EQUIVARIANT BORDISM AND APPLICATIONS

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Bordism theory is one of the main successes of algebraic topology, with the Thom theorem giving a homotopical interpretation of the bordism groups, the Hirzebruch-Riemann-Roch theorem, the first proof of the Atiyah-Singer index theorem, and the proof of the Mumford conjecture. The starting point of this workshop was an old problem in equivariant bordism, which Uribe revived in his 2018 ICM invited lecture [35]: The G-equivariant unitary bordism group $\Omega_*^{U,G}$ is a free Ω_*^U-module on even-dimensional generators whenever G is a compact Lie group...*the evenness conjecture*. This question has been dormant for several years, said Rowlett in 1980. Comezaa in 1996 restated it again over-optimistically, which was thought to be false by Greenlees and May in 1997. If we believe in the evenness conjecture, the natural map from free bordism group $\Omega_*(BG) \to \Omega_*^G$ is trivial since all the elements in $\Omega_*(BG)$ are torsion. In 2018, Uribe came to Oaxaca and asked Segovia: Do all free actions on the oriented surfaces equivariantly bound?...*the free extension conjecture*. After this, Segovia and Domnguez, a bachelor student participating in the 2020 summer research of the Mexican academic of science, brought to light this problem [11] now published in 2022. They showed the free extension conjecture for abelian, symmetric, dihedral, and alternating groups. Immediately, the paper was in the ArXivs in 2020; the following day, Samperton contacted Segovia and showed that he was thinking on similar terms [29]. In 2021, Samperton came up with his counterexample of the free extension conjecture with groups of orders $3^5 = 243$, disproving the free extension conjecture and the evenness conjecture. At that time, it was believed that it might be counterexamples of lower order with a non-trivial Bogomolov multiplier. Finally, Angel-Samperton-Segovia-Uribe [2] in 2021, showed that the Bogomolov multiplier is the complete obstruction to extending finite free actions from closed orientable surfaces to 3-manifolds, which implies that there are counterexamples of orders $2^6 = 64$, $2^7 = 128$ and $2^6 \times 3 = 192$. Since the Thom theorem does not work for equivariant versions of geometric and homotopical cobordism, Uribe thought the homotopical version of the evenness conjecture could be true. Indeed, Uribe asked this question to Landweber on a bordism session in the CLAM 2020. Sophie Kriz [19], at the age of only 16, showed in 2022 a counterexample of the homotopical evenness conjecture constructed for $p > 2$ a prime number, by a p-Sylow subgroup $P \subset \mathrm{GL}_4(\mathbb{F}_p)$. The connection between the geometric and homotopical conjecture is now mysterious. For instance, the Samperton counterexample provides torsion breaking the freeness part, and the Kriz counterexample breaks the evenness part and has a trivial Bogomolov multiplier. There are still open questions about the evenness conjecture that detects rationality in the Noether problem. Recently, on the ArXivs 2023 [9], Segovia, with other bachelor students Cruz-Ortega, posted the solution for the extendability of finite group action over non-orientable surfaces, which opened all the previous questions to the non-orientable context.

1 Overview of the Field

The workshop focused on three main topics: The extension of free actions on surfaces, The unitary evenness conjecture and rationality, and Relations with other bordism theories. We are going to describe in detail these three topics. Additionally, the event had some themes related to the application part. For instance, there were expositions on compositional quantum field theory, factorization algebras and TQFT, mapping class group, knot theory and quantum groups, and twisted K-theory.

1.1 The extension of free actions on surfaces

Obstructions to the extendability of finite group actions over surfaces exist in the literature. For a finite group G, a free surface action of G freely extends, if and only, if the associated class in the Schur multiplier is trivial. The Schur multiplier can be understood as the 2-dimensional free bordism group $\Omega_2^{SO}(BG)$, the 2-dimensional integer homology $H_2(G;\mathbb{Z})$, and the 2-dimensional cohomology with coefficients in the units of the complex $H^2(G;\mathbb{C}^*)$. Recently, Angel-Samperton-Segovia-Uribe [2] showed that the complete obstruction to extend a free surface action to a non-necessarily free action on a 3-manifold, is the Bogomolov multiplier $B_0(G)$. This obstruction is defined as the quotient of the Schur multiplier by the subgroup generated by the toral classes, i.e., principal G-bundles with base space a disjoint union of tori. Cruz-Ortega-Segovia [9] gives the associated generalization of the Schur and Bogomolov multiplier for non-oriented surfaces. Recently, there has been great interest in finding conditions for a surface group action, to extend by a solid handlebody. This is due to the Putman-Wieland conjecture [25]. More precisely, if a free action of a group G on a surface S extends to an action on a handlebody H with $\partial H = S$, then the orbit of every nonzero rational homology class in $H_1(S;\mathbb{Q})$ under the action of lifts of mapping classes on the quotient surface S/G is infinite.

