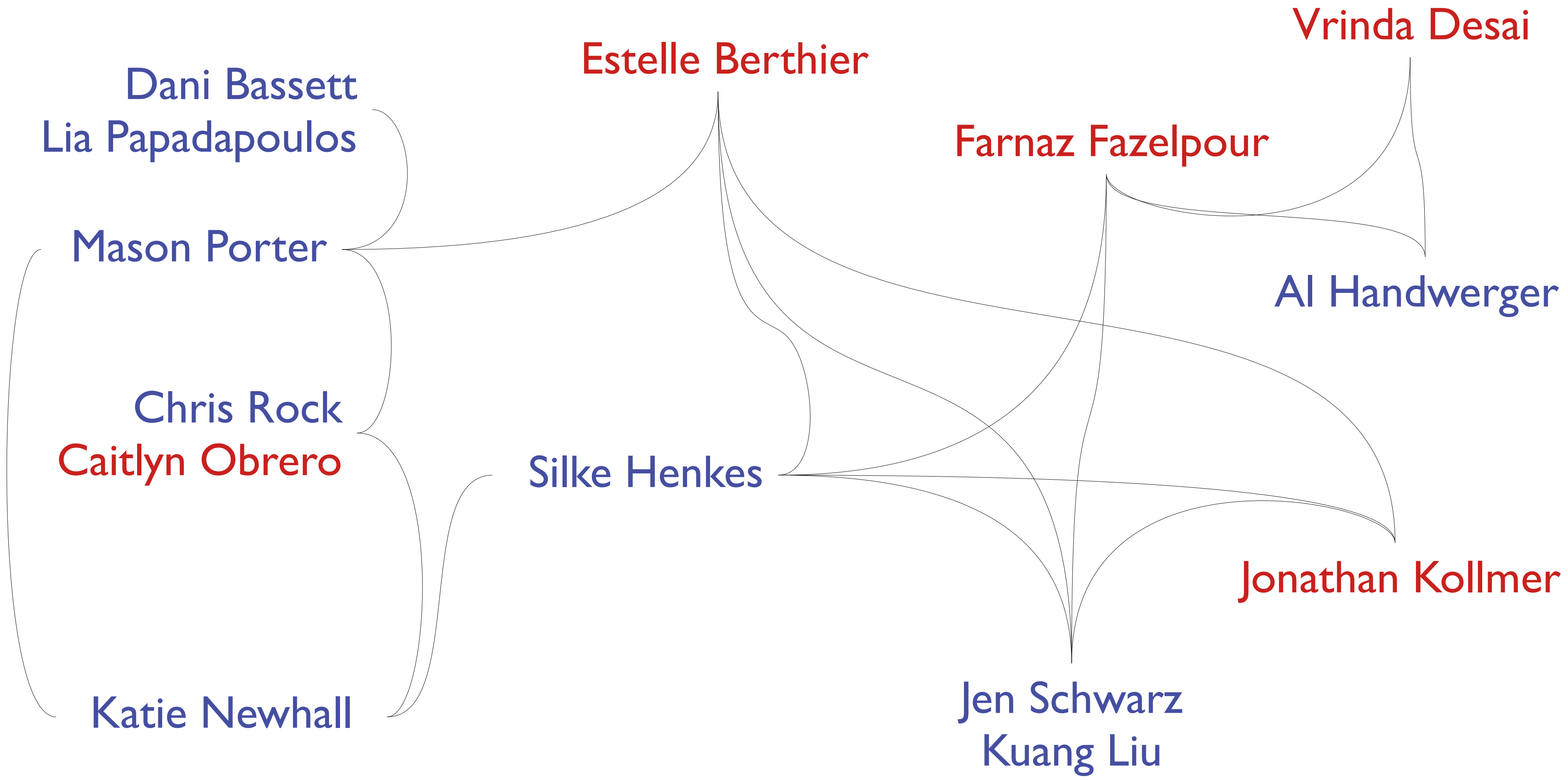




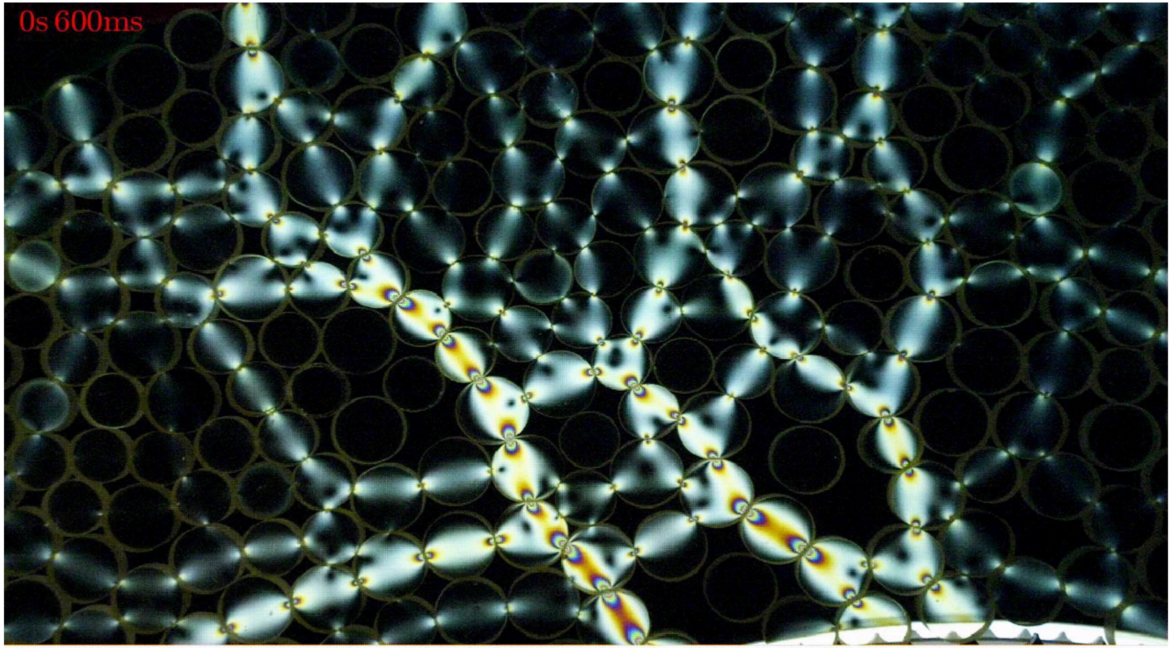


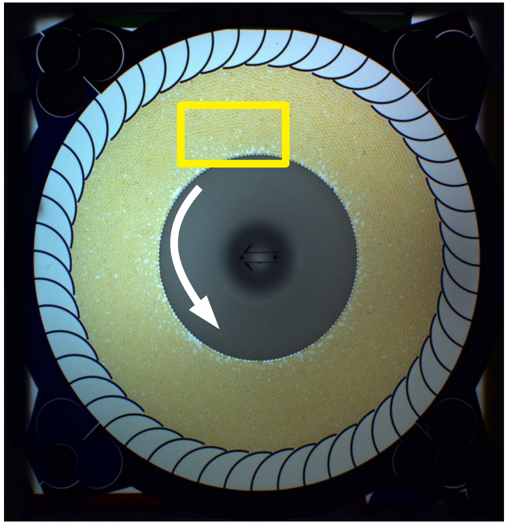

Karen Daniels
Department of Physics
NC State University

# Dramatis Personnae

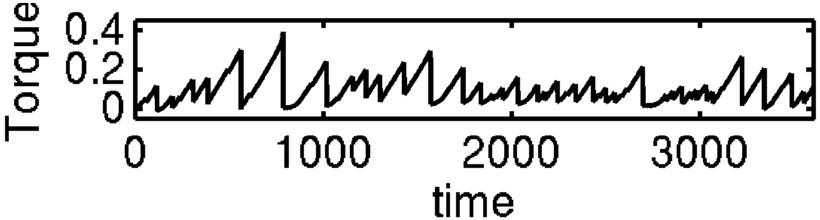


# How do grains resist stresses?



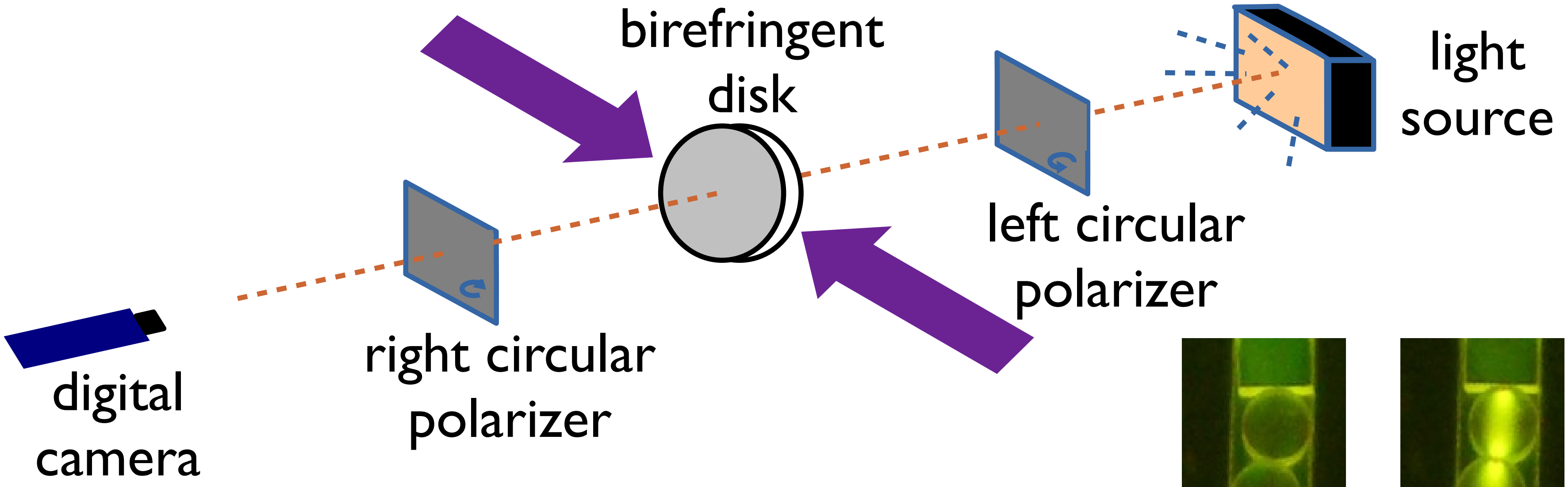



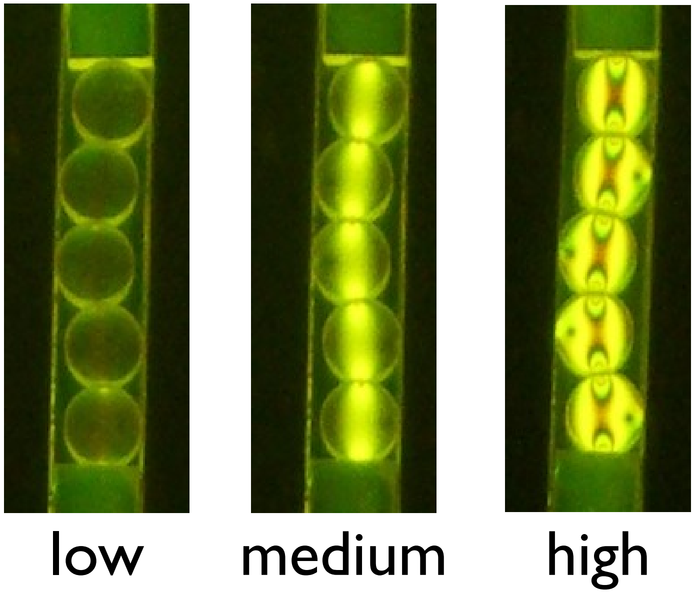
ramped force




Estelle Berthier, Farnaz Fazelpour, Clayton Kirberger

# Measuring Interparticle Contact Forces



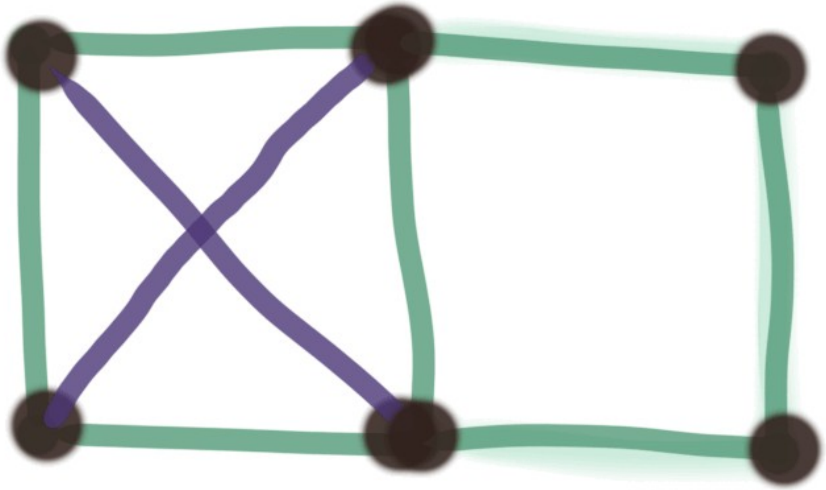




Daniels, Kollmer, Puckett. *Rev. Sci. Inst.* (2017)
Abed-Zadeh, Barés, et al. *Granular Matter* (2019)

# Rigidity

- the ability of a system to resist imposed stresses
- *caveat:* materials often contain rigid & floppy subregions

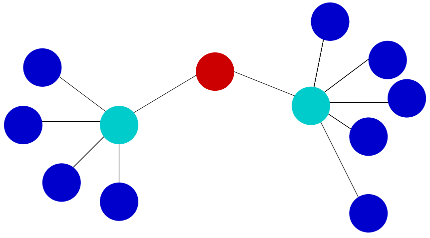
  … are system-wide averages still useful?
- where will failures occur?
- when will failures occur?



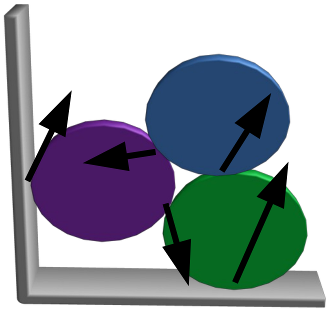
# 3 frameworks

less physics

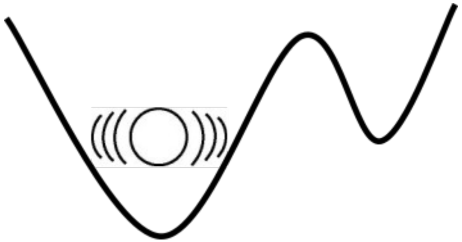
network science



constraint counting



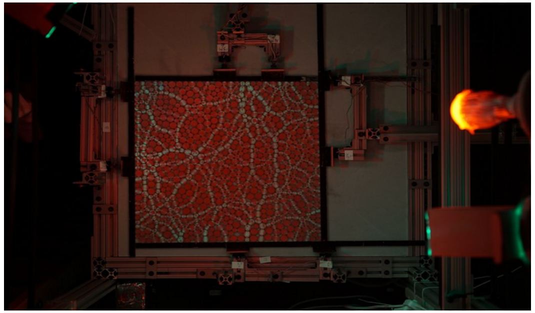
vibrational modes



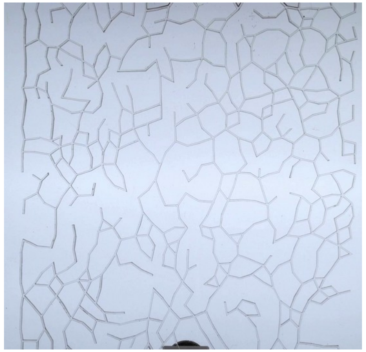
more physics

# 2 materials

frictional grains

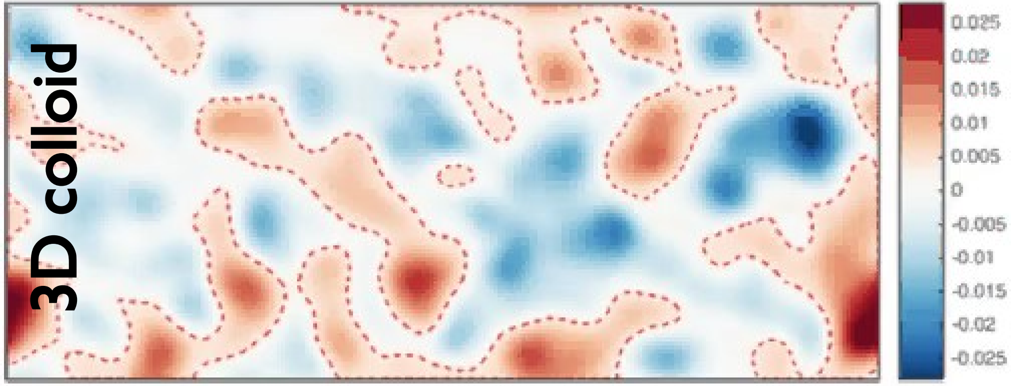


disordered lattices



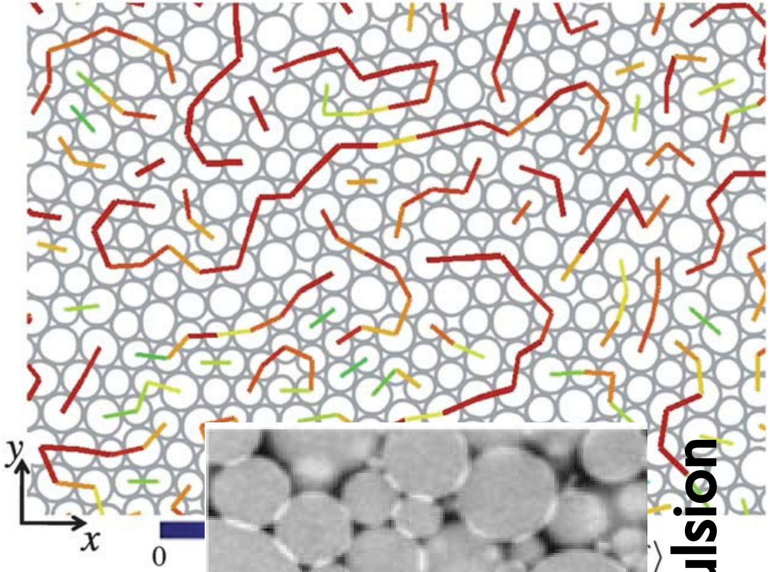
# Force chains

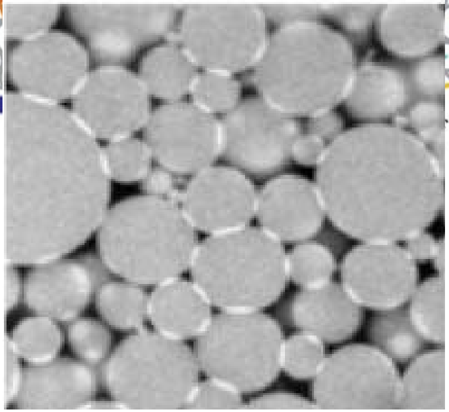
Lin, Bierbaum, Schall, Sethna, Cohen (2016)




Desmond & Weeks. *Soft Matter* (2013)

2D emulsion

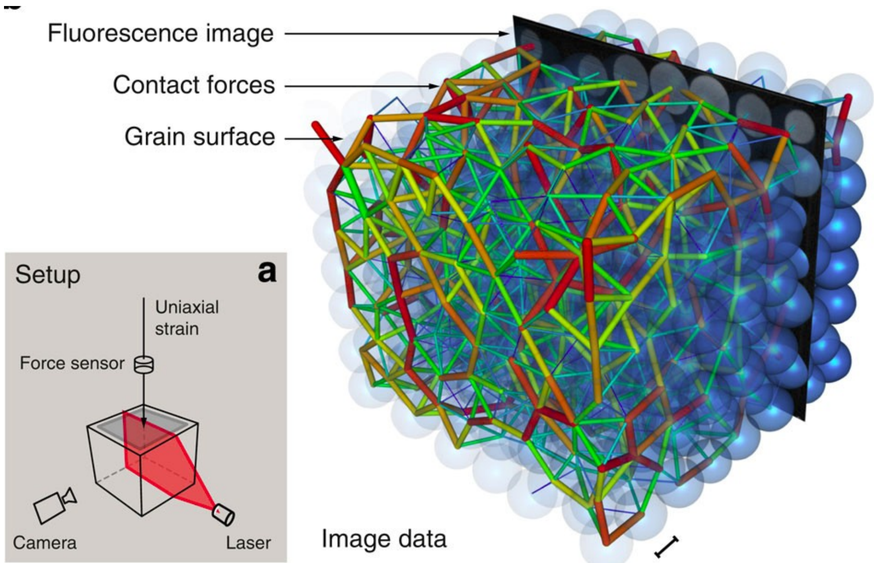


3D emulsion

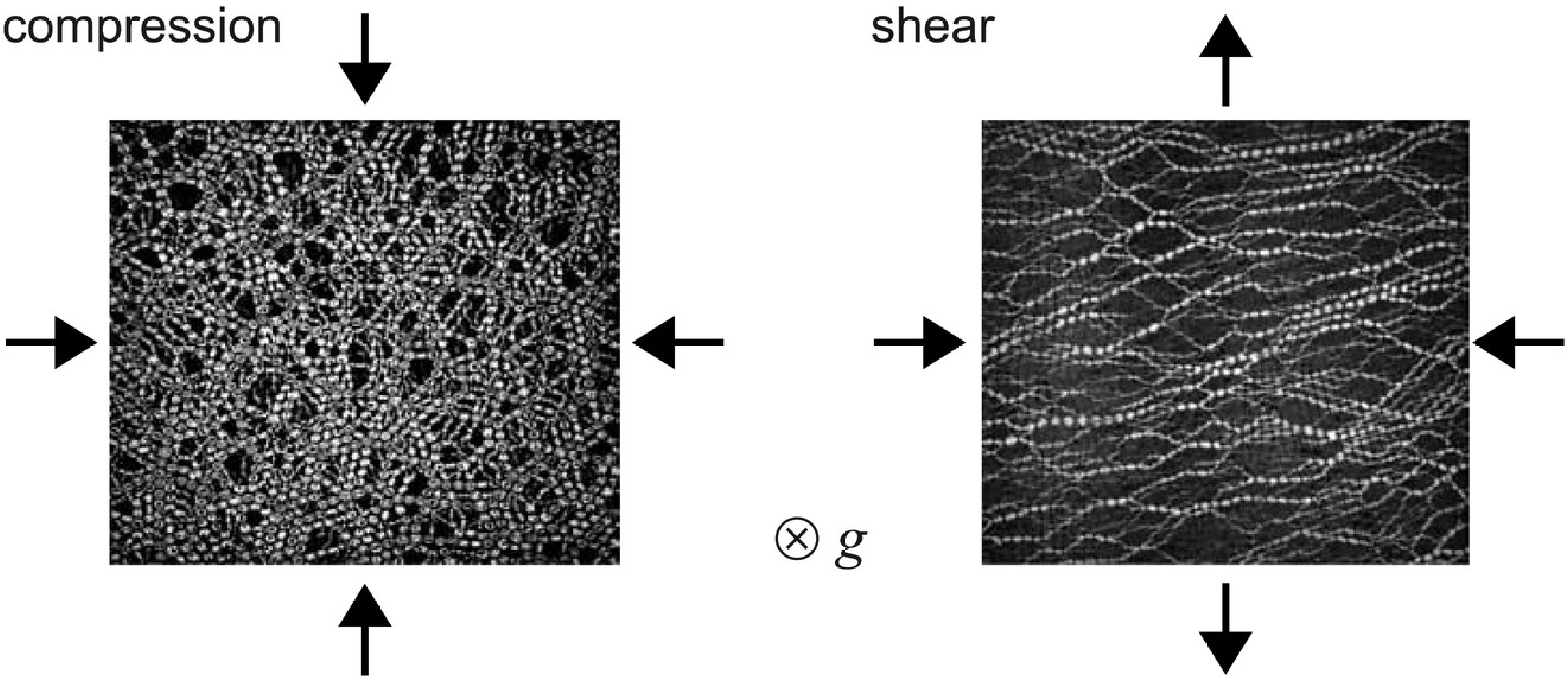
Brujic et al. Physica A (2003)

