Systematic Uncertainties in Neutrino Oscillation analyses

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Banff International Research Station for Mathematical Innovation and Discovery

Introduction

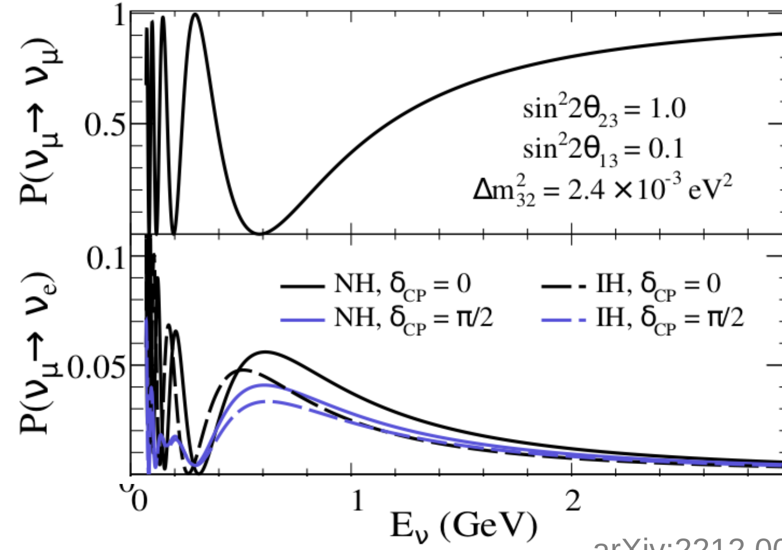
There have been several PhyStats dedicated to neutrinos in the past:

- **IMPMU 2016, Fermilab 2016, CERN 2019 … Somewhere else 202X??**
- **Excellent talk from Chrisophe Bronner at remote workshop PHYSTAT-Systematics 2021**
- If you're interested in learning more about systematics and statistical challenges for neutrino experiments take a look at the summaries.

Disclaimers:

- **This is very focused on the T2K 3-flavour oscillation analysis**
  - **Other analysis techniques exist like NOvA and PRISM techniques for DUNE**
- **I'm not going to mention other interesting statistical areas in neutrinos such as cross-section measurements, BSM searches, reactor neutrinos, atmospheric neutrinos etc.**
- **Very much a Bayesian perspective**

Neutrino Oscillations



arXiv:2212.00809

Neutrinos have a strange property where their mass and weak eigenstates mix.

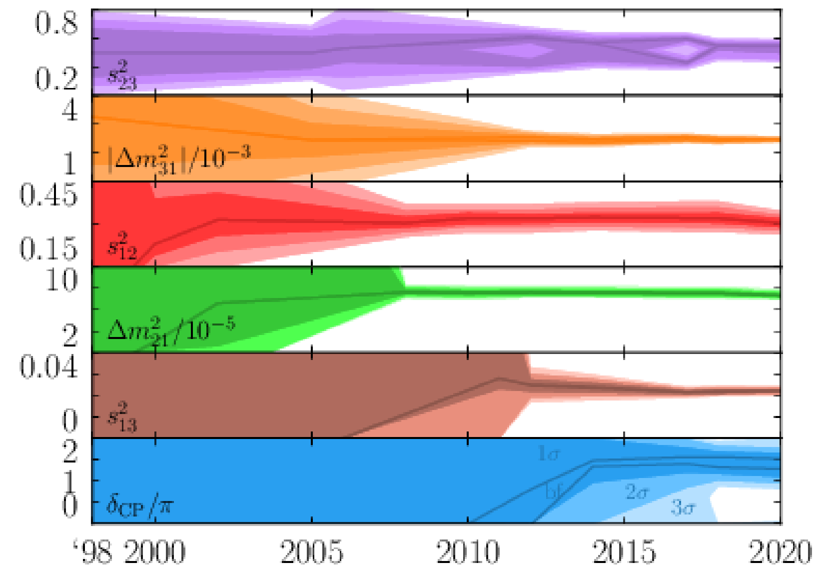
6 parameters which describe 3-flavour neutrino oscillation probability

- Three mixing angles: $\boldsymbol{\theta_{23}, \theta_{13}, \theta_{12}}$
- Two mass splittings: $\boldsymbol{\Delta m^2_{32}, \Delta m^2_{13}}$
- Complex-phase $\boldsymbol{\delta_{CP}}$
- Ordering of mass states also unknown **($\Delta m^2_{32} > 0$?)**

Neutrinos only interact via the weak force

- **Generally thought of as low stats experiments**
- **Huge amount of progress has been made since discovery**

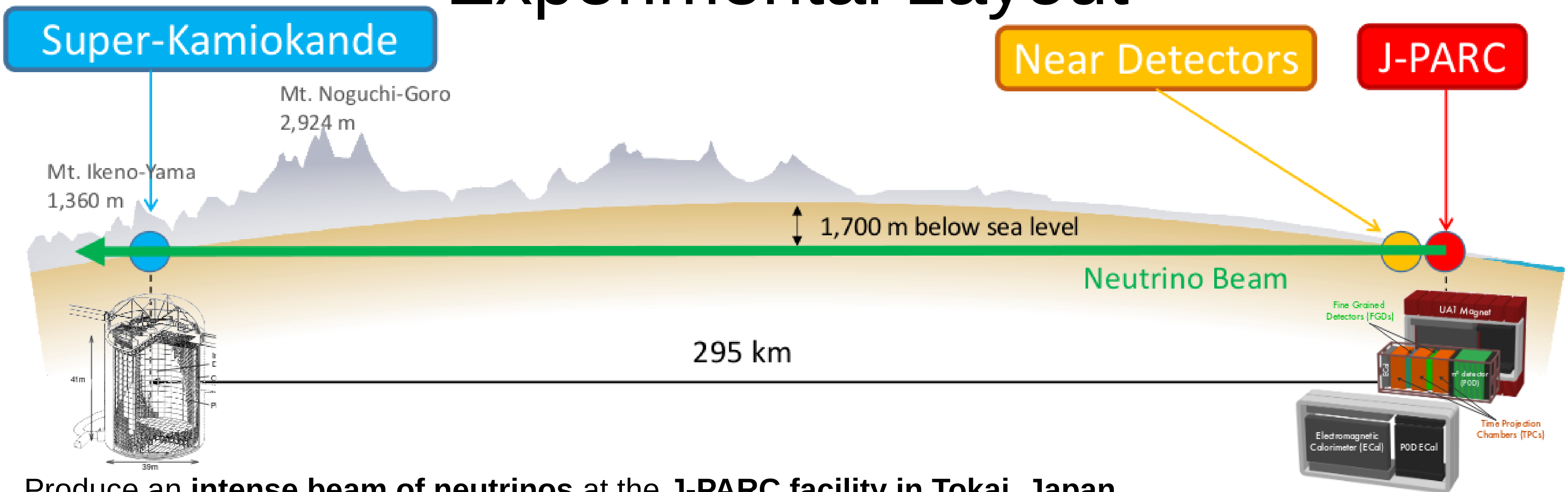
Current Generation long-baseline experiments are **T2K** (Japan) and **NOvA** (US).



Future experiments being built: Hyper-K (Japan) and DUNE (US)

- These experiments aim to make **precision measurements of oscillation parameters**
- Hopes to rule out particular values at 5σ e.g. $\boldsymbol{\delta_{CP} \neq 0}$
- **Understanding systematics is going to be key!**

Experimental Layout



- Produce an **intense beam of neutrinos** at the **J-PARC facility in Tokai, Japan**
- **Two detectors**: one **Near** the neutrino source, **Far** from the neutrino source
- **Far detector:**
  - Measures neutrinos after they have oscillated → this is where we **measure signal parameters**
  - **Lower statistics** due to distance from neutrino source (T2K FD has **~1,000 selected data events** total)
  - (High statistics atmospheric neutrino samples)
- **Near Detector**:
  - Measures unoscillated neutrino beam
  - **High(er) statistics**, **constrains sources of systematics uncertainty** (**~200,000 selected data events**)

$$N^{i}_{pred} = \int_{E_{\min}}^{E_{\max}} \overbrace{P(\nu_\alpha \to \nu_\beta)}^{\text{signal}} \times \underbrace{\overbrace{\Phi(E_\nu)}^{\text{beam}} \times \overbrace{\sigma(E_\nu), \overrightarrow{x})}^{\text{cross section}} \times \overbrace{\epsilon(\overrightarrow{x})}^{\text{detector}}}_{\text{nuisance parameters}} dE_\nu$$

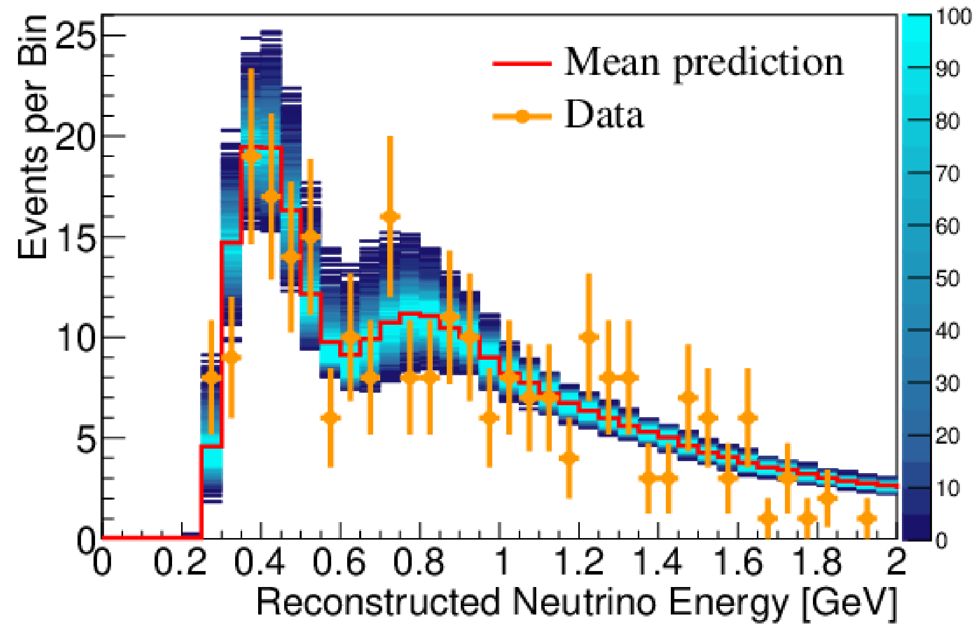
- **N^{i}_{pred}** = number of predicted events in a bin, **E_ν** is neutrino energy, **x** are the kinematics quantities of particles in the detector (lepton momentum, Q^2 etc.)
- **Signal: $P(\nu_\alpha \to \nu_\beta)$** is the oscillation probability for a set of oscillation parameters

Sources of nuisance parameters:

- **Beam**: how many neutrinos did we produce? What was their energy? What flavour?
- **Cross section / Interaction model**: probability of neutrino interacting? Energy, type and number of particles produced in the interaction?
- **Detector**: momentum scale, PID, acceptance, efficiency etc.
  - Different for ND and FD

- **Factorise** nuisance parameters

Total number of parameters **~700 of which 6 are signal**

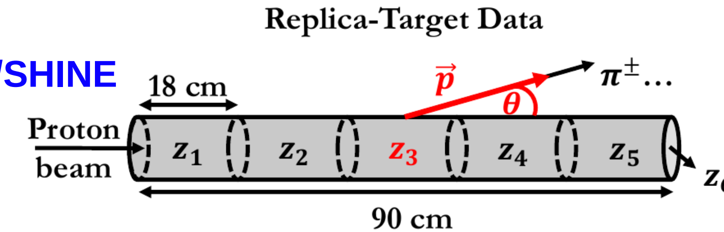


The good, the bad and the ugly

The good(ish)…

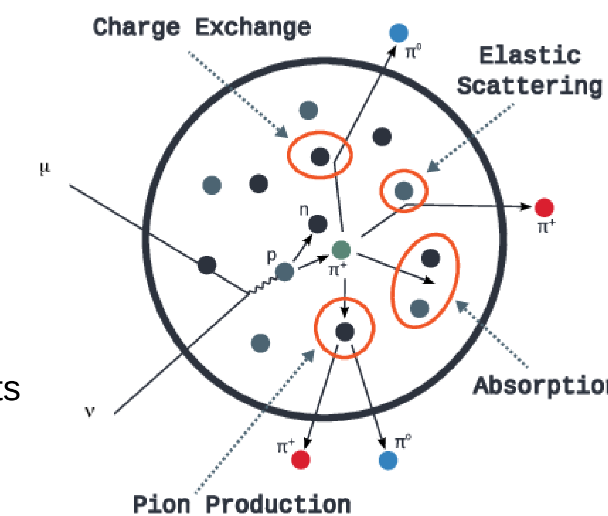
- **Beam systematics: ~100 nuisance parameters**
  - constrained by T2K beam monitors, a dedicated ND (INGRID) and external data
  - Underlying "physics parameters" such as hadron scattering not fitted in analysis
  - Throw underlying parameters to produce a distribution of events in an energy range
  - Build a covariance matrix from all these throws
- **Detector systematics: ~500 nuisance parameters**
  - Calibration and control samples
  - Again, don't directly fit parameters but throw toys to produce covariance matrix
  - Apply these uncertainties to each analysis bin
  - **N.B. moving away from this method and will fit detector systematics directly**

NA61/SHINE



The bad and the ugly…

- **Neutrino interaction systematics: ~50 nuisance parameters**
  - Not a lot of external datasets to constrain systematics
    - Often measurements taken with different neutrino beam energy, different target nuclei
  - Some systematics interpolate between different models
  - Often we add ad-hoc uncertainties motivated by differences seen in external datasets
  - Examples are uncertainties on nucleon form factors, nuclear effects
  - See NuSTEC white paper on Neutrino Interaction systematics for more information



Likelihood function

$$\mathcal{L} = \prod_{bins} \left(\frac{Poisson(N^i_{obs}, N^i_{pred}(o, f))}{Poisson(N^i_{obs}, N^i_{obs})} \right) \times \mathcal{L}_{penalty}(o, f)$$

Poisson likelihood ratio for ND and/or FD, o = signal parameters, f = nuisance

- (N.B. include MC stat uncertainty based on Barlow-Beeston method)

We evaluate this Likelihood using two different techniques:

Simultaneous ND+FD fit

- Use MCMC to sample Likelihood.
  - Samples all ~700 parameters
- Marginalise across nuisance parameters to produce credible intervals in 1D and 2D
- Can perform ND-only fits as well

Sequential fit

1) Fit ND data using MINUIT based fitter. Gives covariance matrix describing ND constraint and correlations between all nuisance parameters

2) Hybrid-frequentist fitter uses matrix to make marginalisation toys of systematics

3) Build confidence intervals using $\Delta\chi^2$ method

Near Detector constraint

ND constrains systematics relative to priors.
See some shifts away from prior central values.

- Flat priors on some parameters
- Some systematics deliberately not fitted at ND, unconstrained uncertainty propagated to far detector

Cross-section and flux parameters become **highly (anti-)correlated**

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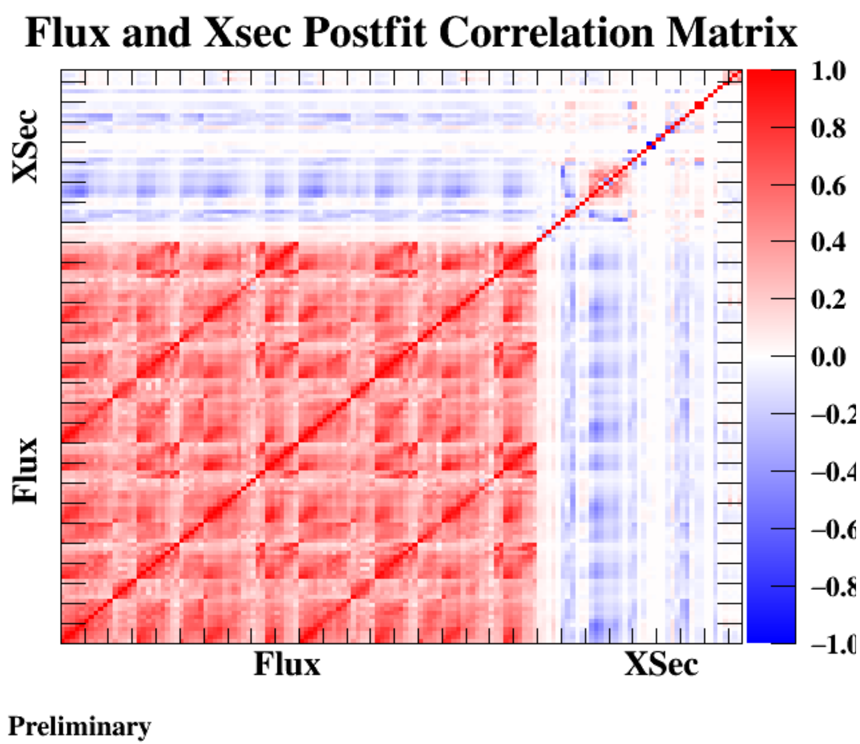
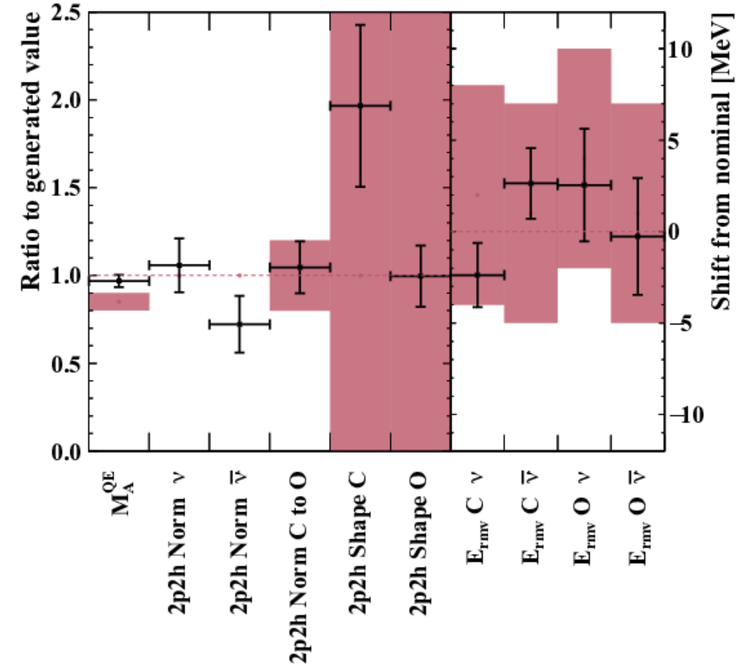
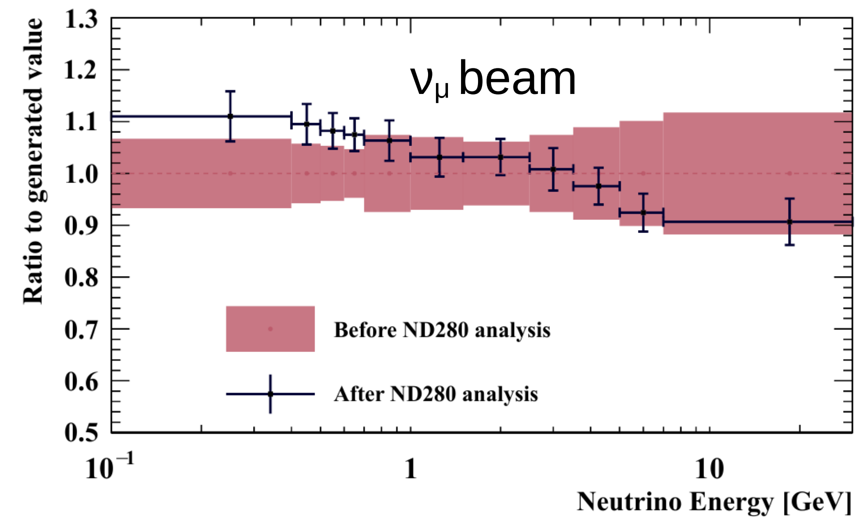
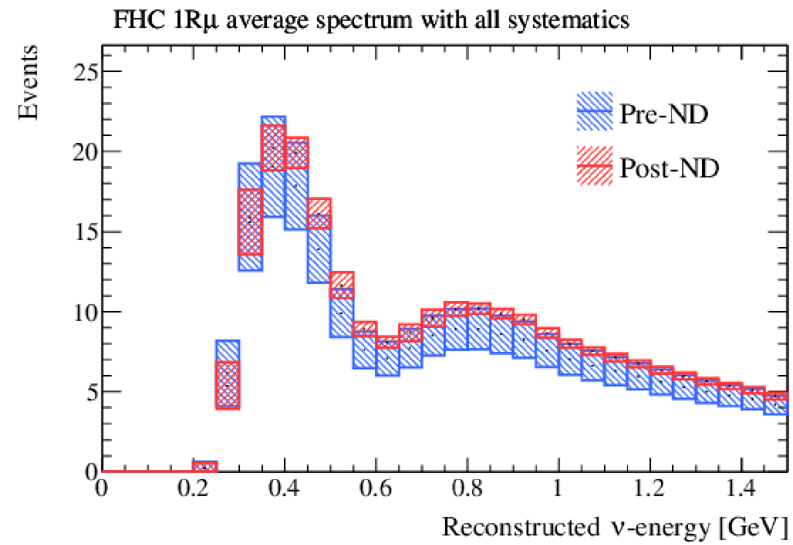


