Joint tensor modeling of single cell 3D genome and epigenetic data with Muscle

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- Far apart in genomic distance, but physically close



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- Hi-C : Genome-wide physical 3D contact level
- Contact map ~ adjacency matrix (weighted graph)





• Bulk Hi-C : 23 (or 20) matrices

Chr1

Chr2

Chr22

ChrX



• Bulk Hi-C : 23 (or 20) matrices

Main objectives

Cell type specific 3D genome structure!

- Cell type clustering
- Contact map of each cell type (TAD / AB comp)
- Multi modal data integration including Hi-C

• Tool : tensor/matrix decomposition

Contributions

Methodology

- Joint decomposition of multiple tensor objects

(Common parameter / Semi-nonnegative Tensor / Data balancing)

- Optimality properties for the Alternating Least Squares algorithm

Interpretation

- Unification of estimation target and parameter of interests

(e.g., Mean contact pattern, cell type information)

- **Direct** / does not require complex modification on parameters

• Estimation of signal tensor(\mathcal{M}) with the existence of noise

(Chr=1, R=2 case)

$$\mathcal{Y} = \mathcal{M} + \mathcal{E}$$
$$= \sum_{r=1}^{R} (A_r B_r^T) \circ c_r + \mathcal{E},$$

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• Estimation of signal tensor(\mathcal{M}) with the existence of noise

(Chr=1, R=2 case)

$$= \mathcal{M} + \mathcal{E}$$

$$= \mathbf{c} + \mathbf{c}$$

2

$$\mathcal{Y} = \mathcal{M} + \mathcal{E}$$
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• Estimation of signal tensor(\mathcal{M}) with the existence of noise

(Chr=1, R=2 case)

$$= \mathcal{M} + \mathcal{E}$$

$$= \mathbf{c} + \mathbf{c} + \mathbf{c}$$

$$\mathcal{Y} = \mathcal{M} + \mathcal{E}$$
$$= \sum_{r=1}^{R} (A_r B_r^T) \circ c_r + \mathcal{E},$$

Joint BTD

- Use all chromosome(tensor)'s cell information
 - Tensor decomposition with common cell loading (

Joint BTD

• Model

$$\begin{split} \min \sum_{chr=1}^{23} \|\mathcal{Y}_{chr} - \mathcal{M}_{chr}\|_{F}^{2} \\ \text{s.t.} \quad \mathcal{M}_{chr} = \sum_{r=1}^{R} (A_{chr,r} B_{chr,r}^{T}) \circ c_{r} \\ \quad A_{chr,r}, B_{chr,r} : \text{Loci loadings specific for each chr} \\ \quad c_{r} \geq 0 : \text{Cell loading common for all chr (Non-negative)} \end{split}$$

DNA methylation represses gene expression !

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Multimodality and Muscle

- DNA methylation & scHi-C sequenced at the same cell level
- MUSCLE : A semi-non negative joint decomposition of MUItiple

Single CelL tEnsors

Model

$$\begin{aligned} \mathcal{Y}_{chr} &= \mathcal{M}_{chr} + \mathcal{E}_{chr}, \quad \epsilon_{i,j,c,chr} \stackrel{i.i.d}{\sim} N(0,\sigma_1^2), \quad \forall chr \in [Chr], \\ Y^k &= M^k + E^k, \quad \epsilon_{l,c}^k \stackrel{i.i.d}{\sim} N(0,\sigma_2^2), \quad \text{for } k \in \{CG, CH\}, \\ \text{s.t. } \mathcal{M}_{chr} &= \sum_{r=1}^R \left(A_{chr,r} B_{chr,r}^T \right) \circ c_r, \quad M^k = \sum_{r=1}^R v_r^k \circ c_r, \quad \text{for } k \in \{CG, CH\}, \quad \forall chr \in [Chr], \\ c_r \geq 0, \quad \|c_r\| = 1, \quad \forall r \in [R], \text{ and } \frac{\sigma_1^2}{\sigma_2^2} = \frac{N_h}{N_m}, \end{aligned}$$