3D gel beads



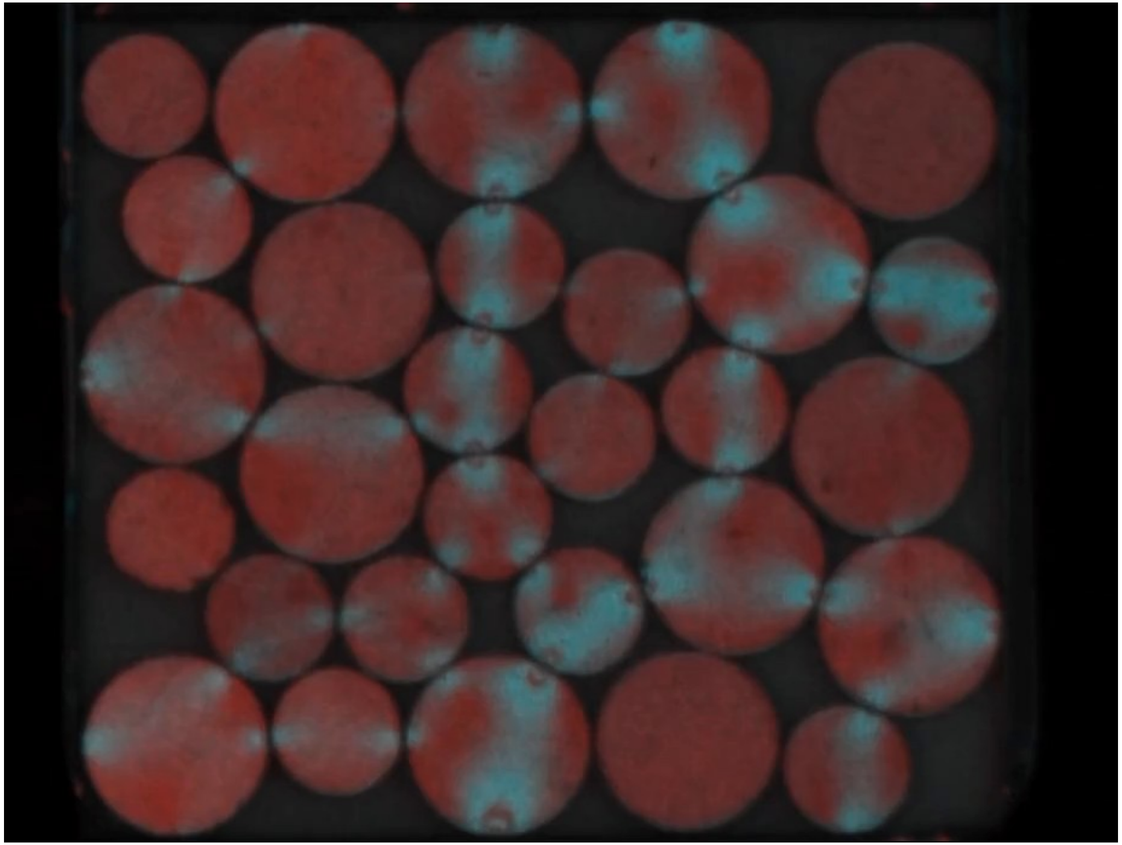

Brodu, Dijksman, Behringer. *Nat Comm.* (2015)

# Force chains record history




Majmudar & Behringer *Nature* (2005)

# Sensitivity to Small Changes

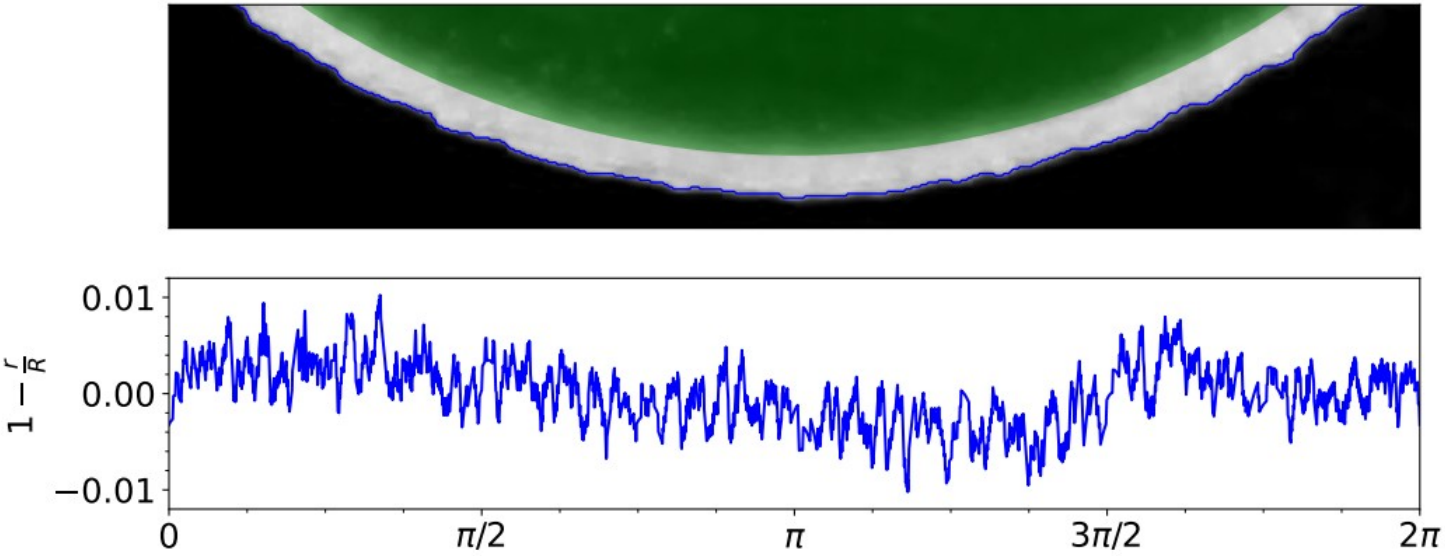


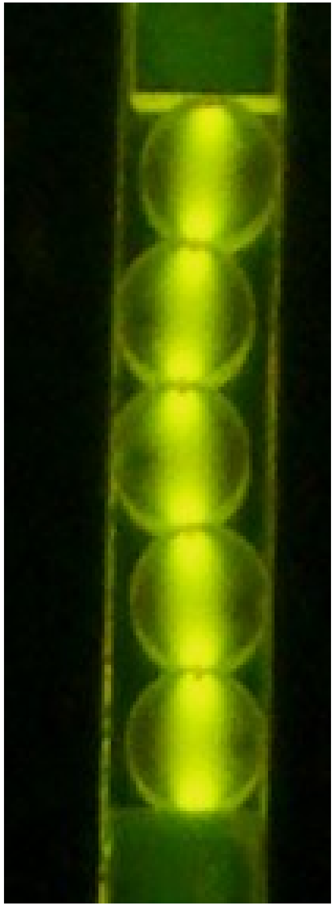
Jonthan Kollmer

"movie" of images taken of the same, regenerated configuration

Kollmer & Daniels. *Soft Matter* (2019)

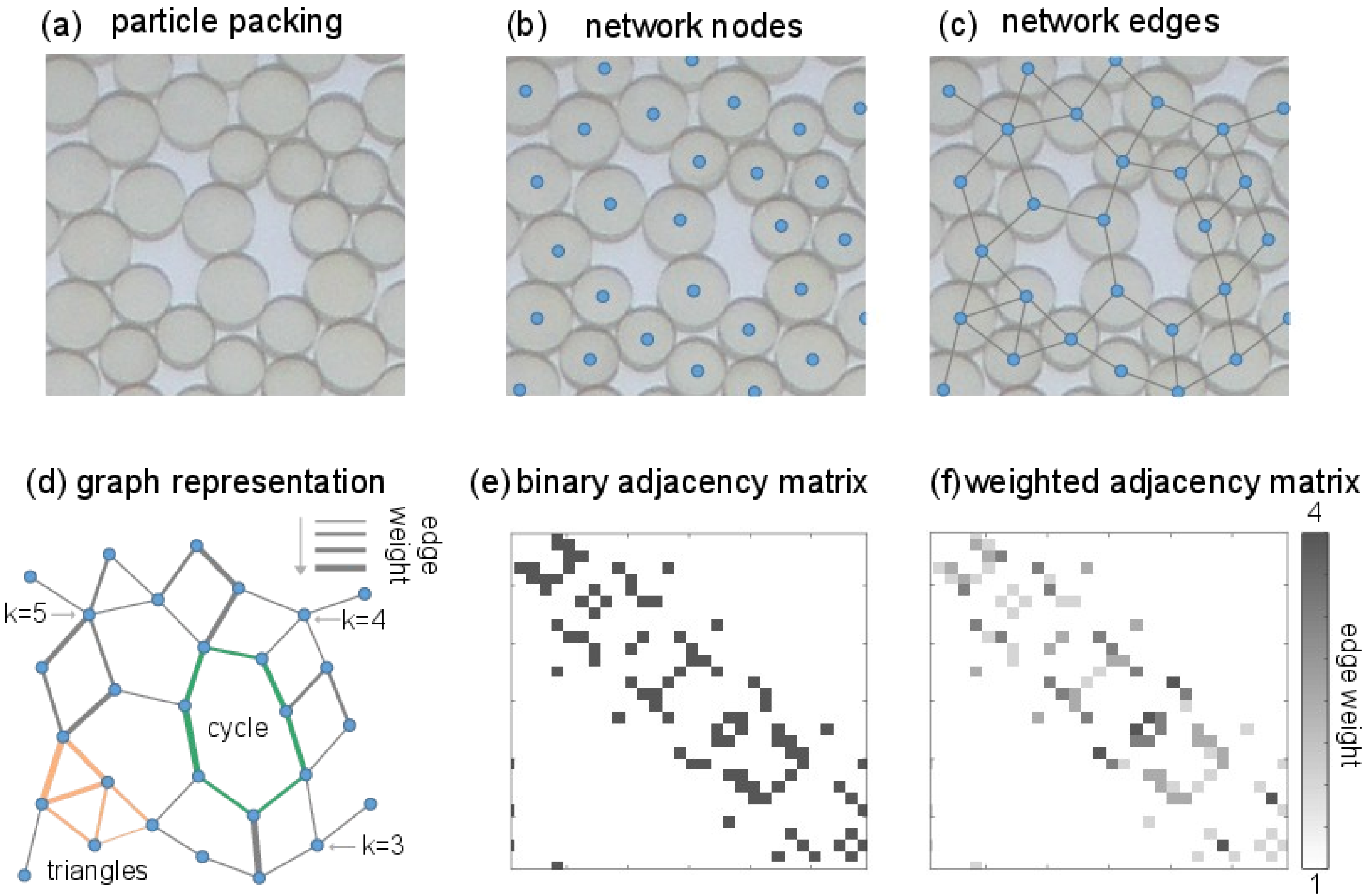
# Real particles are rough



Kool, Charbonneau, Daniels, arXiv: 2205.06794

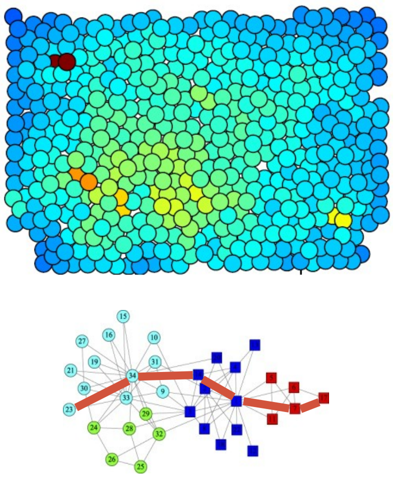
# Configurations → Adjacency Matrix




Papadapoulous, Daniels, Porter, Bassett. *J. Complex Networks* (2018)

# Network science metrics for different scales

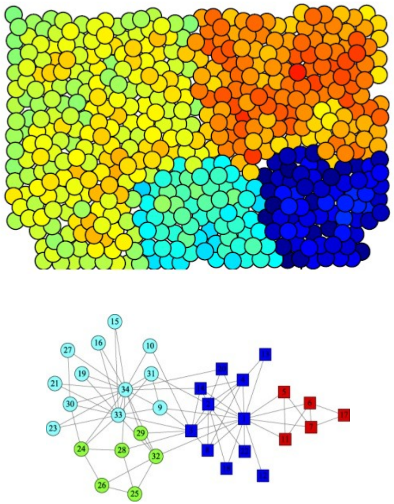
## System



Global Efficiency

- Efficiency of global signal transmission

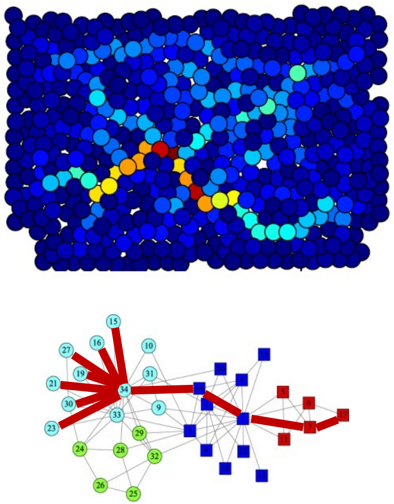
## 2D: Domains



Modularity

- Local geographic domains

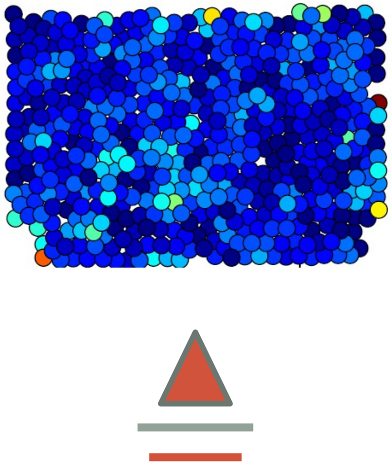
## 1D: Curves



Geodesic Node Betweenness

- Bottlenecks or centrality

## 0D: Particles

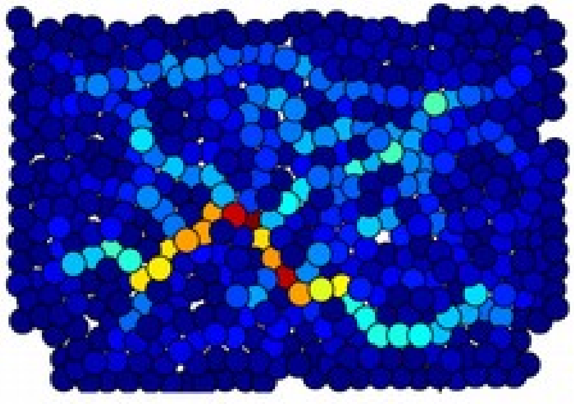


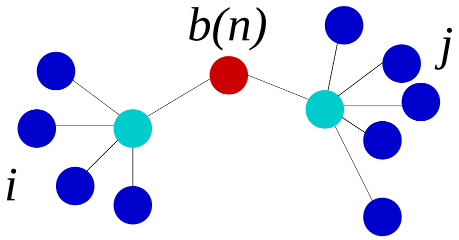
Clustering Coefficient

- Local loop structures

# Betweenness Centrality

$$b(n) = \sum_{i \neq n \neq j} \frac{s_{ij}(n)}{s_{ij}}$$

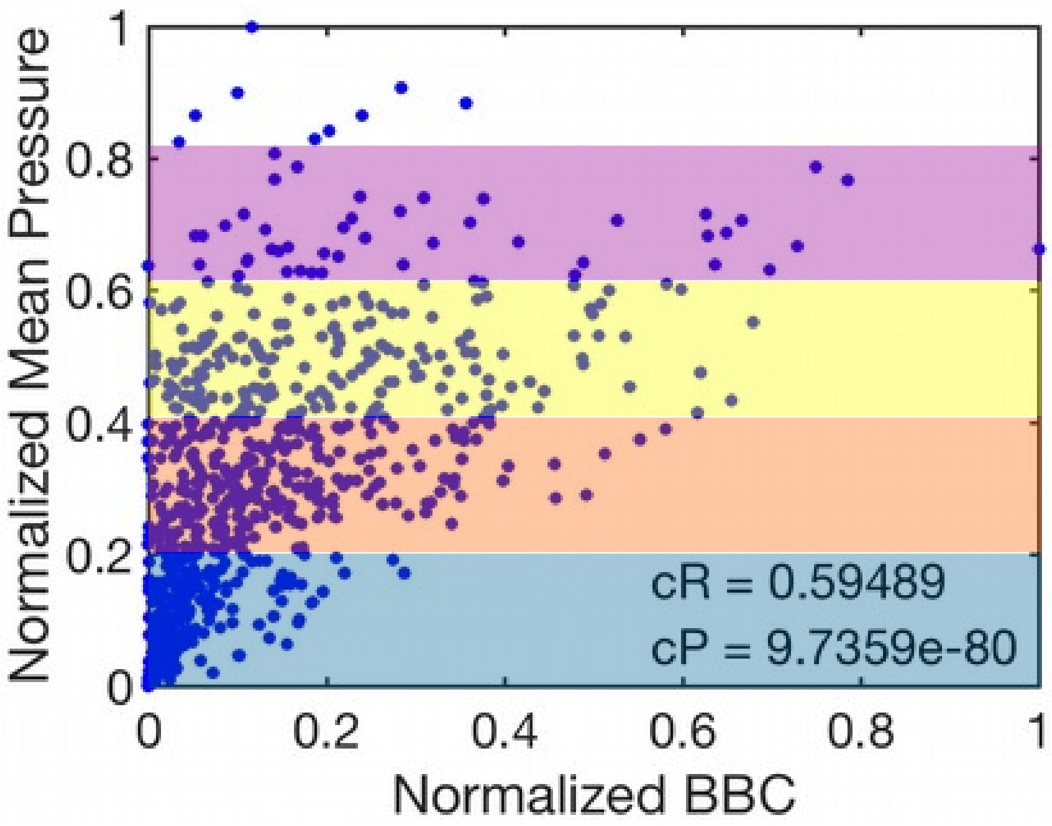




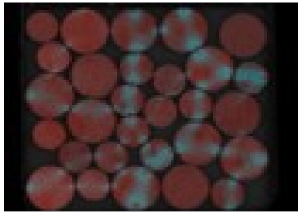

- $s_{ij}$ = shortest path between particles $i,j$
- can be either # of hops or weighted
- $b(n)$ = fraction of total # of shortest paths that go through particles $n$
- high $b(n)$ ~ "airline hubs"

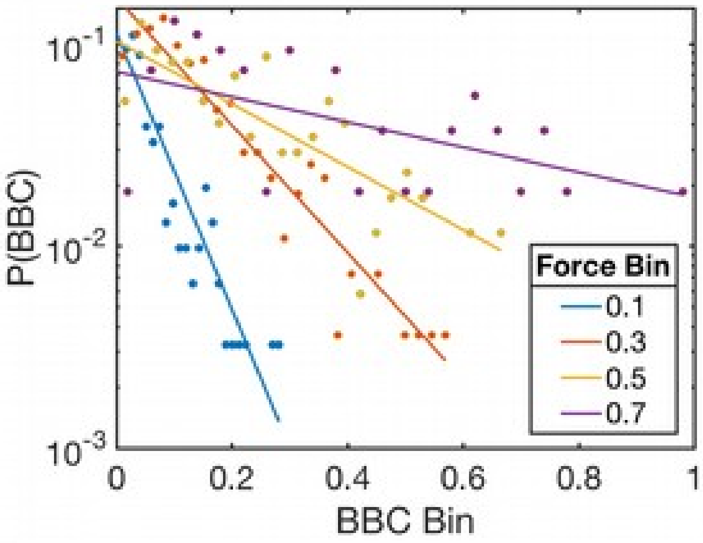
http://www.brain-connectivity-toolbox.net/