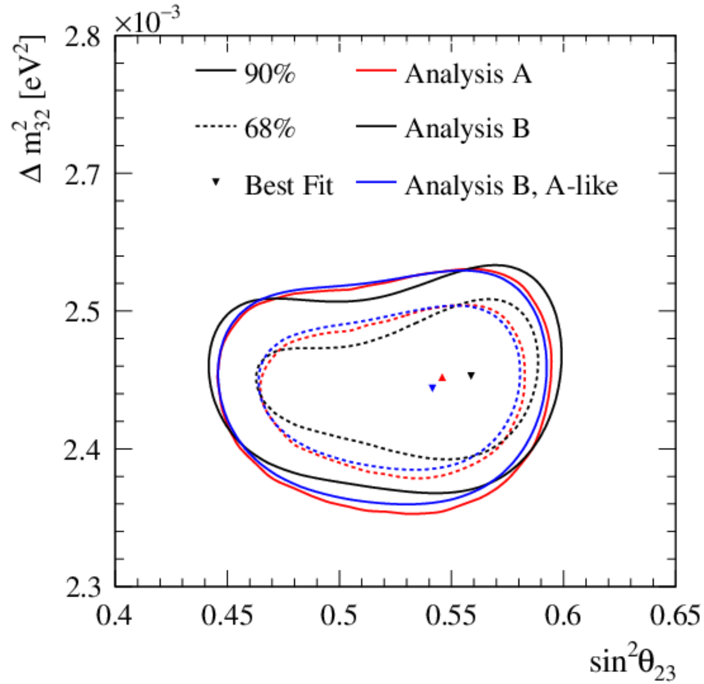




ND constraint at FD

- The Near detector constraint significantly improves measurement at FD
- FD barely constrains any nuisance parameters
- See slight change in oscillation parameter constraints in simultaneous vs. sequential fits
  - Analysis A is simultaneous MCMC fit
  - Analysis B is sequential fit using Hybrid fitter
  - Analysis B, A-like is if Hybrid fitter throws toys from ND-only MCMC posteriors
- Statistics limited at far detector but choice of how propagate nuisance parameters does have a visible effect on our contours
  - Not a systematic uncertainty as such
  - Both results are officialised





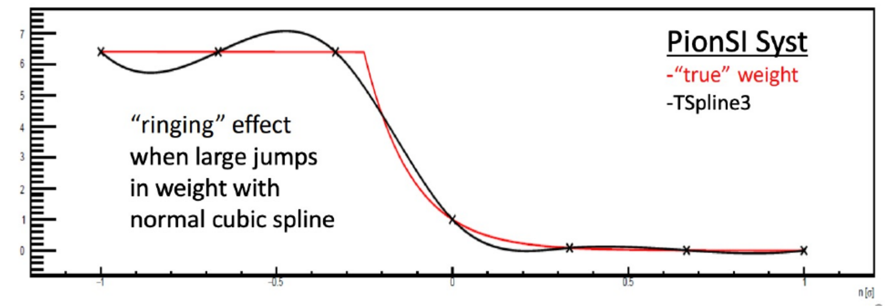
Implementation of nuisance parameters in fitters

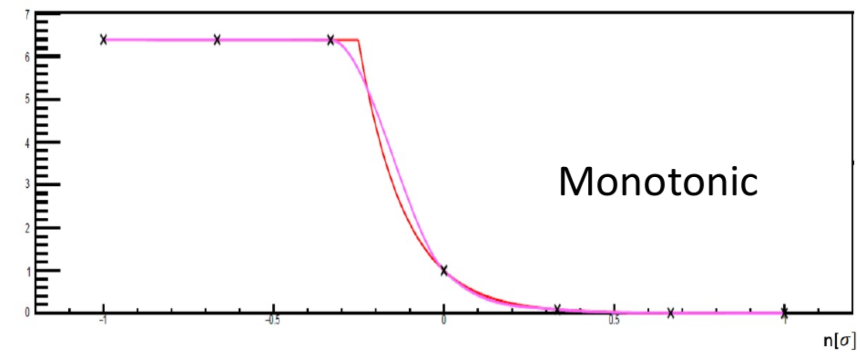
- Implementation of nuisance parameters for all fitters fall into three broad types:
  - **Normalisation**: simple weight applied to some bin, events of particular types or a range of a kinematic parameter
    - Beam and detector systematics implemented like this
  - Splined **response functions**:
    - Most neutrino interaction systematics
  - **Kinematics shifts**:
    - directly modify individual MC events reconstructed quantities
    - Some specific systematics
  - **Reweight MC event-by-event**

Splined response functions

- Change parameters and evaluate change relative to nominal prediction
  - Typically evaluate change up to **3 sigma** of prior uncertainty
- **Interpolate** between these points using cubic splines
  - Cubic response not always ideal, can lead to “ringing”
  - Moving more to **monotonic cubic splines**
- Then have two choices for the splines:
  - **Bin all your splines** to create a mean response function for a given analysis bin
  - **Keep all splines for all events**
- At the Near Detector evaluate all splines for every event for every change of nuisance parameters
  - For T2K MC (**~2M events**) this is not a problem
  - On average 3.5 splines per event gives 1.3GB of RAM
  - Accelerate this on a GPU. Evaluation time is very small (**~0.05s**)
  - **Is this feasible for future experiments with O(100M) MC events?**
- At the Far detector, less worried about the averaging splines since constraint on systematics is negligible.
  - Use a binned the splined response per analysis bin, per systematic, per interaction type

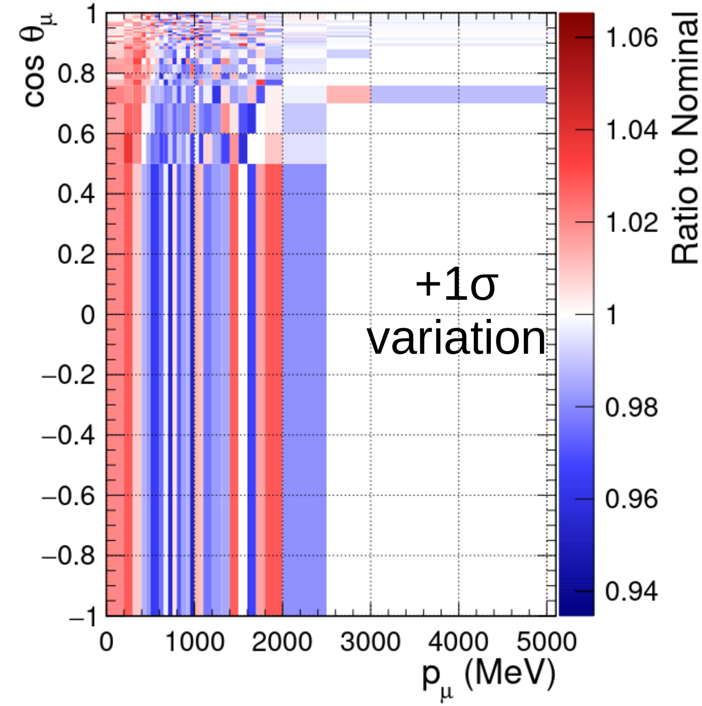
Ewan Miller

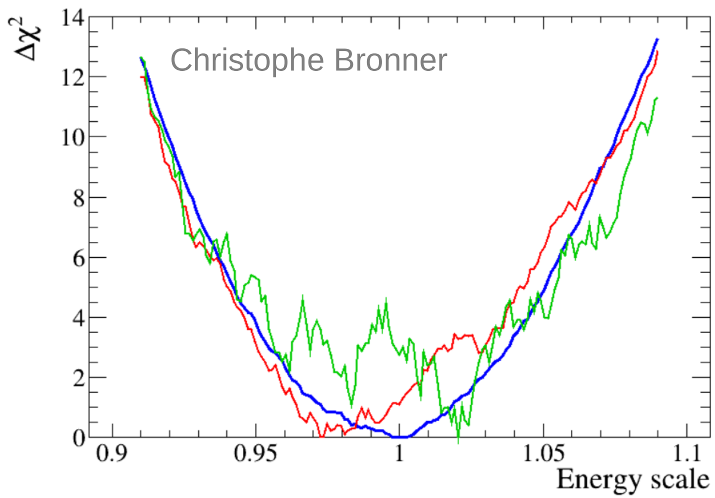




Kinematic Shifts

- Momentum scale systematics and Nuclear effects impact reconstructed variables directly
- Implement these systematics by directly modifying reconstructed variable
  - $X_{simulated}$ += $F(f_i)$
- Individual MC events migrate across analysis bins
- Finding the bin an event migrates to can be computationally expensive
  - Cache the original bin an MC event falls in
  - After shift, first check adjacent bins only then
  - Computationally expensive to find bin for every MCMC step
- Bin migration will cause discontinuities in your likelihood
  - Gradient based fitters have to find alternative implementations
    - Such as a splined responses, regularisation of bin widths
  - Metropololis-Hastings algorithm for MCMC doesn't care if likelihood is discontinuous as acceptance probability is a ratio of likelihoods



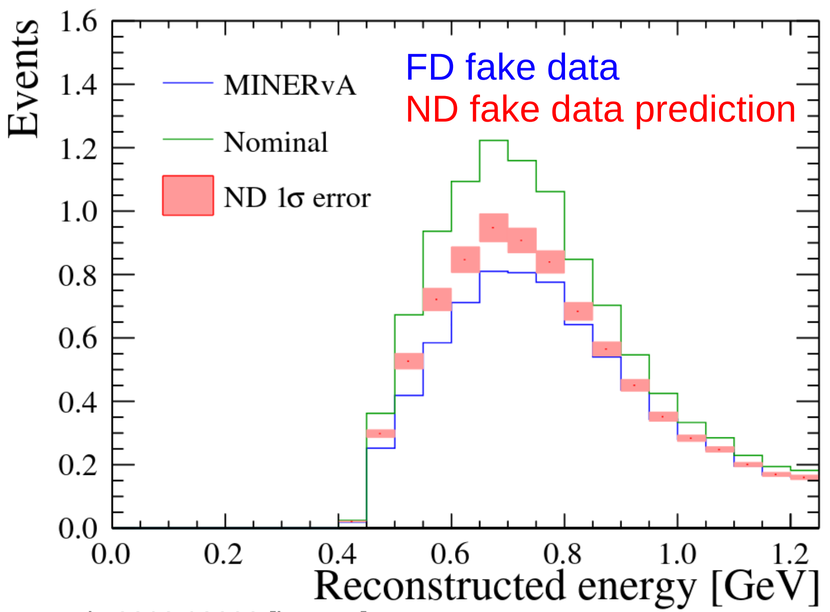


Fake Data Studies

We want to check our systematic model is robust to discrete changes to our model.

On T2K we have a home brewed procedure to do this:

- Create "fake data" at ND and FD with change to our model
- Fit this fake data at the ND and propagate to FD
- If our systematic model is robust we still extract the oscillation parameters with a small bias
- If 1D interval on signal parameters change by more than 50% of our systematic uncertainty "action" is taken
  - We might add in an ad-hoc parameter to inflate our systematic uncertainty
  - We might smear our final contours
- Calculate systematic uncertainty as $\sqrt{(\sigma_{total}^2 - \sigma_{stat}^2)}$ … not reliable to do around physical boundaries
- If any of our statements on excluding values also has to be true in the fake data studies
  - e.g. dCP = 0 is excluded at 3σ has to be true in all our fake data studies as well as the real data fits



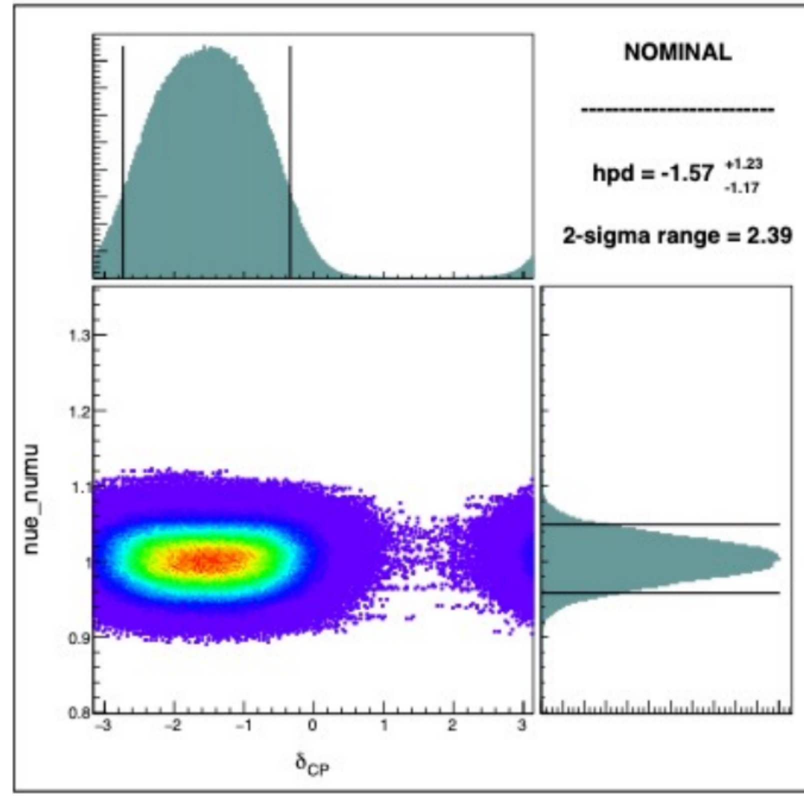
arXiv:2303.03222 [hep-ex]

Simulated data set	Relative to	$\sin^2\theta_{23}$	Δm^2_{32}	δ_{CP}
CCQE 3-comp nom.	Total	1.0%	0.4%	0.8%
	Syst.	2.5%	1.1%	3.1%
CCQE 3-comp high	Total	1.3%	0.7%	0.3%
	Syst.	3.2%	1.8%	1.1%
CCQE 3-comp low	Total	0.7%	0.2%	0.2%
	Syst.	1.7%	0.6%	0.8%
CCQE z-exp nom.	Total	2.5%	0.2%	0.6%
	Syst.	6.1%	0.6%	2.2%
CCQE z-exp high	Total	0.3%	2.1%	0.4%
	Syst.	0.7%	5.7%	1.7%
CCQE z-exp low	Total	3.1%	0.2%	0.1%
	Syst.	7.5%	0.6%	0.6%

Shrink and Pull studies

- From the MCMC analysis, we then have a large posterior to study
- One way to assess the impact of key systematics is to reweight steps in the Markov chain to have a tighter prior
  - Weight = p_{new} / p_{old}
  - i.e. "what we happen is constraint on systematics was tighter?"
  - Caveat: we can only do this for single systematics at a time due to MCMC statistics
- Shows how oscillation parameter constraints change for particular systematics changes

P. Dunne



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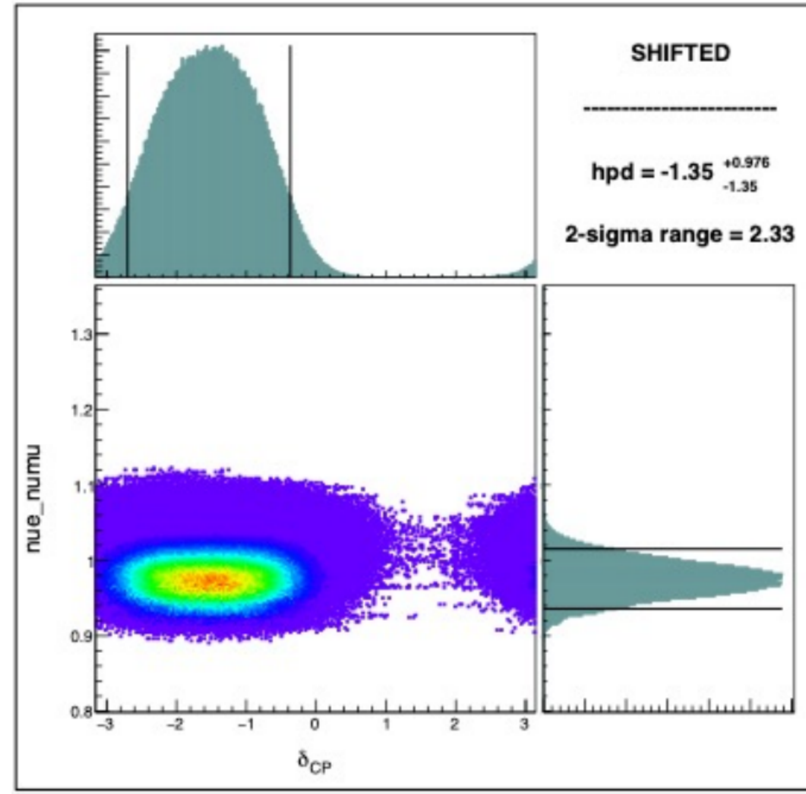