1.2 The unitary evenness conjecture and rationality

The equivariant unitary bordism groups are a module over the unitary bordism ring. It was expected that it was a free module in even-dimensional generators (this is the Evenness Conjecture), and the explicit calculation of the equivariant unitary bordism groups of surfaces showed that torsion groups appear and that they are precisely the groups that are called Bogomolov multipliers in algebraic geometry. The Bogomolov multiplier is the nonramified Brauer group associated with a finite group. Whenever this multiplier is non-trivial, the quotient variety associated with a faithful representation is not rational. Rationality problem in algebraic geometry is a very difficult problem, which tries to decide which algebraic varieties are birrational to a projective space. The same invariant appears in two different setups, bordism of surfaces and birational geometry, and its relation is intriguing. The complete relation of the equivariant unitary bordism invariants and the birational invariants associated with finite groups is still mysterious but interesting.

1.3 Relations with other bordism theories

An interesting bordism theory is made by a cut-paste surgery process denoted by SK_* by the German words *Schneiden and Kleben*. The importance of these theories lies in the fact that they characterize important invariants in each dimension. For instance, the cut-paste invariants are generated by the Euler characteristic and the signature. There is a variant where the error terms are added to different ways of realizing the cutting by a diffeomorphism, denoted by SKK_*, where the additional K is due to *Kontrollierbar*. Fixing the dimension n, the homomorphism from SKK_n to the positive reals classify invertible n-dimensional topological quantum field theories up to some kernel [27]. The interaction with equivariant bordism is the reduced version denoted by $\overline{SK}_n(BG)$, defined by the kernel of the map $SK_n(X) \to SK_n$, where there is the isomorphism with the Bogomolov multiplier $\overline{SK}_2(BG) \cong B_0(G)$. Hoekzema-Merling-Murray-Rovi-Semikina [15] gives a new perspective on these groups using recent advances in algebraic K-theory. Another bordism theory is provided by the combination of two theories given by stratifolds [18] and $\mathbb{Z}_k$-manifolds [23]. This solves the Steenrod problem with $\mathbb{Z}_k$-coefficients [3] to represent by a geometric gadget any cycle in homology with $\mathbb{Z}_k$-coefficients.

2 Recent Developments and Open Problems

We insist again on the history of disproving the free extension conjecture [11] and hence also of the evenness conjecture [35] by Samperton [30]. Initially, the first counterexamples given by Samperton [30] were finite groups of odd order with non-trivial Bogomolov multipliers. These are three groups of order $3^5 = 243$ in the same isoclinic class. After the whole characterization of the extendability of finite free surface group action by Angel-Samperton-Segovia-Uribe [2], we know that a finite free surface group action non-necessarily freely extends if and only if its associated class inside the Bogomolov multiplier is trivial. Thus, there are counterexamples of lower order $2^6 = 64$, $2^7 = 128$ and $2^6 \times 3 = 192$ with a non-trivial Bogomolov multiplier. Recently, Cruz-Ortega-Segovia [9] provided the complete obstruction for finite surface group actions in the non-orientable context. In this sense, the Schur multiplier is just the homology or cohomology with $\mathbb{Z}_2$-coefficients, i.e., the group $H_2(G;\mathbb{Z}_2) \cong H^2(G;\mathbb{Z}_2)$, and to obtain the Bogomolov multiplier we quotient not only the toral classes, if not the classes with base space, a disjoint union of torus, Klein bottles, and projective spaces. A distinguished element in this non-orientable context is the trivial G-bundle over the projective space, which is not extendable since we have a non-trivial action of the 2-dimensional non-orientable bordism group $\Omega_2^O \cong \mathbb{Z}_2$. These elements are dropped from the 2-dimensional bordism group $\Omega_2^O(BG)$ in order to define the Schur multiplier and the Bogomolov multiplier.

