BIRS Workshop on Noncommutative Geometry

Alain Connes (Collège de France and I.H.E.S.),
Joachim Cuntz (Münster),
George Elliott (Toronto)
Masoud Khalkhali (Western Ontario),
Boris Tsygan (Northwestern)

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Noncommutative geometry is a rapidly growing new area of mathematics with links to many disciplines in mathematics and physics. This is a highly interdisciplinary subject which draws its intuitions, ideas and methods from various areas of mathematics and physics and at the same time contributes successfully to the resolution of many of the standard problems and conjectures in these areas. Examples of such interactions and contributions include: operator algebras (K-theory and KK-theory of C^*-algebras and their classification); topology (successful resolution of the Novikov conjecture and the Baum-Connes Conjecture for large classes of groups); global analysis and geometry (various new formulations of index theory problems beyond their classical realms, on singular spaces like the space of leaves of a foliation); algebra (algebraic K-theory computations through topological Hochschild and cyclic homology, the idempotent conjecture for group algebras, the theory of quantum groups and Hopf algebras and their homological invariants); number theory (Connes' new approach to the Riemann hypothesis, relations between Hecke algebras and quantum statistical mechanics via noncommutative geometry, emerging relations with Arakelov theory, hidden quantum symmetries in the space of modular forms recently discovered by Connes and Moscovici); physics, in particular high energy physics (solid state physics of quasi crystals, new formulations of the standard model, relations with string theory, gauge theory, and M(atrix) theory).

It is fair to say that the subject of noncommutative geometry, as we understand it now, started in 1980 with Alain Connes' classic papers on noncommutative differential geometry. His earlier work on the classification of von Neumann algebras, regarded as a kind of noncommutative measure theory, as well as his work on index theory on foliated spaces, showed that there are many situations in mathematics where the spaces one wants to study are badly behaved even as a topological space, let alone being smooth. One of his key observations was that in all these situations one can attach a noncommutative algebra (through a crossed product construction, or variations thereof, called groupoid algebras) that captures most of the relevant information. Examples include: the space of leaves of a foliation, the unitary dual of a noncompact (Lie) group, and the space of Penrose tilings. Thus a pervasive idea in noncommutative geometry is to treat (certain classes) of noncommutative algebras on the same footing as spaces and to try to extend the tools of commutative mathematics (topology, geometry, analysis, commutative algebra) to this new setting.

We should emphasize, however, that this extension has never been straightforward and always involves surprises. An illustration of this is the theory of the flow of weights and the corresponding modular automorphism group in von Neumann algebra theory which has no counterpart in classical measure theory. Similarly, the extension of de Rham homology of manifolds to cyclic cohomology of noncommutative algebras is highly nontrivial.

A new trend in noncommutative geometry is the emergence of a new Hopf algebra as the hidden quantum symmetries of diverse mathematical and physical structures. It is remarkable that this Hopf algebra, namely the Connes-Moscovici Hopf algebra $\mathcal{H}_1$, appears in a natural way as the quantum symmetries of the transverse space of codimension one foliations, as well as the space of modular Hecke algebras in number theory. A closely related Hopf algebra, namely the Connes-Kreimer Hopf algebra, is used to provide a mathematically rigorous treatment of renormalization in quantum field theory.

In the following we will try to summarize the current state of the subject, as reflected in the talks during the workshop, and especially its challenges and also potential for further progress. We will divide this into different subsections.

1. The Baum-Connes conjecture

This conjecture, in its simplest form, is formulated for any locally compact topological group. There are more general Baum-Connes conjectures with coefficients for groups acting on C*-algebras, for groupoid C*-algebras, etc., that for the sake of brevity we don't condsier here. In a nutshell the Baum-Connes conjecture predicts that the K-theory of the group C*-algebra of a given topological group is isomorphic, via an explicit map called the Baum-Connes map, to an appropriately defined K-homology of the classifying space of the group. In other words invariants of groups defined through noncommutative geometric tools coincide with invariants defined through classical algebraic topology tools. The Novikov conjecture on the homotopy invariance of higher signatures of non-simply connected manifolds is a consequence of the Baum-Connes conjecture (the relevant group here is the fundamantal group of the manifold). Major advances were made in this problem in the past seven years by Higson-Kasparov, Lafforgue, Nest-Echterhoff-Chabert, Yu, Puschnigg and others. The talks by P. Baum, S. Echterhoff, R. Meyer, and G. Yu covered various aspects of the Baum-Connes conjecture.