# Betweenness centrality predicts forces



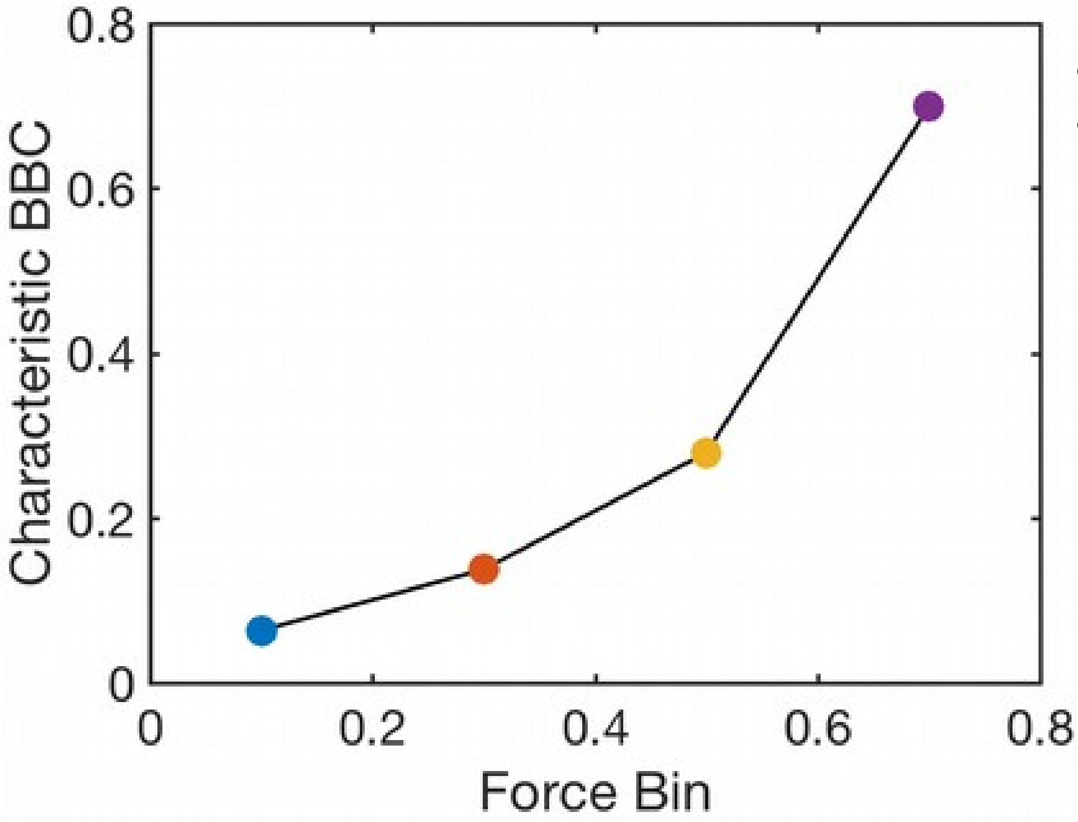

824 particles
80 cycles



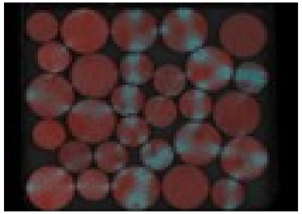


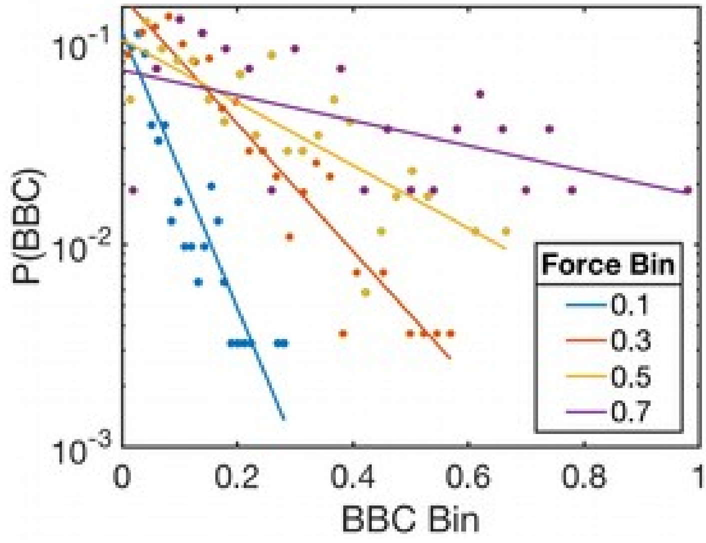

Kollmer & Daniels. *Soft Matter* (2019)

# Betweenness centrality predicts forces

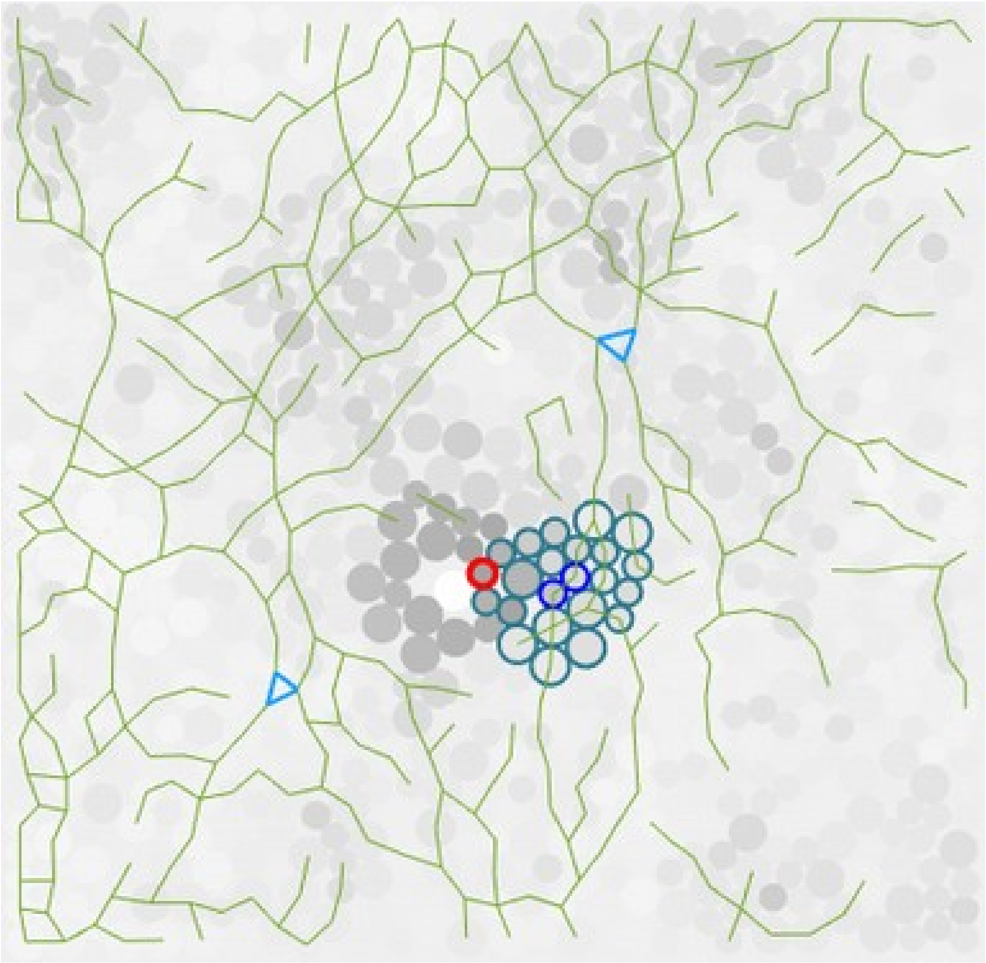



824 particles
80 cycles



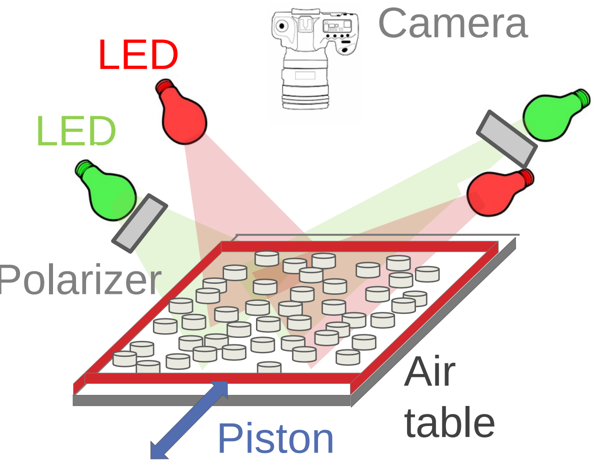



Kollmer & Daniels. *Soft Matter* (2019)

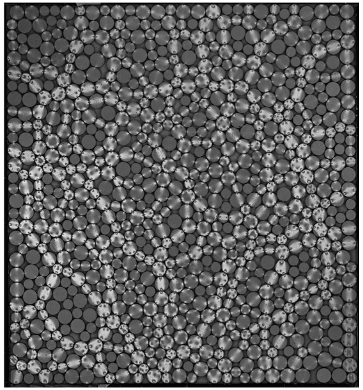


# Can we forecast failure locations?

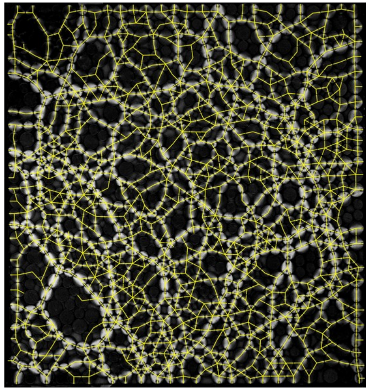
# Simplify! → Disordered lattices




granular force chains

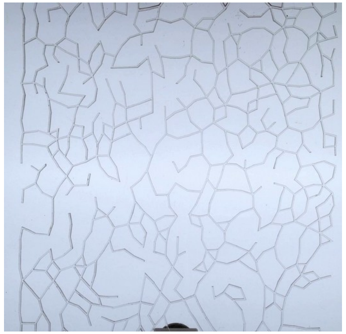


contact network



Estelle Berthier

laser-cut lattice



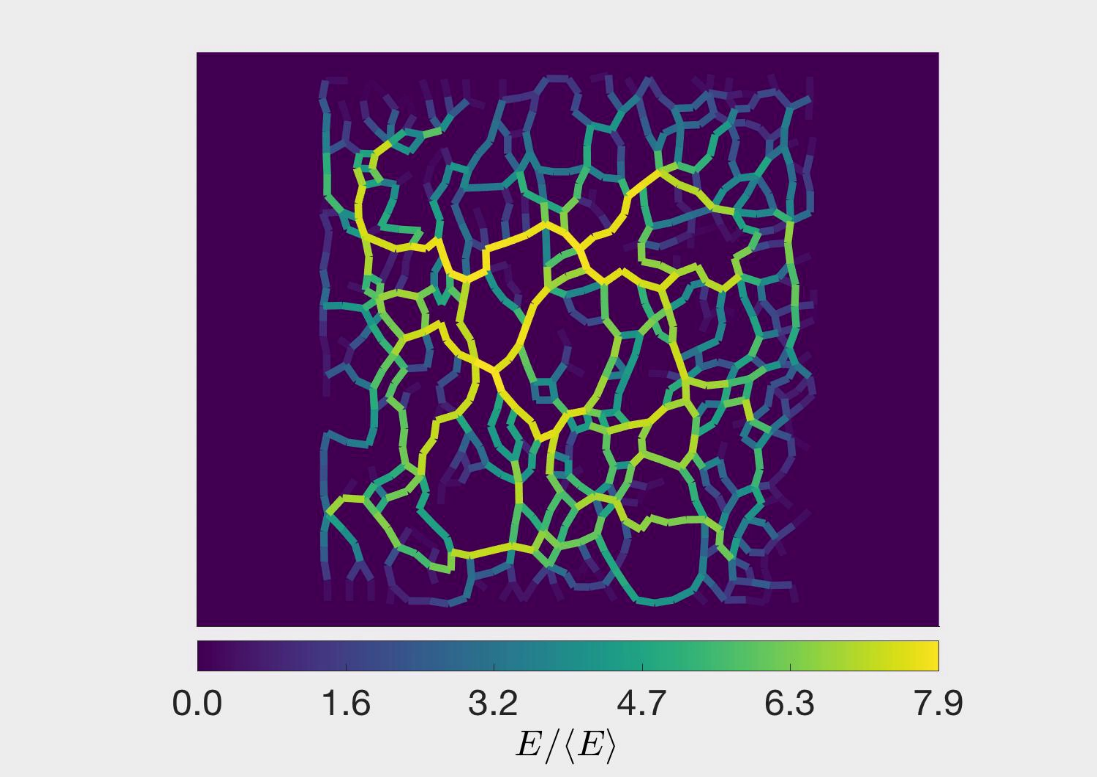
Berthier, Porter, Daniels. *PNAS* (2019)
Berthier, Kollmer, Henkes, Liu, Schwarz, Daniels. *Phys. Rev. Mat.* (2019)

# Failure Locations & Betweenness

About 77% of failing edges have
$E^{failed} > \langle E \rangle$

About 37% of the network edges have
$E > \langle E \rangle$

Shared property
of all networks
for small damage event



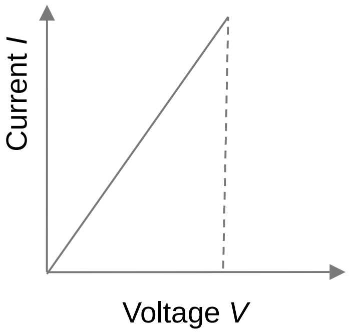

$E_{ij}/\langle \mathbf{E} \rangle$

Berthier, Porter, Daniels. *PNAS* (2019)

# Better model → better prediction

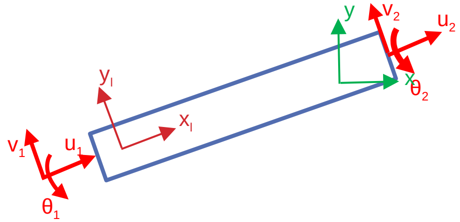
**Fuse Model**

Each beam → fuse
Fails at some current

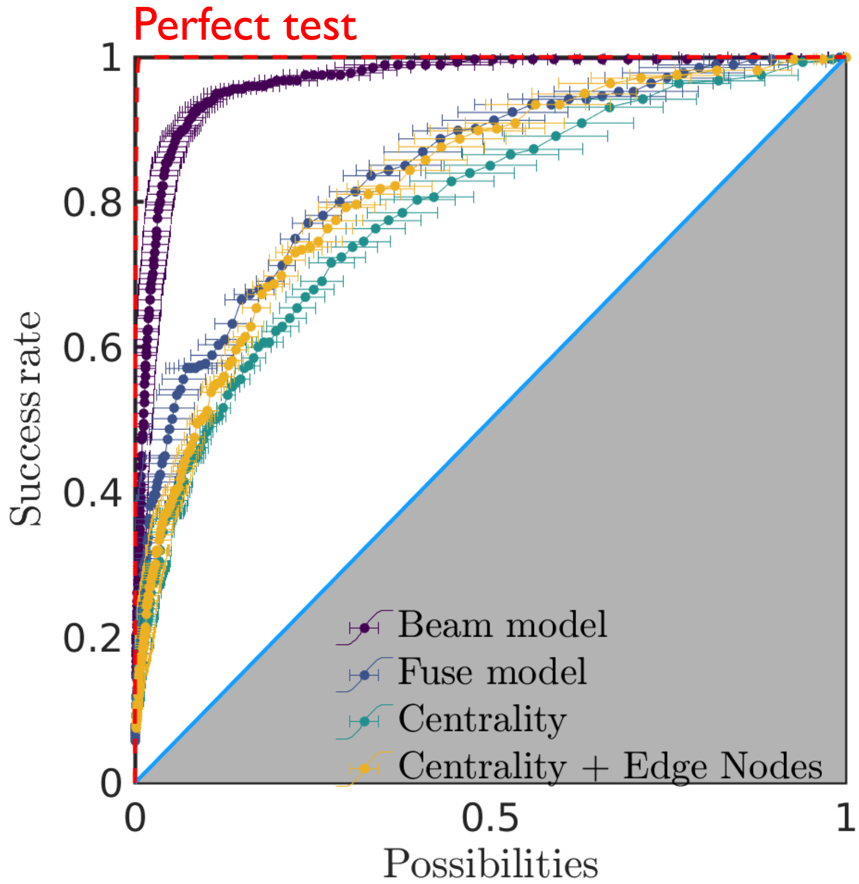



de Arcangelis et al.
*J. Physique Lett.* (1985)
Duxbury et al. *PRB* (1987)

**Beams + Moments**

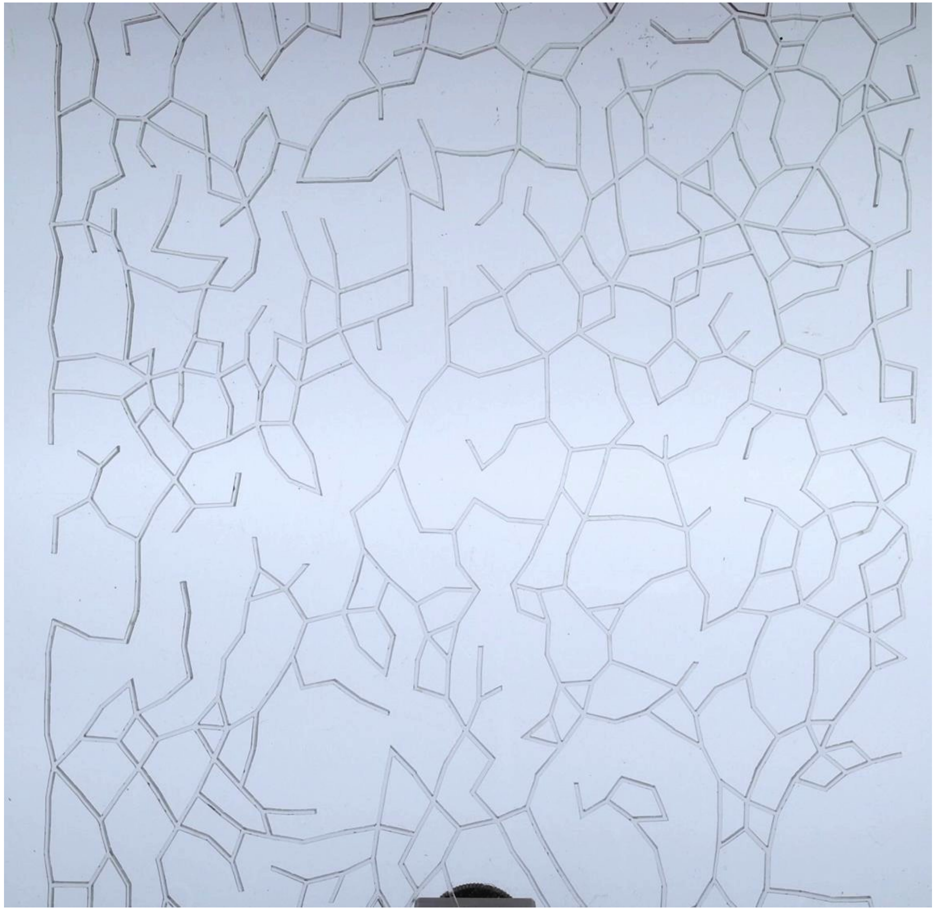



Nukala Zapperi, Alava,
Šimunović. *PRE* (2008)



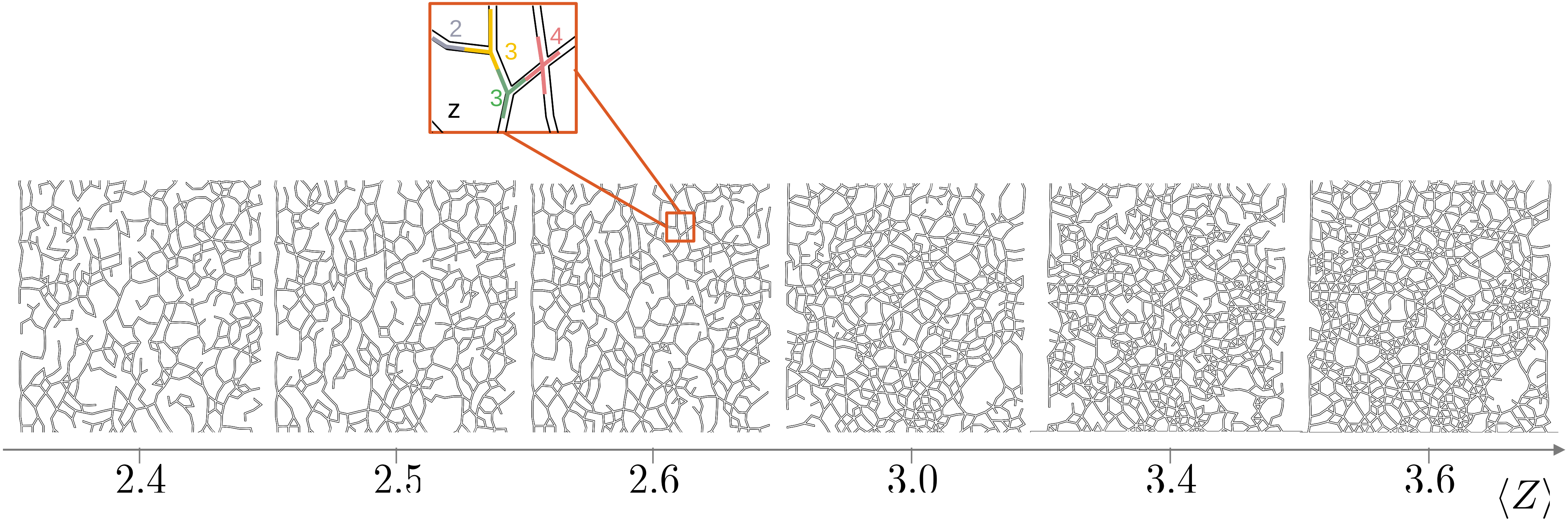

Berthier, Porter, Daniels. *PNAS* (2019)

# Lattice Fracture



Continuous Cast Acrylic
Thickness = 3.17 mm
Beam width = 1.5 mm

# Vary mean coordination number *z*

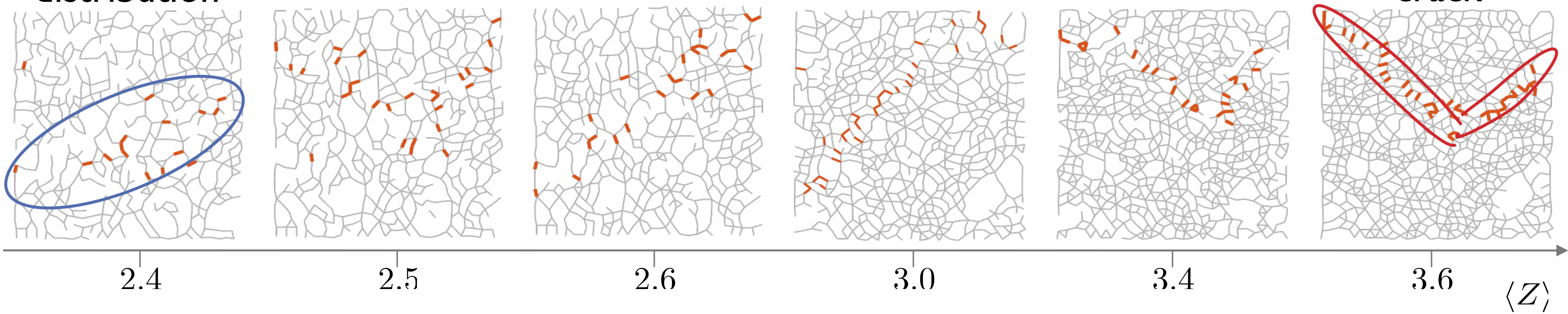



Berthier, Kollmer, Henkes, Liu, Schwarz, Daniels. *Phys. Rev. Mat.* (2019)