There is a special interest in finding the obstruction to extendability of finite free group actions in dimension three. We know the classification of closed 3-dimensional manifolds by Heegaard diagrams from Thurston, see [13]. To give a geometric characterization of the principal G-bundles over three-manifolds, we need to find an algebraic characterization using the Miller relations in dimension two [22]. More precisely, since they are free actions, we take pairs of representations of principal G-bundles that can be trivialized by Miller relations and together with an identification that extends to an equivariant homomorphism. The algebraic counterpart should be the theory of double extensions; for example, see [26, 12, 16]. Recall for a group presentation $G = \langle F : R \rangle$, we have the Hopf formula $H_2(G) \cong R \cap [F,F]/[F,R]$ assigned to the following central extension:

$$1 \to \frac{R}{[F,R]} \to \frac{F}{[F,R]} \to G \to 1\,.$$

In the case of double extensions for a double presentation $G = \langle F : R_1, R_2 \rangle$, we have the commutative square (2-central extension):

$$\begin{tikzcd} F/[R_1 \cap R_2, F][R_1, R_2] \arrow[r] \arrow[d] & F/R_2 \arrow[d] \\ F/R_1 \arrow[r] & G\,. \end{tikzcd}$$

We obtain the Hopf formula $H_3(G) \cong R_1 \cap R_2 \cap [F,F]/[F, R_1 \cap R_2][R_1, R_2]$ found by Brown-Ellis [6]. The open problem is finding the appropriate Miller relations related to the Heegaard diagrams of principal G-bundles over 3-manifolds with the theory of double presentations. To find the Bogomolov multiplier in dimension three, we need to find here the toral classes using the Samperton exact sequence:

$$\cdots \to H_{n-1}(L^C BG) \xrightarrow{F} H_n(BG) \to H_n(BG_C) \to H_{n-2}(L^C BG) \to \cdots \tag{1}$$

associated with $C \subset G$, a conjugacy invariant subset. The toral classes are given by the image of the map F. For instance, in the case $n = 2$, the Bogomolov multiplier is the cokernel of the map F. For $n = 3$, the toral classes are principal G-bundles with base space of the form $S^1 \times S$ with S a closed surface.

The calculation of the equivariant unitary bordism of surfaces, as well as the proof that the homotopic equivariant unitary bordism groups for certain groups is not free over the bordism ring, showing that the equivariant unitary bordism groups, both geometric and homotopic, still encode topological information about the finite groups that we do not know. Open problems are many, starting with the complete description of the equivariant unitary bordism groups; although this problem might be somewhat impossible (it will be related to the classification of finite groups), smaller but still quite interesting problems abound: Equivariant unitary bordism groups of 3D manifolds, 4D manifolds, equivariant oriented bordism groups or surfaces, 3D and 4D manifolds, equivariant bordism groups with different tangential structures, etc. And the relation with birational invariants of finite groups. Recently, Cruz-Ortega-Segovia showed the complete obstruction to extending a finite free group action for non-orientable surfaces. In this direction, there are two immediate problems: We know the module structure in the non-oriented bordism-group, see Dold [10], where we obtain the isomorphism $\Omega_*^O \cong \mathbb{Z}_2[x_2, x_4, x_5, x_6, x_8, \cdots]$ with x_i for $i \neq 2^k - 1$. An open problem is the analogy of the evenness conjecture in the non-orientable context. Cruz-Ortega-Segovia showed that the group $\mathbb{Z}_8 \rtimes Q_8$ of order 64 has a non-trivial Bogomolov multiplier (non-orientable version). This will violate the possibility of freeness module action of Ω_*^O over the G-bordism group $\Omega_*^{O,G}$, implying that the image $\Omega_*^O \to \Omega_*^{O,G}$ is not trivial. The second open question is about the associated rationality in the non-oriented context. For instance, the new definition of the Bogomolov multiplier in the non-orientable case lacks of an interpretation as a birrational for algebraic varieties. Naively, the possibility of a birrational invariant can be associated with a "real" representation in the sense of Atiyah [4] according to the G-equivariant action. All the contexts involving the Bogomolov multiplier in the non-oriented case are open problems.