2. Cyclic cohomology and KK-theory

A major discovery made by Alain Connes in 1981, and independently by Boris Tsygan, was the discovery of cyclic cohomology as the right noncommutative analogue of de Rham homology and a natural target for a Chern character map from K-theory and K-homology. Coupled with K-theory, K-homology and KK-theory, the formalism of cyclic cohomology fully extends many aspects of classical differential topology like Chern-Weil theory to noncommutative spaces. It is an indispensable tool in noncommutative geometry. In recent years Joachim Cuntz and Dan Quillen have formulated an alternative powerful new approach to cyclic homology theories which brings with it many new insights as well as a successful resolution of an old open problem in this area, namely establishing the excision property of periodic cyclic cohomology.

For applications of noncommutative geometry to problems of index theory, e.g. index theory on foliated spaces, it is necessary to extend the formalism of cyclic cohomology to a bivariant cyclic theory for topological algebras and to extend Connes's Chern character to a fully bivariant setting. The most general approach to this problem is due to Joachim Cuntz. In fact the approach of Cuntz made it possible to extend the domain (and definition) of KK-theory to very general categories of topological algebras (rather than just C*-algebras). The fruitfulness of this idea manifests itself in the V. Lafforgue's proof of the Baum-Connes conjecture for groups with property T, where the extension of KK functor to Banach algebras plays an important role.

A new trend in cyclic cohomology theory is the study of the cyclic cohomology of Hopf algebras and quantum groups. Many noncommutative spaces, such as quantum spheres and quantum homogeneous spaces, admit a quantum group of symmetries. A remarkable discovery of Connes and Moscovici in the past few years is the fact that diverse structures, such as the space of leaves of a (codimension one) foliation or the space of modular forms, have a unified quantum symmetry. In their study of transversally elliptic operators on foliated manifolds Connes and Moscovici came up with a new noncommutative and non-cocommutative Hopf algebra denoted by $\mathcal{H}_n$ (the Connes-

Moscovici Hopf algebra). $\mathcal{H}_n$ acts on the transverse foliation algebra of codimension n foliations and thus appears as the quantized symmetries of a foliation. They noticed that if one extends the noncommutative Chern-Weil theory of Connes from group and Lie algebra actions to actions of Hopf algebras, then the characteristic classes defined via the local index formula are in the image of this new characteristic map. This extension of Chern-Weil theory involved the introduction of cyclic cohomology for Hopf algebras. The talks by P. Hajac, D. Perrot, B. Rangipour, A. Thom, C. Valqui, and C. Voigt covered various aspects of cyclic cohomology theory and KK-theory.

3. Index theory and noncommutative geometry

The index theorem of Atiyah and Singer and its various generalizations and ramifications are at the core of noncommutative geometry and its applications. A modern abstract index theorem in the noncommutative setting is the local index formula of Connes and Moscovici. A key ingredient of such an abstract index formula is the idea of an spectral triple due to Connes. Broadly speaking, and neglecting the parity, a spectral triple (A, H, D) consists of an algebra A acting by bounded operators on the Hilbert space H and a self-adjoint operator D on H. This data must satisfy certain regularity properties which constitute an abstraction of basic elliptic estimates for elliptic PDE's acting on sections of vector bundles on compact manifolds. The local index formula replaces the old non-local Chern-Connes cocycle by a new Chern character form $Ch(A, H, D)$ of the given spectral triple in the cyclic complex of the algebra A. It is a local formula in the sense that the cochain $Ch(A, H, D)$ depends, in the classical case, only on the germ of the heat kernel of D along the diagonal and in particular is independent of smooth perturbations. This makes the formula extremely attractive for practical calculations. The challenge now is to apply this formula to diverse situations beyond the cases considered so far, namely transversally elliptic operators on foliations (Connes and Moscovici) and the Dirac operator on quantum SU_2 (Connes). The talks by A. Gorokhovsky, J. Phillips, and R. Ponge centred around index theory and noncommutative geometry.