# Connectivity controls failure mode

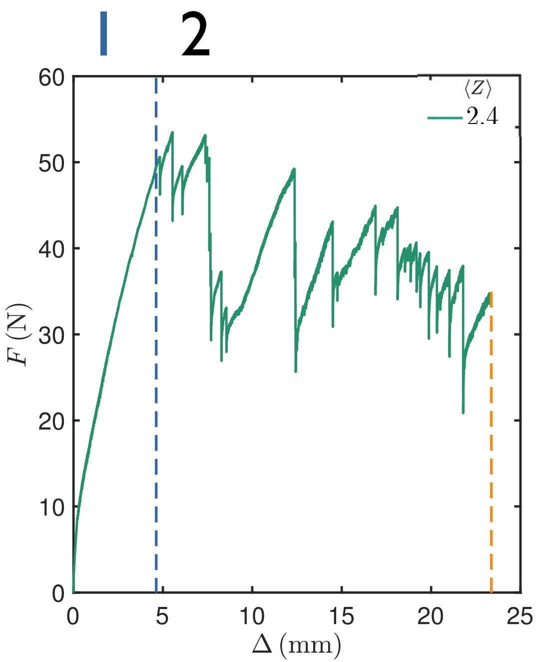
ductile-like: broad distribution

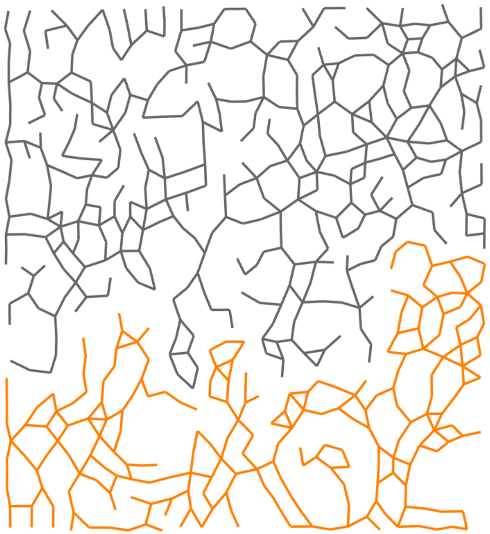
brittle-like: narrow crack



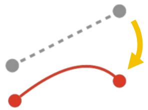

Berthier, Kollmer, Henkes, Liu, Schwarz, Daniels. *Phys. Rev. Mat.* (2019)

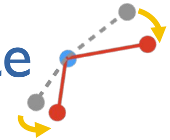
# Low-*z* response & failure



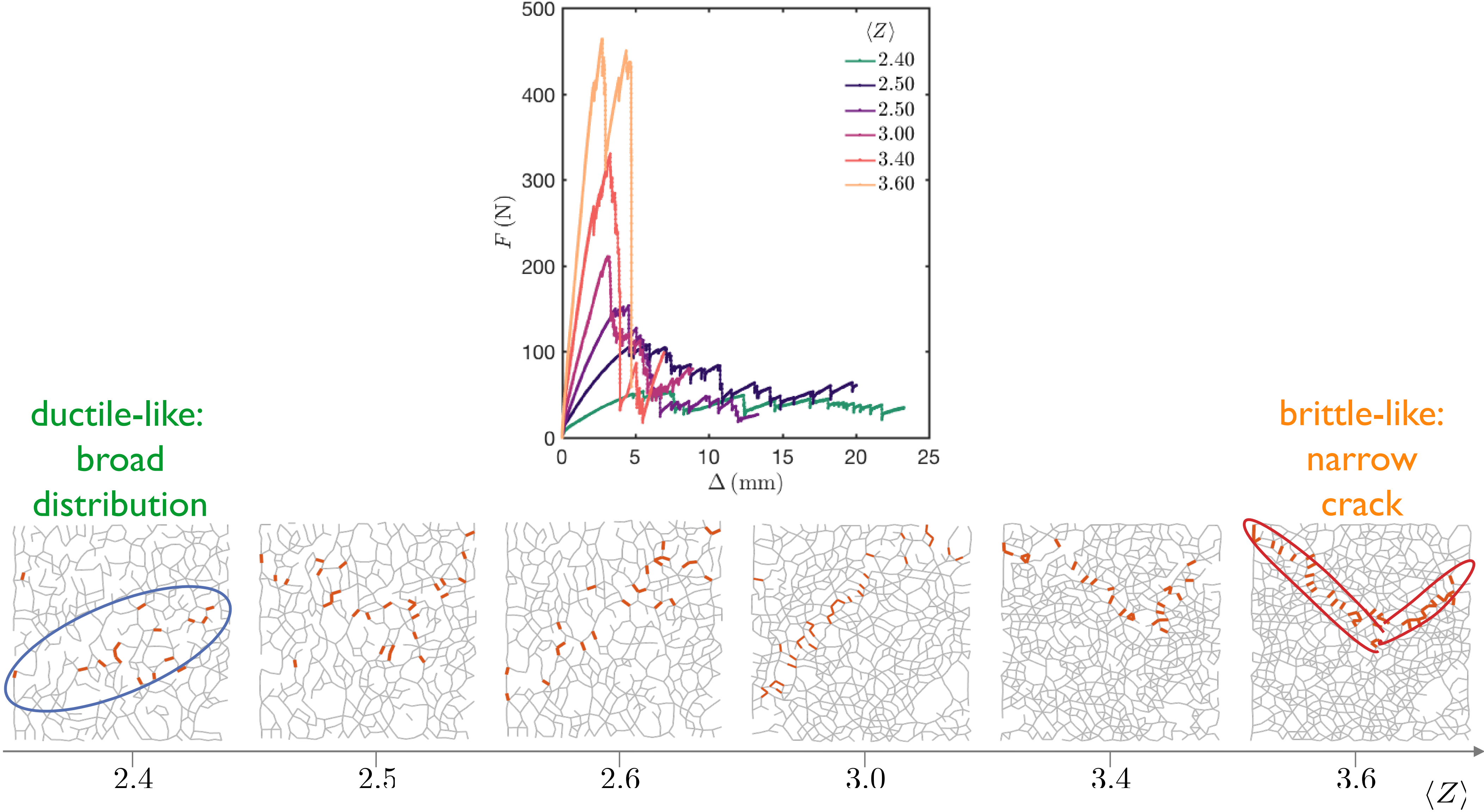



- Phase 1: Elastic response
  - Beams compress & stretch
  - Intersections rotate
- Phase 2: Successive Failures
  - Progressive damage
  - Distributed damage
- End result: spanning crack





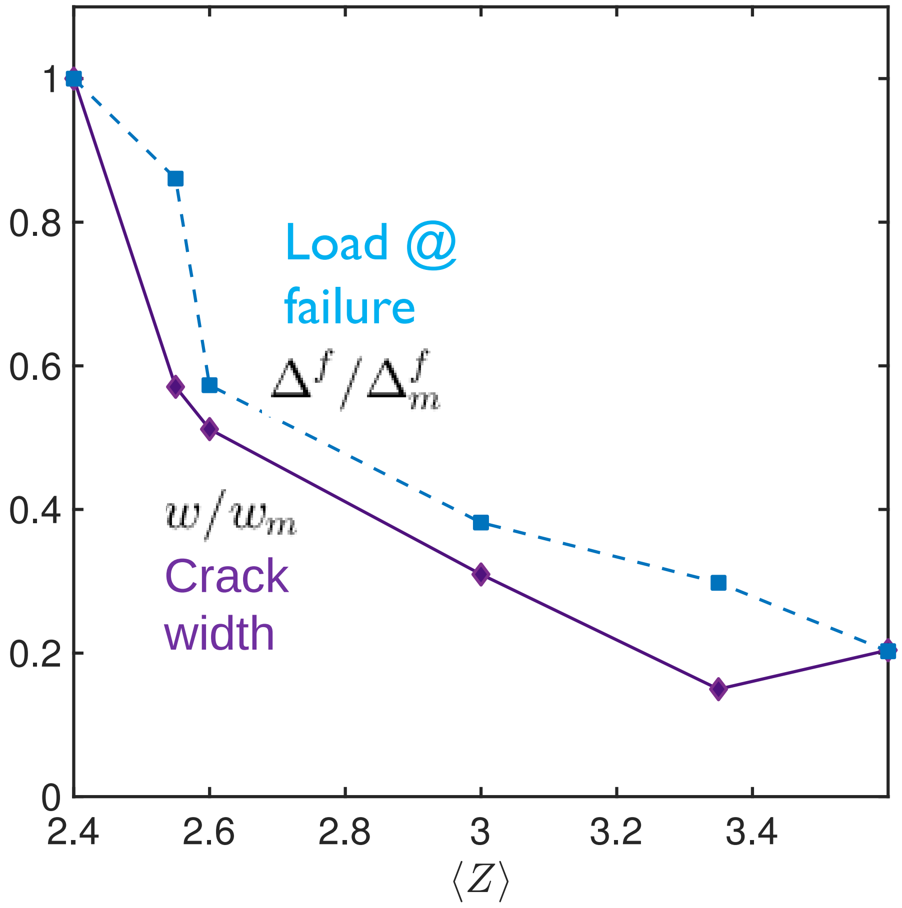
Berthier, Kollmer, Henkes, Liu, Schwarz, Daniels. *Phys. Rev. Mat.* (2019)



500
400
300
200
100
0
F (N)
0
5
10
15
20
25
Δ (mm)
⟨Z⟩
2.40
2.50
2.50
3.00
3.40
3.60
ductile-like:
broad
distribution
brittle-like:
narrow
crack
2.4
2.5
2.6
3.0
3.4
3.6
⟨Z⟩

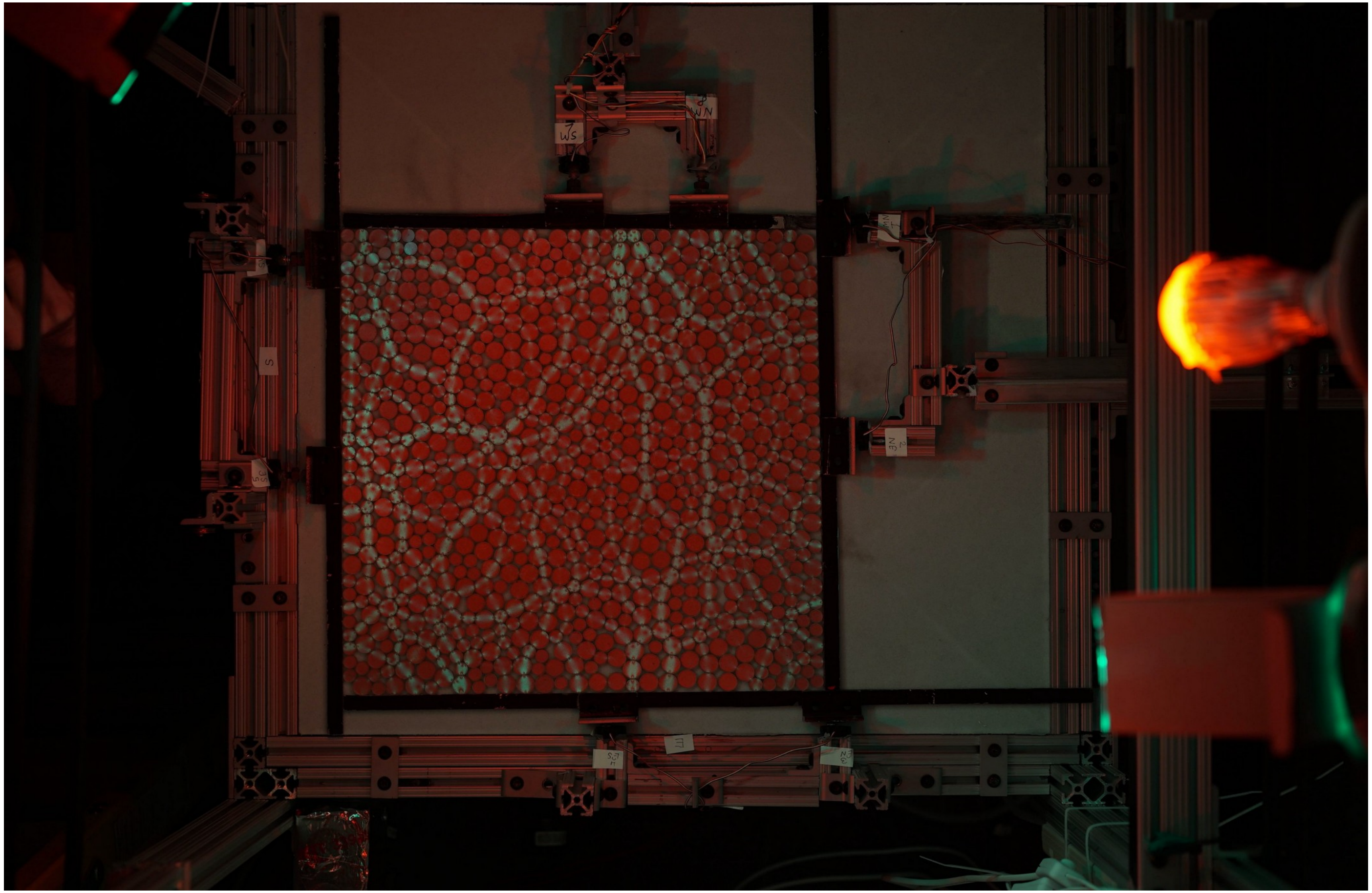
# Changes in behavior with <z>

**Ductile-like:**
- Diffuse
- Progressive
- High deformation

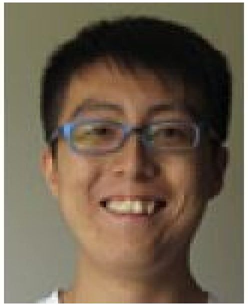



**Brittle-like:**
- Localized
- Catastrophic
- Low deformation

# Rigidity in granular experiments

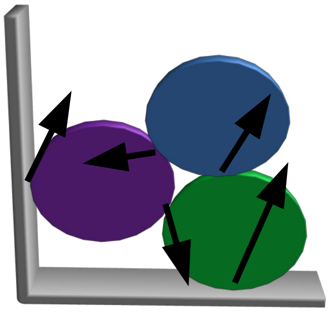


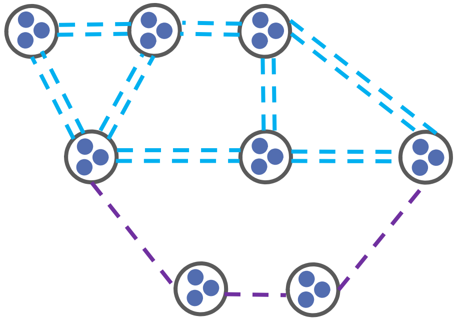
Jonthan
Kollmer

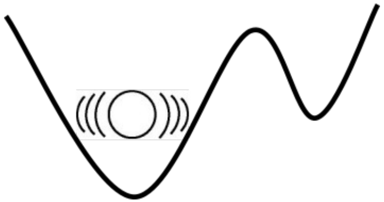


Kuang Liu

# Constraint Counting ::: Vibrational Modes







$$\delta \ddot{r}^{i}_{\alpha\beta} = -D^{ij}_{\alpha\beta}\,\delta r^{j}_{\beta} + \text{dissipation}(\delta \dot{r}) + O(\delta r^2),$$

$$D^{ij}_{\alpha,\beta} = \frac{1}{\sqrt{m_{i,\alpha} m_{j,\beta}}} \frac{\partial^2 V_{ij}}{\partial r_{i,\alpha} \partial r_{j,\beta}}.$$

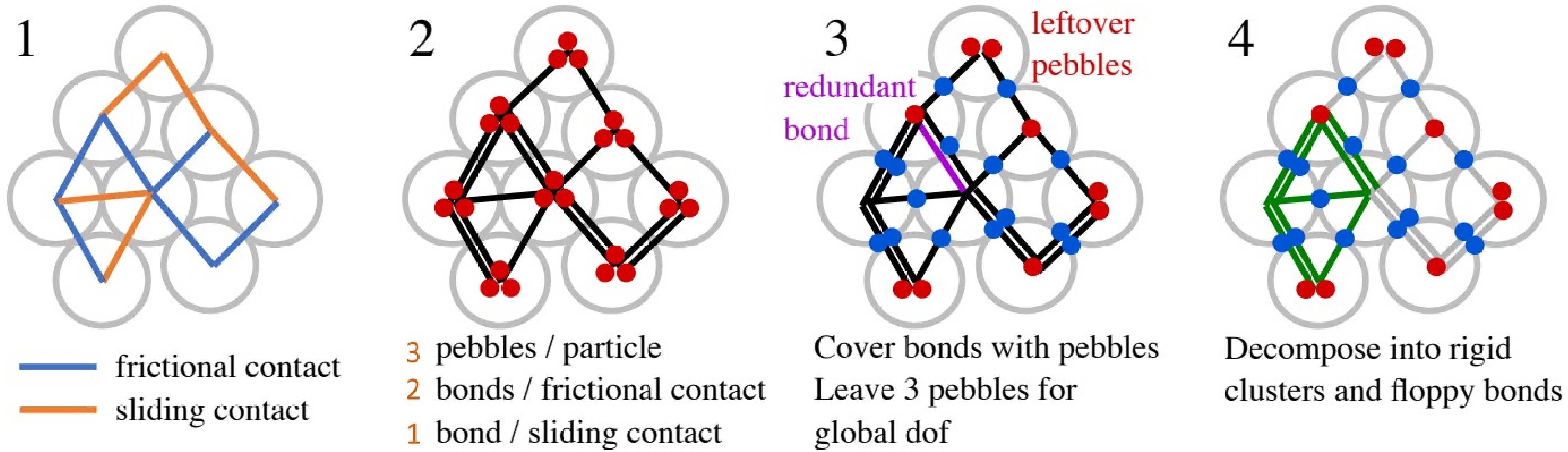
less physics ⟷ more physics

- torque and force balance
- degrees of freedom
- look for clusters where constraints are satisfied

- consider (frictional, dissipative) particles as being in energy wells
- look for regions of low-frequency modes

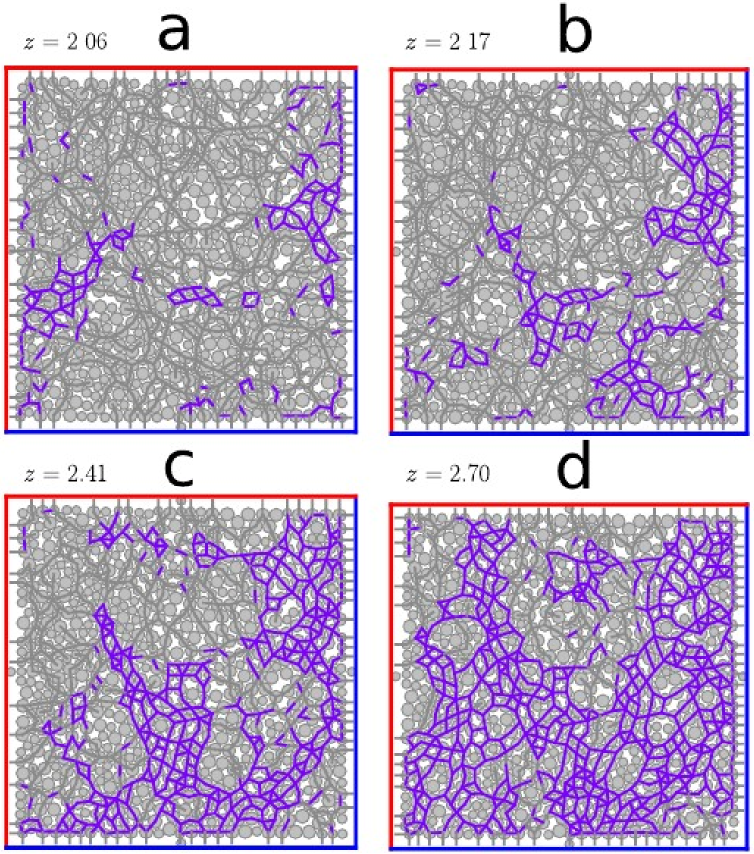
Liu, Kollmer, Daniels, Schwarz, Henkes *PRL* (2021)

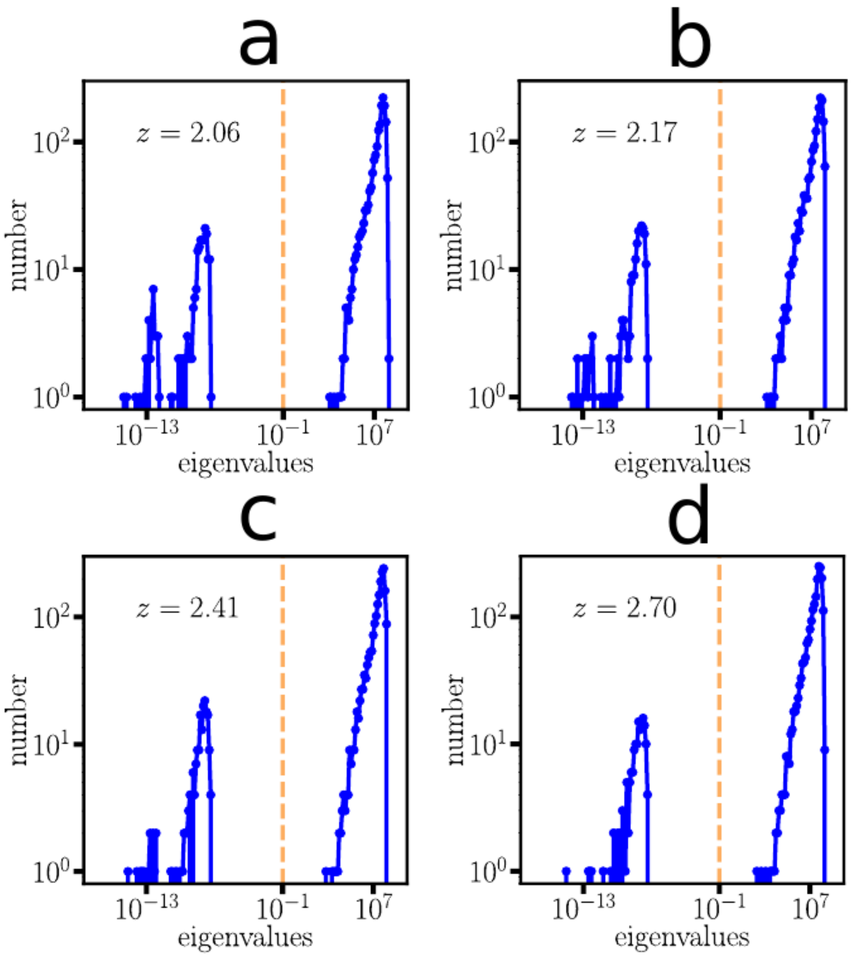
# Pebble game reveals rigid clusters




Jacobs & Thorpe. *PRL* (1995)
Henkes, Quint, Fily, Schwarz. *PRL.* (2016)