- Variances are not ancillary to mean parameters
- Proportional variance model

Model

$$\begin{aligned} \mathcal{Y}_{chr} &= \mathcal{M}_{chr} + \mathcal{E}_{chr}, \quad \epsilon_{i,j,c,chr} \stackrel{i.i.d}{\sim} N(0,\sigma_1^2), \quad \forall chr \in [Chr], \\ Y^k &= M^k + E^k, \quad \epsilon_{l,c}^k \stackrel{i.i.d}{\sim} N(0,\sigma_2^2), \quad \text{for } k \in \{CG, CH\}, \end{aligned}$$

s.t.
$$\mathcal{M}_{chr} &= \sum_{r=1}^R \left(A_{chr,r} B_{chr,r}^T \right) \circ c_r, \quad M^k = \sum_{r=1}^R v_r^k \circ c_r, \quad \text{for } k \in \{CG, CH\}, \quad \forall chr \in [Chr], \end{aligned}$$

$$c_r \geq 0, \quad \|c_r\| = 1, \quad \forall r \in [R], \text{ and } \frac{\sigma_1^2}{\sigma_2^2} = \frac{N_h}{N_m}, \end{aligned}$$

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Model

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- Variances are not ancillary to mean parameters
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• MLE equivalent problem

$$\min_{\substack{A_{chr,r},B_{chr,r},C_r \ge 0 \\ v_r^{CG}, v_r^{CH}, \|c_r\|_2 = 1}} \left\{ \frac{1}{N_h} \sum_{chr=1}^{23} \left\| \mathcal{Y}_{chr} - \sum_{r=1}^R (A_{chr,r} B_{chr,r}^T) \circ c_r \right\|_F^2 + \frac{1}{N_m} \sum_{k \in \{CG, CH\}} \left\| Y^k - \sum_{r=1}^R v_r^k \circ c_r \right\|_F^2 \right\}$$

$$scHi-C$$

$$ScHi-C$$

$$Methylation$$

$$matrices$$

• MLE equivalent problem

Data common cell loading c_r contains the shared information across modalities

$$\min_{\substack{A_{chr,r},B_{chr,r},c_{r}\geq 0\\v_{r}^{CG},v_{r}^{CH},\|c_{r}\|_{2}=1}} \left\{ \frac{1}{N_{h}} \sum_{chr=1}^{23} \left\| \mathcal{Y}_{chr} - \sum_{r=1}^{R} (A_{chr,r}B_{chr,r}^{T}) \circ c_{r} \right\|_{F}^{2} + \frac{1}{N_{m}} \sum_{k \in \{CG,CH\}} \left\| Y^{k} - \sum_{r=1}^{R} v_{r}^{k} \circ c_{r} \right\|_{F}^{2} \right\}$$

$$scHi-C$$

$$ScHi-C$$

$$Methylation$$

$$matrices$$

Muscle – ALS algorithm

Algorithm 1 Muscle ALS algorithm

- **Input:** scHi-C tensors $\mathcal{Y}_{chr} \in \mathbb{R}^{l_{chr} \times l_{chr} \times C}$, methylation matrices $Y^{CG}, Y^{CH} \in \mathbb{R}^{\sum_{chr} l_{chr} \times C}$, scHi-C loci loading rank $K_{chr}, \forall chr \in [Chr]$, data modality common rank R.
- **Output:** scHi-C loci loadings $A_{chr,r}, B_{chr,r} \in \mathbb{R}^{l_{chr} \times K_{chr}}$, methylation loci loadings $v_r^{CG}, v_r^{CG} \in \mathbb{R}^{\sum_{chr} l_{chr}}$, and data modality common cell loading vector $c_r \in \mathbb{R}^C_+$, $\forall chr \in [Chr]$ and $\forall r \in [R]$.
- 1: Initialize the decomposition objects $\tilde{\mathcal{Y}}_{chr} \leftarrow \mathcal{Y}_{chr} \forall chr \in [Chr], \tilde{Y}^{CG} \leftarrow Y^{CG}, \tilde{Y}^{CH} \leftarrow Y^{CH}$.

2: for
$$r = 1$$
 to R do

- 3: Initialize $\hat{A}_{chr,r}$, and $\hat{B}_{chr,r}$ by rank here that the core tensor size is absorb Pools all the loci loading information across the data
- 4: Initialize \hat{v}_r^k by first left singular vector of \tilde{Y}^k (singular value is absorbed), $\not \approx k \in \{CG, CH\}$.
- 5: while the convergence criterion is not met do