4. Noncommutative geometry and number theory

Current applications and connections of noncommutative geometry to number theory can be divided into four categories. (1) The work of Bost and Connes, where they construct a noncommutative dynamical system (B, σ_t) with partition function the Riemann zeta function $\zeta(\beta)$, where β is the inverse temperature. They show that at the pole $\beta = 1$ there is an spontaneous symmetry breaking. The symmetry group of this system is the group of idéles which is isomorphic to the Galois group $Gal(Q^{ab}/Q)$. This gives a natural interpretation of the zeta function as the partition function of a quantum statistical mechanical system. In particular the class field theory isomorphism appears very naturally in this context. This approach has been extended to the Dedekind zeta function of an arbitrary number field by Cohen, Harari-Leichtnam, and Arledge-Raeburn-Laca. All these results concern abelian extensions of number fields and their generalization to non-abelian extensions is still lacking. (2) The work of Connes on the Riemann hypothesis. It starts by producing a conjectural trace formula which refines the Arthur-Selberg trace formula. The main result of this theory states that this trace formula is valid if and only if the Riemann hypothesis is satisfied by all L-functions with Grössencharakter on the given number field k. (3) The work of Connes and Moscovici on quantum symmetries of the modular Hecke algebras $\mathcal{A}(\Gamma)$ where they show that this algebra admits a natural action of the transverse Hopf algebra $\mathcal{H}_1$. Here Γ is a congruence subgroup of $SL(2, Z)$ and the algebra $\mathcal{A}(\Gamma)$ is the crossed product of the algebra of modular forms of level Γ by the action of the Hecke operators. The action of the generators X, Y and δ_n of $\mathcal{H}_1$ corresponds to the Ramanujan operator, to the weight or number operator, and to the action of certain group cocycles on $GL^+(2, Q)$, respectively. What is very surprising is that the same Hopf algebra $\mathcal{H}_1$ also acts naturally on the (noncommutative) transverse space of codimension one foliations. (4) Relations with arithmetic algebraic geometry and Arakelov theory. This is currently being pursued by Consani, Deninger, Manin, Marcolli and others. The lectures by Alain Connes, Katia Consani and Marcelo Laca during the workshop covered interactions between noncommutative geometry and number theory.

5. Deformation quantization and quantum geometry

The noncommutative algebras that appear in noncommutative geometry usually are obtained either as the result of a process called noncommutative quotient construction or by deformation quantization of some algebra of functions on a classical space. These two constructions are not mutually exclusive. The starting point of deformation quantization is an algebra of functions on a Poisson manifold where the Poisson structure gives the infinitesimal direction of quantization. The existence of deformation quantizations for all Poisson manifolds was finally settled by M. Kontsevich in 1997 after a series of partial results for symplectic manifolds. The algebra of pseudodifferential operators on a manifold is a deformation quantization of the algebra of classical symbols on the cosphere bundle of the manifold. This simple observation is the beginning of an approach to the proof of the index theorem, and its many generalizations by Elliott-Natsume-Nest and Nest-Tsygan, using cyclic cohomology theory. The same can be said about Connes's groupoid approach to index theorems. In a different direction, quantum geometry also consists of the study of noncommutative metric spaces and noncommutative complex structures. The lectures by H. Bursztyn, J. Kaminker, G. Landi, H. Li, I. Nikolaev, A. Polishchuk, I. Putnam, M. Rieffel, and B. Tsygan covered various aspects of deformation quantization and quantum geometry.

Reports of individual speakers

The BIRS Workshop on Noncommutative Geometry took place at a crucial moment in the development of our subject. With many of the leading experts on various aspects of noncommutative geometry attending the conference, the participants and in particular younger researchers got a very good chance to communicate and exchange their ideas. In the following we are attaching abstracts of talks presented during the meeting by individual speakers. These abstracts are written by the speakers themselves.

Paul Baum: Local-global principle for Baum-Connes

Abstract: A group G is a BCC group (or BCC is valid for G) if Baum-Connes with arbitrary coefficients is valid for G. Let F be an algebraic number field (i.e. F is a finite degree extension of the rational numbers Q). Let G be an algebraic group scheme defined over F. $G(F)$ denotes the discrete group of F-rational points of G. For each place v of F, F_v denotes the local field obtained by completing F at v, and $G(F_v)$ is the locally compact group of F_v rational points of G.

THEOREM. If BCC is valid for all of the locally compact groups $G(F_v)$, then BCC is valid for the discrete group $G(F)$.

This can be proved by using the Meyer-Nest point of view that Baum-Connes is a derived functor, or it can be proved by a direct argument due to Baum-Millington-Plymen.