# Vibrational modes: set a threshold

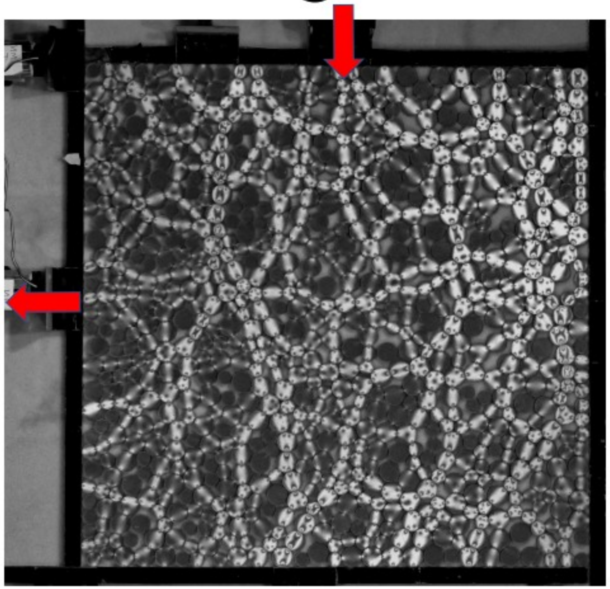


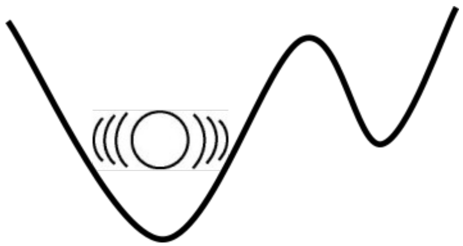

Liu, Kollmer, Daniels, Schwarz, Henkes *PRL* (2021)

# Force Chains

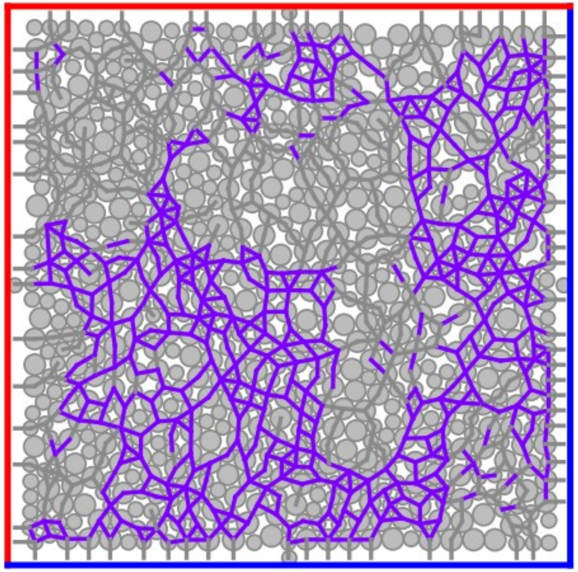
b



# Vibrational

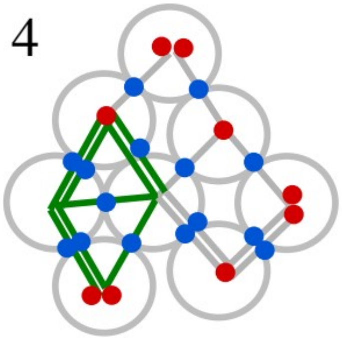


c

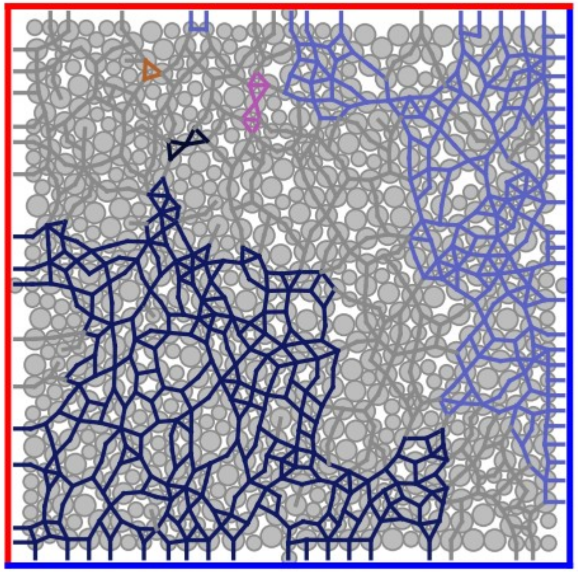


# Constraints

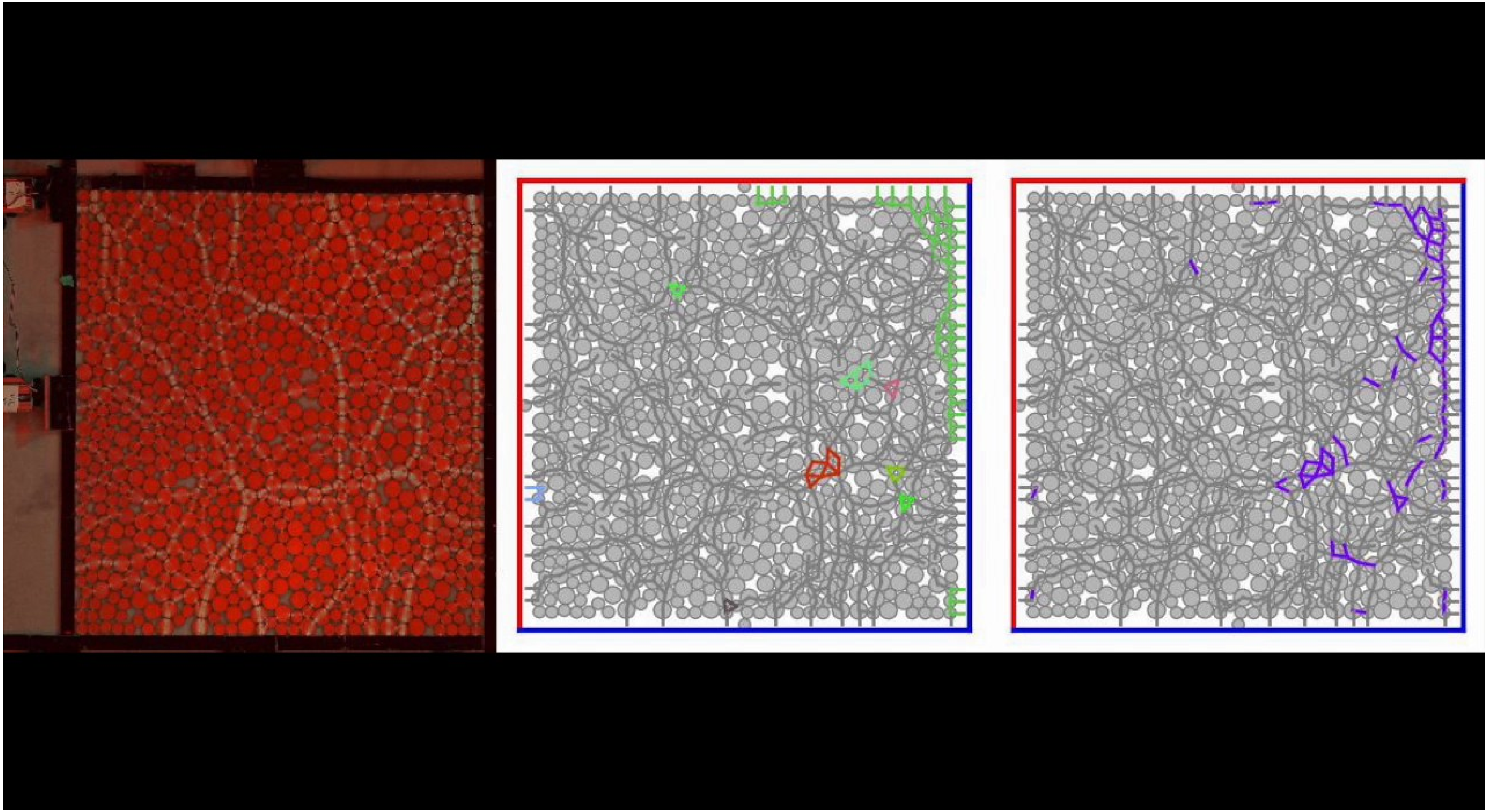
4



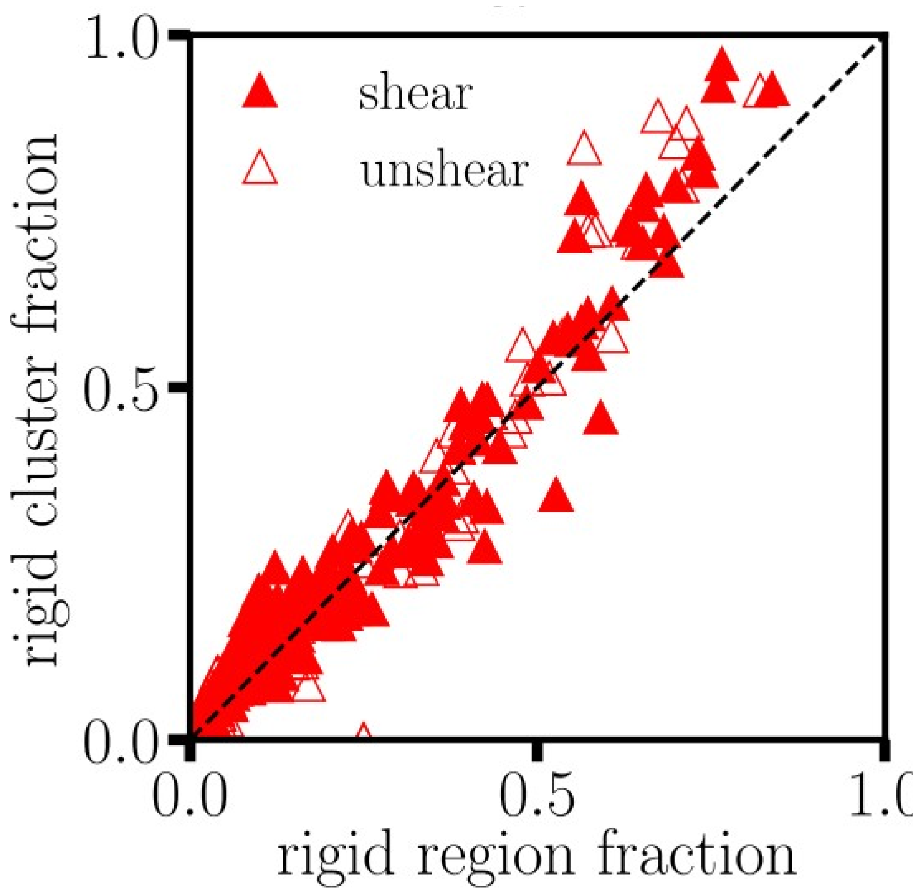
d



# Force Chains Constraints Vibrational

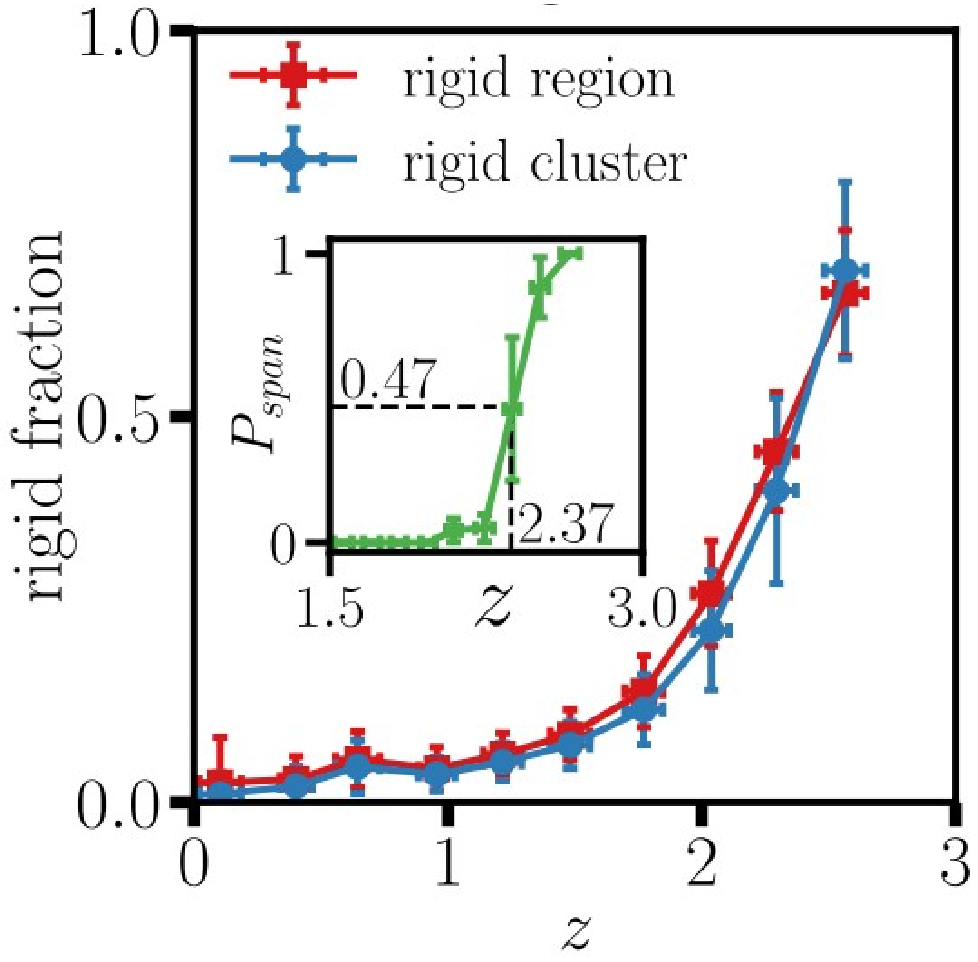


# 2 frameworks tell the same story



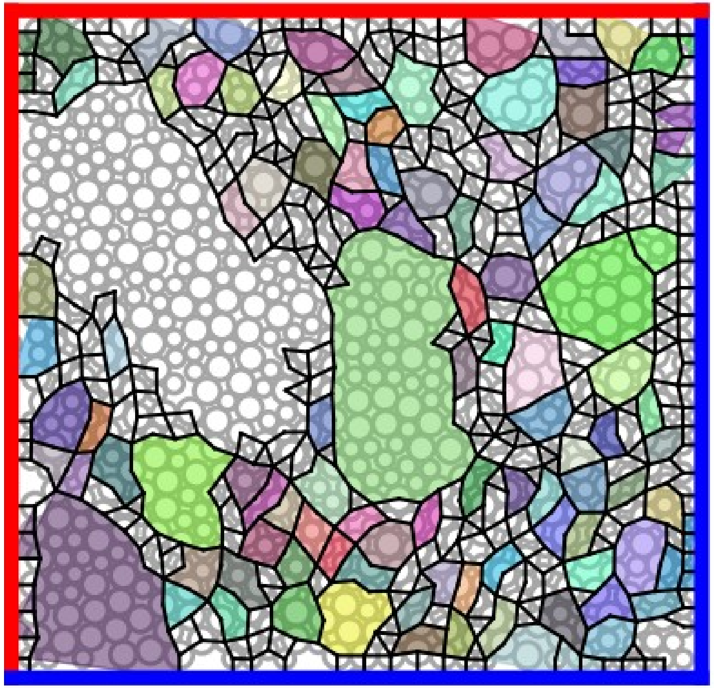

Liu, Kollmer, Daniels, Schwarz, Henkes *PRL* (2021)

# Threshold for global rigidity: $z_c \sim 2.4$

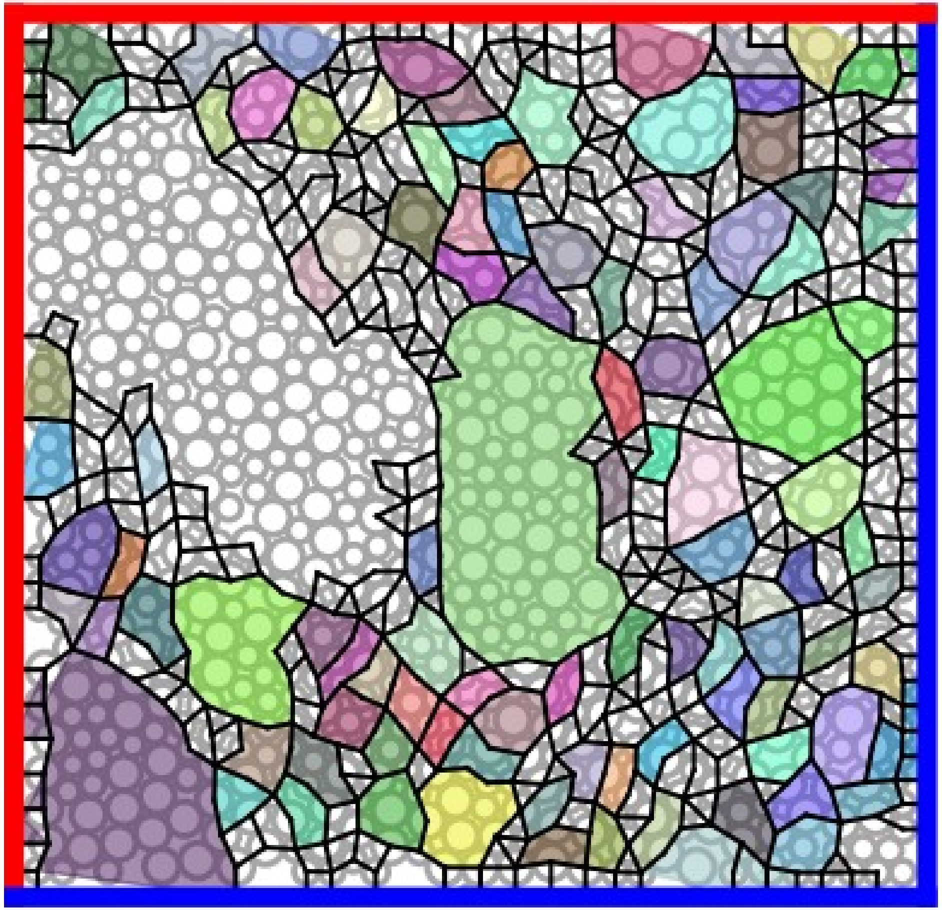



- 50% probability of finding a system-spanning cluster
- no tossing out of rattlers

# Characterizing floppy regions



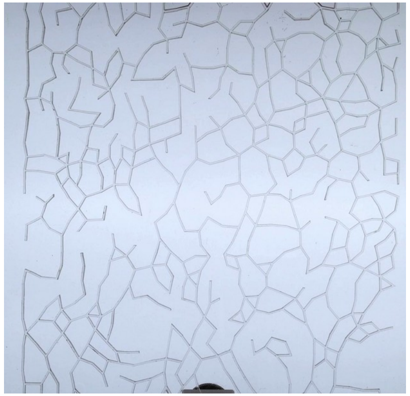

Liu, Kollmer, Daniels, Schwarz, Henkes *PRL* (2021)

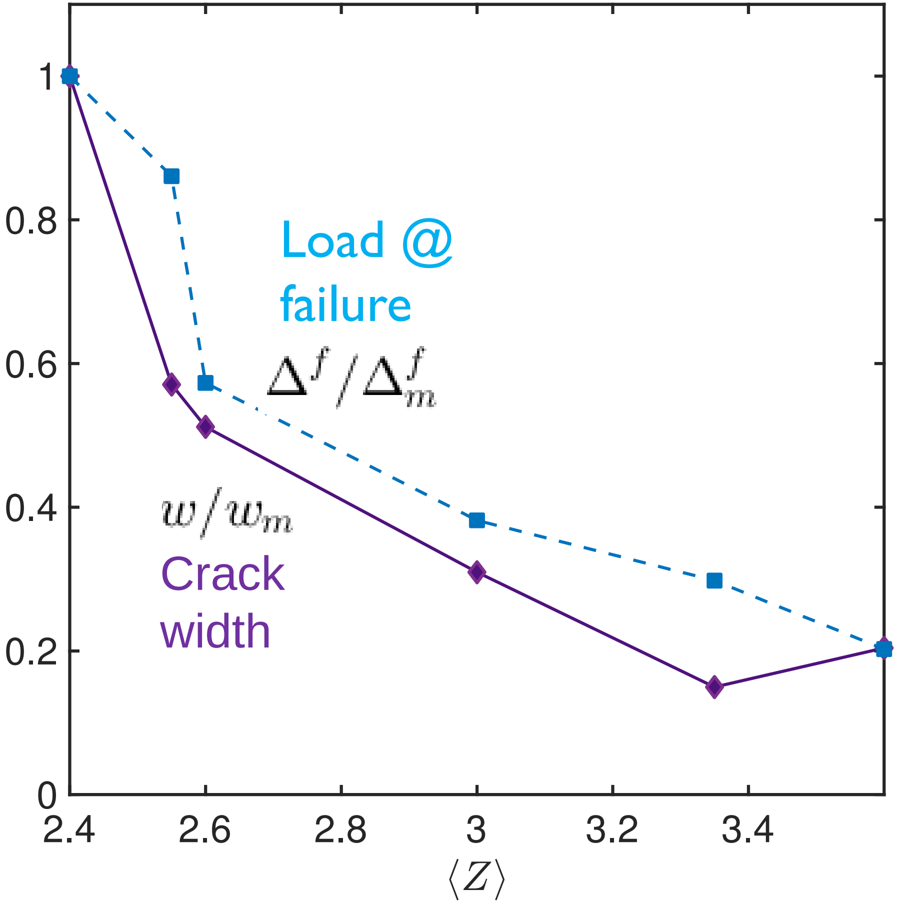


Do floppy regions forecast failure locations?