Summary

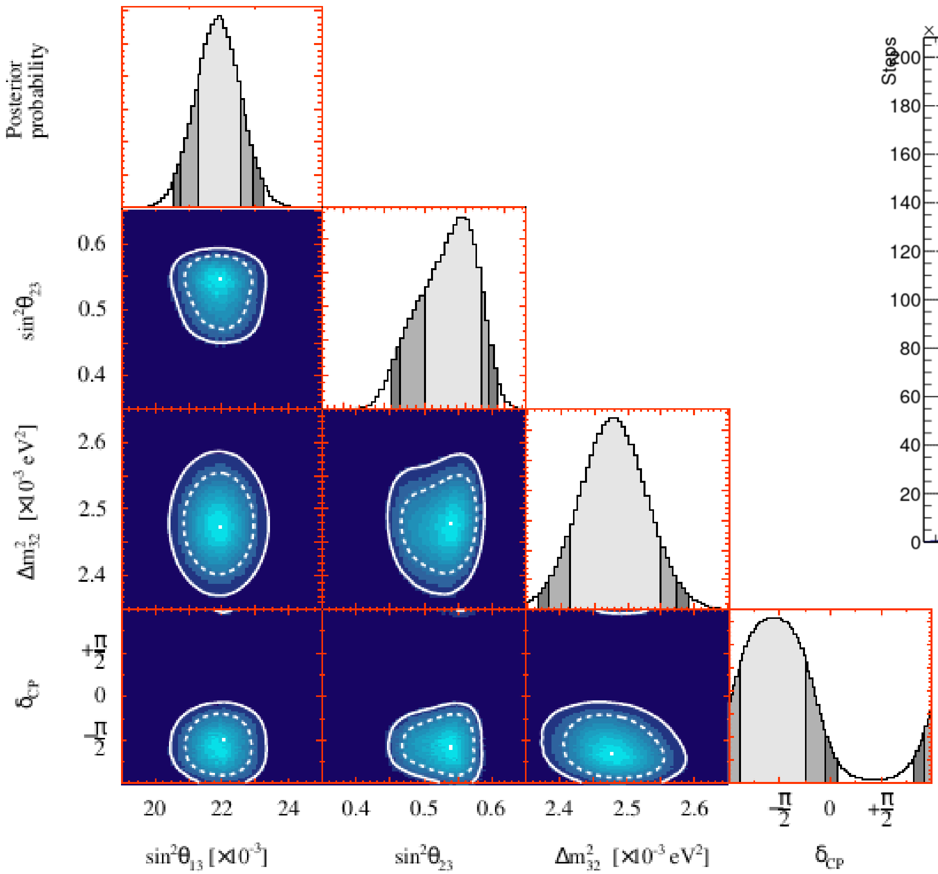
- Neutrino Oscillation analyses
  - Near detector has high statistics and constrains our systematics
  - Far detector has much fewer statistics but is sensitive to signal parameters
  - We have to be **careful how we propagate our systematic constraint** to the far detector
- Systematics parameters in fitters:
  - Event-by-event treatment of systematics parameters
  - Marginalise over large number of nuisance parameters
- Post-fit studies of systematics
  - Fake data studies can be used to check robustness of our systematic model
  - Shrink and pull studies are a nice simplistic method for checking how tighter systematic constraints would affect our result

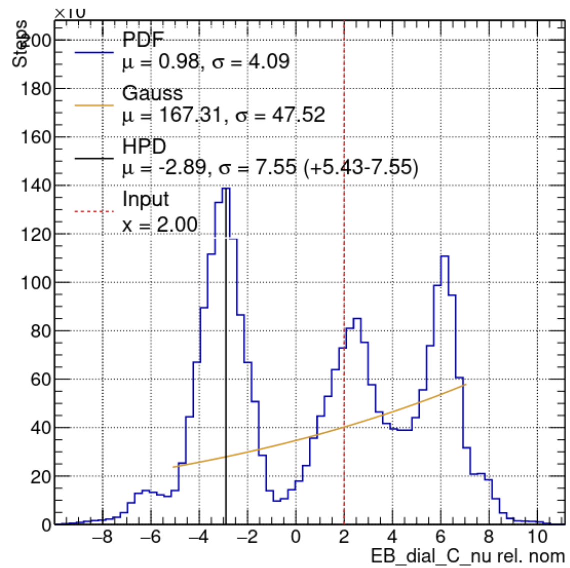
- In the next 5-10 years new experiments will collect 100 times more data.
  - Results will not be statistics limited for much longer!
  - We want to make sure our treatment of systematics and statistical techniques are up to the challenge!

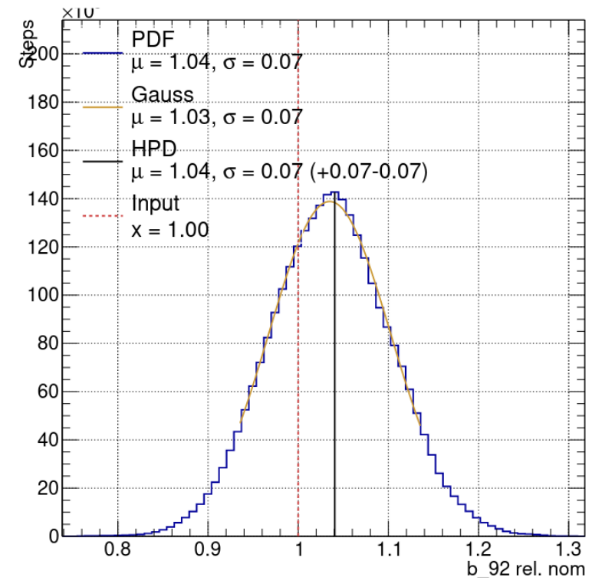
Marginalisation of nuisance parameters

- For all the fitters we marginalise across all nuisance parameters
  - marginalise across ~700 nuisance parameters down to 1D or 2D posteriors on signal param
  - Report a 4D highest posterior density point as well

- Some nuisance parameters are very gaussian others can be very non-gaussian

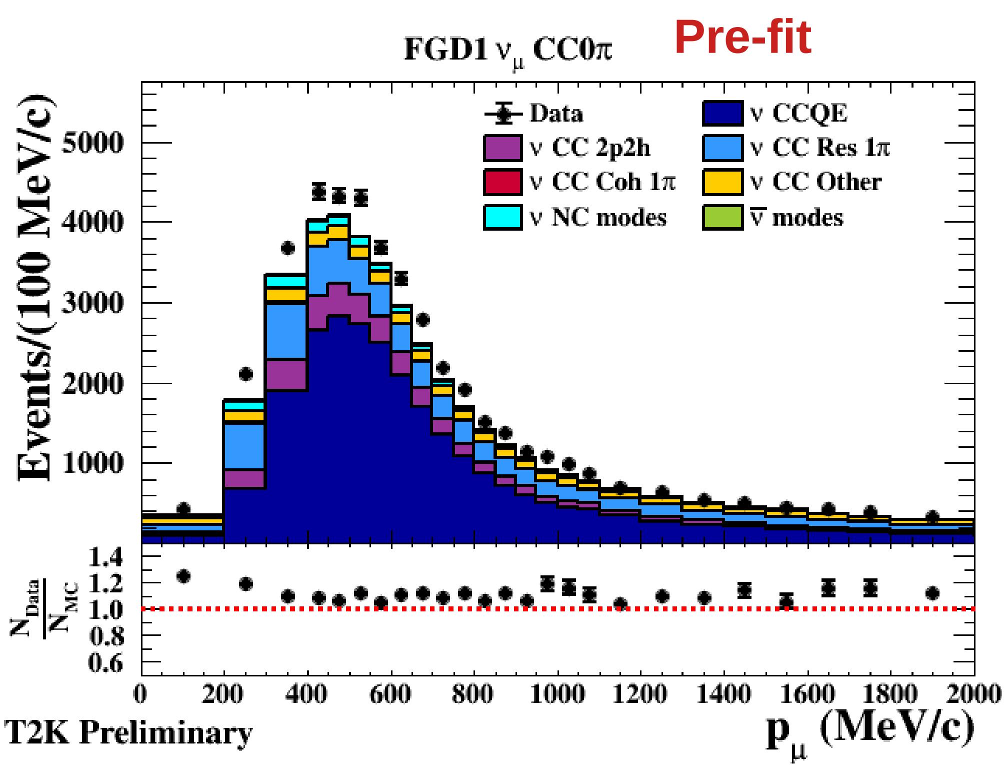


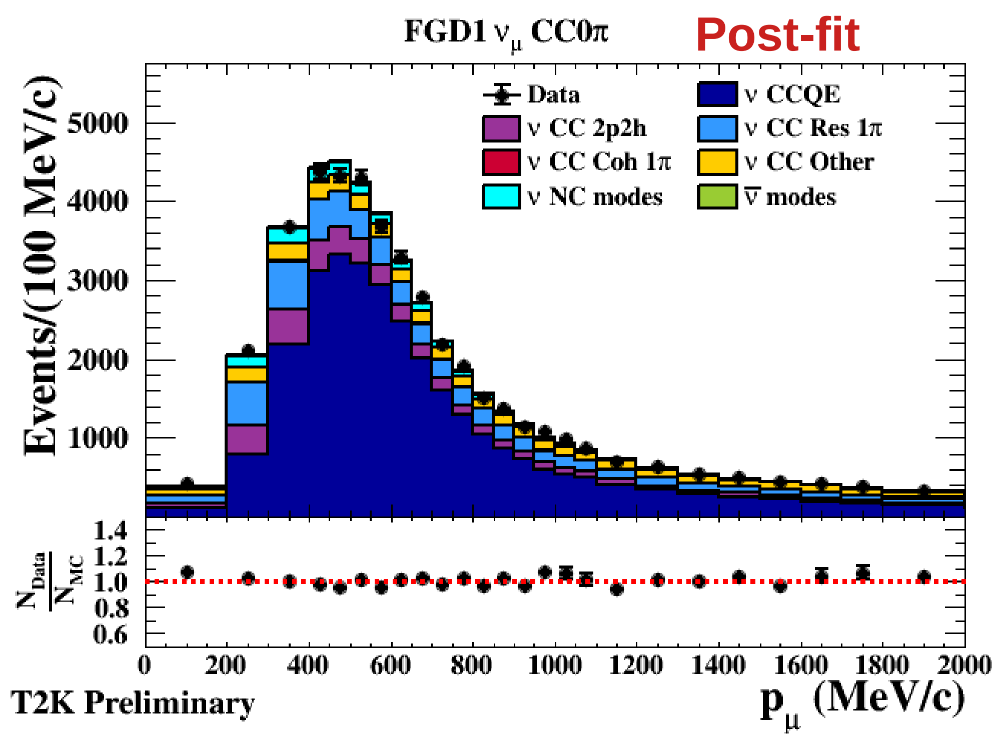




ND280 fit results

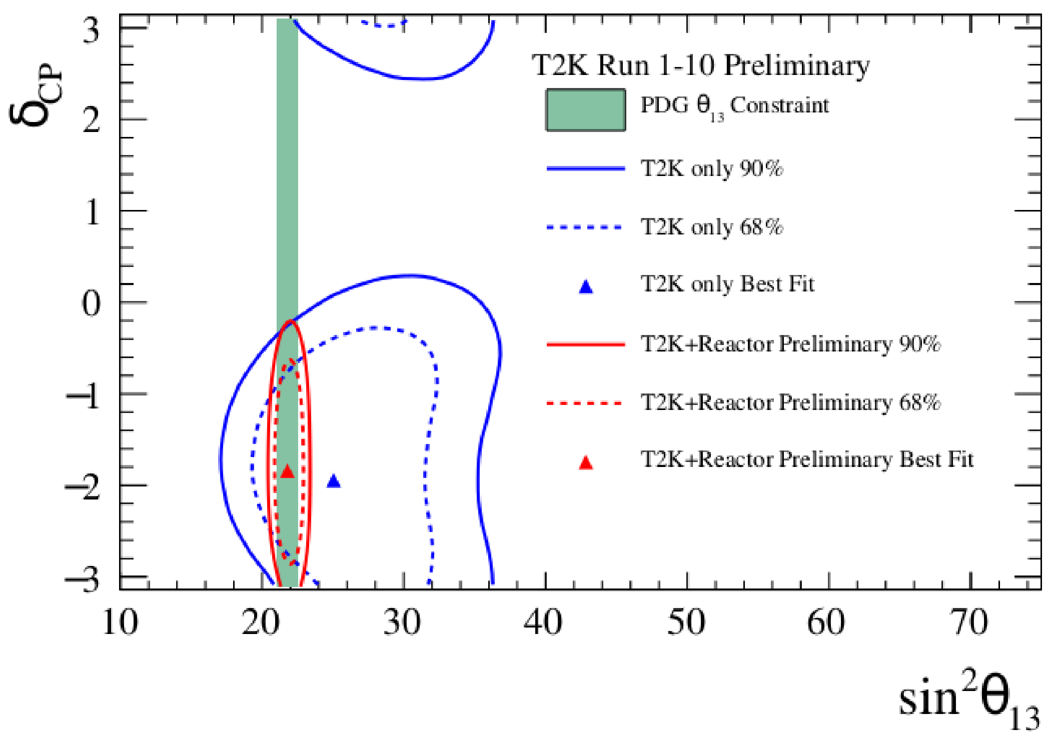
- ND280 data constrains systematic uncertainties before oscillations
- Significantly reduces uncertainty on prediction at SK
- The ND280 fit matches our data well (prior model p-value of 74%)

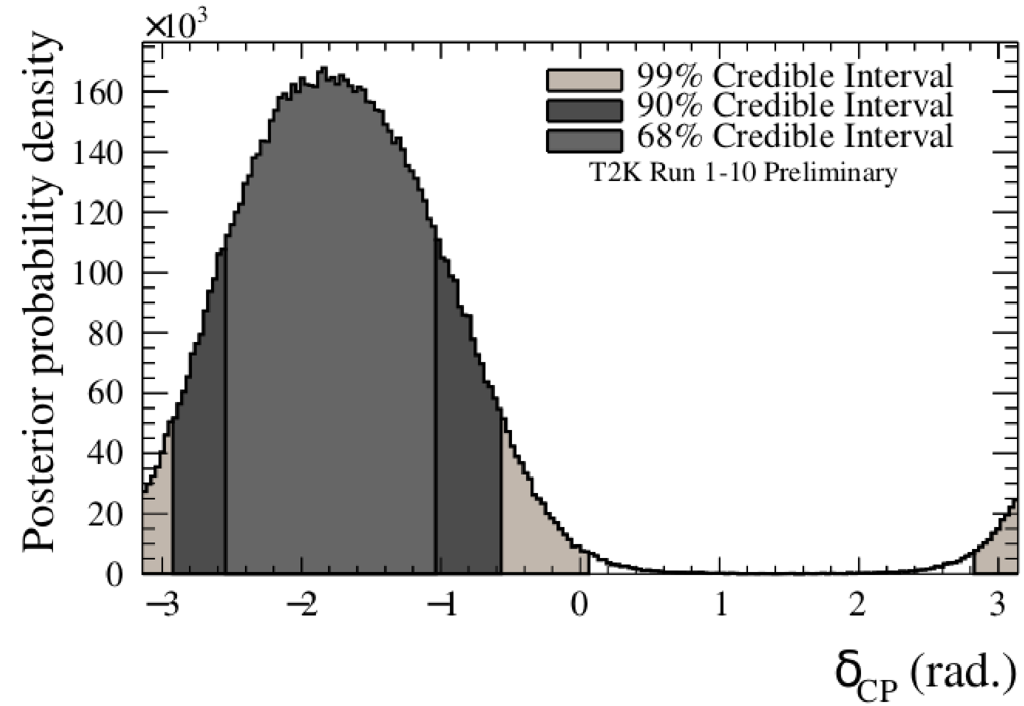




ν_e appearance results

- **T2K prefers value of $\delta_{CP} \approx -\pi/2$**
- **Disfavour** CP conserving values of 0 and π at **90%** confidence





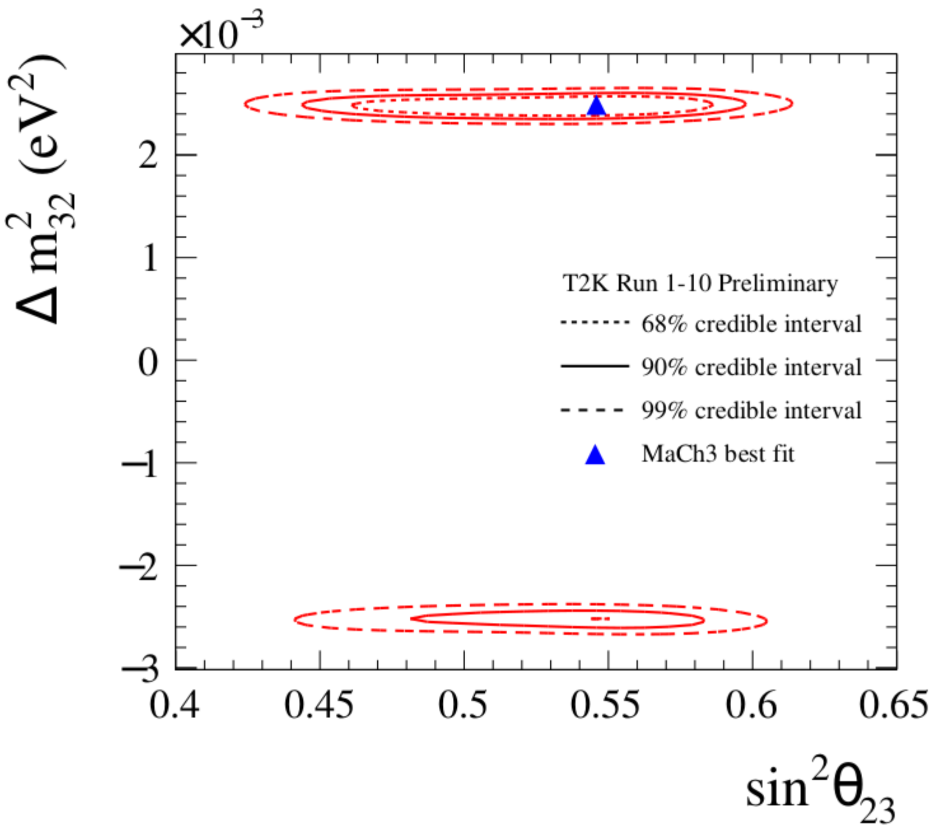
- T2K-only measurement of θ_{13} compatible with PDG average.