Henrique Bursztyn: Picard groups in deformation quantization

Abstract: The notion of Morita equivalence plays a prominent role in noncommutative geometry; the Picard group of an algebra is its group of self-Morita equivalences. In this talk, I discussed the behaviour of Picard groups when algebras are quantized in the sense of formal deformation quantization. In the case of algebras of functions on Poisson manifolds, the change in the Picard groups can be expressed purely in terms of the geometry of the manifold. I will also report on how these ideas lead to interesting results in Poisson geometry.

Alain Connes: Modular Hecke algebras and their Hopf symmetry

Abstract: This is joint work with Henri Moscovici. We associate to any congruence subgroup of $SL(2, Z)$ a 'modular Hecke algebra' extending both the ring of classical Hecke operators and the algebra of modular forms. These are coordinate algebras for the 'transverse space' of lattices modulo the action of the Hecke correspondences. The underlying symmetry is shown to be encoded by the same Hopf algebra that controls the transverse geometry of codimension 1 foliations. The action of its horizontal generator is given by the Ramanujan operator that corrects the usual differentiation by the logarithmic derivative of the Dedekind eta function, and the action of the vertical generator is given by the Euler operator on modular forms. The other generators of the Hopf symmetry are associated to higher derivatives of the classical Hecke operators. The emerging picture is that of a surprisingly close analogy with the foliation case, with the role of the circle as a complete transversal being assumed by the modular elliptic curve X(6) and with a simple Eichler integral replacing the angular variable. The Schwarzian 1-cocycle gives an inner derivation implemented by the level 1 Eisenstein series of weight 4, and leads to a rational projective structure on X(6). The Hopf cyclic 2-cocycle representing the transverse fundamental class provides a natural extension of the first Rankin-Cohen bracket to the modular Hecke algebras. Finally, the Hopf cyclic version of the Godbillon-Vey cocycle gives rise to a 1-cocycle on $SL(2,Q)$ with values in Eisenstein series of weight 2, which when coupled with the 'period' cocycle yields a representative of the Euler class, providing an arithmetic formula for the Euler class of $SL(2,Q)$ in terms of generalized Dedekind sums. We then show how to extend the Rankin-Cohen brackets from modular forms to modular Hecke algebras. More generally, our procedure yields such brackets on any associative algebra endowed with an action of the Hopf algebra of transverse geometry in codimension one, such that the derivation corresponding to the Schwarzian derivative is inner. Moreover, we show in full generality that these Rankin-Cohen brackets give rise to associative deformations.

Katia Consani: Archimedean fibers and non-commutative geometry

Abstract: In Arakelov geometry a completion of an arithmetic surface is achieved by enlarging the group of divisors by formal linear combinations of the *closed fibers at infinity*. If one enriches Arakelov's metric structure on a compact Riemann surface of genus at least 2 by choosing a Schottky uniformization, then this extra datum may be combined with the archimedean cohomology theory on the surface to determine the structure of a non-commutative manifold.

Siegfried Echterhoff: Going-Down functors and topological K-theory

Abstract: In this lecture we report on recent joint work with Jerome Chabert and Herve Oyono-Oyono. If G is a locally compact group, a Going-Down functor on G is a family of functors $\mathcal{F}_H : \mathcal{A}(H) \to \mathbf{Ab}$, where H runs through the closed subgroups of G, $\mathcal{A}(H)$ denotes the category of commutative proper H-algebras and $\mathbf{Ab}$ denotes the caegory of abelian groups, such that $\{\mathcal{F}_H\}_{H<G}$ satisfies certain restriction and inflation axioms. A typical example is the functor $\mathcal{F}_H(C_0(Z)) = KK^G(C_0(Z), A)$ for a fixed G-algebra A. Define $\mathcal{F}(G) := \lim \mathcal{F}_G(C_0(X))$, where X runs through the G-compact subsets of the universal proper G-space $\mathcal{E}(G)$. In the special case $\mathcal{F}_H(C_0(Z)) = KK^G(C_0(Z), A)$, we get $F(G) = K_*^{top}(G; A)$, the topological K-theory of G with coefficient A. We then prove:

Theorem. Suppose that $\mathcal{F}$ and $\mathcal{G}$ are two Going-Down functors on G such that there is a natural transformation from F to G respecting the axioms and such that this transformation induces isomorphisms $\mathcal{F}_K(C_0(V)) \cong \mathcal{G}_K(C_0(V))$ for all compact subgroups K of G and for all euclidean linear K-spaces V; then $F(G) = G(G)$.