Before Samperton [30] came with the disprove of the evenness conjecture [35], it was known that it was true for specific cases: Explicit calculations carried out by Peter Landweber [20] in the cyclic groups, Stong [32] in the abelian p-group, Ossa in [24] generalized this result to any finite abelian group, and for metacyclic groups by Lazarov [21], Rowlet [28] and Ángel-Gómez-Uribe [1]. The main tool used is the equivariant bordism for families $\Omega_*^G(\mathcal{F})$ where the isotropy groups only live on the prescribed family. The Conner-Floyd long exact sequence [7, 8] applied to certain families of all subgroups $\mathcal{A}G$ and the trivial one $\{1\}$, gives the exact sequence

$$\Omega_3^G\{\mathcal{A}G, \{1\}\} \xrightarrow{\partial} \Omega_2(BG) \to \Omega_2^G \to \Omega_2^G\{\mathcal{A}G, \{1\}\}\,, \tag{2}$$

where the Bogomolov multiplier results as the cokernel of ∂. The SK-bordism groups built an important bordism theory by a cut-paste surgery process; see [17]. These groups have the advantage that the calculations on them are easier in a certain sense. We can develop a similar sequence of SK-groups as in (2). An open problem is to find the intertwining relations between this sequence with (2) and also with (1).

We continue with SK groups where Rovi-Schoenbauer provides a relation between SKK invariants and invertible TQFTs, where the additional K is associated with adding error terms that control how we perform a cutting by a diffeomorphism. Indeed, the homomorphism from SKK_n to the positive reals classifies invertible n-dimensional topological quantum field theories up to some kernel. Whenever we talk about invertible TQFT, the category of fractions of the cobordism category is invoked. There are works by Bökstedt-Svane [5] with the "chimera relations" and Segovia [31] with the left calculus of fractions, where the category of fractions has an explicit combinatorial expression. The advantage of these procedures is the feasibility of general cobordism categories defined by cut-pasting, equivariant bundles, or stratified bordisms, among others. An open problem is to verify the results of Rovi-Schoenbauer [27] and the general theory developed by Freed-Hopkins [14]. In the case of Rovi-Schoenbauer [27], we will need the development of a categorification of the SK groups by a cobordism category Cob_{SK} where its connected components return the SK groups. Maybe these ideas can be carried out only at the level of spectra.

Finally, we end with some open questions in stratified bordism. The Steenrod problem asks if every integral homology class is the image of the fundamental class of an oriented manifold. Thom [34] constructed a counterexample with an integral class $\xi \in H_*(L^7 \times L^7)$, where L^7 is the 7-Lens space. Kreck [18] constructed the geometric gadget by the concept of stratifolds representing any integer homology class by the image of the fundamental class of a stratifold. We can ask the same Steenrod problem with $\mathbb{Z}_k$-coefficients, which Angel-Segovia-Torres [3] solved. The theory of $\mathbb{Z}_k$-manifolds [23] is used together with stratifolds, formulating the concept of $\mathbb{Z}_k$-stratifold. A geometric interpretation of the Atiyah-Hirzebruch spectral sequence is provided in order to give counterexamples of the Steenrod problem with $\mathbb{Z}_k$-coefficients. In fact, Angel-Segovia-Torres [3] showed that for every $i \geq 3$, there exists a class $\alpha_{2i} \in H_{2i}(B\mathbb{Z}_p; \mathbb{Z}_p)$, with p an odd prime, that cannot be represented by $\mathbb{Z}_p$-manifolds. An open problem is the development of an algorithm that creates the geometric $\mathbb{Z}_k$-stratifold that represents a certain $\mathbb{Z}_k$ homology class in order to include as input in the Atiyah-Hirzebruch spectral sequence for obtaining the obstruction to representability. For example, an open problem is something said by Sullivan [33]: the generator of $H_8(K(Z,3); \mathbb{Z}_3)$ is not represented by a $\mathbb{Z}_3$-manifold.

3 Presentations

The following is the list of talks that were presented at the workshop. We place them in the order in which they happened.

Equivariant unitary bordism, what we know...

Bernardo Uribe, Universidad del Norte of Barranquilla

In this talk, I will summarize some of the properties of the equivariant unitary bordism group that have been shown lately. I will also present some results that seem to be related to the equivariant bordism group and some open problems in the area.