# Recall: ductile vs. brittle behavior

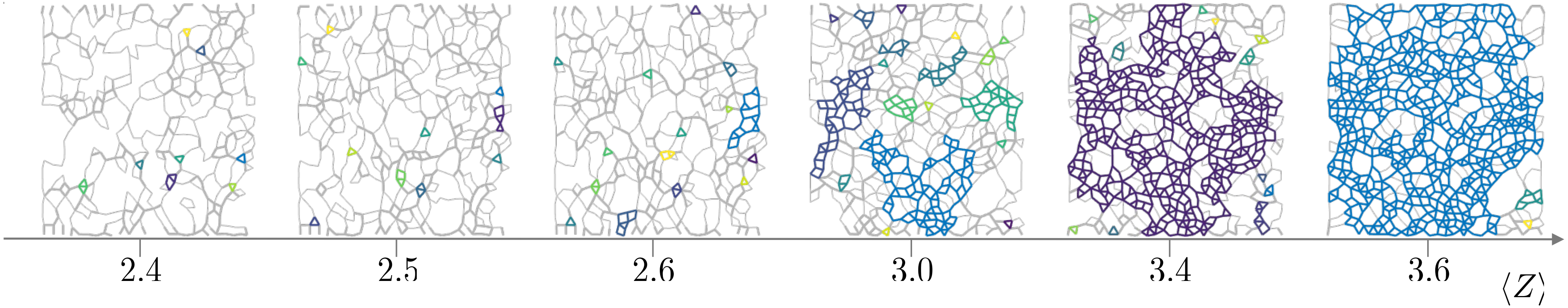
**Ductile-like:**
- Diffuse
- Progressive
- High deformation

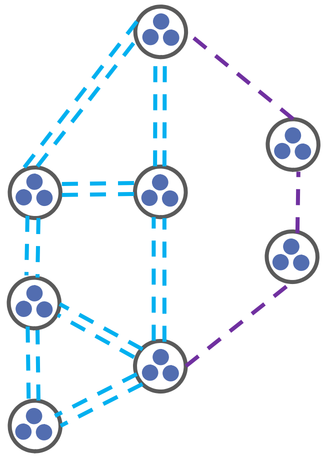




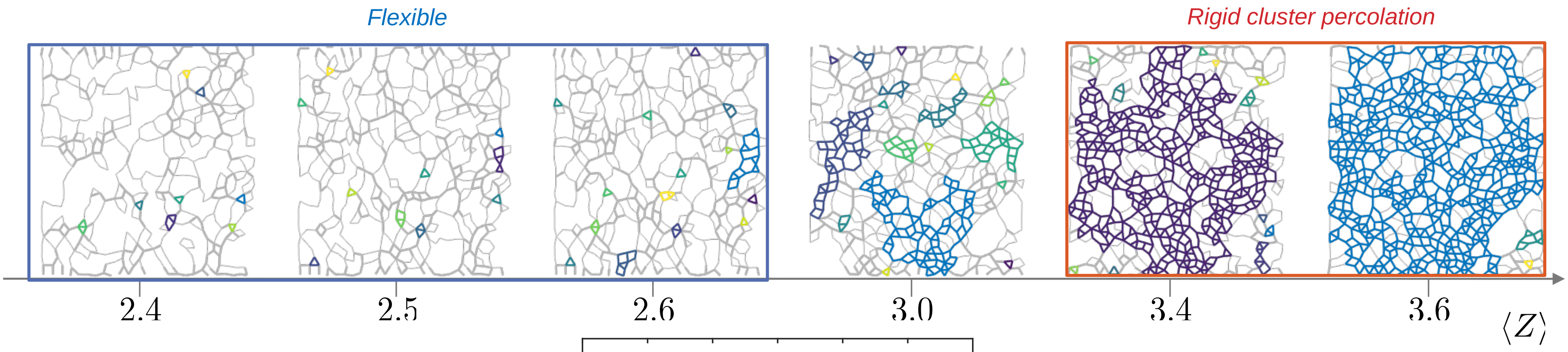

**Brittle-like:**
- Localized
- Catastrophic
- Low deformation

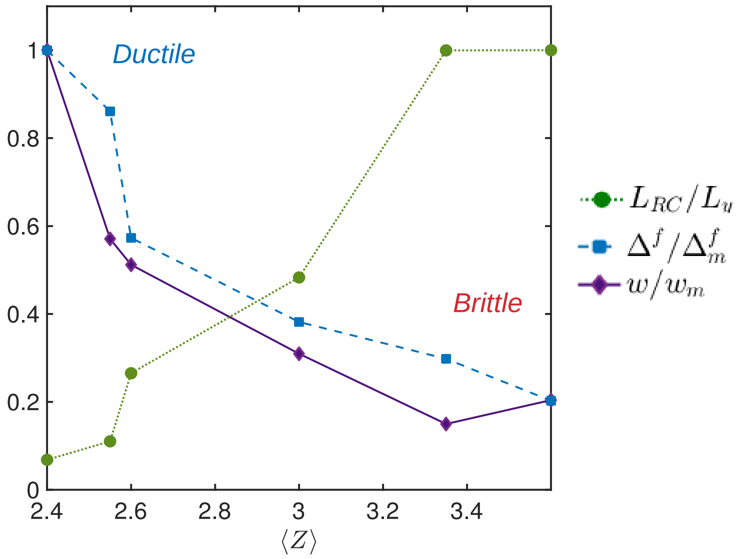
# Identify rigid clusters via pebble game



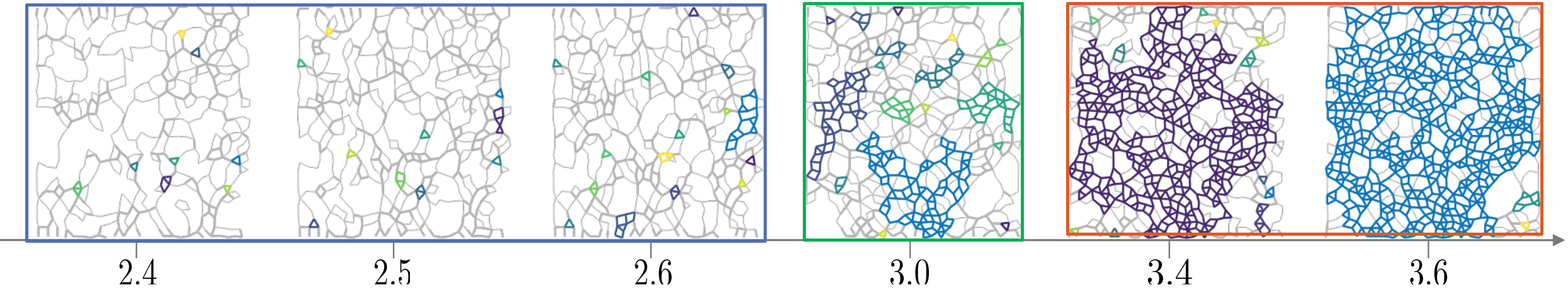
# Compare to failure dynamics



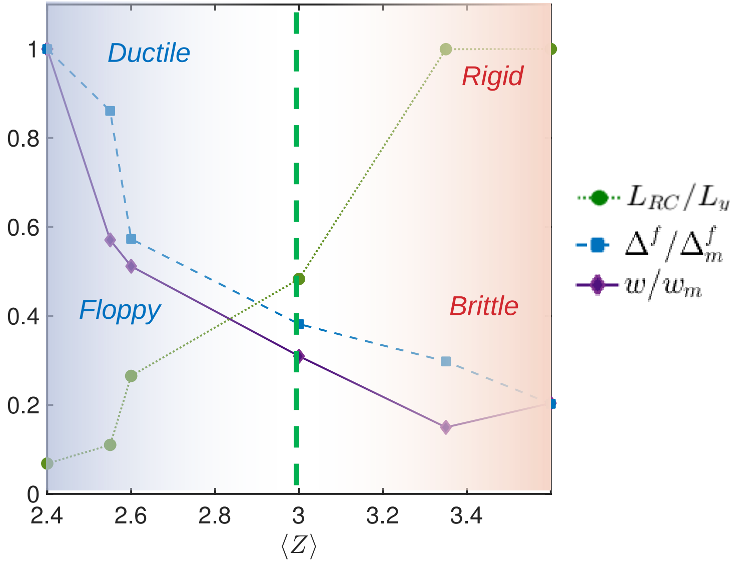




Size of rigid cluster
Force drop
Width of crack

# Lattice failure transition point




Rigidity percolation associated to the failure transition



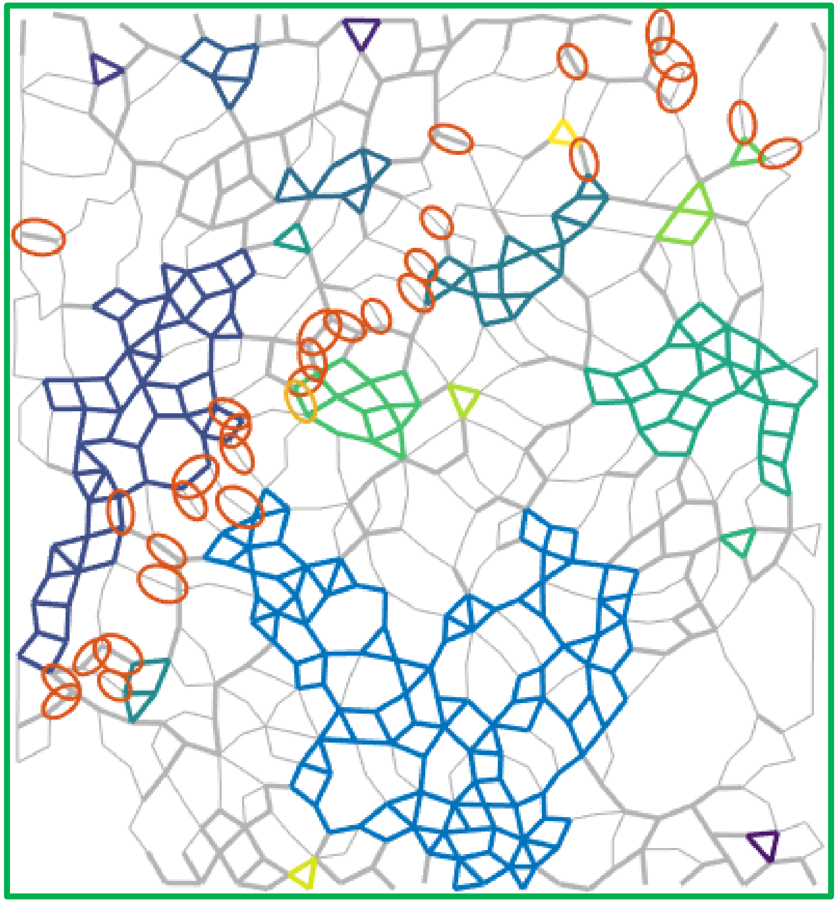

Transition point of lattice failure close to

$$Z_{\text{iso}}^{\mu=\infty}$$

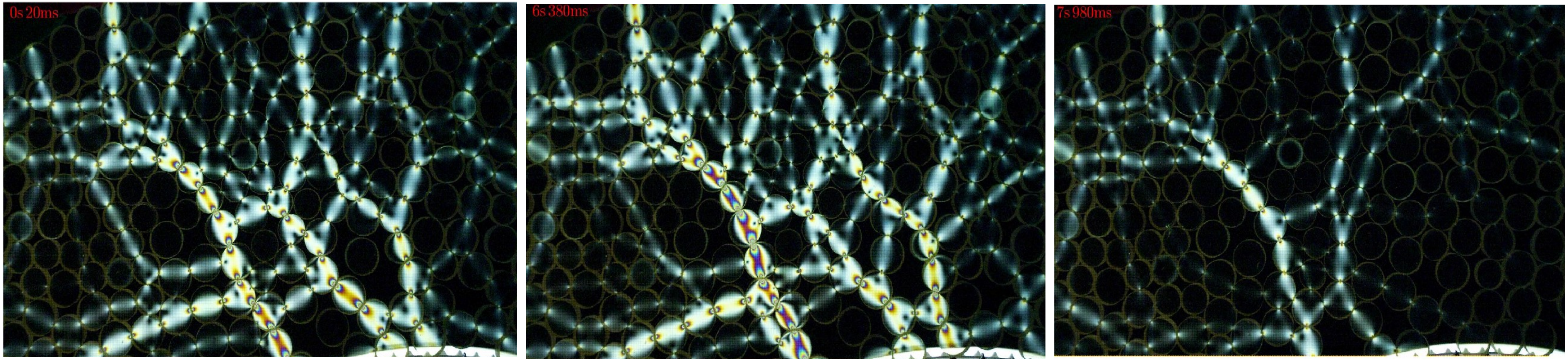
Berthier, Kollmer, Henkes, Liu, Schwarz, Daniels. *Phys. Rev. Mat.* (2019)

# Most failures occur on flexible bonds

<z> = 3.0

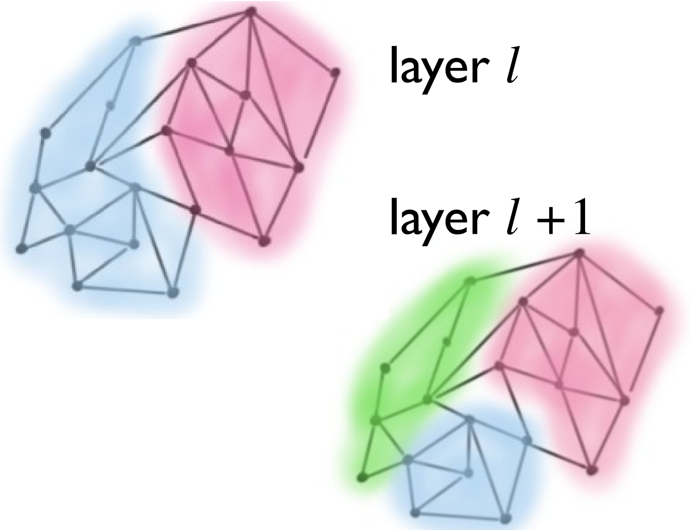


# Forecasting loss of rigidity




- Multilayer community detection
- GenLouvain modularity maximization

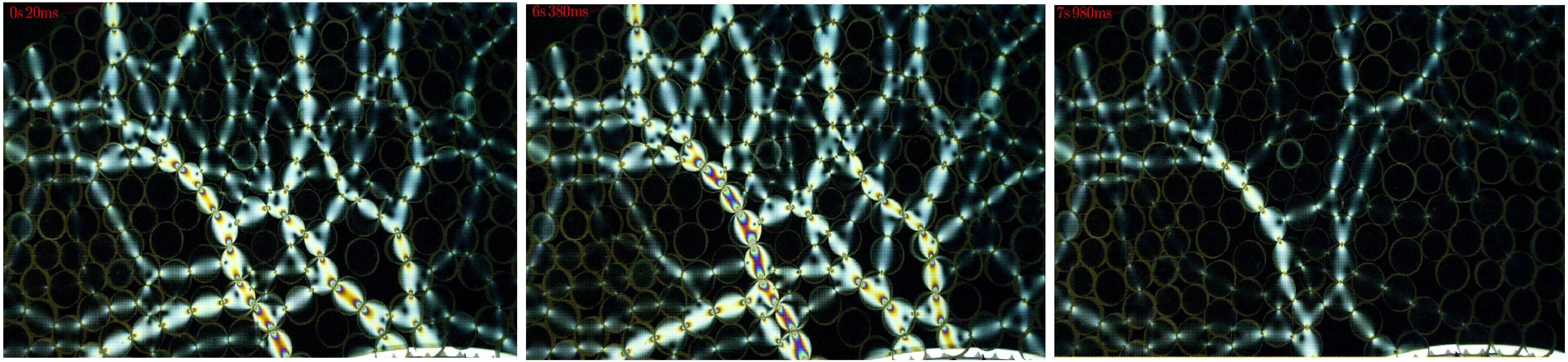
$$Q = \frac{1}{2\mu} \sum_{ij\ell m} [(A_{ij\ell} - \gamma P_{ij\ell})\delta_{\ell m} + \omega_{j\ell m}\delta_{ij}]\delta(c_{i\ell}, c_{jm})$$




Mucha, Richardson, Porter, Onnela, *Science* (2010)

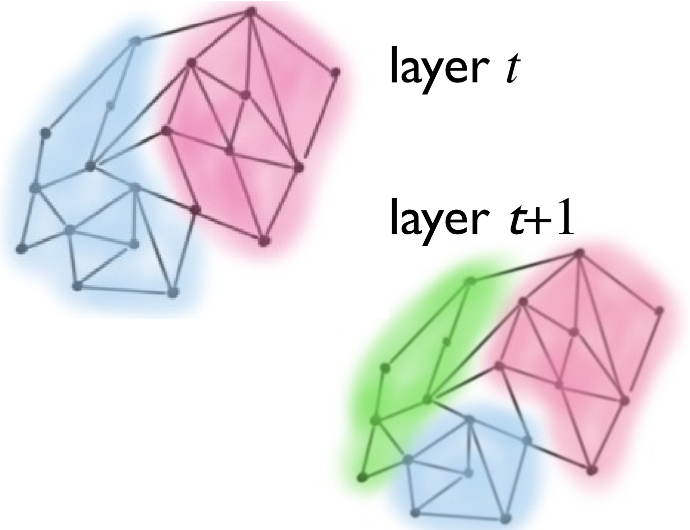
http://netwiki.amath.unc.edu

# Forecasting loss of rigidity

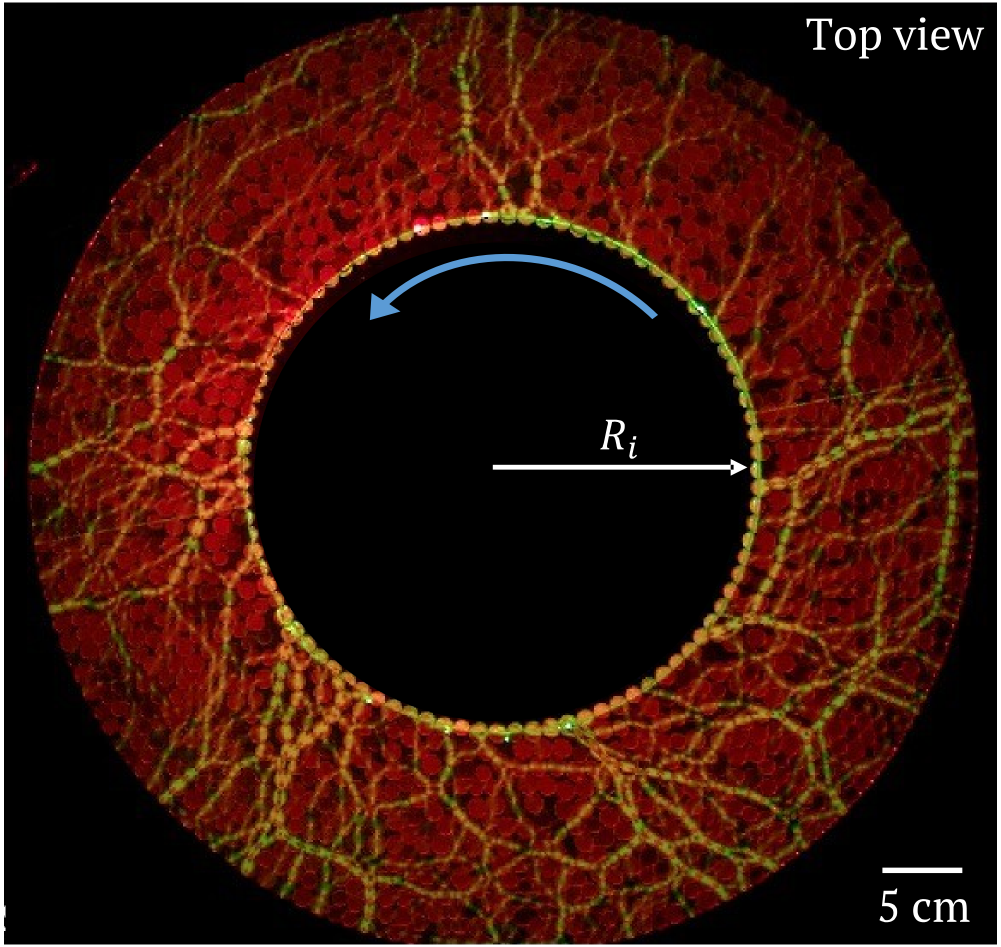


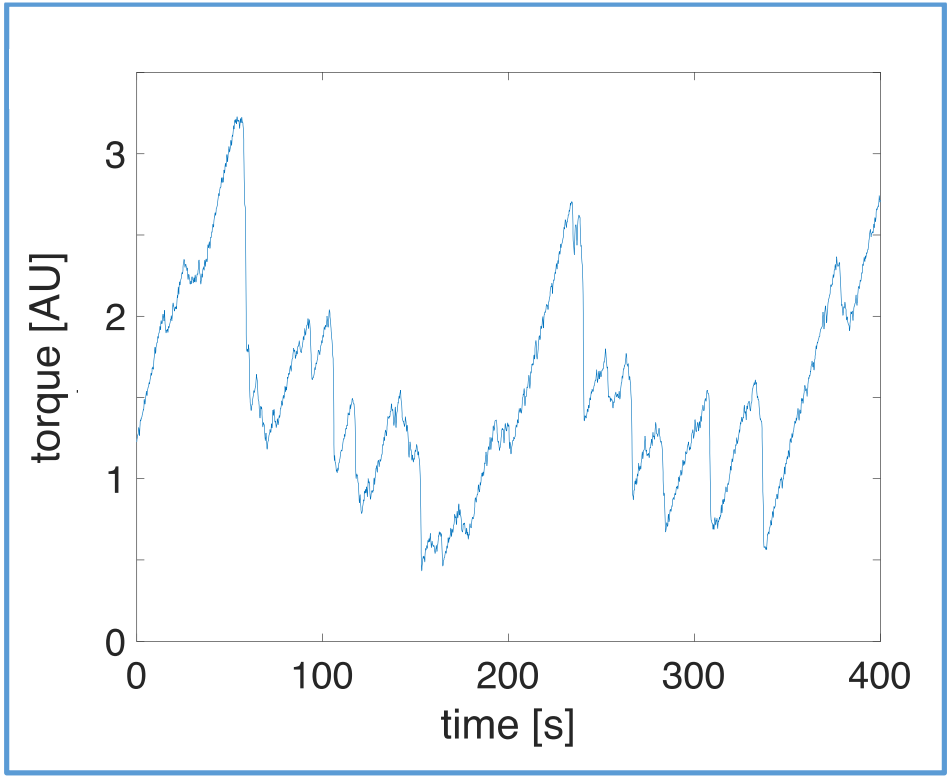

Farnaz Fazelpour

- Size: number of particles in community
- Strength: average interparticle force in community
- Volatility: how much communities change from layer to layer