This result implies, in particular, that any element $x \in KK^G(A, B)$ with $K^G(x) \in KK^K(A, B)$ invertible for all compact $K \subseteq G$ induces an isomorphism $K_*^{top}(G; A) \cong K_*^{top}(G; B)$.

We give several direct applications of the theorem, including a new permanence result of the Baum-Connes conjecture for group extensions and a certain Künneth theorem for topological K-theory and crossed products. As a final application we use our methods to show that the adelic groups $G(\mathbf{A})$ for every linear algebraic group G over a finite extension of $\mathbf{Q}$ satisfies the Baum-Connes conjecture, thus extending an earlier result of Baum, Millington, and Plymen for reductive groups.

Alexnader Gorokhovsky: Local index theory over foliation groupoids

Abstract: This is a report on a work joint with J. Lott. We extend methods of our previous work on a superconnection proof of Connes' index theorem for etale groupoids to the case of foliation groupoids. This allows us to give a more canonical proof of the index theorem for foliations.

Piotr Hajac: Hopf cyclic homology with coefficients

Abstract: The purpose of this talk is to outline constructions of cocyclic modules yielding Hopf-cyclic cohomology with coefficients in stable anti-Yetter Drinfeld modules. This gives a common denominator to known cyclic theories, and allows us to extend the Connes-Moscovici formalism, including the transfer map from the Hopf-cyclic to the usual cyclic cohomology. This is joint work with M. Khalkhali, B. Rangipour, and Y. Sommerhaeuser.

Jerry Kaminker: Noncommutative geometry and hyperbolic dynamics

Abstract: Associated to a hyperbolic dynamical system, consisting of a compact metric space with a self-homeomorphism satisfying the axioms of a Smale Space, are two C*-algebras. These algebras satisfy a form of Spanier-Whitehead duality in K-theory. This was worked out several years ago by Ian Putnam and the speaker. Since then some new applications of these ideas have surfaced. In one direction a class of finitely generated groups, each associated to a compact abelian group admitting an expansive automorphism, can be studied. In many cases one can compactify the countable group by adjoining the original compact group as a boundary. The latter is a hyperbolic dynamical system while the countable group is often solvable. Another direction is the general program of extending the connections between dynamics and homology studied by Bowen, Franks, and many others, to the situation where homology is replaced by the K-theory of the C*-algebras associated to the dynamics.

Marcelo Laca: Hecke algebras from Number Theory

Abstract: We first describe the Hecke algebra of a semidirect product group with respect to an almost normal subgroup of the normal part: it is a semigroup crossed product realizable as a corner in the group C*-algebra of another semidirect product. As an application we use this to study the structure of a Hecke C*-algebra naturally associated to an algebraic number field. This structural knowledge, in turn, enables us to characterize the KMS states of the Hecke algebra. When the number field is purely imaginary of class number 1, we are able to establish that the extreme KMS states at low temperature have symmetry group isomorphic to the Galois group of the maximal abelian extension of the field. The talk is based on joint work with Nadia Larsen on Hecke algebras of semidirect products and on joint work with Machiel van Frankenhuysen on phase transitions on Hecke algebras from number theory.

Giovanni Landi: Fredholm modules on deformed spheres

Abstract: The quantum Euclidean spheres, S_q^{N-1}, are (noncommutative) homogeneous spaces of quantum orthogonal groups, $SO_q(N)$. The $*$-algebra $A(S_q^{N-1})$ of polynomial functions on each of these is given by generators and relations which can be expressed in terms of a self-adjoint, unipotent matrix. We explicitly construct complete sets of generators for the K-theory (by nontrivial self-adjoint idempotents and unitaries) and the K-homology (by nontrivial Fredholm modules) of the spheres S_q^{N-1}. We also construct the corresponding Chern characters in cyclic homology and cohomology and compute the pairing of K-theory with K-homology. On odd spheres (i.e., for N even) we exhibit unbounded Fredholm modules by means of a natural unbounded operator D which, while failing to have compact resolvent, has bounded commutators with all elements in the algebra $A(S_q^{N-1})$. This is joint work with Eli Hawkins.

Hanfeng Li: Metric aspect of theta-deformations

Abstract: We show that Connes and Landi's theta-deformations are compact quantum metric spaces under the deformed Dirac operators. We also introduce a new quantum Gromov-Hausdorff distance which is able to distinguish the algebra structure, and show that the theta-deformations are continuous with respect to this distance.