ν_μ disappearance results

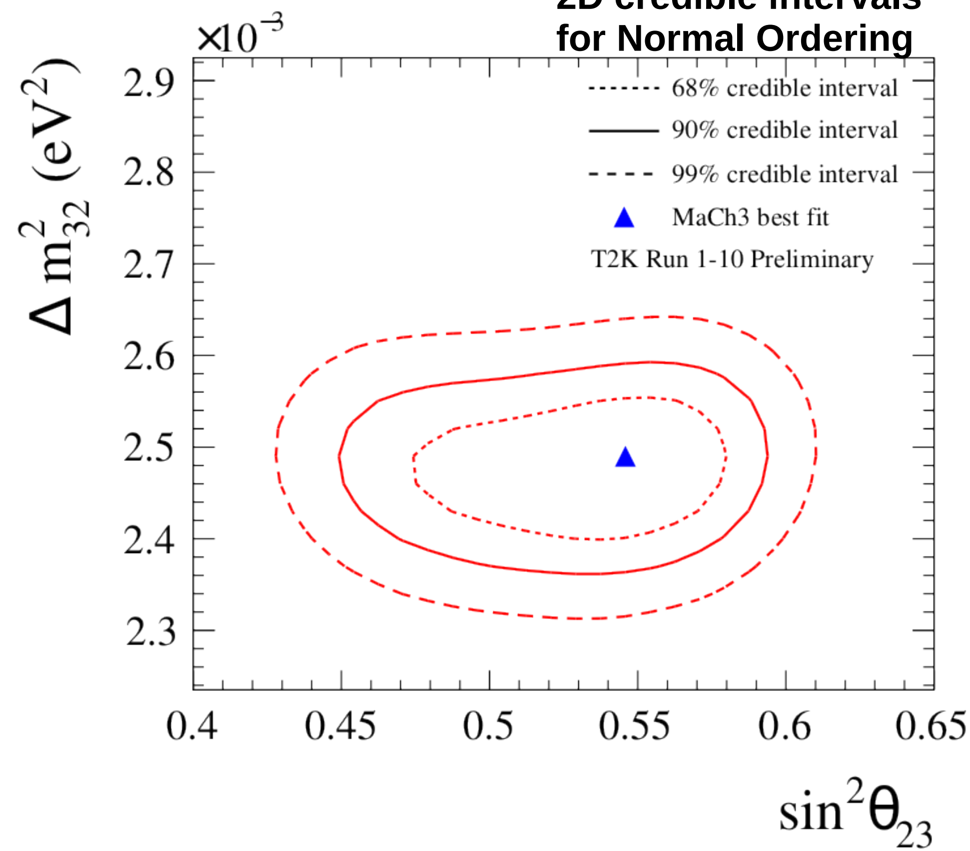
T2K prefers Normal Ordering.

T2K prefers **Upper octant** of $\sin^2\theta_{23}$ and slight preference for **non-maximal $\sin^2\theta_{23}$.**

Results shown here are **using the PDG reactor constraint.**

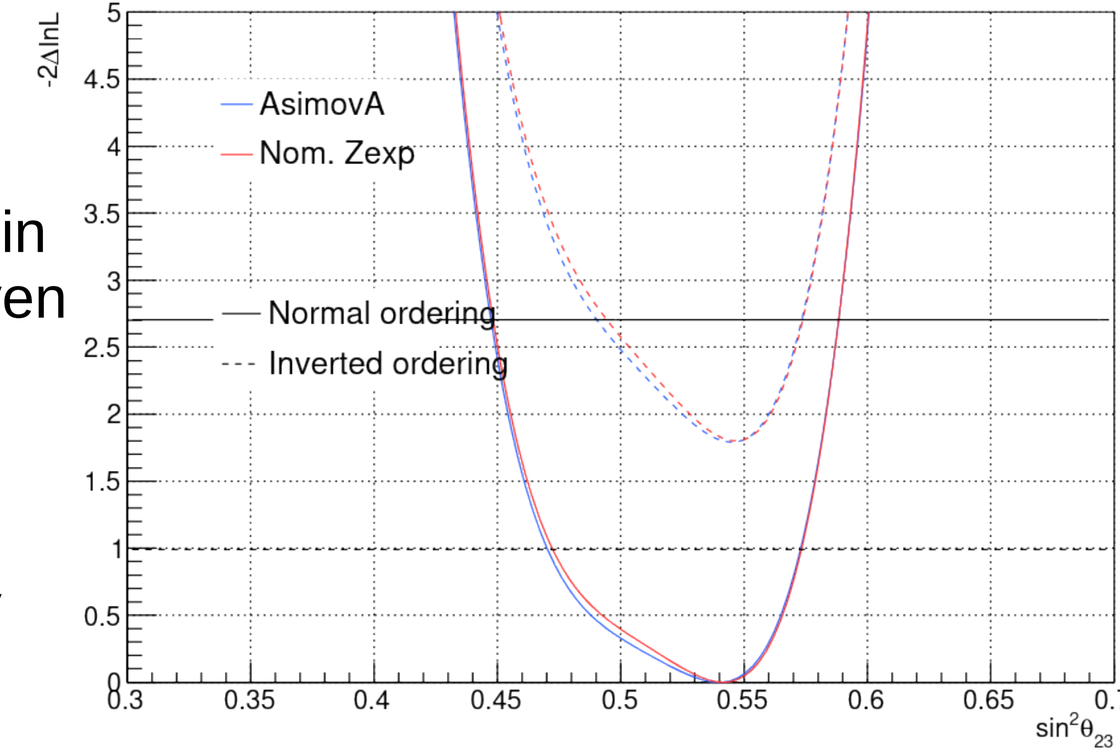


2D credible intervals for Normal Ordering



Robustness Studies

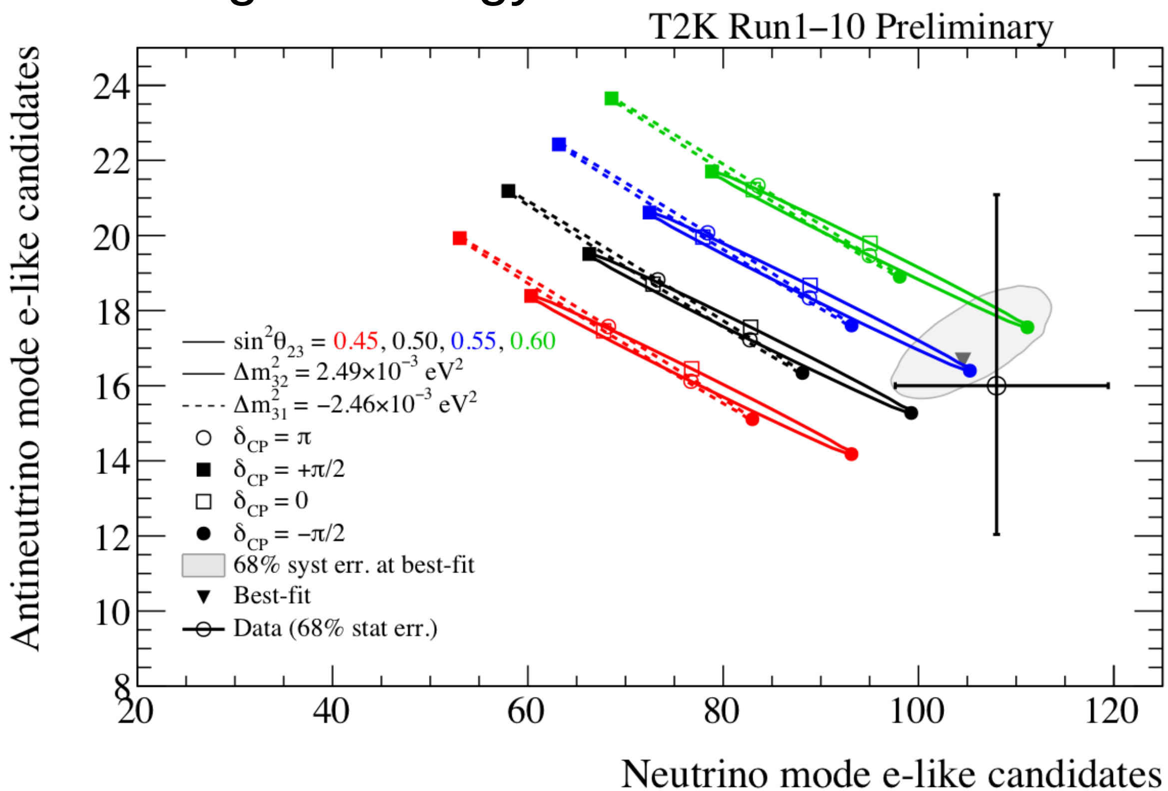
- Want to check analysis is robust to choice of MC.
- Simulate data using alternative interaction models e.g. alternate form factors for CCQE, change in pion production model, data-driven changes to the model
- Small changes in δ_{CP} limits. Largest bias causes left (right) edge of 90% interval to move by 0.073 (0.080)
- Apply smearing to Δm^2_{32} contours of 8.65 x 10^{-6} eV^2/c^4 from largest bias seen.

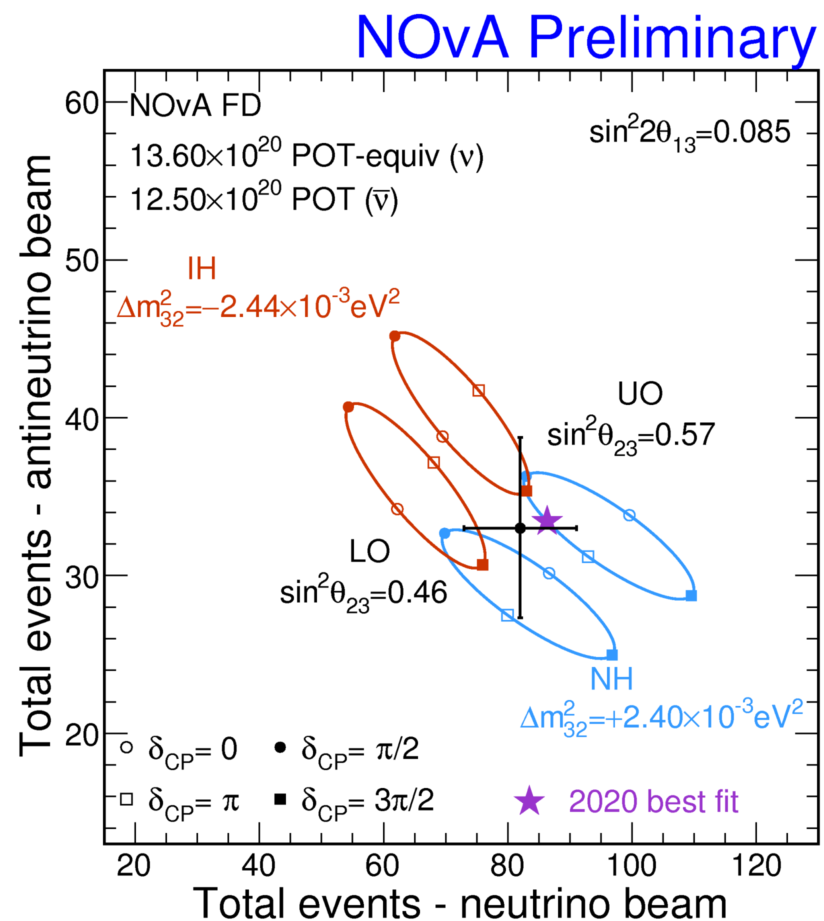


$\sin^2 \theta_{23}$ with reactor constraint

Comparison of results to NOvA

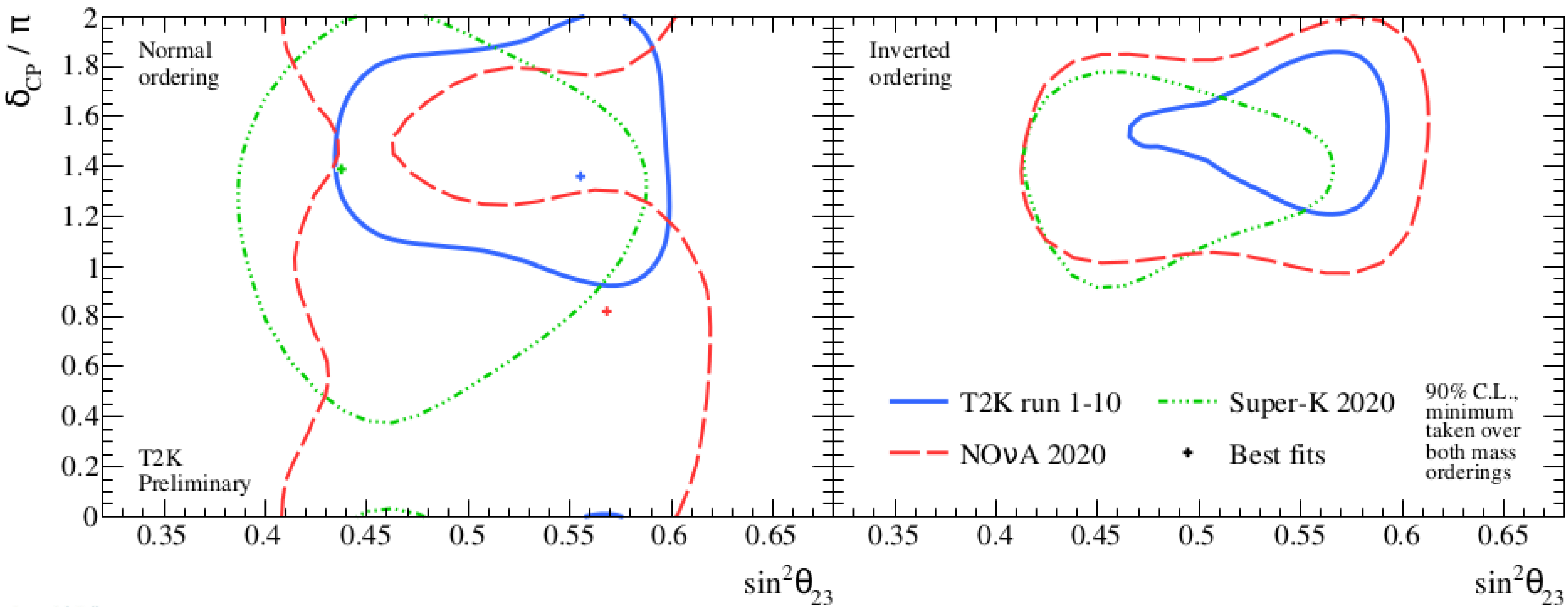
- NOvA experiment is a long-baseline neutrino experiment in the USA. See Erika's talk next!
  - Baseline of 810 km
  - Higher energy and broader neutrino flux





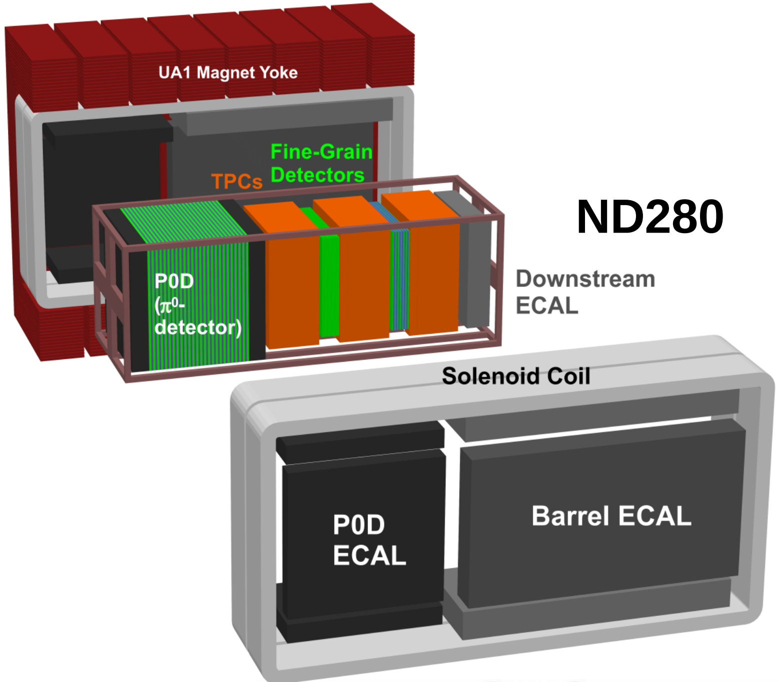
Comparison of results to NOvA

- T2K prefers $\delta_{CP} \approx -\pi / 2$ and NOvA disfavours this region slightly.
- In Normal Ordering slight disagreement. Inverted Ordering agrees well.
- **Reminder**: both experiments have different sensitivities and both experiments still statistics limited.

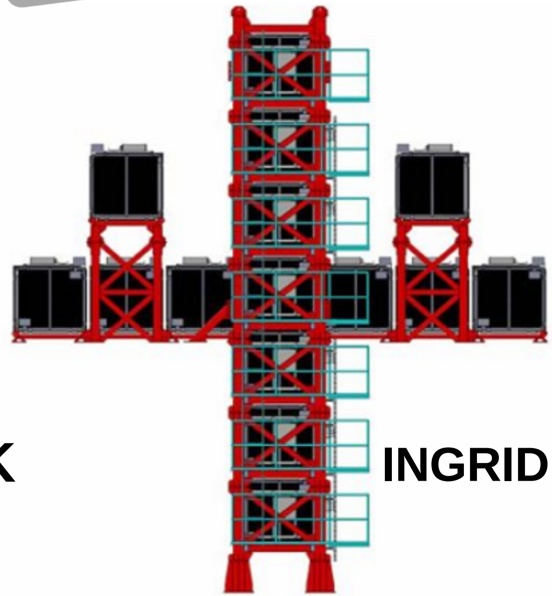


Near Detectors

- **Near Detector at 280m (ND280)** is situated 280m downstream of neutrino production point
  - Fine Grain Detectors (FGDs) - Plastic scintillator based
  - Time Projection Chambers (TPCs) – measures momentum and gives excellent PID
  - All inside UA1 magnet provides 0.2 T field
- **Interactive Neutrino Grid (INGRID)** monitors neutrino beam position and direction. Made from 14 scintillator modules
- **Measure neutrino beam characteristics before oscillations**
- **Very active cross-section measurement program at T2K**