6: Update
$$\hat{c}_r \leftarrow \frac{\left(\frac{1}{N_h}\sum_{chr}Y_{chr}^T X_{chr} + \frac{1}{N_m}(\tilde{Y}^{CG})^T v_r^{CG} + \frac{1}{N_m}(\tilde{Y}^{CH})^T v_r^{CH}\right)_+}{\left\|\left(\frac{1}{N_h}\sum_{chr}Y_{chr}^T X_{chr} + \frac{1}{N_m}(\tilde{Y}^{CG})^T v_r^{CG} + \frac{1}{N_m}(\tilde{Y}^{CH})^T v_r^{CH}\right)_+\right\|_2},$$

- with $Y_{chr} = \text{unfold}_3(\tilde{\mathcal{Y}}_{chr}) \in \mathbb{R}^{(l_{chr}*l_{chr})\times C}$ and $X_{chr} = [(\hat{A}_{chr,r} \odot \hat{B}_{chr,r})\mathbf{1}_{K_{chr}}] \in \mathbb{R}^{(l_{chr}*l_{chr})}$. Update $(\hat{A}_{chr,r}, \hat{B}_{chr,r}) \leftarrow \text{Eigen}_{K_{chr}}(\tilde{\mathcal{Y}}_{chr} \times_3 \hat{c}_r), \forall chr \in [Chr]$. Note here that the eigenvalues are absorbed
- 7: Update $(A_{chr,r}, B_{chr,r}) \leftarrow \operatorname{Eigen}_{K_{chr}}(\mathcal{Y}_{chr} \times_3 \hat{c}_r), \forall chr \in [Chr].$ Note here that the eigenvalues are absorbed into $\hat{A}_{chr,r}$.
- 8: Update methylation loci loadings $v_r^k \leftarrow \tilde{Y}^k \hat{c}_r$. $\forall k \in \{CG, CH\}$.
- 9: end while

10: Update
$$\tilde{\mathcal{Y}}_{chr} \leftarrow \tilde{\mathcal{Y}}_{chr} - (\hat{A}_{chr,r}\hat{B}_{chr,r}^T) \circ \hat{c}_r$$
 and $\tilde{Y}^k \leftarrow \tilde{Y}^k - \hat{v}_r^k \circ \hat{c}_c$ for $\forall k \in \{CG, CH\}$.

11: end for

Muscle – ALS algorithm

Optimality properties for ALS algorithm

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- **Input:** scHi-C tensors $\mathcal{Y}_{chr} \in \mathbb{R}^{l_{chr} \times l_{chr} \times C}$, methylation matrices $Y^{CG}, Y^{CH} \in \mathbb{R}^{\sum_{chr} l_{chr} \times C}$, scHi-C loci loading rank $K_{chr}, \forall chr \in [Chr]$, data modality common rank R.
- **Output:** scHi-C loci loadings $A_{chr,r}, B_{chr,r} \in \mathbb{R}^{l_{chr} \times K_{chr}}$, methylation loci loadings $v_r^{CG}, v_r^{CG} \in \mathbb{R}^{\sum_{chr} l_{chr}}$, and data modality common cell loading vector $c_r \in \mathbb{R}^C_+$, $\forall chr \in [Chr]$ and $\forall r \in [R]$.
- 1: Initialize the decomposition objects $\tilde{\mathcal{Y}}_{chr} \leftarrow \mathcal{Y}_{chr} \forall chr \in [Chr], \tilde{Y}^{CG} \leftarrow Y^{CG}, \tilde{Y}^{CH} \leftarrow Y^{CH}$.

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$$\hat{c}_r \leftarrow \frac{\left(\frac{1}{N_h} \sum_{chr} Y_{chr}^T X_{chr} + \frac{1}{N_m} (\tilde{Y}^{CG})^T v_r^{CG} + \frac{1}{N_m} (\tilde{Y}^{CH})^T v_r^{CH}\right)_+}{\left\| \left(\frac{1}{N_h} \sum_{chr} Y_{chr}^T X_{chr} + \frac{1}{N_m} (\tilde{Y}^{CG})^T v_r^{CG} + \frac{1}{N_m} (\tilde{Y}^{CH})^T v_r^{CH}\right)_+ \right\|_2},$$