Examples and counterexamples in 2-d equivariant bordism

Eric Samperton, Purdue University of Indiana

Fix a finite group G and an oriented surface S. Given an action of G on S, when does this action extend to an action on a 3-manifold M with boundary S? What can we say about M or the qualities of the action of G on M? Ill quickly review the concrete answer that Angel, Segovia, Uribe, and I gave to the first question. Ill then report on various examples and counterexamples regarding the second question; this part of the talk is based on joint work with Marco Boggi and Carlos Segovia.

Equivariant real algebraic models

Karl Heinz Dovermann, University of Hawai'i at Mānoa

Problem 1 Let G be a compact Lie group and M a closed smooth G manifold. Does there exist a nonsingular real algebraic G variety X equivariantly diffeomorphic to M? so that all equivariant vector bundles over X are strongly algebraic (classified by entire rational maps)?

For the trivial group this problem was posed by J. Nash and resolved positively by Tognoli (including the bundle question by Benedetti and Tognoli). The problem reduced to a bordism question that had been solved previously. In the equivariant setting the algebraic realization problem reduces to the following equivariant bordism problem.

Problem 2 Does every class in $\mathcal{N}_*^G(\mathfrak{G})$ have an algebraic representative?

Here $\mathcal{N}$ denotes unoriented bordism and $\mathfrak{G}$ is a product of equivariant Grassmannians. An *algebraic representative* of a class in $\mathcal{N}_*^G(\mathfrak{G})$ is a nonsingular real algebraic G variety X with an equivariant entire rational map $f : X \to \mathfrak{G}$.

Solutions to the bordism problem provide answers to the algebraic realization problem. There are some efficient tools that help analyze equivariant bordism problem, such as blow–ups and reduction to 2 groups. There are some computations of equivariant bordism groups that come in handy, like for elementary abelian 2 groups. We did our own computations that yield algebraic realization results for cyclic groups, and groups with cyclic Sylow 2 subgroups.

On the other hand, manifolds of positive dimension have an uncountable number of birationally inequivalent algebraic models. In the equivariant setting only homogeneous spaces have unique algebraic structure.

Orientation Theory in Mackey Cohomology and Disproving the Evenness Conjecture for Homotopical Equivariant Complex Cobordism

Sophie Kriz, University of Michigan

I will discuss the properties of orientation classes in Mackey cohomology and their relationship to Borel cohomology. As an application, I will describe a non-derived completion theorem for certain equivariant ring and module spectra. I will also discuss an application of these results to finding examples of finite groups G whose G-equivariant complex cobordism Thom spectra coefficients are not flat modules generated in even degrees over non-equivariant complex cobordism. This gives counterexamples to the homotopical version of the evenness conjecture for equivariant complex cobordism.

Compositional Quantum Field Theory and homeomorphism extension

Juan Orendain, Case Western Reserve University

Compositional Quantum Field Theory (CQFT) is an axiomatic framework for quantum theories that focuses on spacetime locality and compositionality. I will explain what the axioms for CQFT mean, how they can be encoded as involutive symmetric monoidal functors with extra structure, and I will explain how every CQFT contains a criterion for when the action by homeomorphisms of a group on a closed oriented hypersurface extends to an action on a region bounding it.

Relating cut and paste invariants and TQFTS

Carmen Rovi, Loyola University

In this talk, we will be concerned with a relation between TQFTs and the controlled cut-and-paste invariants introduced by Karras, Kreck, Neumann, and Ossa. The controlled cut-and-paste invariants (SKK invariants) are functions on the set of smooth manifolds whose values on cut-and-paste equivalent manifolds differ by an error term depending only on the gluing diffeomorphisms. I will present a natural group homomorphism between the group of invertible TQFTs and the group of SKK invariants and describe how these groups fit into a split exact sequence. We conclude in particular that all positive real-valued SKK invariants can be realized as restrictions of invertible TQFTs.

Cut and paste invariants of manifolds

Renee Hoekzema, Free University of Amsterdam

Cut and paste or SK groups of manifolds are formed by quotienting the monoid of manifolds under disjoint union by the relation that two manifolds are equivalent if I can cut one up into pieces and glue them back together to form the other manifold. Cobordism cut and paste groups are formed by moreover quotienting by the equivalence relation of cobordism. We categorify these classical groups to spectra and lift two canonical homomorphisms of groups to maps of spectra. This is joint work with Mona Merling, Laura Murray, Carmen Rovi and Julia Semikina.