ND280

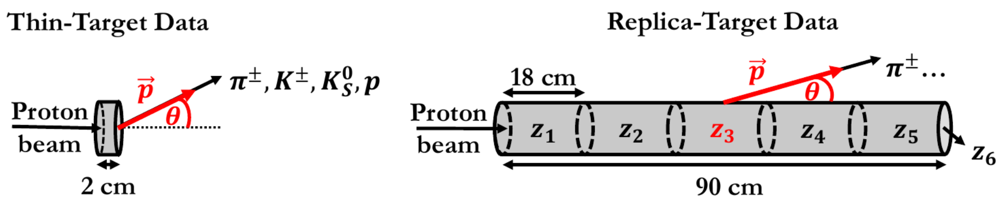


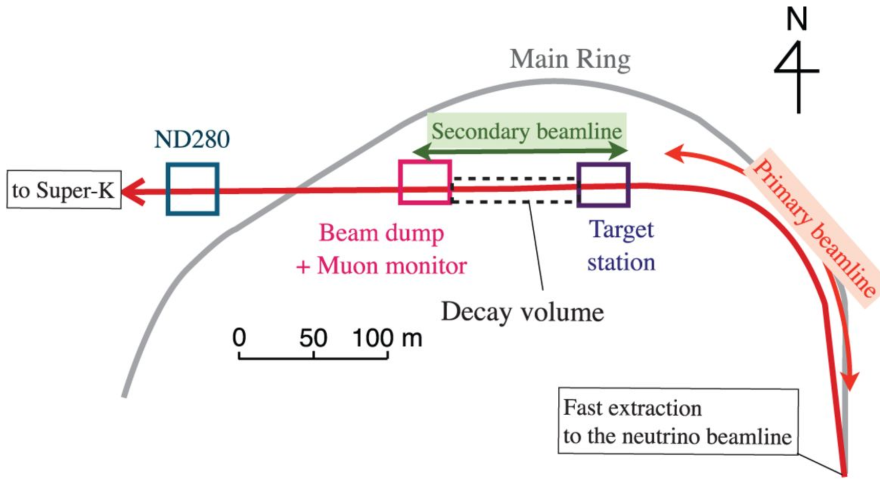
INGRID

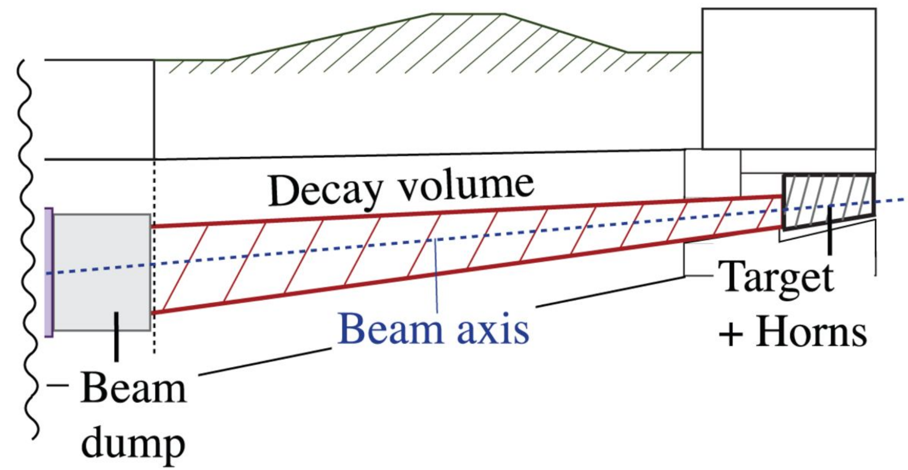
Neutrino Flux

- Neutrino beam is produced by colliding protons from J-PARC facility with graphite target
- Many hadrons are produced in collision
- Hadrons are focussed by a series of **magnetic horns**
- These hadrons (mainly π, K) **decay** to produce neutrinos
- Ideally we would like a pure muon (anti-)neutrino beam
- Can run in **neutrino mode** and **anti-neutrino** mode by changing direction of field in horns
- Proton beam and neutrino beam are measured by a series of **beamline monitors**

- **External constraints** on production of hadrons on/in target used from **NA61 experiment**



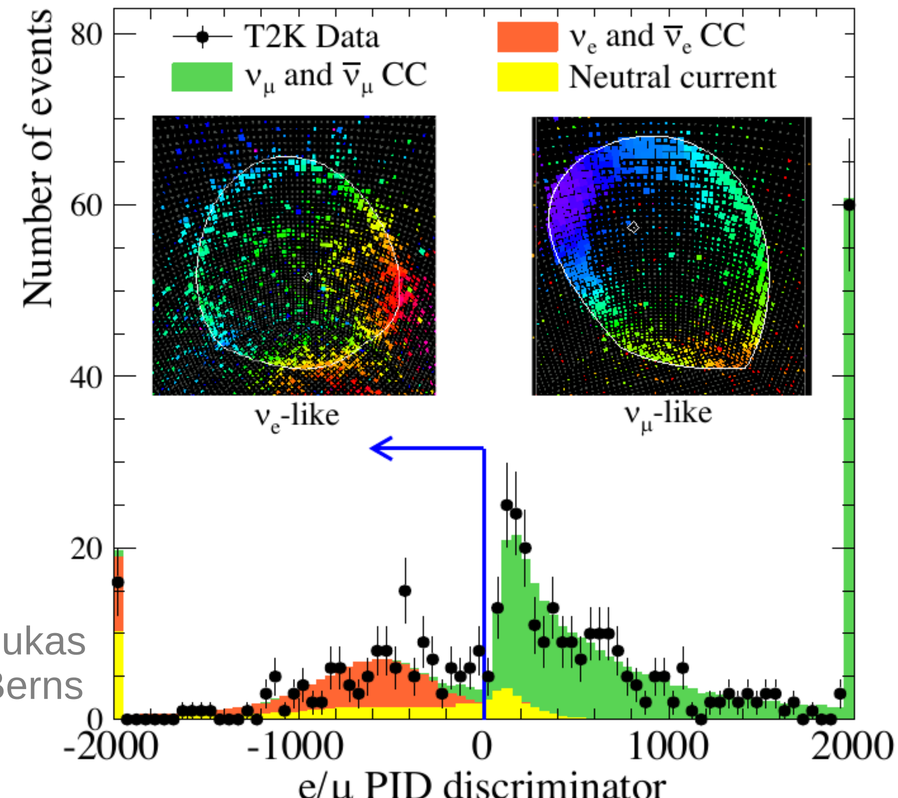




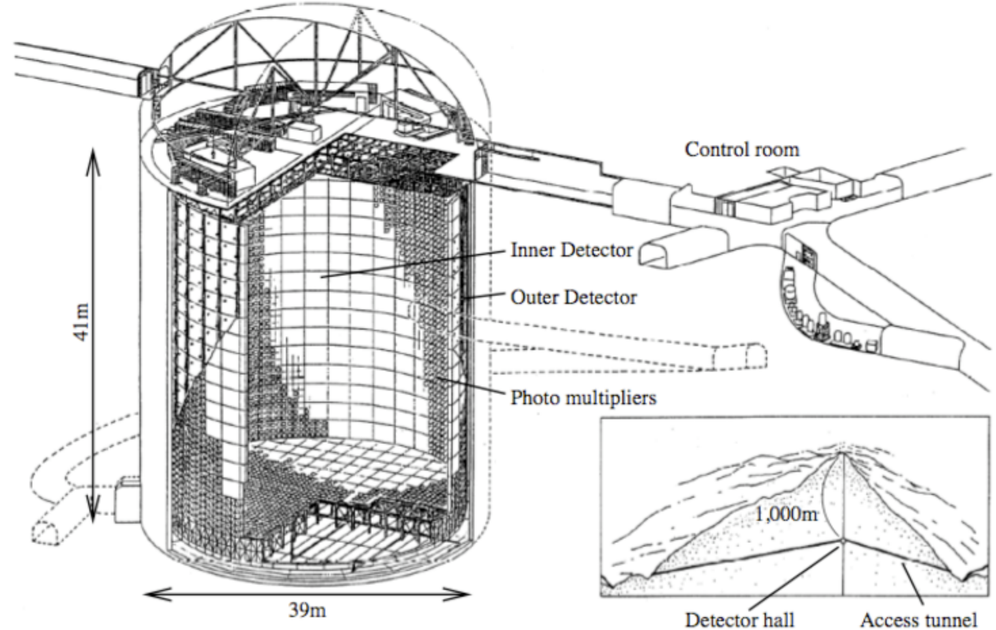
Super-Kamiokande

- **50 kt water-Cherenkov detector**
- Split into two regions: inner and outer detector
- Instrumented with **PMTs**

Adapted from arXiv:1910.03887v2



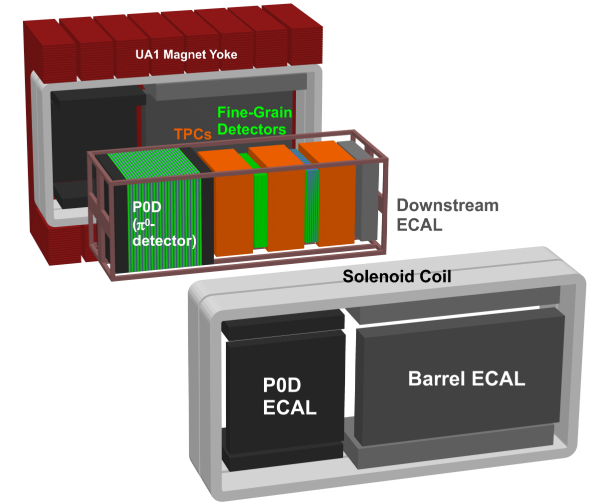
Lukas Berns



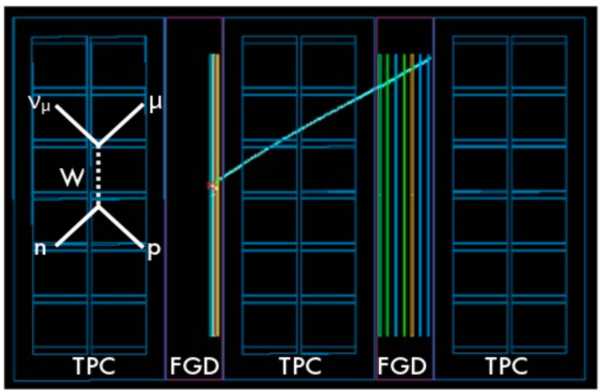
- **Particles are identified by their Cherenkov rings**
  - **Muons** produce **sharp** Cherenkov rings
  - **Electrons** scatter more so produce **"fuzzier"** rings
- Pions tagged by looking for **Michel electrons**

ND280 data samples

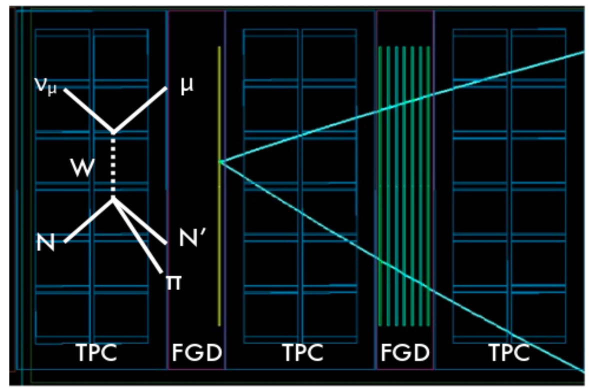
Always require one reconstructed muon.
Select events in FGD1 or FGD2.
Three topologies based on number of pions.



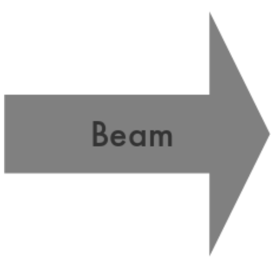
CC0π

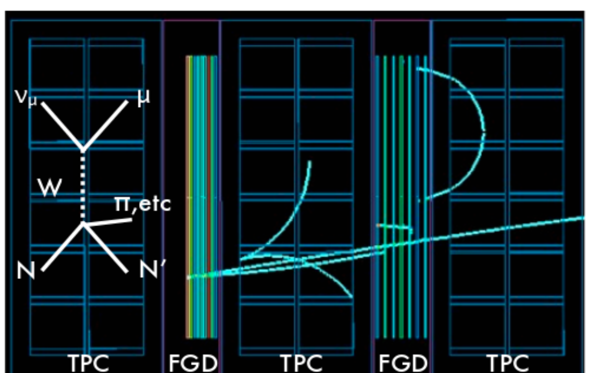


$CC1\pi^+$



CC other





Asher Kaboth

CC0π – no π in the final state

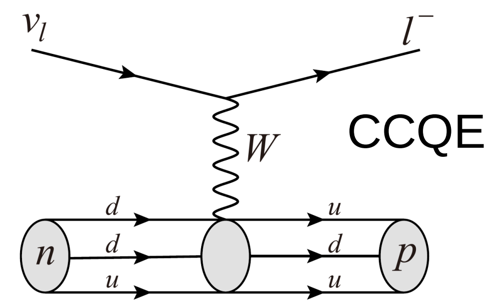
$CC1\pi^{+(-)}$ – a charged pion in the final state

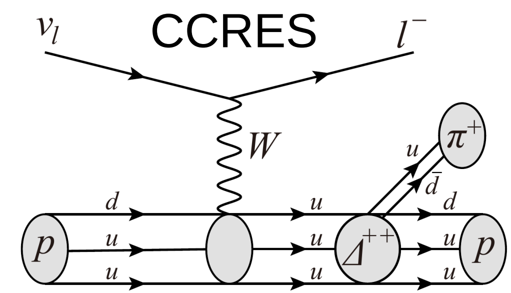
CC-Other – everything else! Multiple πs, gammas, π^0…

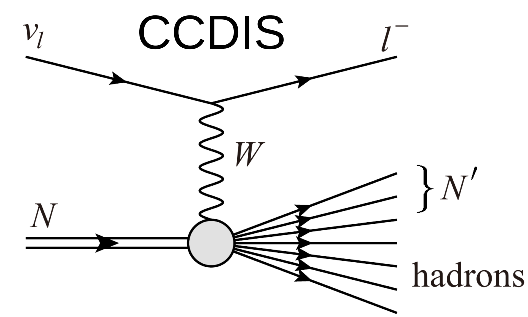
Selections in neutrino and anti-neutrino mode; 18 in total.

Neutrino interaction modelling

- Important to understand how neutrino interact otherwise we can’t accurately reconstruct neutrino energy
- Interactions occur within a nucleus, propagation of particles through nucleus also needs to be modelled. Commonly referred to as Final State Interactions (FSI)
- At T2K energies, Charged Current (CC) Quasi-Elastic (QE) interactions are most dominant type, significant number of multi-nucleon interactions (2p-2h) and resonant pion production (RES). Some Deep Inelastic Scattering (DIS)
- T2K uses the NEUT (5.4.0) neutrino event generator for simulations
- Prior uncertainties motivated by external data sets (e.g. bubble chamber data) and theory





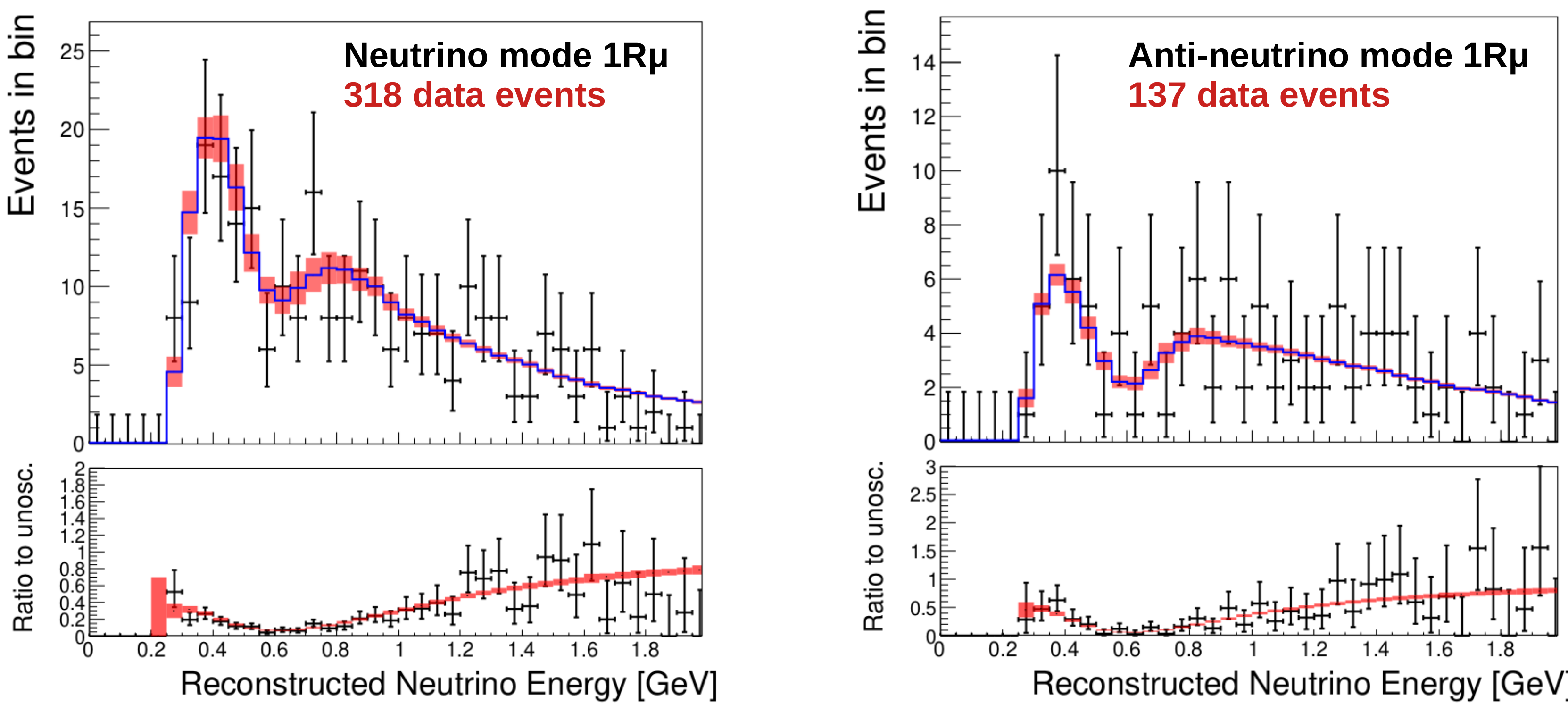


SK data fit results

Two samples with 1 muon-like cherenkov ring: neutrino mode and anti-neutrino mode.

Systematic uncertainty band is given by red band and statistical uncertainty on data given by error bars.

Systematic uncertainty on rate is 3% for neutrino mode and 4% for anti-neutrino mode.



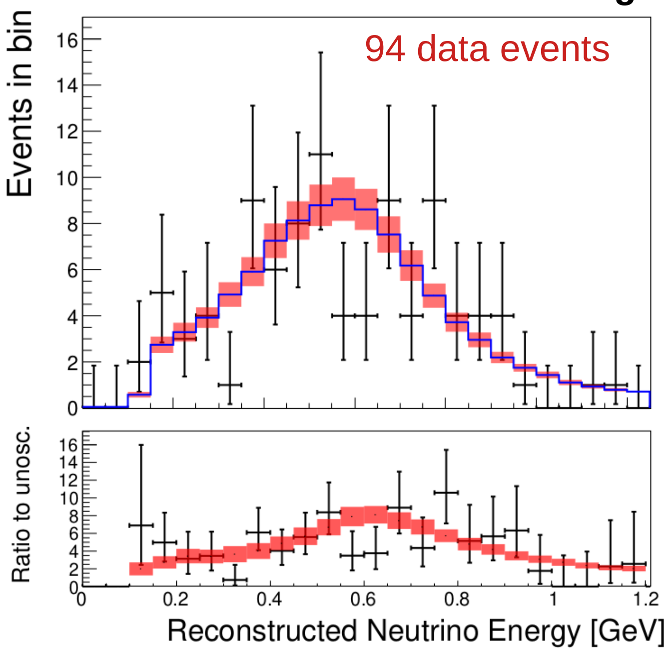
SK data fit results

Three samples with e-like cherenov rings:

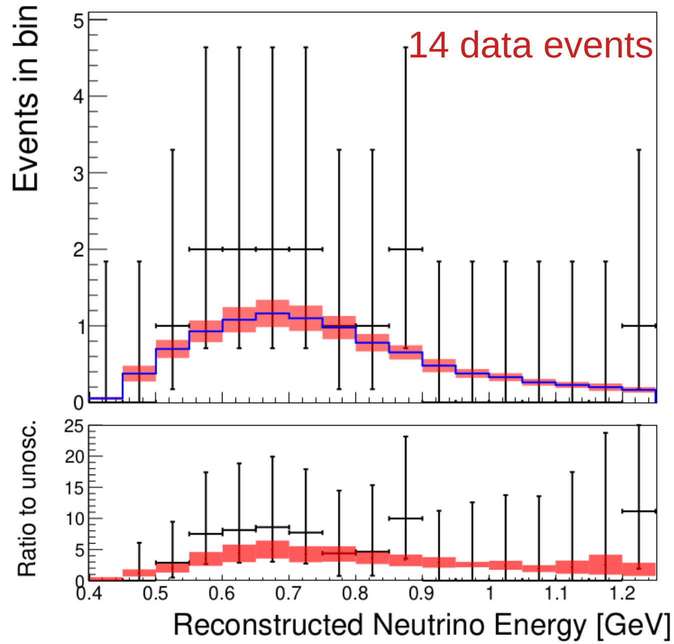
- Two samples with one e-like ring; one in neutrino mode and one in anti-neutrino mode
- One sample with one e-like ring and Michel electron from pion below cherenkov threshold

Uncertainty on rate is 4.7%-5.9% for single ring e-like samples and 14.3% for Michel electron sample.

