Semi-Classical-Fourier-Integral-Operator-Valued Pseudodifferential Operators and Scattering in a Strong Magnetic Field

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Introduction

The Model The Scattering Operator The Scattering Amplitude A Representation of the Scattering Amplitude

Idea of Proof

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For $n \ge 3$ and b > 0 let $H(b) = H_0(b) + bV$, where the electric potential V satisfies

$$V = V(x, y, z) \in C_c^{\infty}(\mathbb{R}^n)$$
$$H_0(b) = \sum_{j=1}^d \left[\left(D_{x_j} - \frac{b\mu_j}{2} y_j \right)^2 + \left(D_{y_j} + \frac{b\mu_j}{2} x_j \right)^2 \right] - \Delta_z$$

We consider the case 0 < 2d < n. $\mu_j > 0, j = 1, \dots, d$.

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We consider the case 0 < 2d < n. $\mu_j > 0, j = 1, \dots, d$. b — the strength of the magnetic field Let $h = \frac{1}{\sqrt{b}}$ be the semi-classical parameter.

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The wave operators for the pair $(H_0(b), H(b))$ defined by

$$W_{\pm}= ext{s-}\lim_{t
ightarrow\pm\infty}U(t)U_0(-t) ext{ in }L^2(\mathbb{R}^n)$$

exist and are complete, where

$$U(t) = e^{-rac{i}{\hbar}tH(b)}, \ U_0(t) = e^{-rac{i}{\hbar}tH_0(b)}, \ t \in \mathbb{R}$$

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We can therefore define the scattering operator S by setting

$$S=W_+^*W_-.$$

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For E > 0, there exists an operator

$$S(E,b): L^2\left(\mathbb{R}^{2d} \times \mathbb{S}^{n-2d-1}\right) \to L^2\left(\mathbb{R}^{2d} \times \mathbb{S}^{n-2d-1}\right)$$

such that

$$S(E, b)\mathcal{F}_0(E) = \mathcal{F}_0(E)S(b),$$

where $\mathcal{F}_0(E)$ is a version of the Fourier transform.

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where $\mathcal{F}_0(E)$ is a version of the Fourier transform. $\mathcal{T}(\lambda b, b) = S(E, b) - I$ is called the scattering amplitude at energy E

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$$\begin{split} \mathcal{T}(\lambda b, b) &= \frac{2\pi}{h^3} \tilde{U}^* \tilde{\mathcal{F}}_0\left(\frac{\lambda}{h^2}\right) R_- \left[h^2 \Delta_z, \chi_1\right] \int_0^{T_0} e^{-\frac{i}{h}t \mathcal{P}_N(\lambda)} dt \\ &\left[h^2 \Delta_z, \chi_2\right] R_+ \tilde{\mathcal{F}}_0\left(\frac{\lambda}{h^2}\right)^* \tilde{U} + \mathcal{O}\left(h^{N-2}\right), \end{split}$$

where $T_0 >> 0$ and \tilde{U} is a metaplectic (i.e., a change of variable) operator.

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We construct a parametrix (i.e., an approximation) to $e^{-\frac{i}{\hbar}t\mathcal{P}_N}$ for small t of the form

$$\frac{1}{(2\pi h)^{n-2d}}\int_{\mathbb{R}^{n-2d}}e^{\frac{i}{h}(\phi(t,y,z,\eta;\theta)-z'\cdot\theta)}\upsilon(t,y,z,\eta;\theta)d\theta$$

where ϕ has certain properties such that for every (y, η) this a semi-classical Fourier integral operator (h-FIO).

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where ϕ has certain properties such that for every (y, η) this a semi-classical Fourier integral operator (h-FIO). It is a symbol $\tau(y, \eta)$ in (y, η) , which is quantized as $\tau^w(y, h^2 D_y)$. Thus $\tau \in S^{\frac{1}{2}}$ and it is an h-FIO-valued symbol.

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