Cut and paste spectrum of manifolds: relation to BCob and semicharacteristic

Julia Semikina, University of Muenster

We will take a close look at the $K(Mfd)$ spectrum whose zeroth homotopy group recovers the classical cut and paste group of manifolds SK_n. I will show how to relate the spectrum $K(Mfd)$ to the algebraic K-theory of integers, and how this leads to the Euler characteristic and the Kervaire semicharacteristic when restricted to the lower homotopy groups. Further, I will explain how to construct the maps relating $BCob$, $K(Mfd)$ and $K^{cube}(Mfd)$ that spectrify the natural group homomorphisms relating SKK, SK, and the cobordism group.

Fine structure of the third cohomology

Alexei Davydov, Ohio University

Drinfeld doubles are certain braided tensor categories associated to finite groups. 3-cocyles of group cohomology can be used to modify)or twist) Drinfeld doubles. We use this categorical interpretation of 3-cocyles to define a filtration on the third cohomology, measuring the strength of "non-triviality" of a cocycle.

Equivariant factorization algebras and TQFT on bordisms with rigid geometry

Laura Murray, Providence College

Given a Lie group G acting on a manifold M, one can consider manifolds built from this local model for rigid geometries with isometry group G; i.e. one can look at manifolds with an atlas of charts into open subsets of M, where the transition functions for these charts are given by the G-action on M. One use for this category of manifolds equipped with a rigid geometry is as input data for factorization algebras, a model of observables of a field theory. I show that factorization algebras on the category of all manifolds equipped with a rigid geometry given by the pair (M, G) are equivalent to equivariant factorization algebras on M. This is related to work of Dwyer-Stolz-Teichner on bordism categories of manifolds equipped with rigid geometry and topological quantum field theories constructed from these. I will briefly sketch the relationship to this work.

Irrationality of quotient varieties

Urban Jezernik, University of Ljubljana

The rationality problem in algebraic geometry asks whether a given variety is birational to a projective space. We will gently introduce the problem and take a look at some recent advances, principally in the direction of negative examples constructed via cohomological obstructions. Special focus will be set on quotient varieties by linear group actions.

The non-orientable Bogomolov multiplier

Carlos Segovia González, CONAHCYT-UNAM-Oaxaca

We present the non-orientable version of the Schur and Bogomolov multiplier associated with a finite group G. They serve as obstructions to extending finite free actions from non-orientable surfaces to 3-manifolds. We provide the Miller description of the Schur multiplier regarding universal relations for squares. This allows us to define the non-orientable Bogomolov multiplier as the quotient of the Schur multiplier by the group generated by 1-tori, 1-Klein bottle, and 1-projective space. We show that every finite free action over non-orientable closed surfaces, different from the trivial G-bundle over the projective space, always extends for abelian and dihedral groups.

Calculations for non-orientable Schur multiplier

Omar Alexis Cruz Castillo, CIMAT

We present calculations for the non-orientable Schur multiplier, particularly for the cyclic and dihedral groups. We explicitly give each case's elements and what this means in the non-orientable Bogomolov Multiplier. Finally, we show an example of a non-trivial element in the Bogolomov multiplier that is trivial in the non-oriented case.

Homological, Cohomological interpretations of the non-orientable Schur multiplier

Gustavo Ortega Fernández, UNAM

We find an analogous of Hopf formula for the non-oriented Schur multiplier $\mathcal{N}(G)$. We also give an interpretation of $\mathcal{N}(G)$ in terms of a certain class of group extensions.

Homology of covers of surfaces

Marco Boggi, Universidade Federal Fluminense

Let $S \to S'$ be a finite, possibly ramified, cover of closed oriented topological surfaces. A problem which recently has received much attention is how to generate the first homology group of S in terms of 1-cycles supported on elevations of curves on S' via the given covering map. I will explain recent advances (joint work with A. Putman and N. Salter) and the connection with the problem of extending finite group actions from surfaces to handlebodies (joint work with E. Samperton and C. Segovia).