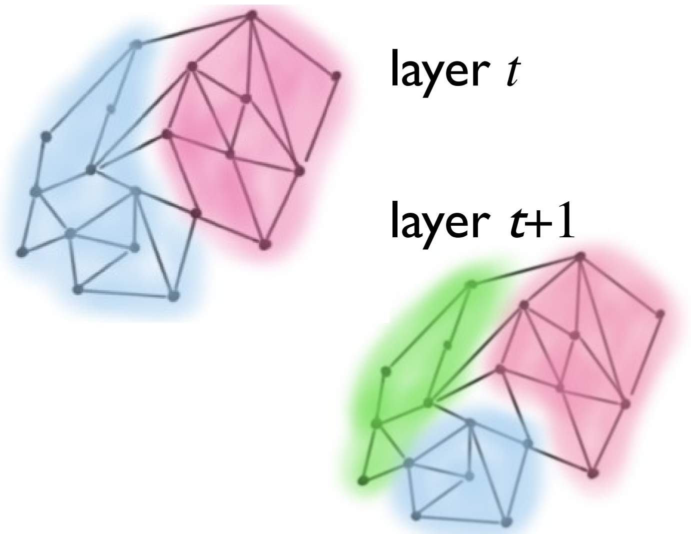


# Examine a series of stick-slip failures

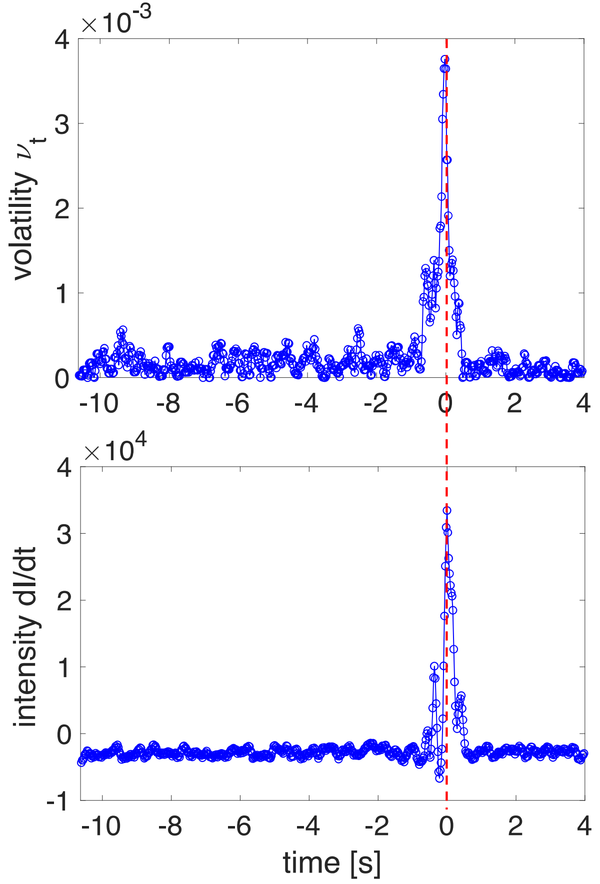


# Volatility changes precede image intensity changes?



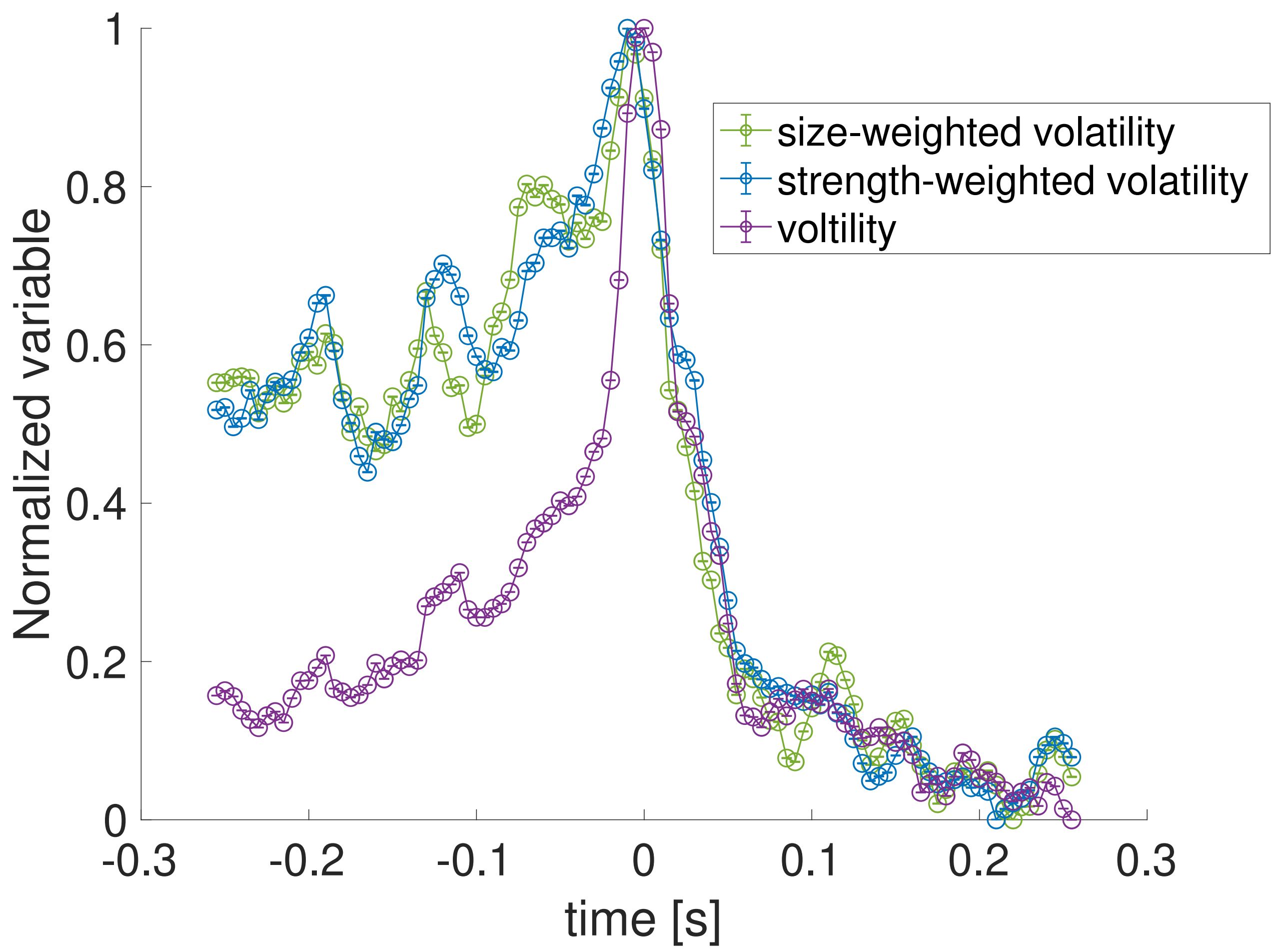

$$\upsilon_{tc} = \frac{|c_t \Delta c_{t+1}|}{|c_t \cup c_{t+1}|}$$

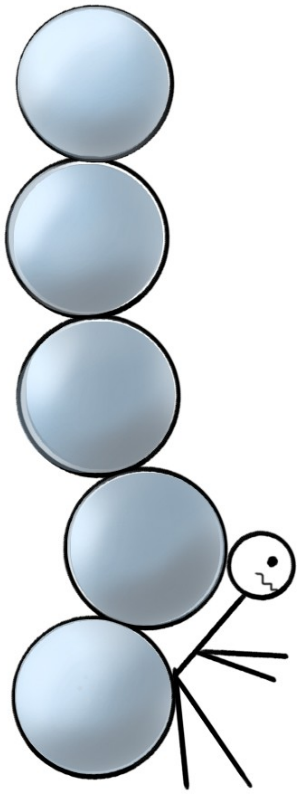



$|c_t \Delta c_{t+1}|$ : distinct nodes

$|c_t \cup c_{t+1}|$ : total nodes

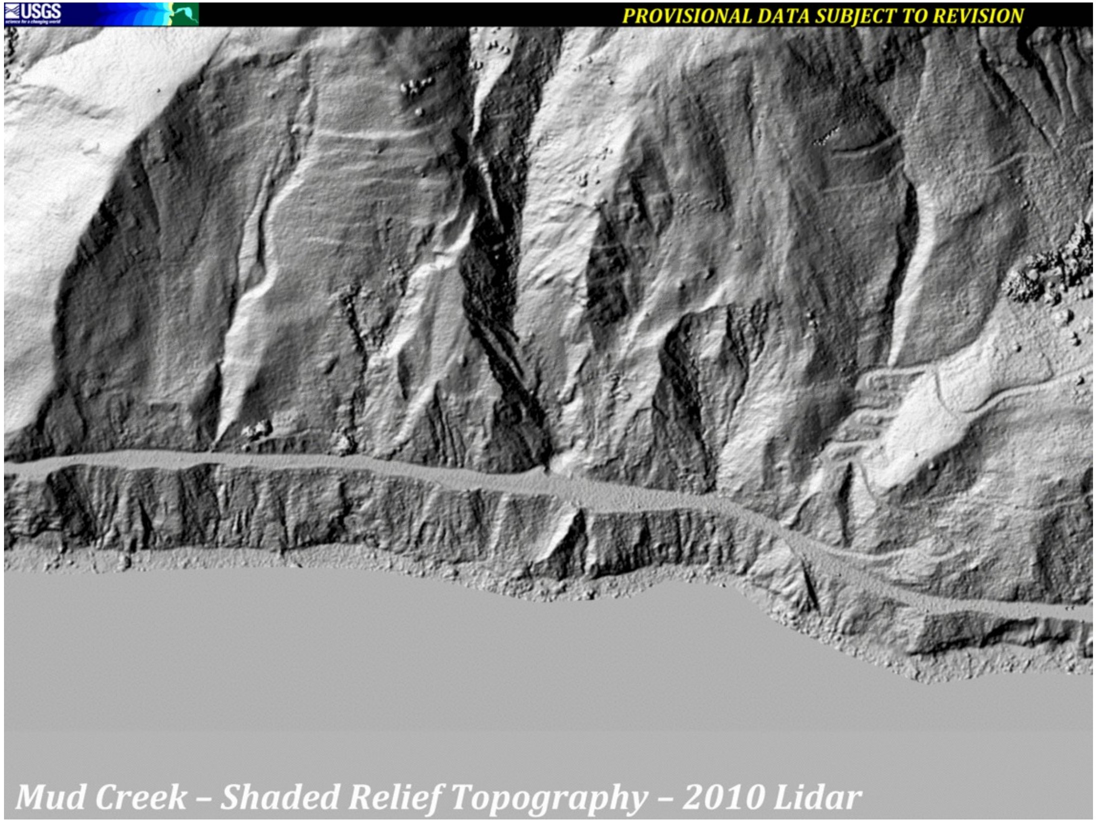
# Weak chains matter



# How about for real landslides?

Handwerger et al. *Scientific Reports* (2019)

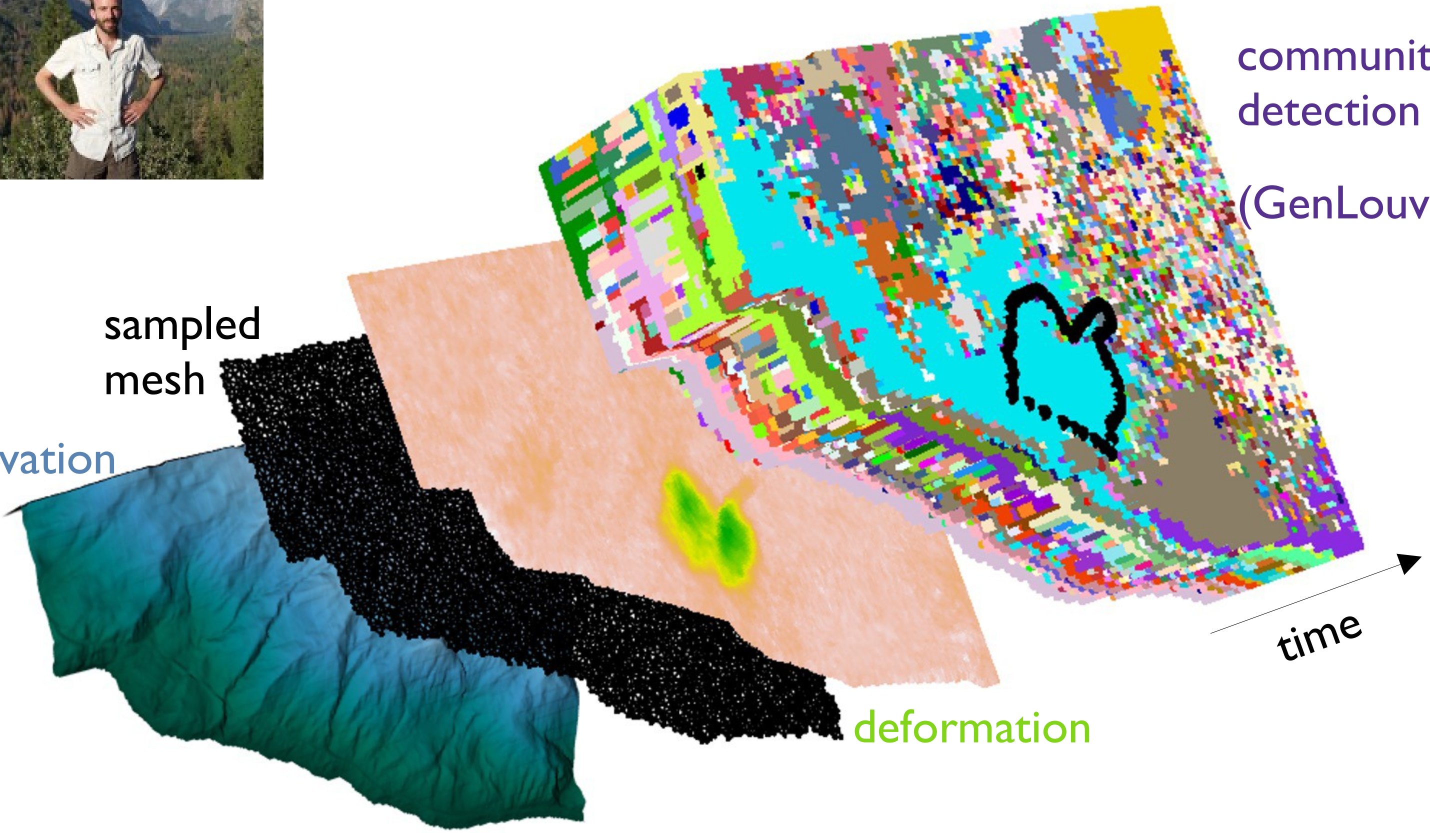



https://www.usgs.gov/media/images/mud-creek-shaded-relief-topography-2010-2017

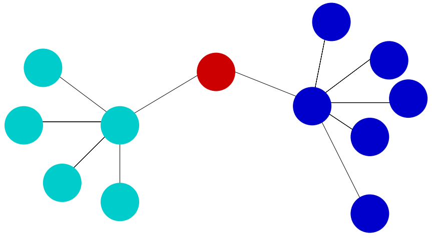
# Forecasting loss of rigidity

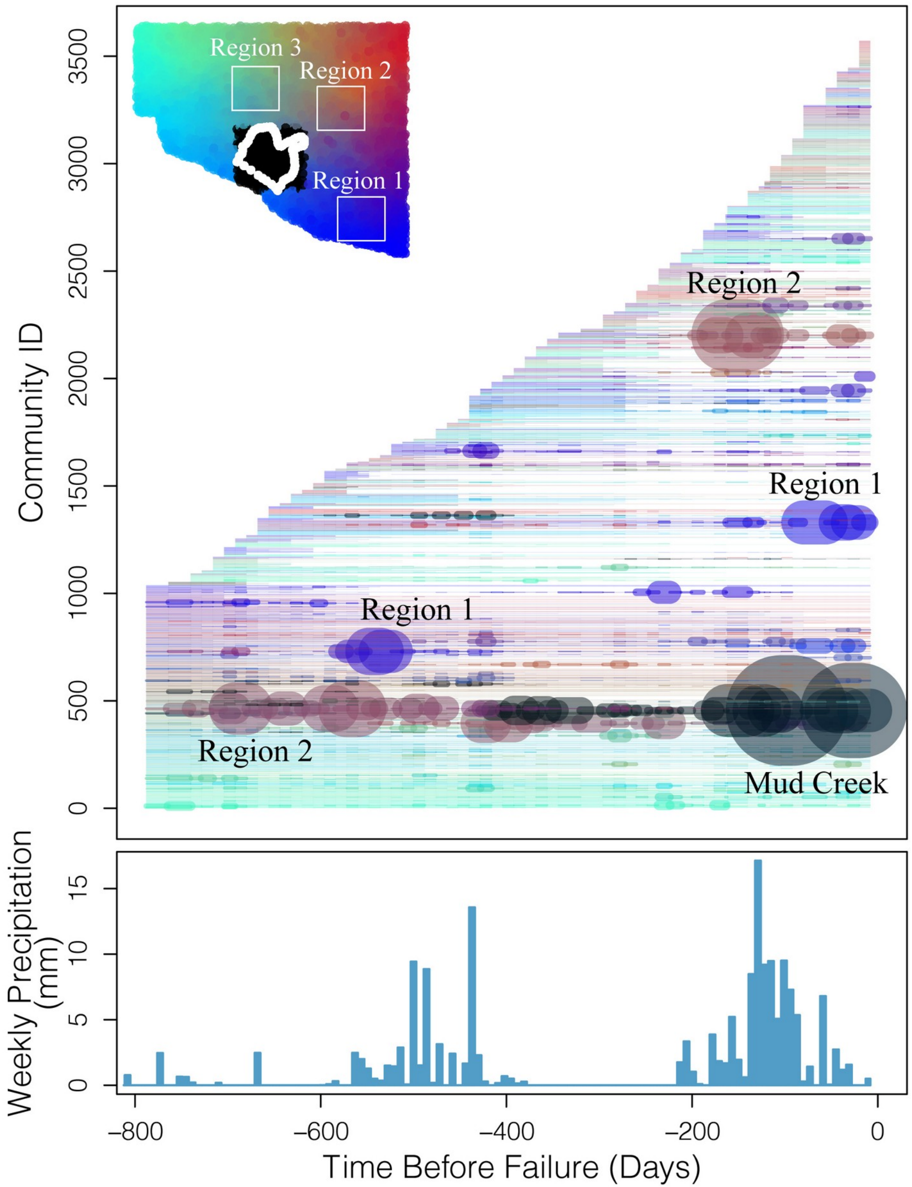
Vrinda Desai

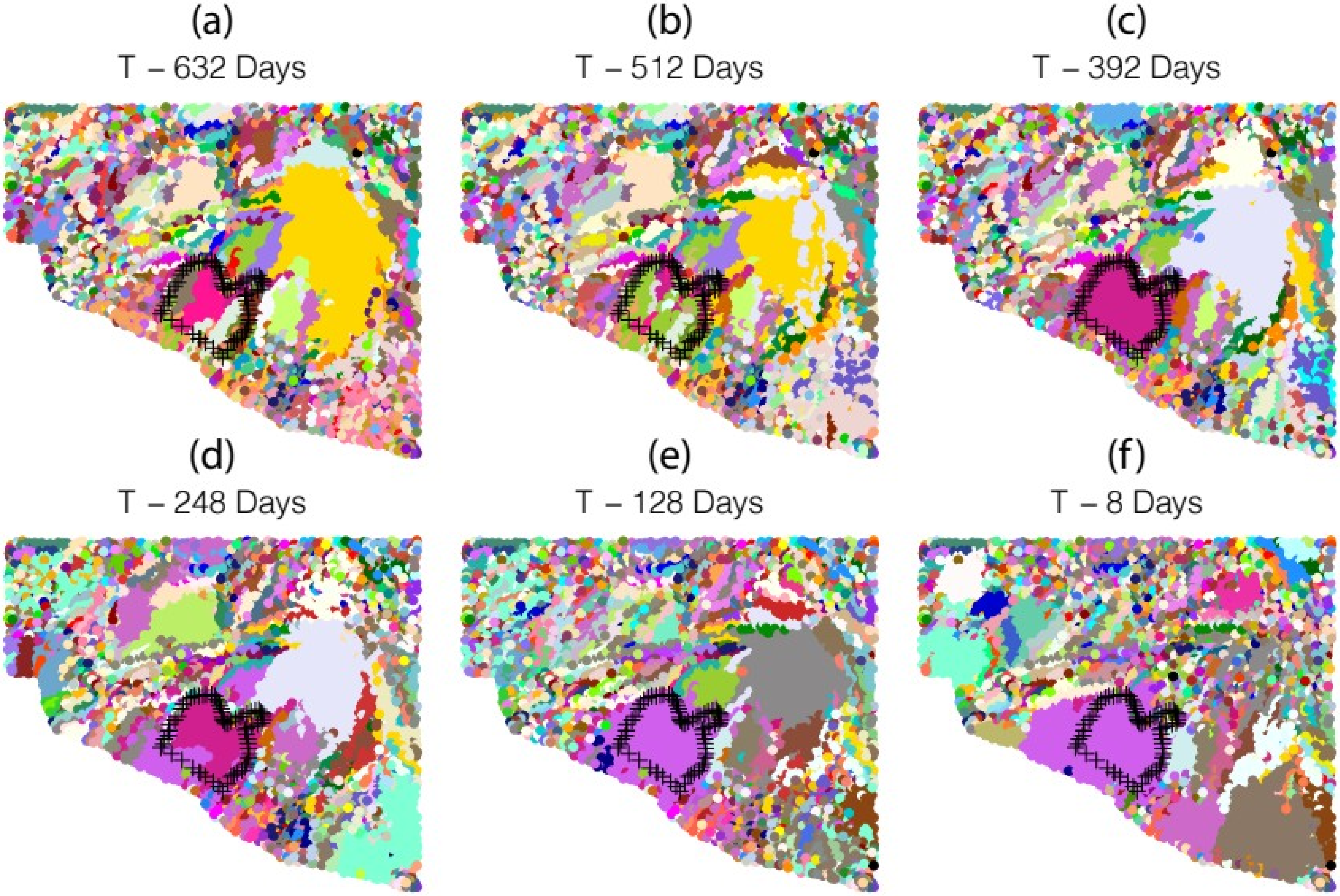
Al
Handwerger (JPL)



# Which locations have reliable community detection?






(a)
T – 632 Days
(b)
T – 512 Days
(c)
T – 392 Days
(d)
T – 248 Days
(e)
T – 128 Days
(f)
T – 8 Days