Ralf Meyer: The Baum-Connes conjecture via derived functors

Abstract: This is joint work with Ryszard Nest. We develop a new approach towards the Baum-Connes assembly map that uses the framework of derived categories. One goal of this project is to formulate analogues of the Baum-Connes conjecture for quantum groups. The topological K-theory $K^{\mathrm{top}}(G, A)$ appears as the derived functor of the K-theory of the crossed product, and the assembly map as the natural transformation from a derived functor to the original functor. The main result needed for this is the existence of an analogue of the Dirac element of the Dirac-dual Dirac method. This is proved using representability theorems in triangulated categories.

Igor Nikolaev: Geodesic laminations and noncommutative geometry

Abstract: Measured geodesic laminations is a remarkable abstraction (due to W. P. Thurston) of many otherwise unrelated phenomena occurring in differential geometry, complex analysis and geometric topology. In this talk we focus on connections of geodesic laminations with the inductive limits of finite-dimensional semi-simple C*-algebras (AF C*-algebras). Our main result is a bijection between the combinatorial presentations of such C*-algebras (the so-called Bratteli diagrams) and measured geodesic laminations on compact surfaces. This link appears helpful indeed as it provides insights to the Teichmuller spaces, Thurston's theory of surface homeomorphisms, Stallings' fibrations to the one side, and noncommutative geometry to the other.

Denis Perrot: Characteristic classes of algebraic KK-theory

Abstract: We propose a definition of algebraic KK-theory based on the category of finitely summable quasihomomorphisms, and construct a Chern character with values in the bivariant cyclic cohomology. One recovers as a particular case the negative Chern character of Hood and Jones. Moreover, the secondary characteristic classes of the corresponding algebraic K-homology are related to the BRS cohomology classes appearing in Quantum Field Theory.

John Phillips: Spectral Flow of Unbounded Self-Adjoint Fredholm Operators

Abstract: We study the gap (= "projection norm" = "graph norm") topology of the space of all (not necessarily bounded) self-adjoint Fredholm operators in a separable Hilbert space. We use the Cayley transform to obtain a uniformly homeomorphic model for our space as a subset of the unitary operators. Using this model, we are able to show the surprising result that this space is path-connected: for example we can connect $+1$ to -1 by a path of self-adjoint Fredholm operators (some of which are necessarily unbounded)! This is in striking contrast to the bounded case where Atiyah and Singer showed that there are three path-components in the space of **bounded** self-adjoint Fredholm operators with $+1$ and -1 being in different contractible components. Moreover, we present a rigorous definition of spectral flow of a path of such operators (actually alternative but mutually equivalent definitions) and prove the homotopy invariance. Thus, as in the bounded case, spectral flow defines a surjective homomorphism from the fundamental group of our space to $\mathbf{Z}$. Unfortunately, we have been unable to determine if this homomorphism is one-to-one, or more generally, whether this space is a classifying space for the functor, K_1. As examples, we investigate paths of Dirac type operators on manifolds with boundary. This article has recently been accepted for publication in the *Canadian Journal of Mathematics*.

Alexander Polishchuk: Holomorphic bundles on noncommutative tori

Abstract: In this talk I define the notion of a holomorphic bundle on a noncommutative complex torus and propose a conjecture that relates the category of such bundles to the derived category of coherent sheaves on the associated commutative complex torus.

Raphael Ponge: A new short proof of the local index formula of Atiyah and Singer

Abstract: In this talk it was attempted to give a new short proof of the local index formula of Atiyah and Singer. It combines the Getzler rescaling with Greiner's approach of the heat kernel asymptotics. The latter uses a pseudodifferential representation of the heat kernel and thereby allows a differentiable heat kernel proof of the local index formula. In turn we can easily compute the CM cocycle for Dirac spectral triples, in both the even and odd cases. So we can bypass the use of the asymptotic pseudodifferential calculus of the previous computations of the CM cocycle.

Ian Putnam: Orbit equivalence for Z^2 minimal Cantor systems

Abstract: I will give a short overview describing joint work with Thierry Giordano (Ottawa) and Christian Skau (Trondheim) on the problem of orbit equivalence for Cantor minimal systems. This is a natural extension to the topological category of the program of H. Dye. Specifically, we are now able to enlarge our classification to include certain minimal free actions of Z^2.

