"Alberta Number Theory Days XV" 24w2020 (Banff)

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1   Overview of the Field

Number theory is a broad and central area of research with many connections and applications to other areas of mathematics and science. Canada is one of the major hubs for number theory in the world, and western Canada has always been one of the major Canadian centres for number theory. Alberta number theorists are working in a wide range of subdisciplines, including analytic number theory, arithmetic geometry, automorphic forms, the Langlands programme, computational number theory, algebraic dynamics and diophantine geometry, and the interactions of number theory with mathematical physics and with applications in cryptography.

2   Scientific Highlights

Alberta number theorists are working in a wide range of subdisciplines, including analytic number theory, arithmetic geometry, modular forms and L-functions, the Langlands programme, computational number theory, arithmetic statistics, algebraic dynamics and diophantine geometry, and the interactions of number theory with mathematical physics and cryptography.

There were a total of 14 presentations given: four 50-minute plenary talks, one 45-minute talk, seven 20-minute talks by postdoctoral fellows and PhD students, and two 15-minute talks by master's students. 12 of these talks were delivered in person from TCPL 201 at BIRS and 2 were delivered online (one speaker opted for an online talk closer to the event due to family restrictions, and the other was a plenary speaker and he had to change it last minute as his flight got cancelled due to bad weather conditions). Out of all presentations, 4 video recordings and 12 talk slides were shared to the public. We aimed to actively include several early-career researchers, post-doctoral fellows, and graduate students, in our speaker list. We also asked speakers for their talks to be graduate-student-friendly. We are proud to say that graduate students and postdoctoral fellows account for more than 64% of the speakers at this event.

2.1   Plenary Lectures

We invited 3 speakers from outside of Alberta and 2 speakers from within Alberta to give presentations on their research. These speakers were carefully selected: it was important to ensure that the research presented fell under the umbrella of research topics supported by Alberta institutions. The 4 plenary speakers (and approximate research area) were Alvaro Lozano-Robledo (elliptic curves and Galois representations), Chantal David (analytic number theory), and Jason Bell (online, algebraic dynamics) and Ha Tran (algebraic number theory). We had a special talk by Jean-Francois Biasse (computational number theory), a visiting professor at the University of Calgary from the University of South Florida. The titles and abstracts of their talks are included here:

- **Ha Tran (Concordia University of Edmonton)**
  Title: *Well-rounded ideals of cyclic cubic and quartic fields*

Abstract: A well-rounded (WR) ideal lattice or a WR ideal is an ideal of a number field for which the associated lattice is well-rounded. WR ideal lattices can be used to investigate various problems such as kissing numbers, sphere packing problems, and Minkowski's conjecture. They also have a variety of applications to coding theory. In this talk, we show criteria for when cyclic cubic and cyclic quartic fields have well-rounded ideal lattices. We prove that every cyclic cubic field has at least one well-rounded ideal. We also prove that there exist families of cyclic quartic fields which have well-rounded ideals and explicitly construct their minimal bases. In addition, for a given prime number p, if a cyclic quartic field has a unique prime ideal above p, then we provide the necessary and sufficient conditions for that ideal to be well-rounded.

- **Alvaro Lozano-Robledo (University of Connecticut)**
  Title: *Advances in the theory of Galois representations attached to elliptic curves*

  Abstract: In this talk we will give an overview of recent advances in the theory of Galois representations attached to elliptic curves. In particular, we will discuss recent results of the speaker and collaborators (Enrique González-Jiménez, Asimina Hamakiotes, Benjamin York) in the special case of Galois representations attached to elliptic curves with complex multiplication, where we can give a complete classification of the ℓ-adic representations that occur, together with the arithmetic applications of such classification.