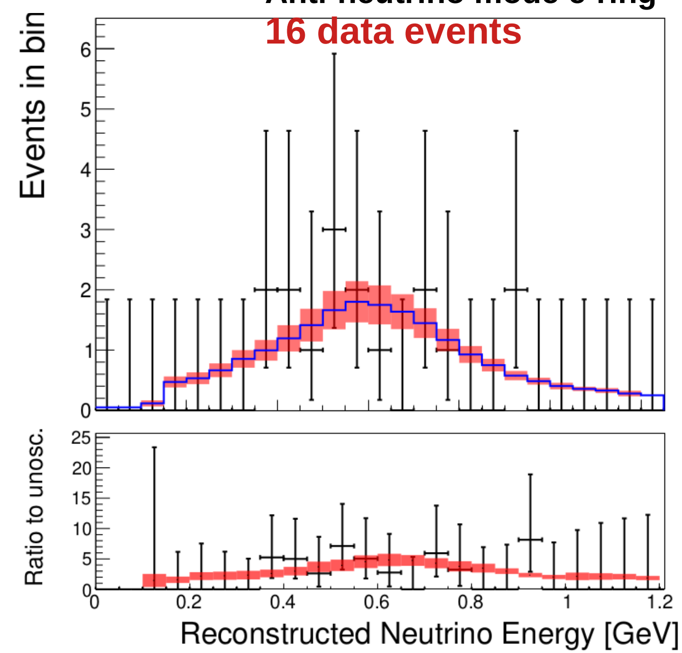
Neutrino mode e-ring



Neutrino mode e-ring and michel electron

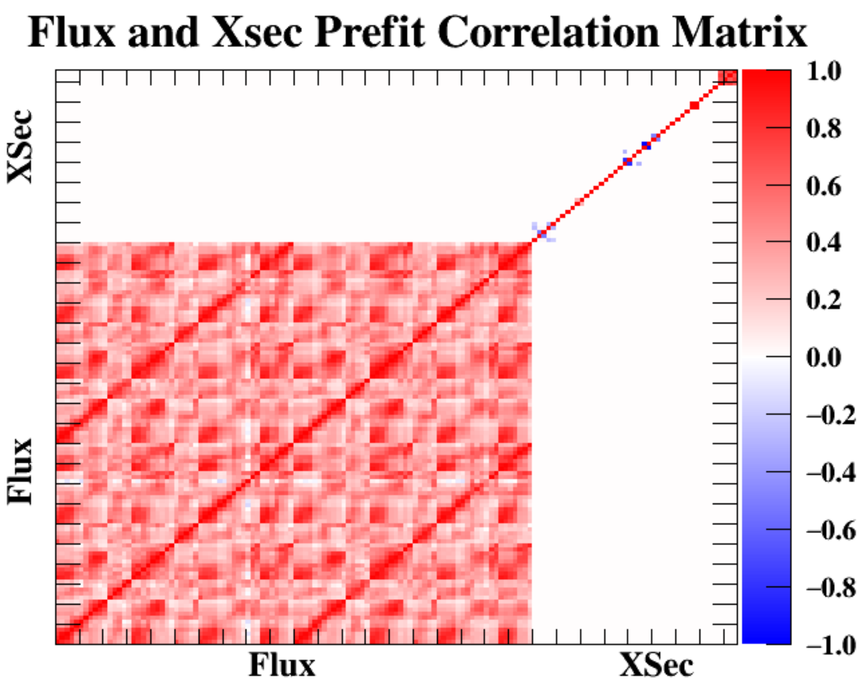


Anti-neutrino mode e-ring

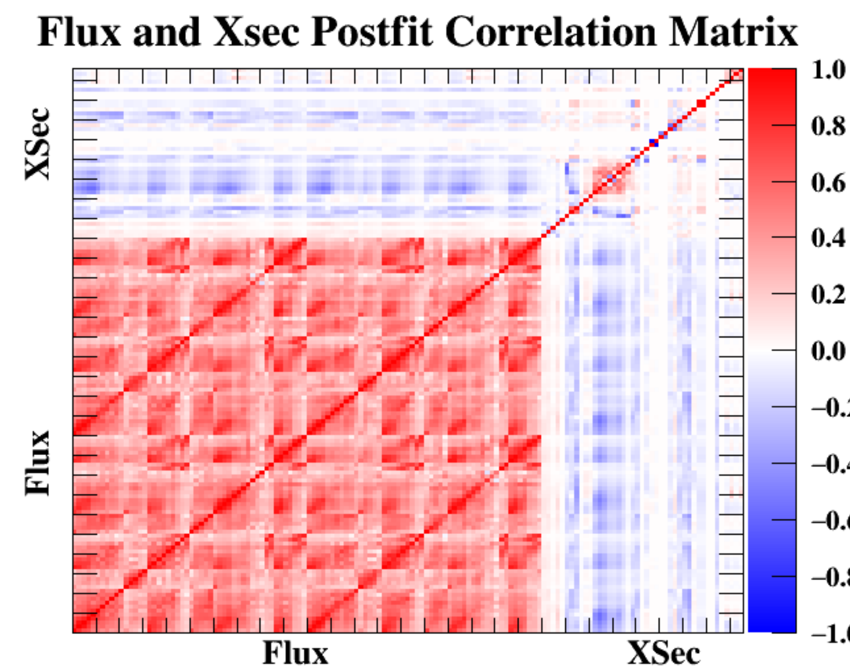


ND280 fit results

- ND280 data constraints uncertainties on neutrino interactions and neutrino flux before oscillations have occurred
- Significantly reduces uncertainty on prediction at SK
- ND280 constrains systematics to the ~3% level
- The ND280 fit matches our data well (prior model p-value of 74%)

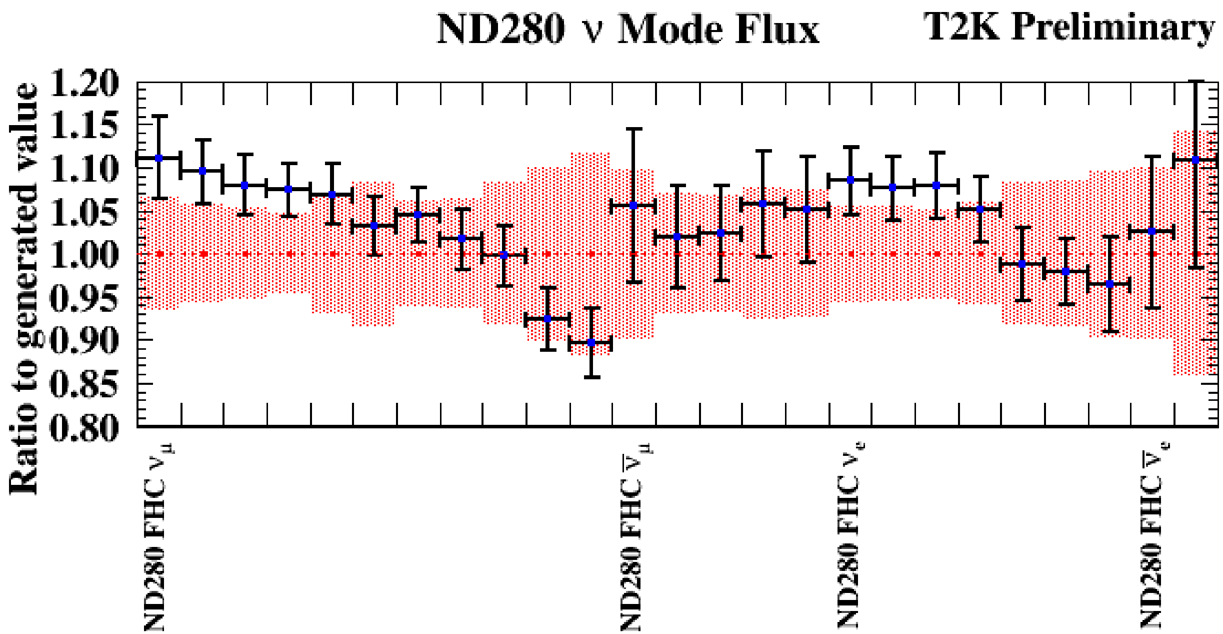


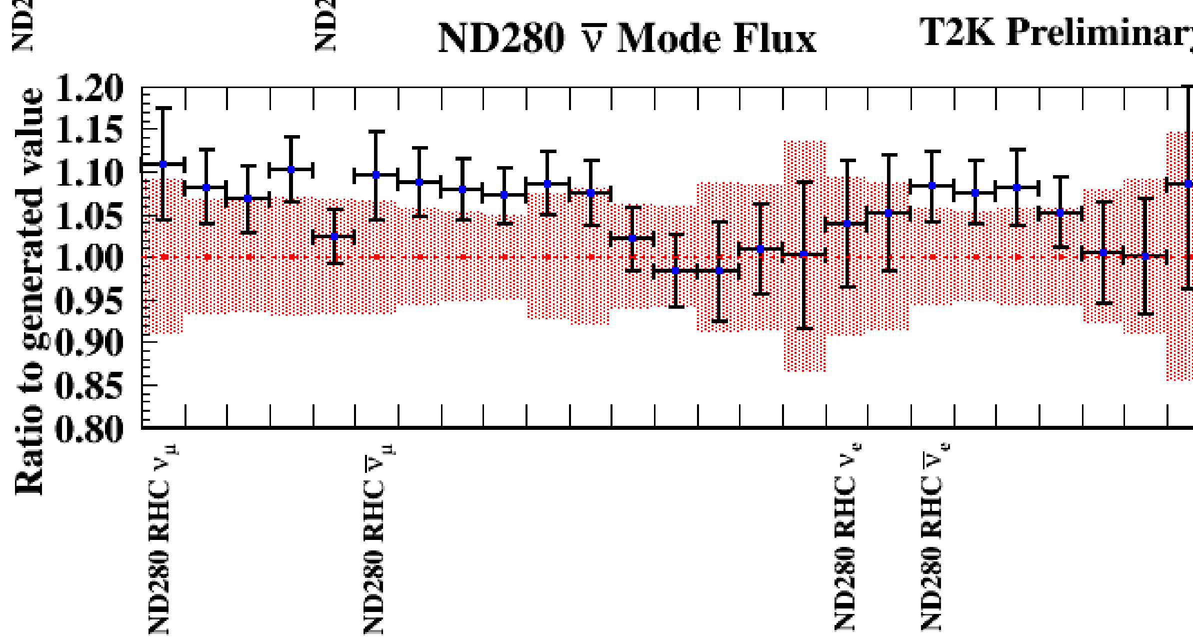
T2K Preliminary

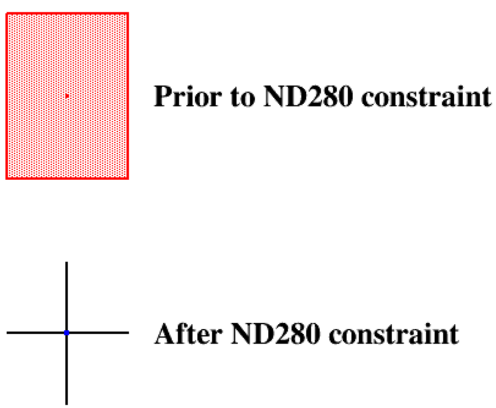


T2K Preliminary

ND280 post-fit parameters: flux







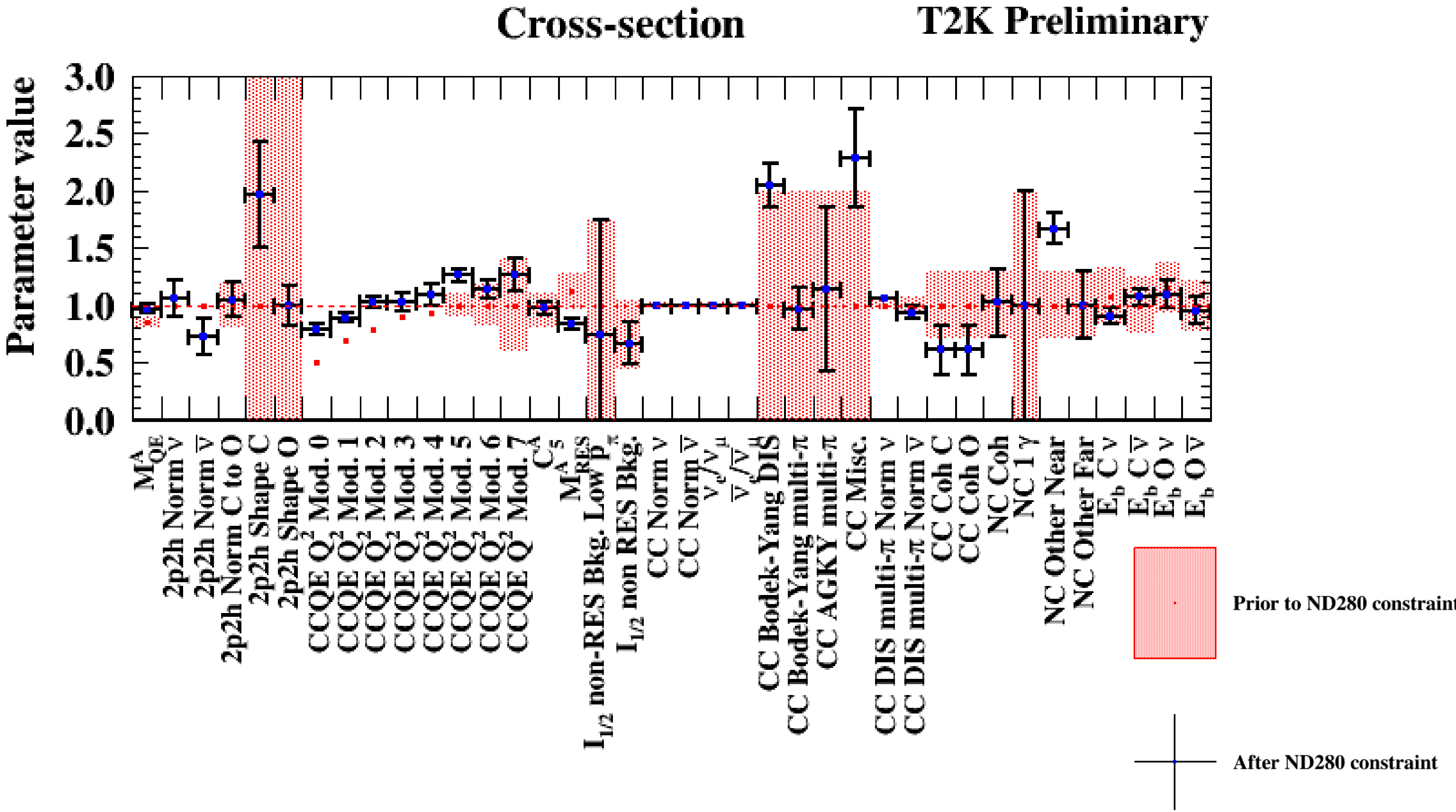
Systematic uncertainty at SK

Systematic Uncertainty					
	Neutrino Mode			Anti-neutrino Mode	
	1 ring μ-like	1 ring e-like	1 ring e-like 1 d.e.	1 ring μ-like	1 ring e-like
Before ND280 fit	11.1%	13.0%	18.7%	11.3%	12.1%
After ND280 fit	3.0%	4.7%	14.3%	4.0%	5.9%

Sources of uncertainty before ND280 fit

	1Rμ		1Re			
Error source (units: %)	FHC	RHC	FHC	RHC	FHC CC1π^+	FHC/RHC
Flux	5.1	4.7	4.8	4.7	4.9	2.7
Cross-section (all)	10.1	10.1	11.9	10.3	12.0	10.4
SK+SI+PN	2.9	2.5	3.3	4.4	13.4	1.4
Total	11.1	11.3	13.0	12.1	18.7	10.7

ND280 post-fit parameters: xsec

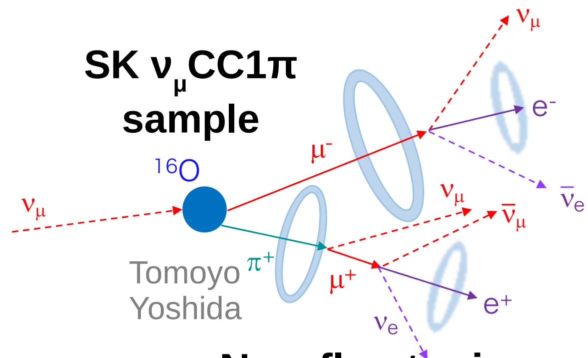


Future plans at T2K

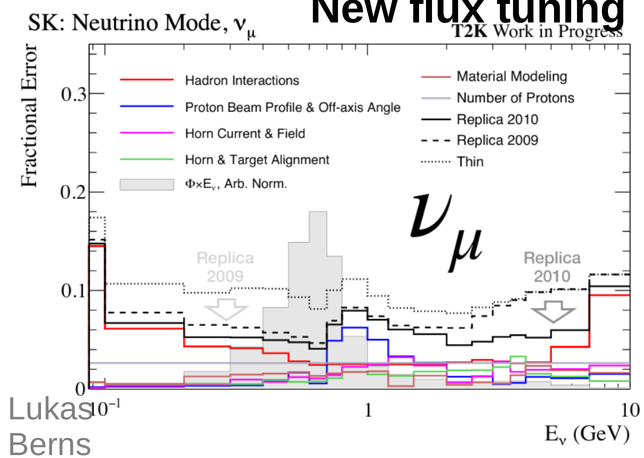
T2K

- More data samples at ND280 and SK; **muon-like sample with pion** at SK, ND280 samples using **proton and photon tagging**
- Improved systematics; **new neutrino flux tuning and neutrino interaction model**
- Cross-section measurements with multiple Near Detectors

SK ν_μCC1π sample



New flux tuning

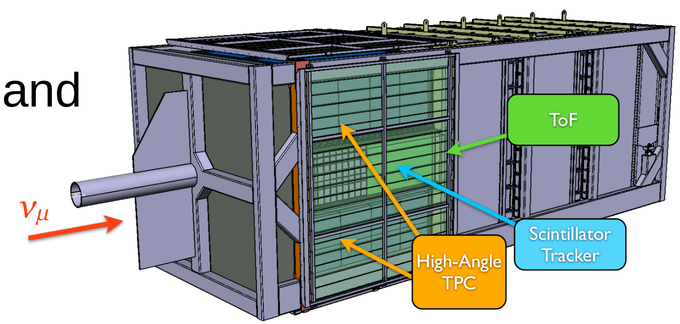


T2K phase-II

- **Upgraded ND280** – high angular coverage, 3D scintillator readout, better hadron tagging and reconstruction
- **SK being doped with Gd** – neutron tagged samples for oscillation analysis
- J-PARC beam upgrade to **0.75 MW and then 1 MW**