Bahram Rangipour: Cyclic cohomology of extended Hopf algebras

Abstract: This is joint work with Masoud Khalkhali. Extended Hopf algebras are a variation of the concept of Hopf algebroids. Hopf algebroids are quantizations of groupoids exactly in the same way that Hopf algebras and quantum groups are quantizations of groups. The definition of an extended Hopf algebra, proposed in our joint paper, is motivated by the need to define a cyclic cohomology theory for them and to extend the formalism of cyclic cohomology of Hopf algebras, defined by Connes and Moscovici, to extended Hopf algebra. The whole theory is motivated by the introduction, by Connes and Moscovici, of the extended Hopf algebra H_{FM} for any smooth manifold M and its cyclic cohomology theory.

Marc Rieffel: Hyperbolic group C^*-algebras as compact quantum metric spaces

Abstract: Let ℓ be a length function on a group G, and let M_ℓ denote the operator of pointwise multiplication by ℓ on $\ell^2(G)$. Following Connes, M_ℓ can be used as a "Dirac" operator for $C_r^*(G)$. It defines a Lipschitz seminorm on $C_r^*(G)$, which defines a metric on the state space of $C_r^*(G)$. We show that if G is a hyperbolic group and if ℓ is a word-length function on G, then the topology from this metric coincides with the weak-$*$ topology (our definition of a "compact quantum metric space"). We show that a convenient framework is that of filtered C^*-algebras which satisfy a suitable " Haagerup-type" condition.

Andreas Thom: Connective E-theory and bivariant homology

We analyze the homotopy theory of sub-homogenous algebras. A connective version of Connes' and Higson's E-theory is presented. It satisfies universal properties analogous to ordinary E-theory. We give the definition of a bivariant homology theory generalizing cellular homology and analyze the algebra of cohomology operations. It turns out that it serves as a measure of the failure of Bott periodicity in connective E-theory. Finally we give some applications to the homotopy theory of sub-homogenous algebras and matrix bundles.

Boris Tsygan: Remarks on modules over deformation quantization algebras

Abstract: The theory of modules over algebras of differential operators plays a central role in partial differential equations, representation theory, and geometry. Recently, a wide class of algebras has been studied extensively. These algebras are given by deformation quantization of the algebra of functions on a manifold. They are a far-reaching generalization of the algebra of differential operators. Therefore the structure of modules over them is a natural question. In this talk, we present first results on holonomic modules over a deformation quantization of a symplectic manifold with a metaplectic structure. When the manifold is a cotangent bundle of another manifold, the deformation quantization is closely related to the algebra of differential operators, and our construction yields an asymptotic version of the Hormander's module of distributions whose wave front is the given Lagrangian submanifold. In general, we hope that the category of modules, suitably refined along the lines suggested in the talk, would be related to the objects arising in the theory of Lagrangian intersections, in particular to the Fukaya category of the symplectic manifold.

Guoliang Yu: The coarse Novikov conjecture and convexity

Abstract: In this talk, I explain how the concept of uniform convexity of Banach spaces can be used to study the coarse Novikov conjecture.

Christian Valqui: A categorical approach to excision in bivariant periodic cyclic cohomology

Abstract: This is Joint work with Guillermo Cortinas. We extend the excision theorem of Cuntz and Quillen to algebras and pro-algebras in arbitrary Q-linear categories. For this we define a Waldhausen category structures for various complexes and pro-complexes relevant to cyclic homology and show that this added structure is preserved by them. We establish then a Wodzicki excision theorem for abstract algebras. At last we use the Goodwillie theorem and techniques of the original Cuntz-Quillen proof for periodic cyclic cohomology to get excision in a very general setting.

Christian Voigt: Equivariant cyclic homology

Abstract: We present a general framework in which cyclic homology can be generalized to the equivariant setting. From a conceptual point of view our construction is a (delocalized) noncommutative version of the Cartan model in classical equivariant cohomology. In this talk we focus on actions of discrete groups. We explain in detail the definition of bivariant equivariant periodic cyclic homology HP^G_* in this case. It turns out that HP^G_* is homotopy invariant, stable and satisfies excision in both variables. Moreover analogues of the Green-Julg theorem and its dual hold. We illustrate the general theory by considering group actions on simplicial complexes. In this case equivariant periodic cyclic homology is closely related to a bivariant equivariant cohomology theory which was introduced by Baum and Schneider. As a consequence we see in particular that HP^G_* behaves as expected from the Baum-Connes conjecture.