# Community Persistence $\Pi$

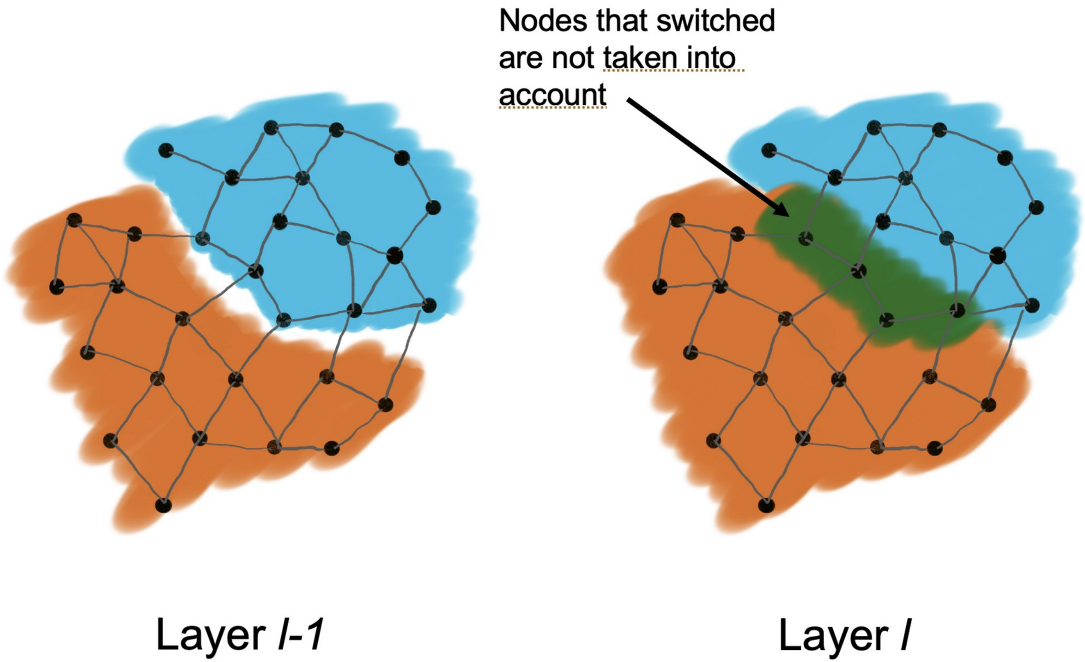
A measure based on the stability of nodal composition for each community in relation to community size for each layer *l*

$$\Pi = \frac{1}{N} \sum_{c} \frac{|c_{l-1} \cap c_l|}{n_{c,l}}$$

*N* : Total number of nodes

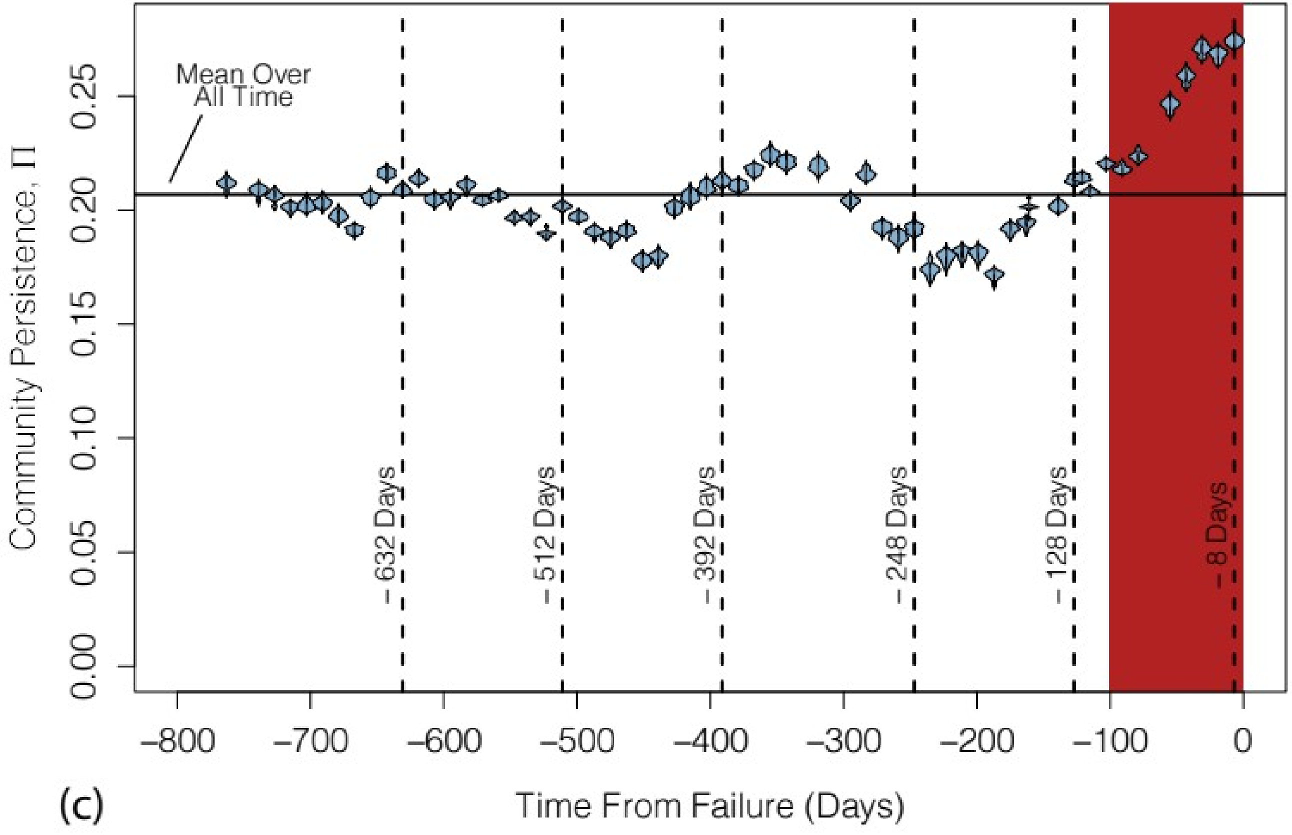
$n_{c,l}$ : Number of nodes in community *c* at layer *l*

$|c_{l-1} \cap c_l|$ : number of nodes present in community *c* in both layers *l* and *l-1*

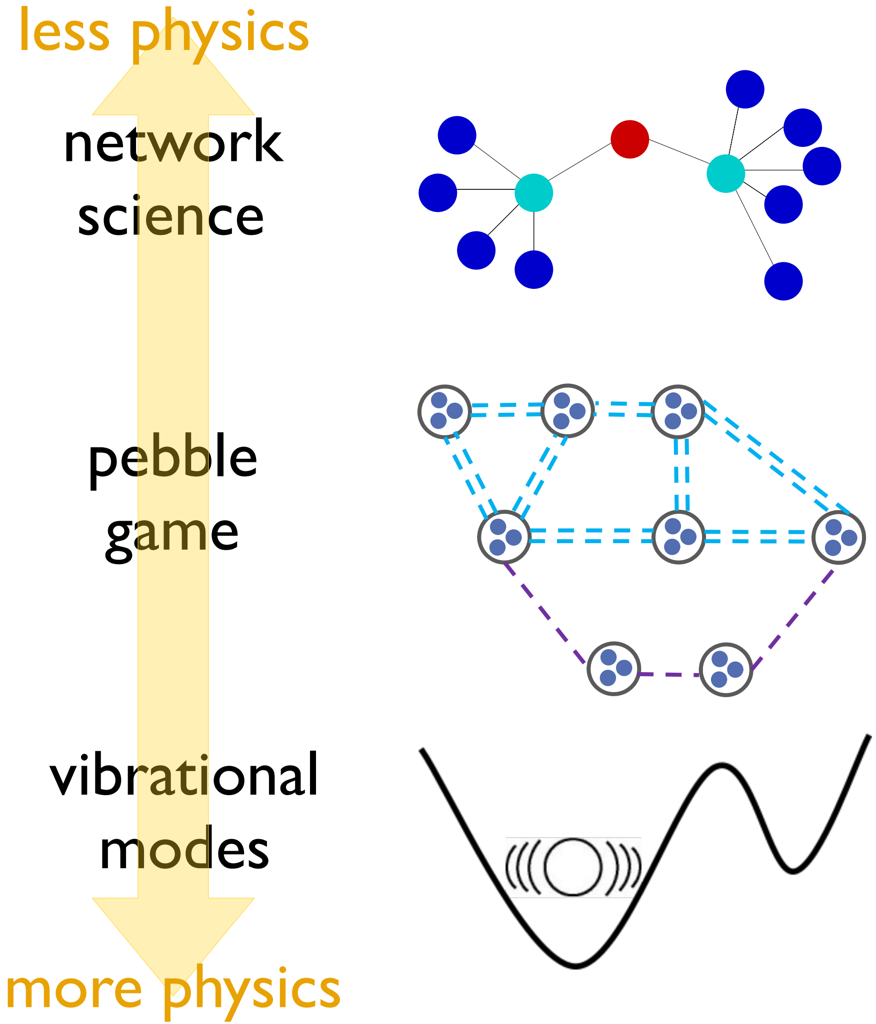



NC STATE UNIVERSITY

# Increased community persistence forecasts failure



# General Conclusions

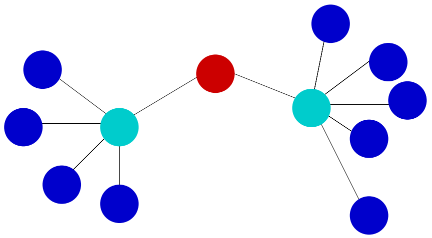



- more physics gives you better predictions and a better understanding …
- but simple models are surprisingly effective
- sometimes topology is a strong control

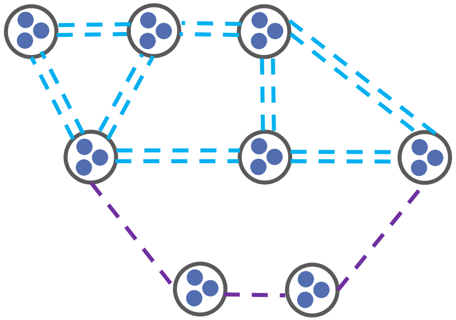
# Conclusions

less physics

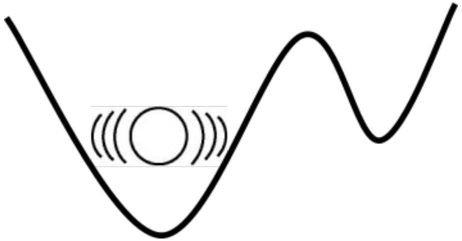
network science



pebble game



vibrational modes



more physics

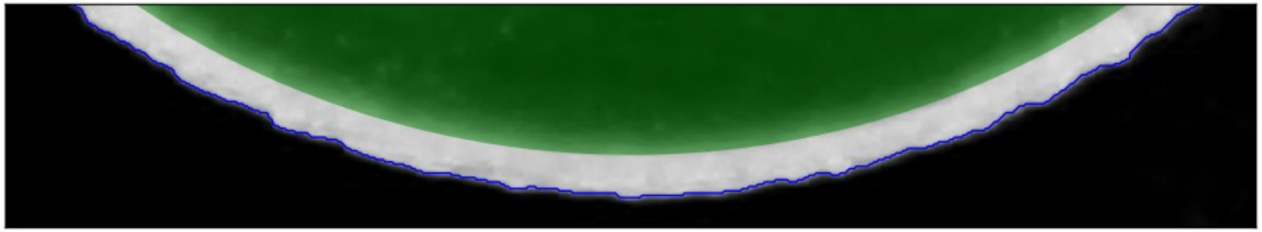
- centrality identifies likely force chain locations, lattice failure locations
- communities-detected change character ahead of failure
- granular packings: pebble game identifies same rigid areas as vibrational modes
- floppy areas may be more prone to failures (at least for some lattices)

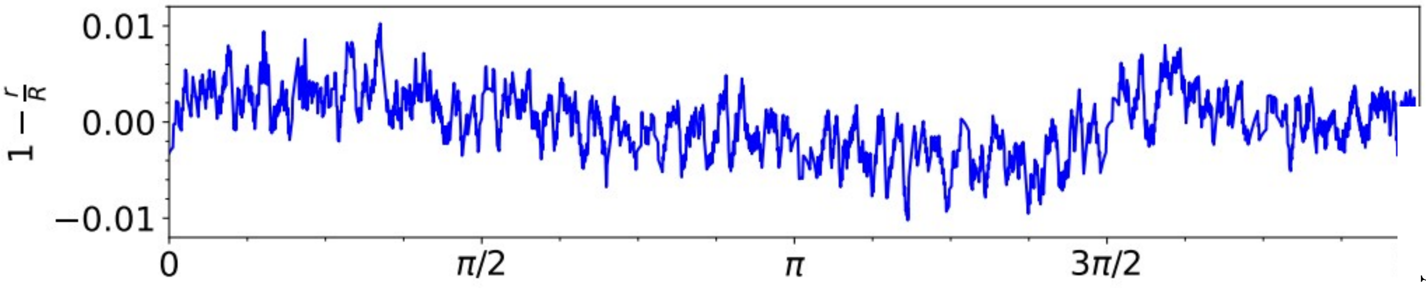
# Open Science Tools

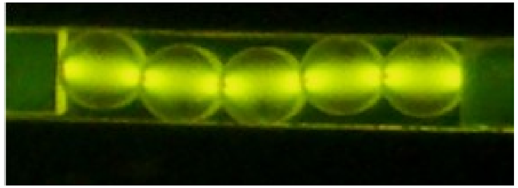
- Data from our papers: http://datadryad.org
- Photoelastic Granular Solver: Jonathan Kollmer
  github.com/jekollmer/PEGS
- Rigidity Toolbox: Silke Henkes
  https://github.com/silkehenkes/RigidLibrary
- NetWiki: Mason Porter, Peter Mucha
  http://netwiki.amath.unc.edu/
- Brain Connectivity Toolbox: Mikail Rubinov, Olaf Sporns
  http://www.brain-connectivity-toolbox.net/

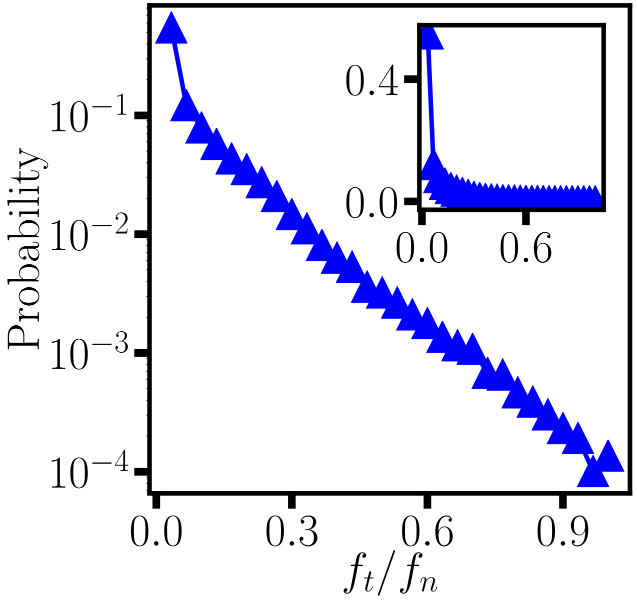
# Real particles

Kool, Charbonneau, Daniels, arXiv
Liu, Kollmer, Daniels, Schwarz, Henkes *PRL* (2021)



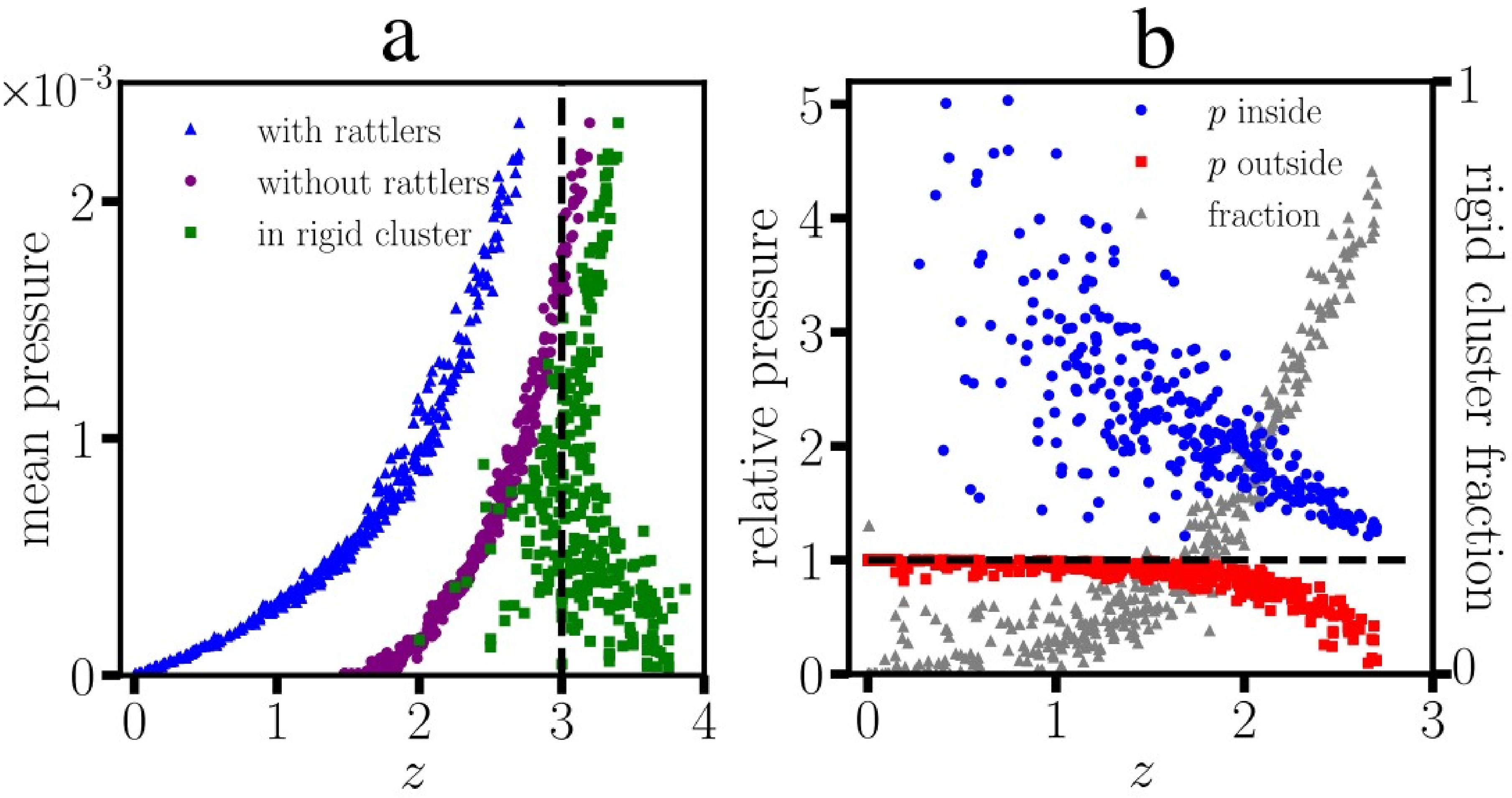


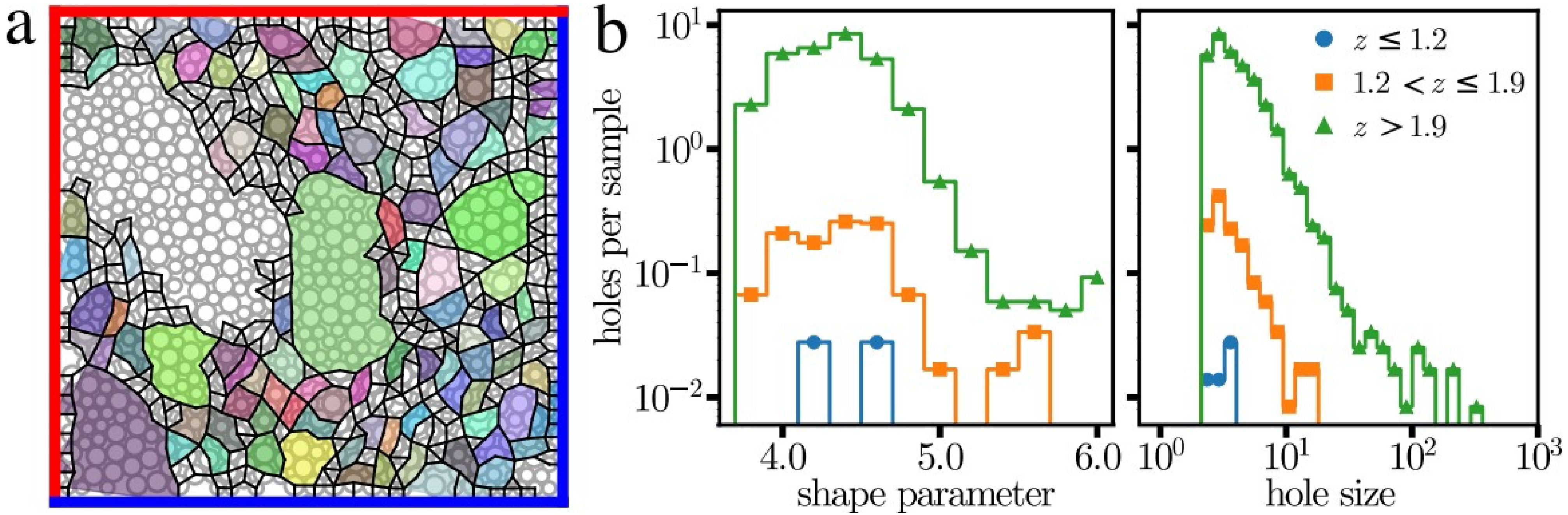



- are rough
- deform elastically
- are dissipative
- might not have μ = constant

# Rigid vs. floppy clusters



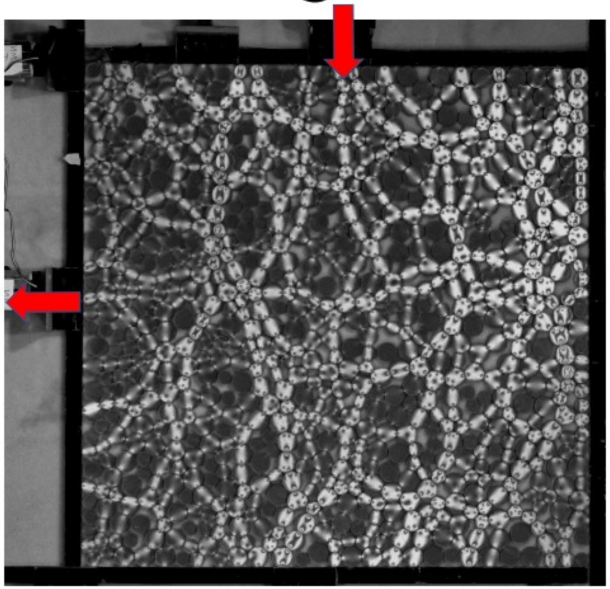
# Characterizing floppy regions



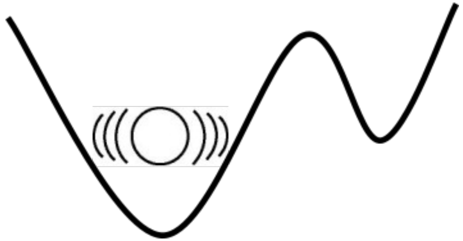

Liu, Kollmer, Daniels, Schwarz, Henkes *PRL* (2021)

## Force Chains

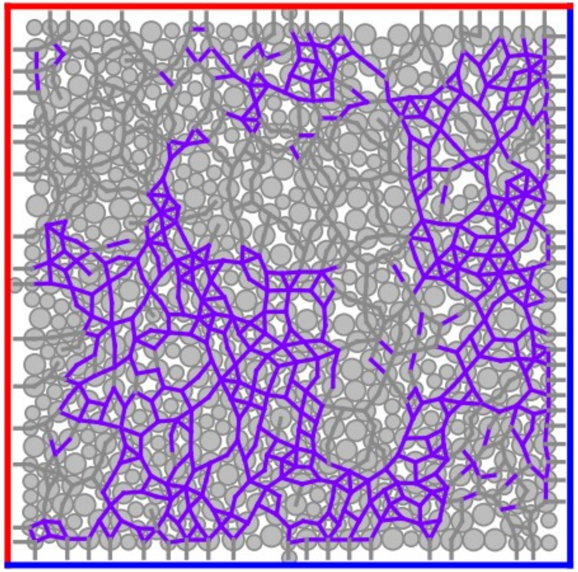
b



## Vibrational

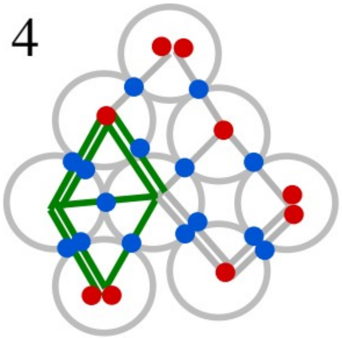


c



## Constraints

4



d