- **Jason Bell (University of Waterloo) **Virtual Talk****
  Title: *Transcendental dynamical degrees of birational maps*

  Abstract: The degree of a dominant rational map $f : \mathbb{P}^n \to \mathbb{P}^n$ is the common degree of its homogeneous components. By considering iterates of f, one can form a sequence $\deg(f^n)$, which is submultiplicative and hence has the property that there is some $\lambda \geq 1$ such that $(\deg(f^n))^{1/n} \to \lambda$. The quantity λ is called the first dynamical degree of f. We'll give an overview of the significance of the dynamical degree in complex dynamics and describe an example of a birational self-map of $\mathbb{P}^3$ in which this dynamical degree is provably transcendental. *(This is joint work with Jeffrey Diller, Mattias Jonsson, and Holly Krieger.)*

- **Chantal David (Concordia University)**
  Title: *Equidistribution of quartic Gauss sums at primes arguments*

  Abstract: Gauss sums are fundamental objects in number theory. Quadratic Gauss sums were studied by Gauss who gave a simple formula depending only on the argument of the Gauss sums modulo 4. Higher degree Gauss sums behave differently. It was conjectured by Kummer that the cosines of the angles of (normalized) cubic Gauss at prime arguments are not equidistributed, and exhibit a bias towards positive values. This was disproved by Heath-Brown and Patterson in 1979, and they showed that (normalized) cubic Gauss at prime arguments are equidistributed. This was later generalized by Patterson to general nth-order Gauss sums. We explain in this talk what is involved in proving those results, and how to improve the results of Patterson for the distribution of quartic Gauss sums at prime arguments. (*This is joint work with A. Dunn, A. Hamieh and H. Lin.*)

And a Special Long-talk:

- **Jean-Francois Biasse (University of Calgary & University of South Florida)**
  Title: *Norm relations and computational problems in number fields*

  Abstract: In this talk, I will discuss recent work on how to reduce computational problems to subfield calculations via the framework of norm relations. For example, this enables efficient class group and S-unit group computations in large degree number fields. This also enables the search for small generators of principle ideals in fields of large degree, which has applications to the computation of approximate short vectors in ideal lattices (a topic with applications to cryptography).

2.2   Junior research highlights

We invited junior speakers from each institution who had not had the opportunity to share their research in a previous ANTD meeting. These talks exhibited the breadth of research being conducted in Alberta: topics

included Analytic, Algebraic, and Computational Number Theory, Automorphic Forms, and Representation Theory. We scheduled 9 short talks. We gave postdocs and PhD students 20 minutes, plus 5 for questions. Master's students gave 15-minute presentations, plus 5-minute dedicated questions time. Of the 9 talks, 5 were given by postdocs, 2 were by advanced PhD students, and 2 were by Master's students, talking about their current work.

Talks by **Master's students**:

- **Sreerupa Bhattacharjee (University of Lethbridge)**
  Title: *Parity Bias in Partitions and Restricted Partitions*

- **Golnoush Farzanfard (University of Lethbridge)**
  Title: *Zero Density for the Riemann zeta function*

Talks by **Ph.D. Students**:

- **Fatemeh Jalalvand (University of Calgary)**
  Title: *Geometry of Log-unit lattices*

- **James Steele (University of Calgary)**
  Title: *Categorical Structure in the Local Langlands Correspondence for p-adic Groups*

Talks by **Postdoctoral Fellows**:

- **Kübra Benli (University of Lethbridge)**
  Title: *Sums of proper divisors with missing digits*

- **Erik Holmes (University of Toronto, last year U of Calgary)) **Virtual Talk****
  Title: *Shapes of lattices in number theory*

- **Abbas Maarefparvar (University of Lethbridge)**
  Title: *On Divisibility of Class Numbers of Cubic Fields by Three*

- **Greg Knapp (University of Calgary)**
  Title: *Exponential Relations Among Algebraic Integer Conjugates*

- **Sarah Dijols (University of British Columbia, last year U of Calgary))**
  Title: *Parabolically induced representations of p-adic G_2 distinguished by SO_4*

The question sessions were very active and many continued into the breaks/scheduled social times. It seems that many collaborations had formed during this weekend conference.