On normalizers and commensurators of abelian subgroups of mapping class groups

Rita Jiménez Rolland, Instituto de Matemáticas UNAM-Oaxaca

Let $\mathrm{Mod}(S)$ be the mapping class group of a connected surface S of finite type with negative Euler characteristic. In joint work with León lvarez and Sánchez Saldaa, we show that the commensurator of any abelian subgroup of $\mathrm{Mod}(S)$ can be realized as the normalizer of a subgroup in the same commensuration class. As a consequence, we give an upper bound for the virtually abelian dimension of $\mathrm{Mod}(S)$. In this talk we will introduce the necessary definitions and explain how these results are obtained.

Binary group actions, their orbits and classifying spaces

Quitzeh Morales Meléndez, CONAHCYT-Universidad Pedaggica Nacional

Binary G-actions are a generalization of usual (left) G-actions on topological spaces. In this cases a group G acts through invertible binary continuous operations on a topological space. Usual notions as orbits are not easily generalized to this context. We will show how to give a universal construction for these spaces in whenever there exists a universal space for usual G-actions. Also, it will be shown some concrete examples of types of orbits of such spaces. This is joint work with Pavel S. Gevorgyan.

Geometric twistings for Borel equivariant K-theory

Jose Maria Cantarero Lopez, CIMAT-Mérida

Given a discrete group G acting on a compact CW-complex M, we introduced the notion of a derivation of line bundles over M, which can be used to define a projective bundle over the Borel construction. There is a spectral sequence converging to the twisted Borel equivariant K-theory of M in terms of the K-theory of M, the derivation and group cohomology. In this talk, I will begin describing the work of Harju-Mickelsson on twisted K-theory for decomposable twistings, which motivated this work. A generalization for mapping tori of homeomorphisms will be presented next, which will allow me to connect with the general case. This is joint work with Alffer G. Hernndez.

Knot polynomials with genus bounds from quantum groups

Daniel Lpez-Neumann, Indiana University

Quantum invariants are certain topological invariants of knots and 3-manifolds built from representation theory, more precisely, the theory of quantum groups, Hopf algebras and monoidal categories. However, what geometric/topological information of a given knot these invariants contain is still mysterious. In this talk, we will explain how to add group actions/equivariance into the usual picture to obtain knot invariants that do carry some geometric information, namely, lower bounds to the Seifert genus. As a corollary, we get genus bounds for non-semisimple quantum knot polynomials. This is joint work with Roland van der Veen.

The equivariant $RO(G)$-graded James spectral sequence

Mehmet Akif Erdal, Yeditepe University

The James spectral sequence is a generalization of the Atiyah-Hirzebruch spectral sequence and is used for computing generalized homology groups of Thom spectra. This spectral sequence was first introduced by P. Teichner in 1993 and later it has been used in many geometric applications due to its edge homomorphism from the baseline. For a compact lie group G, a G-equivariant stable vector bundle $\xi : X \to BOG(U)$ and an equivariant homology theory h, we give the construction of the equivariant $RO(G)$-graded version of the James spectral sequence by expressing the Thom spectra $M\xi$ as the homotopy colimit of a suitable functor and using the homotopy colimit spectral sequence. Later we discuss some consequences and applications of this spectral sequence. This is a joint work with zgn nl.

The use of Stratifolds to understand the $\mathbb{Z}_k$-Bordism Spectral Sequence

Arley Fernando Torres Galindo, Universidad Externado de Colombia

In this talk, I wanna present some historical background about the Steenrod representation problem and how it relates bordism and singular homology. I present the concept of stratifold developed by Mathias Kreck as a way to resolve the representation problem. After that, I introduce $\mathbb{Z}_k$-stratifolds to resolve the representation problem with $\mathbb{Z}_k$-coefficients.

From Borel-equivariant bordism to the fermionic crystalline equivalence principle

Arun Debray, Purdue University

Freed-Hopkins propose a model for the classification of invertible phases of matter with a symmetry group acting on space using Borel-equivariant bordism. In this talk, I'll discuss a generalization of their ansatz using twisted equivariant bordism to account for cases where the symmetry type mixes nontrivially with the spatial symmetry, such as crystalline phases with spin-1/2 fermions. Using this ansatz, one recovers as a theorem the "fermionic crystalline equivalence principle" predicted in the physics literature; I will discuss this theorem and its consequences in some examples.

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