- with $Y_{chr} = \text{unfold}_3(\tilde{\mathcal{Y}}_{chr}) \in \mathbb{R}^{(l_{chr}*l_{chr})\times C}$ and $X_{chr} = [(\hat{A}_{chr,r} \odot \hat{B}_{chr,r})\mathbf{1}_{K_{chr}}] \in \mathbb{R}^{(l_{chr}*l_{chr})}$. Update $(\hat{A}_{chr,r}, \hat{B}_{chr,r}) \leftarrow \text{Eigen}_{K_{chr}}(\tilde{\mathcal{Y}}_{chr} \times_3 \hat{c}_r), \forall chr \in [Chr]$. Note here that the eigenvalues are absorbed
- 7: Update $(A_{chr,r}, B_{chr,r}) \leftarrow \operatorname{Eigen}_{K_{chr}}(\mathcal{Y}_{chr} \times_3 \hat{c}_r), \forall chr \in [Chr].$ Note here that the eigenvalues are absorbed into $\hat{A}_{chr,r}$.
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 and $\tilde{Y}^k \leftarrow \tilde{Y}^k - \hat{v}_r^k \circ \hat{c}_c$ for $\forall k \in \{CG, CH\}$.

11: end for
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Massively multiplex single-cell Hi-C

Vijay Ramani, Xinxian Deng, Ruolan Qiu, Kevin L Gunderson, Frank J Steemers, Christine M Disteche,

William S Noble, Zhijun Duan 🖂 & Jay Shendure 🖂

Nature Methods 14, 263–266(2017) Cite this article 6287 Accesses 197 Citations 94 Altmetric Metrics

nature methods

ARTICLES https://doi.org/10.1038/s41592-019-05



.i2019

Simultaneous profiling of 3D genome structure and DNA methylation in single human cells

Dong-Sung Lee^{1,5}, Chongyuan Luo^{2,3,5}, Jingtian Zhou^{2,5}, Sahaana Chandran¹, Angeline Rivkin², Anna Bartlett², Joseph R. Nery^{©2}, Conor Fitzpatrick⁴, Carolyn O'Connor⁴, Jesse R. Dixon^{©1*} and Joseph R. Ecker^{02,3*}

Large scale efforts

PLOS COMPUTATIONAL BIOLOGY



RESEARCH ARTICLE Capturing cell type-specific chromatin compartment patterns by applying topic modeling to single-cell Hi-C data

Hyeon-Jin Kim^{1†}, Galip Gürkan Yardımcı^{1†}, Giancarlo Bonora¹, Vijay Ramani^{1,2}, Jie Liu¹, Ruolan Qiu¹, Choli Lee¹, Jennifer Hesson^{3,5}, Carol B. Ware^{3,5} Jay Shendure¹, Zhijun Duan^{4,5}*, William Stafford Noble^{1,6}*



nature methods

BRIEF COMMUNICATIO https://doi.org/10.1038/s41592-019-0502-

Cell an2021

Changes in genome architecture and transcriptional dynamics progress independently of sensory experience during post-natal brain development

Longzhi Tan,^{1,2,*} Wenping Ma,^{3,4,5} Honggui Wu,^{3,4,6} Yinghui Zheng,^{3,4} Dong Xing,^{3,4} Ritchie Chen,¹ Xiang Li,^{3,4,5} Nicholas Daley,^{2,7,10} Karl Deisseroth,^{1,8,9} and X. Sunney Xie^{3,4,11,*}

CellPress



Guogiang Li^{® 19}, Yaping Liu^{23,4,9}, Yanxiao Zhang^{® 1}, Naoki Kubo¹, Miao Yu¹, Rongxin Fang^{1,5}, Manolis Kellis 6,7 and Bing Ren 1,8*

Resource









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A c B

Cell type clustering (only scHi-C tensors)



A c B

Cell type clustering (only scHi-C tensors)





Li 2019



Li 2019



Li 2019









C B

Grand Mean



С

Kim 2020

200

250

150

bin





GM12878



Chr1 $A_7 B_7^T$ GM12878 (Muscle)









Loci clustering (TAD)





Chr1 Bulk HFF



Loci Kim 2020



A/B compartments



c B

A/B compartments



Kim 2020

B

С





A/B compartments and Loci clustering







Integrative inference

















Integrative inference


Integrative inference



Integrative inference





Integrative inference





Take aways and discussion

Muscle provides cell type clustering (
)



Take aways and discussion

Muscle provides cell type clustering (
)

provides TAD, A/B compartments (A)







cell type specific manner



cell type specific manner

Integration with other datasets (Methylation)

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Kim 2020





Kim 2020



Loci



Loci

Lee 2019

Loci



Loci

Lee 2019

Loci



Cell type clustering (only scHi-C tensors)





Cell type clustering (only scHi-C tensors)

