Precise Computation of Quantum Amplitudes

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1 Overview of the Field

Quantum amplitudes are needed to predict probabilities and properties of physical processes such as atomic transitions and scattering and decay of elementary particles. Comparisons of such predictions with measurements improve the knowledge of fundamental constants such as particle masses and strengths of interactions, and probe for new phenomena beyond the current theory.

In order to fully exploit the precision of contemporary experiments, amplitudes have to be computed with high-order of quantum effects. For example, in the language of Feynman diagrams, multiple loops must be included. Recently, significant progress has been achieved in the mathematics involved in such calculations. Tools that are being used include large systems of linear difference equations, asymptotic expansions and singular perturbation theory.

Our group at the University of Alberta is among the world leaders in performing such high-precision calculations. We specialize in the computation of processes involving bound particles. Recently, we made a breakthrough in the calculation of magnetic moment of a bound electron [1]. This workshop was intended to build on this success and prepare methods for the next stage of our research.

2 Recent Developments and Open Problems

The intention of this workshop was to prepare tools for making theoretical predictions to guide and interpret precise experiments. These experiments search for so-called New Physics, that is for phenomena beyond the currently known subatomic models. They are performed primarily in the United States (for example Mu2e at Fermilab) and in Japan (COMET at J-PARC). The subatomic theory group at the University of Alberta specializes in providing such precise predictions for experiments involving bound elementary particles.

One of the objectives was to define observables that should be predicted first. We identified a group of problems related to decays of a muon bound in an atom. We outlined a plan for a series of projects whose aim is to fully characterize the influence of binding on the distributions of decay products.

Binding is interesting from the mathematics point of view because it is a singular perturbation. It introduces a qualitative change in the spectrum of daughter electrons. The reason for this is the presence of the nucleus which is so heavy in comparison with both the muon and the electron that it can absorb momentum almost without taking up any kinetic energy. As a result, the range of energies of the produced electron almost doubles in comparison with decays of a free muon.

3 Outcome of the Meeting

An important outcome was the start of a collaboration with Professor Jamil Aslam, who arrived from Islamabad, Pakistan. He is an expert in few-body hadronic and electromagnetic systems.

As our first project we decided on a rare decay of a bound muon. When a muon bound in an atom decays, there is a small probability that the daughter electron remains bound. We set out to evaluate that probability. Surprisingly, a significant part of the rate turned out to be contributed by the negative energy component of the wave function, neglected in a previous study. We found a simple integral representation of the rate. In the limit of close muon and electron masses, an analytic formula was derived. We published this result in [2].

In the near future we plan to extend this work. We want to reveal the mechanism that enhances the negative energy contribution. We also plan to derive a closed formula in the limit of very small electron masses. Numerical fits indicate that this limit is singular, that is the dependence on the small ratio of electron-to-muon masses is logarithmic, rather than an integer power. This is another example of a singular perturbation: even though the electron mass is much smaller than the muon mass, it cannot be neglected without a qualitative change of the process. Namely, without the finite mass, the electron would not be bound in the atom: the so called Bohr radius is inversely proportional to the electron mass and tends to infinity when the electron mass tends to zero.

References

[1] A. Czarnecki, M. Dowling, J. Piclum, and R. Szafron, *Two-loop binding corrections to the electron gyromagnetic factor*, Phys. Rev. Lett. **120**, 043203 (2018), `1711.00190`.

[2] M. J. Aslam, A. Czarnecki, G. Zhang, and A. Morozova, *Decay of a bound muon into a bound electron*, Phys. Rev. D **102**, 073001 (2020), `2005.07276`.