Joint-fits

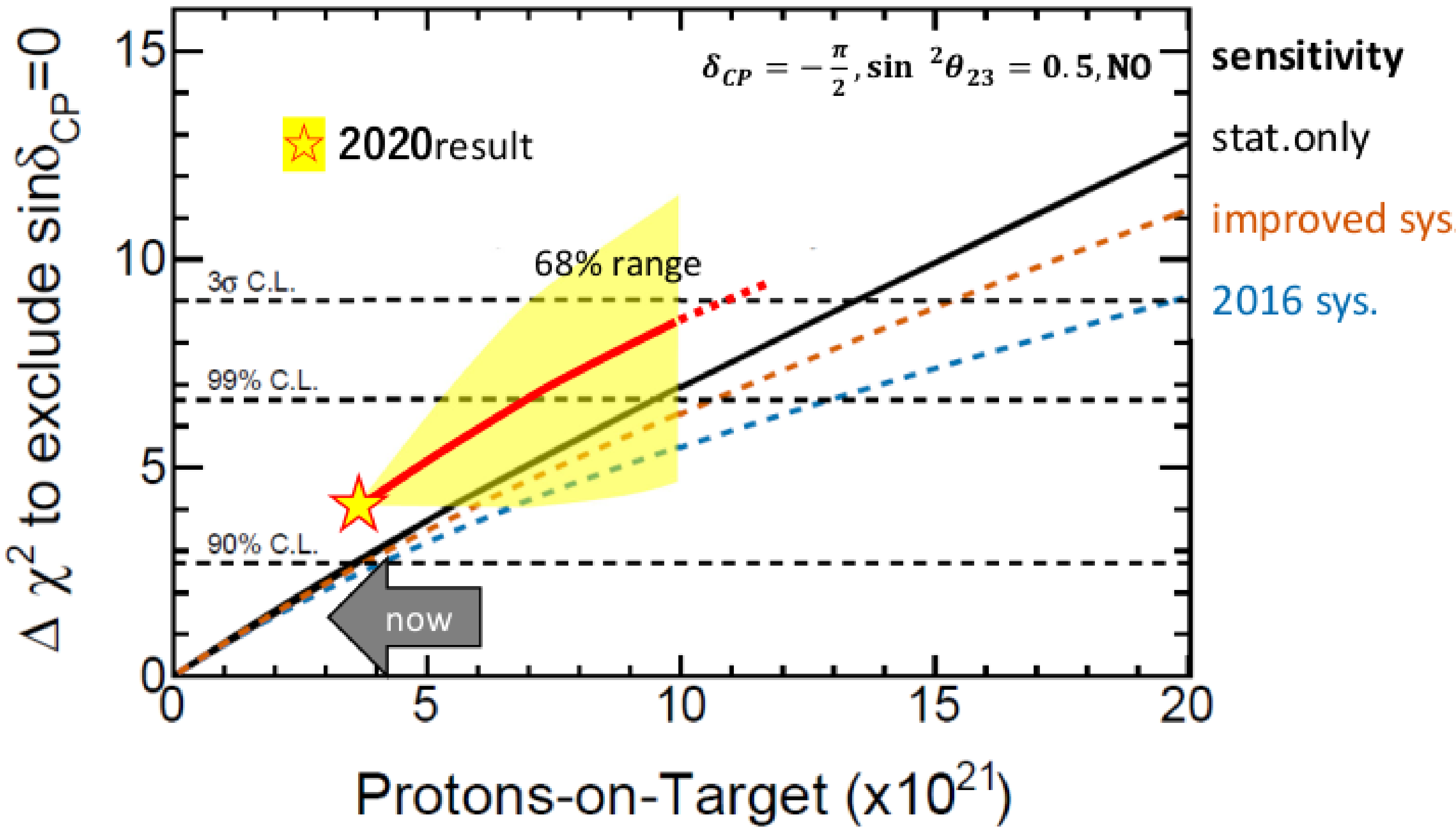
- Joint-fits between T2K and **SK atmospherics** as well as T2K and **NOvA**
- These joint-fits should allow some of the **most precise constraints** on neutrino oscillation parameters.



ND280 Upgrade

T2K Future Sensitivity

Expected evolution of CPV sensitivity for maximal CPV case

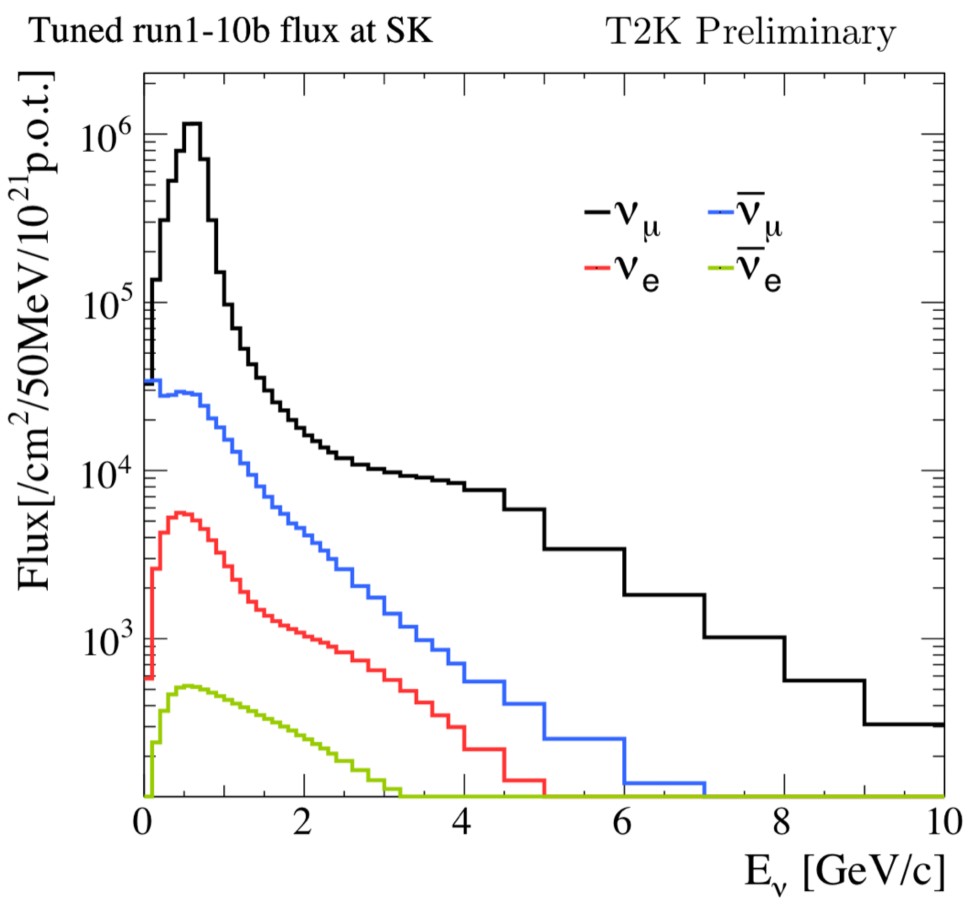


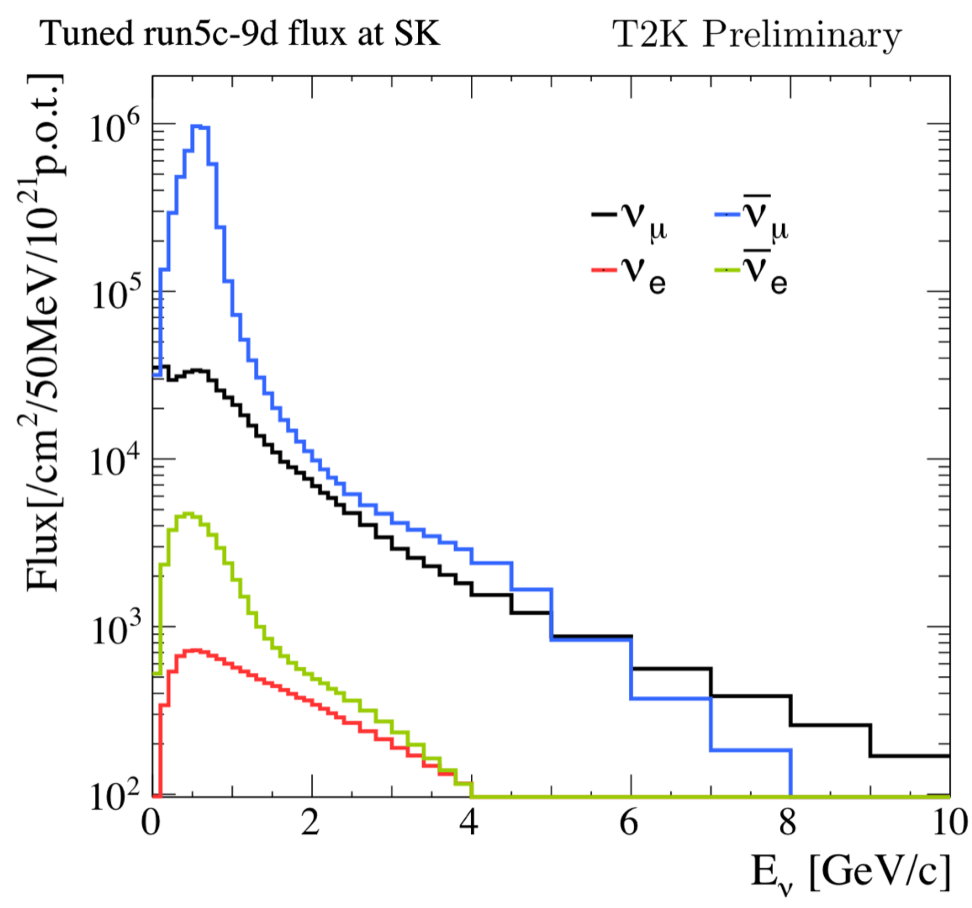
Summary of Data at SK

Selection	Run 1-10 POT	Events in Data
FHC 1Rμ		318
FHC 1Re	19.644×10^{20}	94
FHC 1Re1d.e		14
RHC 1Rμ	16.34556×10^{20}	137
RHC 1Re		16

SK flux prediction

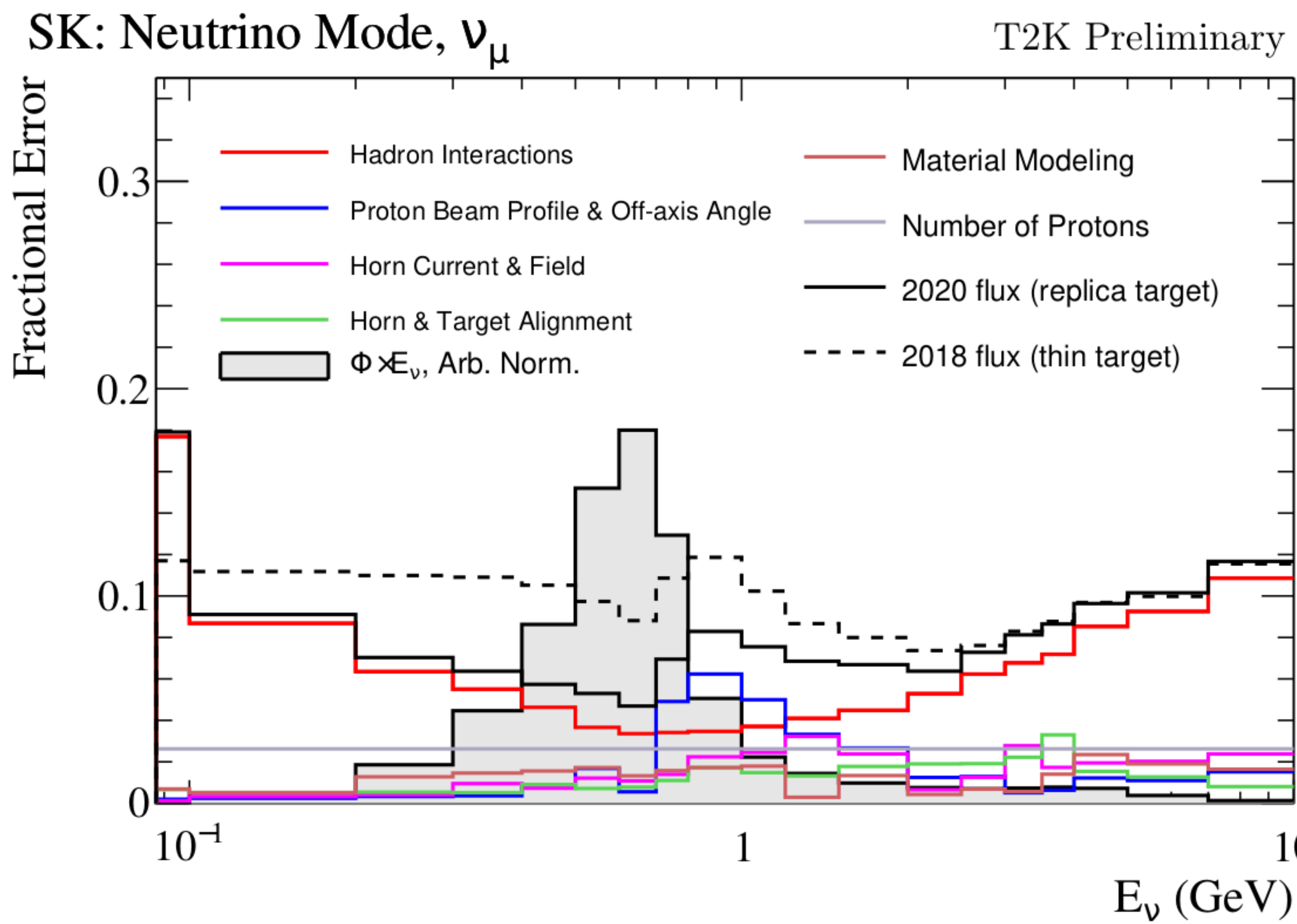
Flux predictions at SK for different flavour components for neutrino mode (left) and anti-neutrino mode (right).





Flux Uncertainties

- Flux uncertainties come from a variety of sources; hadron interactions, proton beam, horn current, target alignament etc.
- Use beam monitors and external data from NA61 to make pre-fit flux prediction.



Neutrino energy reconstruction at SK

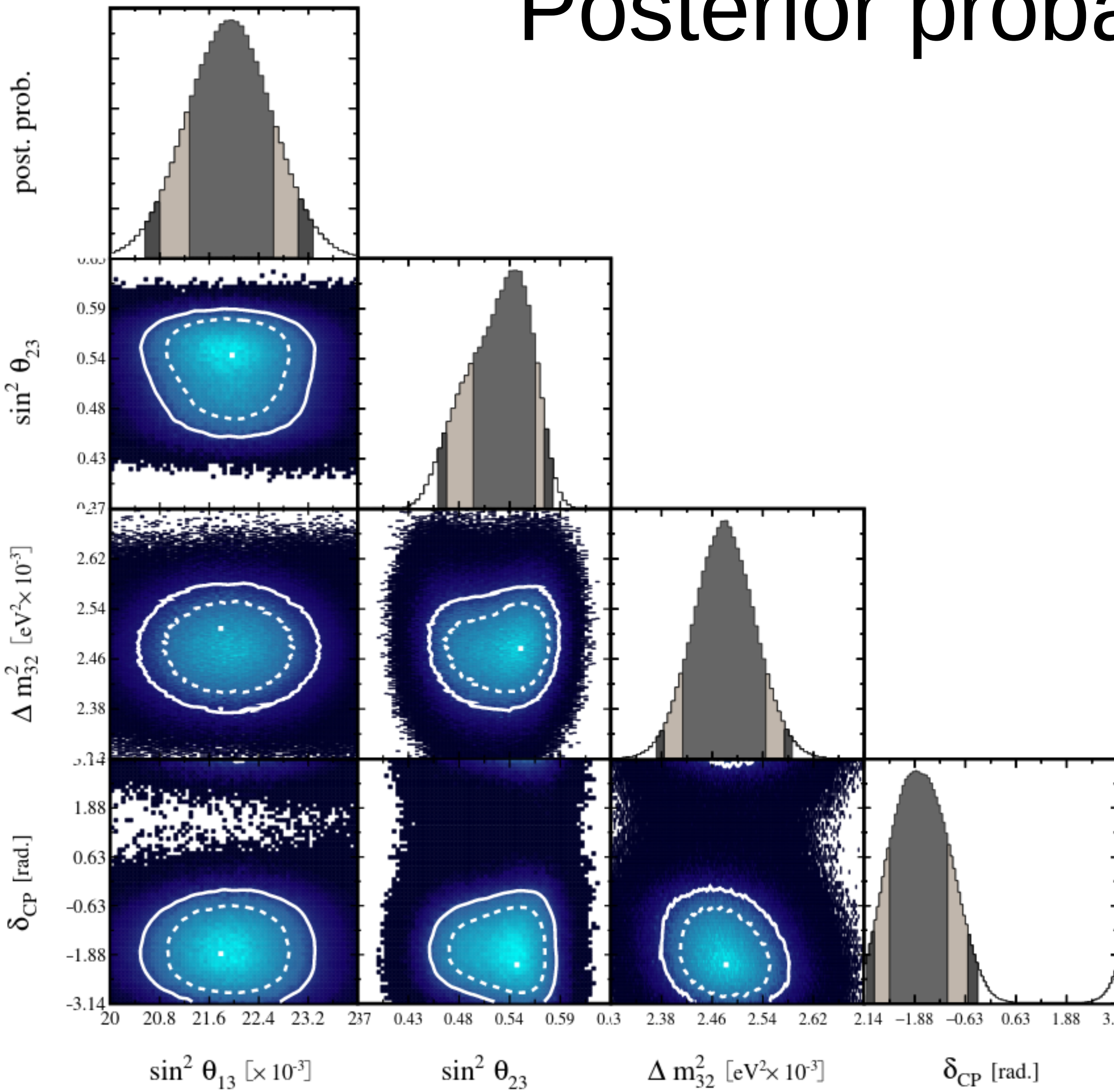
Neutrino energy reconstructed assuming CCQE interaction for single-ring samples.
Only uses lepton kinematics, particle masses and nuclear model.

$$E_{reco} = \frac{m_p^2 - m_n^2 - m_l^2 + 2m_n E_l}{2\left(m_n - E_l + p_l \cos\theta_{\nu l}\right)}$$

For single-ring with 1 michel electron sample, events assumed to have come from delta++ decay.

$$E_{reco} = \frac{m_{\Delta++}^2 - m_p^2 - m_l^2 + 2m_p E_l}{2\left(m_p - E_l + p_l \cos\theta_{\nu l}\right)}$$

Posterior probabilities



Bayesian "triangle plot" of all oscillation parameters.