3   Comments

3.1   Facilitating communication among participants

The structured schedule at BIRS, including coffee breaks and designated social times in the evenings, played a pivotal role in facilitating communication among participants. These intervals provided valuable opportunities for attendees to exchange ideas about their current academic positions, career aspirations, and ongoing

research projects. One of the organizers, who was present in person, took charge of chairing the sessions. This responsibility included facilitating the exchange of questions between speakers and attendees, both in-person and online. The online component, though small, was consistently active and engaged, allowing for direct and meaningful interactions with the speakers, thereby enriching the collaborative spirit of the conference.

3.2   Fostering a welcoming and inclusive environment at BIRS

In alignment with our sponsors' (BIRS, PIMS and NTF) statement on Equity, Diversity, and Inclusion (EDI), this edition of Alberta Number Theory Days diligently upheld principles that foster an open, diverse, and inclusive academic community. This year, 50% of all talks were delivered by female speakers, including 50% of the plenary talks and 40% of those by postdoctoral researchers. Among student presentations, 75% were by women, highlighting our commitment to supporting emerging female mathematicians.

Our conference, constrained to 25 in-person participants by BIRS, achieved a female representation of 44%, reflective of broader systemic issues rather than organizational bias. However, including observers (who were all senior faculty, the overall in-person female attendance rate was approximately 36%. The composition of senior faculty—predominantly male due to factors beyond our control—affected this statistic. The conference format was hybrid, allowing us to include more students from Albertan universities and connect with distant members of the community. In total, the in-person group consisted of 12 students, 8 postdocs, and 5 faculty members, with 80% being early career researchers. Moreover, 64% of in-person participants were part of the visible minority. The visible minority totalled 54.55% of all participants, which is still strictly greater than 50%. Our setup not only adhered to but also propelled our vision for a mathematical community enriched by diverse perspectives and equitable opportunities. Through deliberate planning and dedicated efforts, ANTD provided a supportive and safe environment, where talents from underrepresented groups were encouraged and showcased, thereby enriching the academic and collaborative quality of the conference.

4   Outcome of the Meeting

The 15th edition of the event continued its tradition of showcasing the emerging talents of Calgary, Edmonton, and Lethbridge. A significant number of presentations were delivered by graduate students and postdoctoral researchers, marking the first formal research talk for several speakers. These presentations were met with enthusiastic engagement, constructive feedback, and supportive questions from an interested and welcoming audience. The sessions fostered a lively exchange of ideas, evidenced by extensive discussions that spilled over from formal talks to informal social gatherings, catalyzing new and ongoing collaborations.

This year's program featured four plenary talks by distinguished academics—three professors and one associate professor. While three of these speakers came from outside Alberta, their inclusion was strategic, aimed at extending the local research network and exposing Alberta's early-career researchers to seminal work in their respective fields. Moreover, a special lecture was presented by a professor intimately connected with Alberta's number theory community, currently stationed at the University of South Florida but also holding a visiting position at the University of Calgary in Alberta during the conference. Maintaining and fostering these academic relationships are crucial for the community's growth, particularly in supporting its newer members.

Due to a systematic error at BIRS, only four video recordings from the conference are available to the public, including two of the plenary lectures, which is fewer than in previous years. Nevertheless, the slides for 12 of the 13 presentations that used slides have been made available on the conference website. The conference also successfully recorded and posted the only talk that utilized a blackboard.

ANTD has consistently and traditionally played a crucial role in unifying Alberta's geographically dispersed number theory research groups. This year, we continued to utilize a hybrid model to enhance accessibility. This format not only facilitated the participation of graduate students from Alberta's institutions but also attracted attendees from other PIMS institutions and re-engaged former members of the Alberta number theorists community and beyond. The event underscored the pivotal role of gatherings like ANTD in **building a welcoming and supportive community within number theory across Western Canada**, emphasizing the collective endeavour to advance mathematical research and collaboration.