2D posteriors:

- Dashed lines 68% credible interval
- Solid lines 90% credible interval

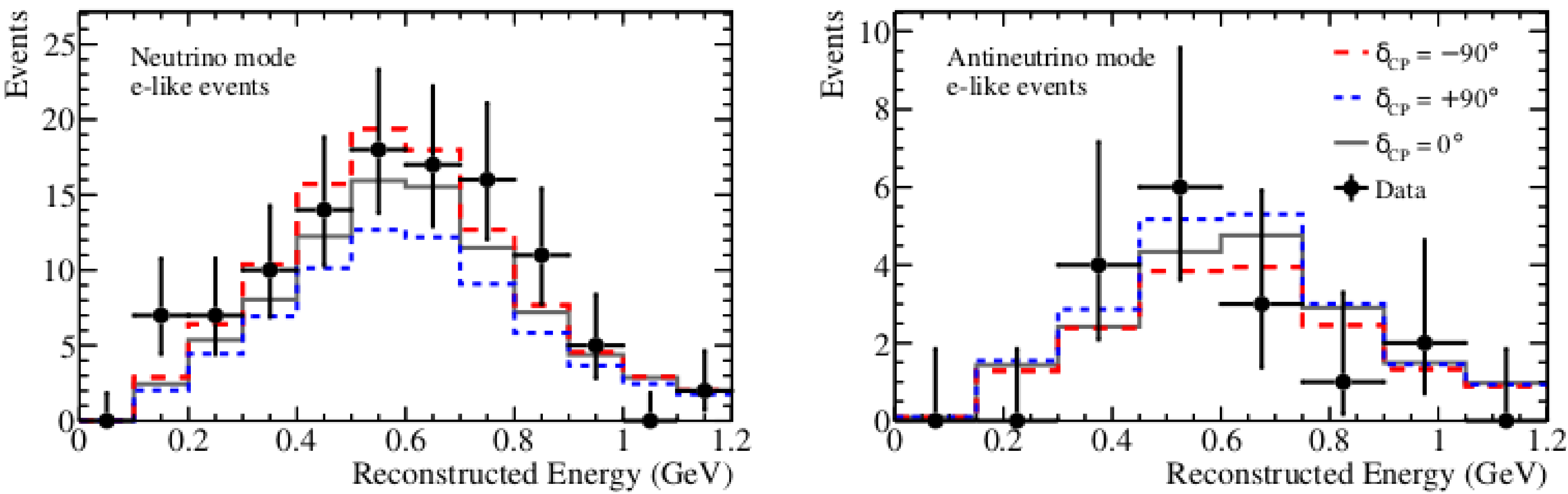
1D posteriors:

- 68%, 90% and 95.4% (2σ)

Appearance dCP comparison

Comparison of 1 e-like ring samples at SK for different values of dCP

Other oscillation parameters set at best-fit values.



SK p-values

SK p-values using reactor constraint.

Sample / p-value	**Shape-based**	**Total Rate-based**
FHC 1Rμ	0.48	0.18
FHC 1Re	0.19	0.49
RHC 1Rμ	0.85	0.74
RHC 1Re	0.61	0.39
FHC 1Re1d.e.	0.86	0.22
Total	0.73	0.30

T2K Analysis

$$
\begin{aligned}
-\ln(P(\vec{\theta}|D)) = & \sum_{i}^{ND280bins} N_i^{ND,p}(\vec{f},\vec{x},\vec{d}) - N_i^{ND,d} + N_i^{ND,d} ln[N_i^{ND,d}/N_i^{ND,p}(\vec{f},\vec{x},\vec{d})] \\
& + \sum_{i}^{SKbins} N_i^{SK,p}(\vec{f},\vec{x},\vec{skd},\vec{o}) - N_i^{SK,d} + N_i^{SK,d} ln[N_i^{SK,d}/N_i^{SK,p}(\vec{f},\vec{x},\vec{skd},\vec{o})] \\
& + \frac{1}{2}\sum_{i}^{osc}\sum_{j}^{osc} \Delta o_i (V_o^{-1})_{i,j} \Delta o_j \\
& + \frac{1}{2}\sum_{i}^{flux}\sum_{j}^{flux} \Delta f_i (V_f^{-1})_{i,j} \Delta f_j \\
& + \frac{1}{2}\sum_{i}^{xsec}\sum_{j}^{xsec} \Delta x_i (V_x^{-1})_{i,j} \Delta x_j \\
& + \frac{1}{2}\sum_{i}^{nd280det}\sum_{j}^{nd280det} \Delta d_i (V_d^{-1})_{i,j} \Delta d_j \\
& + \frac{1}{2}\sum_{i}^{skdet}\sum_{j}^{skdet} \Delta skd_i (V_{skd}^{-1})_{i,j} \Delta skd_j
\end{aligned}
$$

Data at ND280

Data at SK

What we want!!

Oscillation Parameters

Flux

Interaction Model

ND280

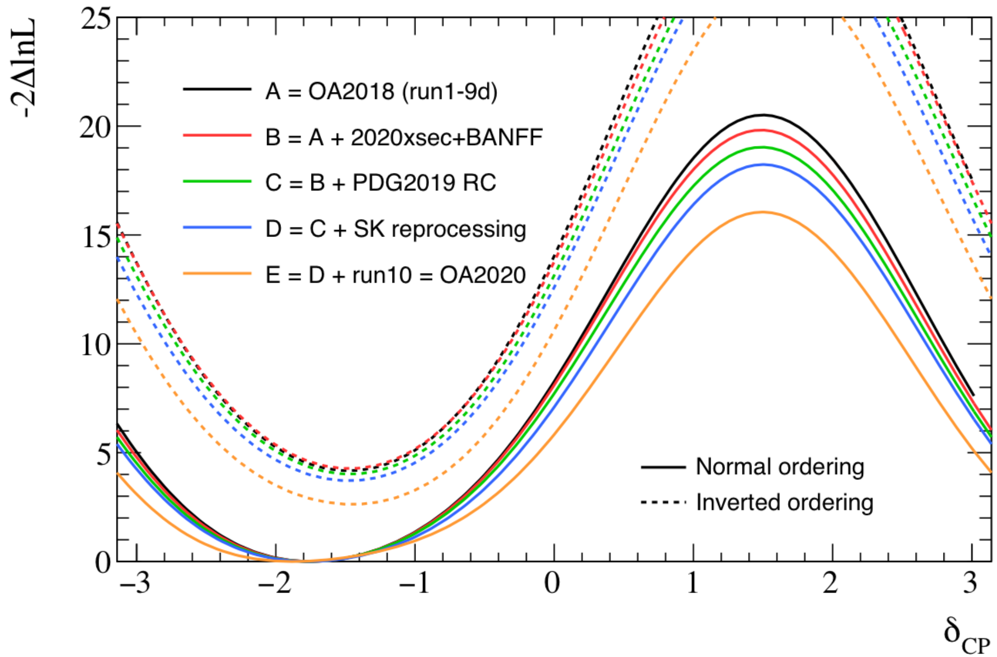
SK Detector

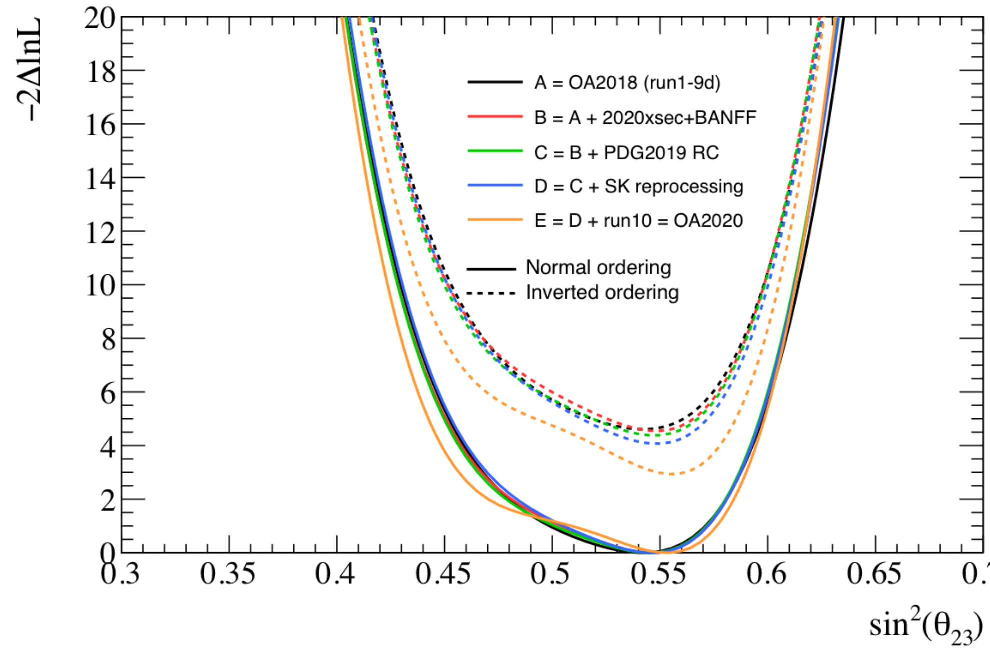
Use priors from various sources

Comparison to previous analyses

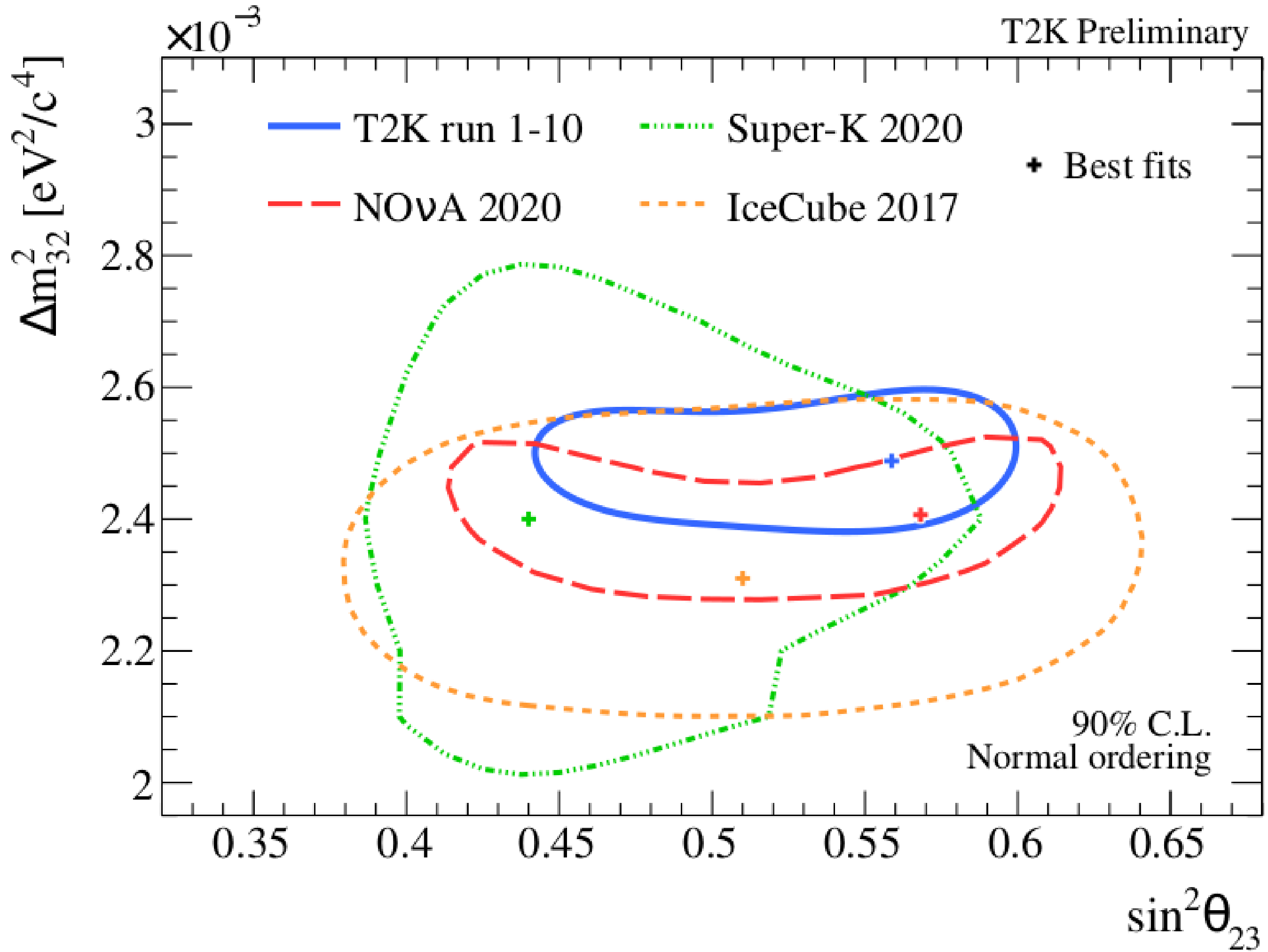
Comparison of 2020 analysis with 2018 analysis, showing the impact of different updates in the analysis on the sensitivity.

- BANFF is the ND280 fit
- SK reprocessing migrates some event due to new calibration
- Addition of new data has largest impact





Comparison to other experiments



Summary of oscillation results

Disappearance best-fit and credible intervals with reactor constraint

	$\sin^2\theta_{23}$	$\Delta m^2_{32}(\times 10^{-3})\text{eV}^2$
2D best fit	0.546	2.49
68% C.I. (1σ) range	0.50 – 0.57	2.408 – 2.548
90% C.I. range	0.460 – 0.587	−2.596 – −2.452 & 2.368 – 2.592

Appearance best-fit and credible intervals with reactor constraint

	$\sin^2\theta_{13}$	δ_{CP}
2D best fit	0.0220	−1.967
68% C.I. (1σ) range	0.0212 – 0.0226	−2.545 – −1.037
90% C.I. range	0.0208 – 0.0231	−2.922 – −0.565
95.4% C.I. range	0.0206 – 0.0234	$-\pi$ – −0.346
99% C.I. range	0.0201 – 0.0237	$-\pi$ – 0.063 & 2.827 – π
99.7% C.I. range	0.0198 – 0.0240	$-\pi$ – 0.346 & 2.545 – π

Posterior probabilities for mass ordering and octant

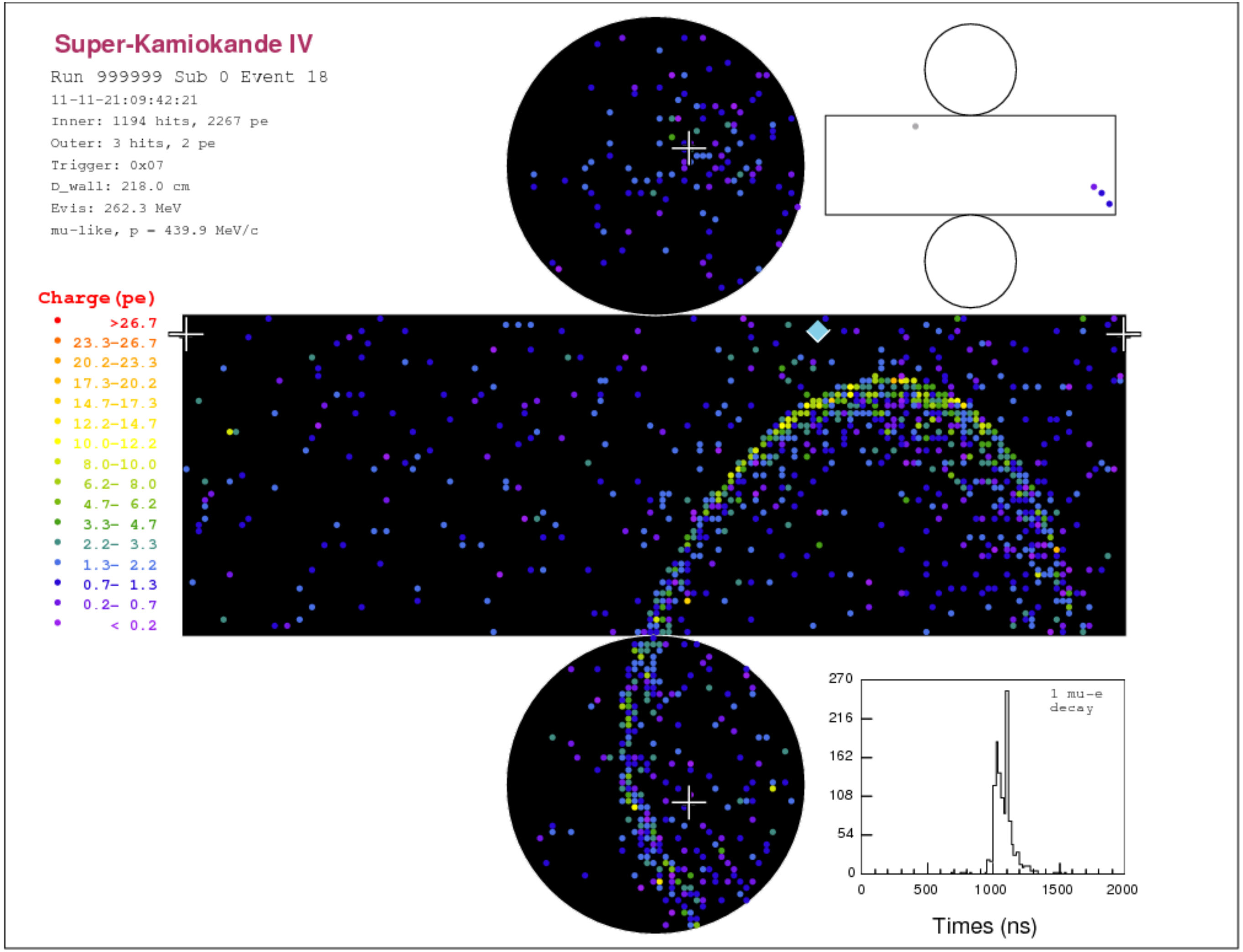
	$\sin^2\theta_{23} < 0.5$	$\sin^2\theta_{23} > 0.5$	Sum
NH ($\Delta m^2_{32} > 0$)	0.195	0.613	0.808
IH ($\Delta m^2_{32} < 0$)	0.034	0.158	0.192
Sum	0.229	0.771	1.000

Different fitters

Summary of the different statistical techniques used by the three fitters at T2K

	Analysis 1	Analysis 2	Analysis 3
Kinematic variables for 1Re sample at SK	Erec-θ	p_e-θ	Erec-θ
Likelihood	Binned Poisson Likelihood Ratio	Binned Poisson Likelihood Ratio	Binned Poisson Likelihood Ratio
Likelihood Optimization	Markov Chain Monte Carlo	Gradient descent and grid scan	Gradient descent and grid scan
Contours/limits produced	Bayesian Credible Intervals	Frequentist Confidence Intervals with Feldman-Cousins (credible intervals supplemental)	Frequentist Confidence Intervals with Feldman-Cousins
Mass Hierarchy Analysis	Bayes factor from fraction of MCMC points in each hierarchy	Bayes factor from likelihood integration	Frequentist p-value from generated PDF
Near Detector Information	Simultaneous joint fit	Constraint Matrix	Constraint Matrix
Systematics Handling	Simultaneous fit then marginalization	Marginalization during fit	Marginalization during fit

SK event display ν_μ



SK event display ν_e